

Digital Communication
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Lecture - 15
Digital Modulation Techniques (Part – 4)

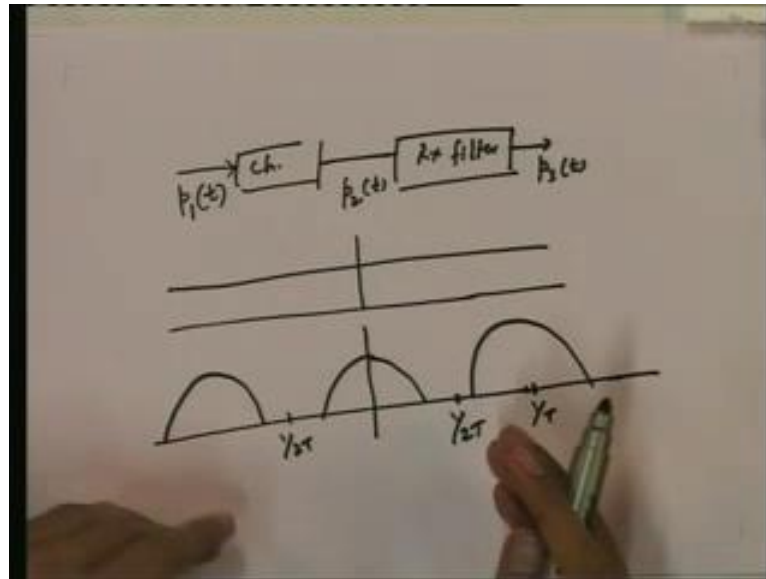
Hello everyone, welcome to the class. We have been discussing digital modulation techniques for few classes now and we will continue discussing digital modulation techniques in this class. So, so far we have discussed first base band representation of band pulse signals and band pulse systems. And then we discussed the first modulation technique in this class that is; pulse amplitude modulation in base band, as well as in pass band.

Then we have discussed demodulation techniques for PAM signal; in that we have seen that match filter demodulation is the best. First we have seen that the sampling at multiples of T at the output of the filter is the best. Then we have seen that match filter itself is the best among all possible filters for demodulation. Then we have seen we have discussed in the last class Nyquist criteria for inter symbol interference free decoding demodulation.

So, there we have seen that for match filter receiver we tried to maximize the SNR. We did not consider inter symbol interference when we tried, when we discussed match filter receiver and when we proved that match filter receiver is the optimum. So, that match filter receiver is the optimum for maximizing SNR. But, different pulses may interfere at the; interfere with each other at the receiver.

So, what is what kind of pulse shapes should we have to have 0 ISI? That is what we discussed in the last class and we have seen that the receipt pulse should satisfy certain condition to have ISI free ISI free demodulation.

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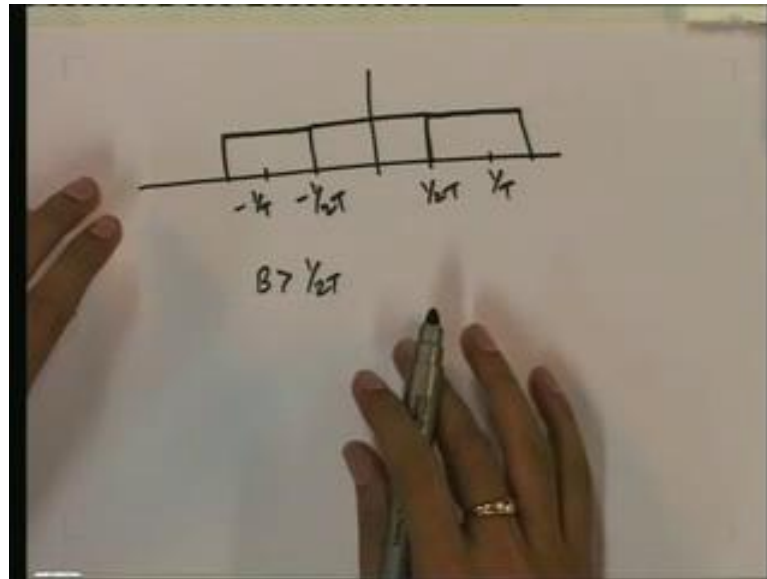


So, what do you mean by receipt pulse. We saw that the original pulse P_t which we also call $P_1(t)$ passes through channel filter, then this becomes $P_2(t)$. Then noise is added and we neglect noise for time being when this signal this pulse passes through the receipt filter. Which is usually the match filter then, this is Rx filter receipt filter. Then this output was called $P_3(t)$ and we saw that, this pulse $P_3(t)$ should satisfy certain conditions to have ISI free demodulation. And what was the condition?

The condition is called the Nyquist criteria and that is as when this signal is sampled at multiples of T seconds then, the resulting discrete time sequence should have this spectrum constant spectrum. So, that meant, that whatever is this pulse. If we take the spectrum of this pulse, if it is suppose like this then what is the discrete time Fourier transform of the sampled version. It will repeat every $1/T$ by; if this is $1/T$ frequency then it will repeat here again and there will be there will be infinitely many copies.

So, this will be the spectrum of the sampled signal obtained from $P_3(t)$. Now, once we if we add all these components then, so this will give us the spectrum of that sampled signal. Now, Nyquist criteria says that to have ISI free ISI free demodulation, we should have this spectrum like this it should add up to like; this it cannot be like this. So, we have seen that if this pulse is band limited; if $P_3(t)$ pulse is band limited to less than $1/2T$ bandwidth then, that is not possible. If we add up all the replicas we will not get this constant magnitude.

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So, on the other hand if we have the bandwidth equal to $1/2T$ then the only choice is that this spectrum. Because, then if we replicate and add this $1/T$ and so on, then we will get constant spectrum. If its band limited its bandwidth is exactly $1/2T$ and its spectrum is something other than this rectangular spectrum then, we will not get constant magnitude spectrum once we add all the replicas. Then we said if the bandwidth is greater than $1/2T$ then, we can have many possibilities many ways we can achieve this 0 ISI.

There are many possible pulses; which when sampled will give us that constant magnitude. So, we will discuss in this class first 1 class of such pulses and that class is called the raised cosine pulses a very popular pulse which is used in practice.

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The image shows a whiteboard with handwritten mathematical equations. At the top, it says $p_3(t) \rightarrow g(t)$. Below that, it says "Raised cosine pulse:". The main equation is
$$g(t) = \frac{\sin(\pi t/T)}{\pi t/T} \cdot \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$
 followed by a simplified version:
$$= \text{sinc}(\pi t/T) \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

So, we have called in the last class this $P_3(t)$ as $g(t)$. And then we will talk about the spectrum of $g(t)$. So, this raised cosine pulse is of this form. So, we are discussing raised cosine pulse this is actually a class of pulses not a single pulse. So, $g(t)$ is $\sin(\pi t/T)$ by T $\pi t/T$, this you will recognize as the sinc function. And then there is an additional term $\cos(\pi \alpha t/T)$ where α is a constant $1 - 4\alpha^2 t^2/T^2$, this is the pulse in time domain.

This can be written as $\text{sinc}(\pi t/T)$ then $\cos(\pi \alpha t/T)$ by $1 - 4\alpha^2 t^2/T^2$. Now, we know that if we take $g(t)$ if we take we take in the Fourier transform domain; in the frequency domain if we take a rectangular pulse the inverse Fourier transform of that pulse in the time domain will be this sinc function. But, here we have taken some thing multiplied to that sinc function we have not taken exactly sinc function.

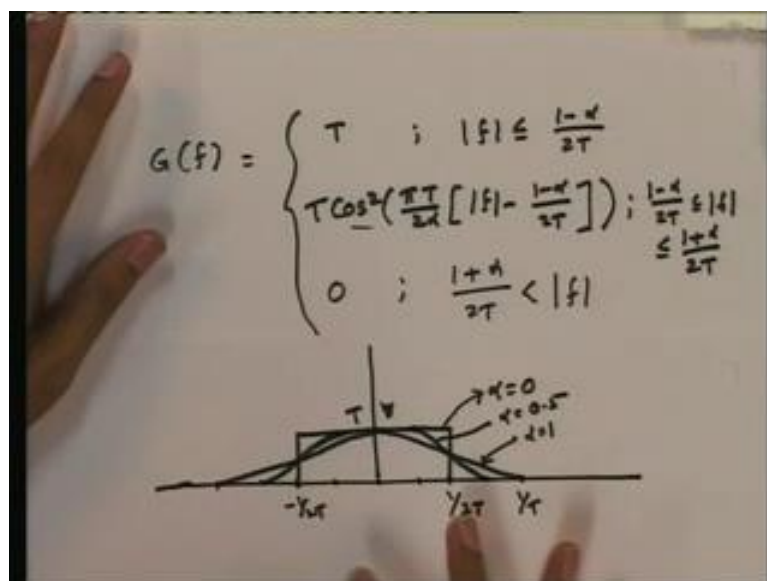
So, this cosine actually we will see in the spectrum also the \cos term will come. So, this will be called this is called raised cosine pulse. Now, what is the Fourier transform of this? We will not drive the Fourier transform, but it is a well know well know Fourier transform is called this.

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$$G(f) = \begin{cases} T & ; |f| \leq \frac{1-\alpha}{2T} \\ T \cos^2\left(\frac{\pi T}{2\alpha} \left[|f| - \frac{1-\alpha}{2T}\right]\right) & ; \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0 & ; \frac{1+\alpha}{2T} < |f| \end{cases}$$

It looks like this, the spectrum $G(f)$ of the $g(t)$ is T in this range. So, for all the frequencies which whose absolute value is less than equal to $\frac{1-\alpha}{2T}$ the value is constant at T . And this is the value in this range $\frac{1-\alpha}{2T}$. So, if the absolute frequency is less than this constant; the value is constant, if it goes greater than that, but it stays less than this quantity; then this way according to this. Then it is 0 for all the frequencies greater than this with absolute value greater than this. So, thus this is this. So, let us draw the spectrum and see how it looks like.

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Let us first draw the rectangular spectrum which is the only way to get satisfy a Nyquist criteria for b equal to $1/2T$. So, that pulse is from $-1/2T$ to $1/2T$ this. Now, this is the energy is taken as one. So, this is this duration is $1/T$ so this magnitude is take as $1/T$. Now, what is this spectrum? This value is T that is same as this in this range when f is between minus of this quantity to plus this quantity so then, somewhere here to here. So, till here the value is T this part.

Then from $1 - \alpha/2T$; $1 - \alpha/2T$ to $1 + \alpha/2T$ see that, $1 - \alpha/2T$ is less than $1/2T$ because α is taken as positive always. So, this is certainly below this it does not go till here it does not cross this limit. Then this quantity $1 + \alpha/2T$ is greater than this $1/2T$. So, somewhere here, so in this range the value is given by this quantity. So, if you draw it will look like this and similarly in the negative side.

Now, it will of course, depend on α this limits will depend on α and this shape itself will depend on α . Now, for example, if you take α equal to α equal to 1 what will you get. This will be $0, 1 - 1/2T$. So, it will it will be this magnitude only at this point it will not be a constant in a non 0 duration it will be $1/T$ magnitude only here. From here onwards it will start decreasing and it will go to till which frequency: $1 + 1/2T$; that is $1/T$. That is the double of this frequency somewhere here and it will look like this.

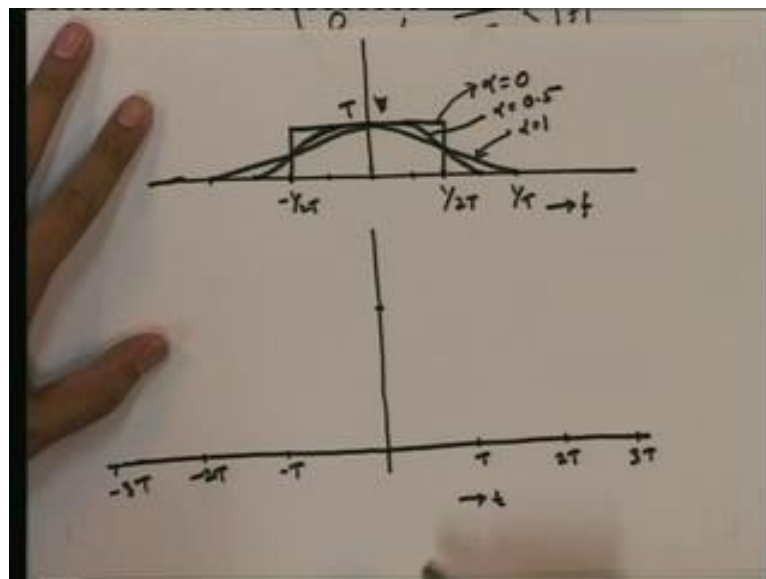
Similarly, on this side also it will look like this. And what is the situation for α equal to zero. Here, if you put α equal to 0 it will be the magnitude T for from $-1/2T$ to $1/2T$ that is from here to here it will be this magnitude the full rectangle. And then what is this magnitude? $1 + 0/2T$. So, this also this point, so there is no where this is valid except for this point and otherwise it will become 0 afterwards. So, it is this rectangle.

So, this rectangle is a special case this is the case remember the second case we discussed to have ISI free pulse that is; the rectangular pulse in a in the in the frequency domain with bandwidth $1/2T$. And that is a special case with α equal to 0 of the raised cosine pulses. And this 1 this 1 is for α equal to 1. And this is something in between let us say α equal to 0.5. So, it is like as from α equal to 0 to 1 usually

the alpha value of alpha is taken in between 0 and 1 so that, the pulse will be somewhere in between this pulse and the rectangular pulse.

So, you can choose your alpha to determine to actually choose how much you wanted to spill over this bandwidth. So, if you want you know more spill over and more smooth pulse you can take greater alpha that is near to 1 and if you take a more sharp pulse you can take alpha small. So, this is a typical this is the typical raised cosine spectrum. And how does the raised cosine pulse look in the time domain. We will draw the pulses in time domain now for different alpha for a few alpha to see how it looks like.

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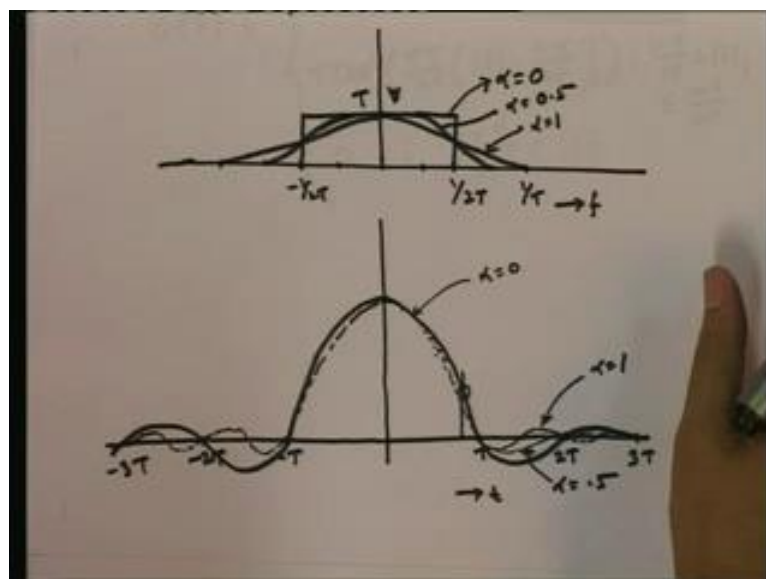
Say in the time domain this is time T , this is time $2T$, this is time $3T$, this is minus t , this is minus $2t$ and this is minus $3t$ and so on. Then for alpha equal to 0 what is the spectrum? For alpha equal to 0 we have seen that the raised cosine pulse is nothing, but nothing, but the sinc function. You can see in the time domain as well as you can see in the frequency domain in the time domain if you see.

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$$\begin{aligned}
 & f_3(t) \rightarrow f(\omega) \\
 & \text{Raised cosine pulse:} \\
 & \frac{g(t)}{g(t)} = \frac{\sin(\pi t/T)}{\pi t/T} \cdot \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2} \\
 & = \text{sinc}(\pi t/T) \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2}
 \end{aligned}$$

See for alpha equal to 0 for alpha equal to 1 for alpha equal to 0 what do we have? This whole thing is 0. So, then cos of 0 is 1 and then alpha equal to 0 gives here also 1. So, we have this whole quantity as one. So, it is a sinc function. So, what is the spectrum of this sinc function? It is the rectangular pulse. So, for alpha equal to 0 the spectrum is a rectangular pulse and the in the time domain the pulse is sinc pulse.

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You can see that, here also for alpha equals to 0 the spectrum is rectangular. So, let us draw first for alpha equal to zero. Alpha equal to 0 it is the sinc function and we all know

the well known shape of the sinc function. This is the pulse in time domain for alpha equal to 0. Now, for alpha equal to 1 that is another boundary value for alpha. It looks like it will be almost like this here, but it will change the shape like this similarly, on the negative side. And then any in between value of alpha will give us a curve between these two. So, it will be somewhere here and then here it will be like this and then after sometime this curve will cross the 0 here.

So, this is for alpha equal to 0. And this curve is for alpha equal to 1 and something in between here is alpha equal to 0.5. So, this is the typical raised cosine pulse in time domain. Now, other than having spectral containment or this you know rectangular pulse is actually, rectangular spectrum though it is possible to have rectangular spectrum and have 0 ISI. It may be difficult to detect such signal because in the time domain this pulse is infinitely long.

It may be difficult to generate such pulse also. If you take a finite duration pulse at the transmitter if you have all finite FIR filters the channel and as a receiver filter also you can never get an infinite length in both sides or infinite length pulse in time domain. So, that is 1 case which may not be possible to have in practice. So, other raised cosine pulses for other values for alpha also it is possible to have 0 ISI.

Another good thing about raised cosine pulse is that see we said that we have to sample at T and that is quite reasonable from this picture also because you can see that, if we sample at T this pulse will not interfere with the next pulse. The next pulse will have peak here at T because the pulses are shifted by T and multiplied by the symbols and then transmitted. So, when you sample here the next pulse this pulse will not interfere because its value here is 0.

Similarly, next pulse will have peak here and this will still not interfere that pulse. So, that way this is also a pictorially this is also a pictorial way of saying that; obviously, this pulse will have 0 ISI if sampled at t . But, what happens if our synchronization at the sampler is not perfect. So, if we instead of sampling here, if we suppose sample somewhere here what happens. This value, if there is a little bit deviation of the sampling instances then, there will not be much error there will not be ISI will not increase drastically.

So, that can be seen from here see that at T at T equal T the sinc function goes to 0 we know that, because this is the sinc function it goes to 0 at T . So, it gives no interference. But, what about other T , so it is deterred that this function decreases very sharply as small t increases. That is this values the amplitude should not be should decrease very fast. So, how fast is the amplitude; that is the peak amplitude let us say because those are the worst cases? If I sample here then this will interfere maximum. So, these peak values how are the peak values decreasing how fast.

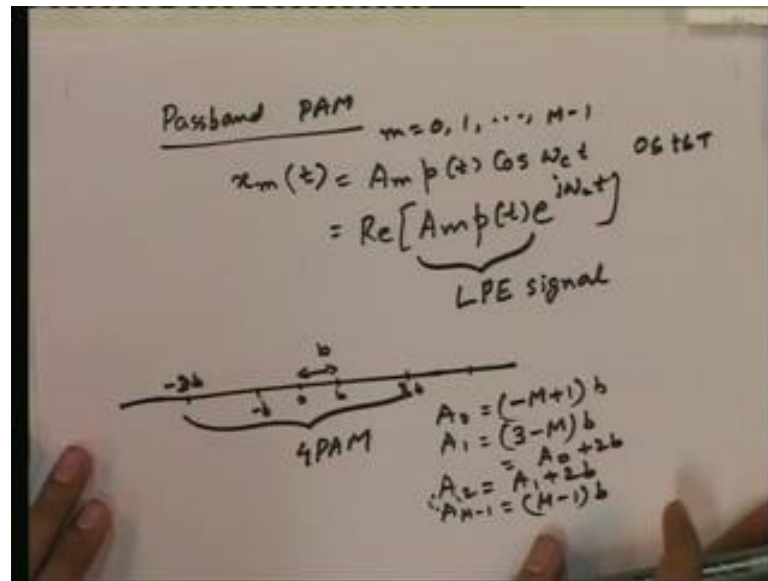
So, that will. So, this envelope of this signal will give us some insight into that. So, envelope is basically if you have pulse width here. So, something like this is a envelope. So, how fast is the envelope decreasing? So, that we can see that there is small t term in the denominator here and there is also, these 2 are sin waves sin and cos waves. So, they are they contribute to the oscillation. Whereas, these things actually contribute to the decrease of this function; decrease of this envelope.

So, how fast is it decreasing that will that we will be able to judge from these quantities? So, this is t this is like it is decreasing as 1 by t . And here, 1 minus four alpha square t T square; so here also there is t . So, this is actually t square. So then, we see that this denominator is varying the denominator has degree tq the maximum degree the degree of the denominator is 3. So, it will decay faster than 1 by t itself that is the sinc function.

So, that is why you can see that this envelope of for alpha equal to 1 is decreasing faster because, these peaks are lower than these this peak. So, that also gives us some advantage because if there is some lack of synchronization with the clock between the sampler and the clock. Then there will be not much not significant damage the ISI will not be very high.

So, so far we have discussed only for discussed all these demodulation techniques only for PAM. But, we will discuss in this class also other modulation techniques and their demodulation techniques like: QAM, PSK all this; standard modulation, detail demodulation techniques. So, in this in the previous classes we have seen: base band modulation and pass band modulation. What we are going to discuss next is we will be also pass band modulation.

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So, we will just summarize what we discussed for pass band PAM. So, pass band PAM, the signal was of this type the transmitted signal for message m . So, m was from 0 to m minus 1 and it was of this form. It is a carrier frequency and $p(t)$ is the pulse shape and A_m is the amplitude that is that gives us amplitude modulation. This can also be written as real part of $A_m p(t) e^{j\omega_c t}$.

So, what is the low pass equivalent signal of this? You can see that from here this is the low pass equivalent signal. In this particular case it turns out to be also real if you take real $p(t)$ which you do because you want this to be here. So, this is low pass equivalent signal of the transmitted signal. And what is the pass band pulse; this $p(t) \cos \omega_c t$ gives actually the pass band pulse. So, what are values of A_m ? We have seen that the common way of doing it is to pick amplitudes like this.

Take suppose this is the 0 value takes here the equidistant this is 1 value this is 1 value then the next value is the same distance here. Here, if we want 4 values then we will take this 4, if you want eight we will have 2 on this side 2 on that side again, so this for 4 PAM, Now, suppose this distance this is suppose this is b this distance 0 to this distance is b then we can express these points as following. A_0 , so A_0 is the first point and then on the left side. So, that is minus M plus 1 b then A_1 is 3 minus M b , that can be written has A_0 plus 2 b .

You can see that it is jumping by 2 b this distance is 2 b. So, every time it is jumping by 2 b A 2 is again a 1 plus 2 b; that can be written as 5 minus M time's b and so on. It will go to Am minus 1 is M minus 1 times b. So, this is just the negative of this one. So, it is on the negative side and this is the on the positive side. So, this is this is b this is minus b this is 3 b this is minus 3 b. These are points you will get if you apply this formula. There are in between lot of points here. And what is the energy of this these signals?

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The image shows a whiteboard with handwritten mathematical derivations. At the top, it defines the energy of a signal $x_m(t)$ as $E_m = \int_0^T x_m^2(t) dt$. A diagram labeled "correlation" shows a pulse of width T with a smaller pulse of width τ inside it. Below this, the energy is expressed as $E_m = \frac{1}{2} A_m^2 \int_0^T p^2(t) dt$, which is then simplified to $E_m = \frac{1}{2} A_m^2 E_p$. Next, a unit energy signal $\phi(t)$ is defined as $\phi(t) = \sqrt{\frac{2}{E_p}} p(t) \cos \omega_c t$, with a note that $\int_0^T \phi^2(t) dt = 1$ (unit energy). Finally, the original signal is expressed as $x_m(t) = \sqrt{\frac{2 E_m}{E_p}} \left(A_m \sqrt{\frac{E_p}{2}} \right) \phi(t)$.

Suppose, if we take the $x_m(t)$ mth signal what is the energy of the mth signal? It is $\int_0^T x_m^2(t) dt$ is function of t. This we can derive and it will turn out to be half $A_m^2 \int_0^T p^2(t) dt$, this is the energy of the pulse E_p . So, this is E_p and in terms of E_p we have E_m in terms of E_p and A_m the amplitude they are used for the mth message; we have expressed the energy of mth signal. So, we can suppose if we take now this $\phi(t)$ is $\sqrt{\frac{2}{E_p}} p(t) \cos \omega_c t$. This basically, in the same line on the same line we these points.

We take 1 appropriate magnitude; this magnitude. Why do I take this magnitude? Because, this particular vector this signal has unit energy. So, this signal is unit energy. And we will express all the signals in terms of this particular signal because, this particular signal is a special signal it has unit energy. And any other signal all the signals can be expressed in terms of this signal.

So, $x_m(t)$ can be written as $\frac{2}{E_p}$ this is $A_m \sqrt{\frac{2}{E_p}}$ because, if you see $x_m(t)$ is nothing but, $A_m \cos(\omega_c t)$. So, $\sqrt{\frac{2}{E_p}}$ is this $\cos(\omega_c t)$ so, you have multiply by $\sqrt{\frac{2}{E_p}}$ to A_m to get this. So, what have we done is what we have done is. We have these points for PAM modulation. We have taken 1 point here which has unit length unit energy means the square of the length is the energy basically. So, unit length we have taken and expressed all these points as just scaled version of this signal.

So, any signal is a scaled version of this signal the scaling is different for different end that is all. And when we draw these points the signal points this way in terms of this point that is unit length point then, this diagram is called the constellation. Set of points this this this they are called this is called the constellation; constellation of the PAM signal set that is used. So, for every PAM signal set we can draw a constellation.

How could you draw? We will take the any PAM any any of the signals. Here we have these signals, we take any of signals and then scale it scale this signal in such a way that it has magnitude it has energy one. So, that is what we have got as $\sqrt{\frac{2}{E_p}}$. This is a energy 1 signal which is a scaled version of this. So, any signal PAM signal can be expressed as a scaled version of this signal and then, we draw these points as scaled version of this point.

So, these points are scaled version of 1 point here which is unit length; this gives us unit length vector 0. So, this diagram will be called the constellation; this set of points actually. So, we have discussed PAM modulation, pass band PAM modulation and its constellation. Now, we will go into another modulation technique where instead of changing the magnitude depending on the message, we will change the phase of carrier signal.


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PSK (Phase shift Keying)

$$p(t) \cos(2\pi f_c t)$$

$$x_m(t) = p(t) \cos\left[2\pi f_c t + \frac{2\pi}{M} m\right]$$

$0 \leq m \leq M-1$
 $0 \leq t \leq T$



$$= \operatorname{Re} \left[p(t) e^{j2\pi f_c t} e^{j2\pi m/M} \right]$$

$$= \cos\left(\frac{2\pi}{M} m\right) p(t) \cos 2\pi f_c t + \sin\left(\frac{2\pi}{M} m\right) p(t) \sin 2\pi f_c t$$

$0 \leq t \leq T$

$$\Sigma = \int_0^T x_m^2(t) dt = \frac{1}{2} \int_0^T p^2(t) dt$$

$= \frac{E_p}{2}$
- constant energy modulation

So, let us suppose we have this $\cos 2\pi f_c t$ $p(t)$ is our pulse. We will not multiply something here, but we will add some phase here depending on the M . So, our $x_m(t)$, so this we are discussing here PSK that is, phase shift keying. This is a kind of phase modulation. So, $x_m(t)$ the modulated signal is $p(t)$ we do not have any magnitude change, but we will have phase change $2\pi f_c t$ plus 2π by M times m , this is 0 to m . This can be written as the real part of just like PA modulation it can be written as, $p(t) e^{j2\pi f_c t} e^{j2\pi m/M}$.

So, what have we done? We are adding phase to this signal. Now, while adding it we know how many points we want and divide the whole phase that is 2π into m number of parts. So, each part is 2π by M . So, take each one. So, if the phase is denoted we have 360 degree phase total we divide it into many parts say suppose, we want M equal to 8 then 8 parts. Then if 0 t is this point 0 phase 2π by M time 0 is 2π by M times 1 so this point and so on.

So, this can, so this is this can also be written as this can also be written as $\cos 2\pi$ by M m $p(t) \cos 2\pi f_c t$ plus $\sin 2\pi$ by M m $p(t) \sin 2\pi f_c t$. Now, what is the energy of this signals? First of all you can see here that energy of each point is basically same because, energy is the square of the distance from 0 to this point this point. So, energy of each signal should be same and let us verify that fact from here directly calculating the energy.

So, what is the energy of x_{mt} ? The energy is x_{mt}^2 that is, this is the energy. If you put this here we will get this. So, they because \cos^2 something is basically half half \cos^2 times because $\cos^2 a$ is equal to $\frac{1}{2} (1 + \cos 2A)$. So, from the using that you can get this. So, we see here as we expected from this constellation that energy of each signal is same. So, this modulation scheme that is PSK is a constant energy modulation scheme. So, constant constant energy modulation.

And you remember that here if you want to add more points that is, if you want transmit more bits in 1 symbol then we can simply do that by increasing number of points on the circle. So, then we have to divide the 360 degree angle into more parts, if you want to transmit 3 bits, we need to divide into 8 parts. If you want to do transmit 4 bits we want to divide the 360 angle into 16 parts. And as a result what will happen is the distance between the points will decrease.

Then, once the channel introduces noise the the points will shift a little bit and then we will not able to distinguish which point was transmitted because they are so close that, it may shift to near to that other point. So, you will think that this was transmitted instead of the 1 which was actually transmitted. So, probability of error will be more if we put more points near by it.

So, if you do not want to transmit if you do not want more probability of error if you want keep the probability of error approximately same and still want to increase number of points what should we do using PSK. We can simply increase the size of the circle we can increase the radius of the circle and then take some increase number of points. Then the we can keep the distance between the point same and then that will give us approximately same probability of error.

So, if you want to introduce sixteen points on the circle we will take a bigger circle, if you do not want more probability of error. So, then we can take 16 points. You can see that approximately the distance between 2 points here and 2 points consecutive points here are same. So, but what have we compromised here instead of using this circle we have used this circle; bigger circle.

Then now the energy of each signal has increased so that means, we will need to transmit more power to transmit more bits per symbol. So, previously we transmitted the same

power to increase the number of bits by simply introducing more points here because, the distance is decreased and as the result probability of error increase.

And now we keep the distance same, but increase the radius of circle and this will result in the in a same in the same probability of error but, increased power transmission. So, for a mobile set for example, this really matters because then this will exhaust the battery power faster and then probably you have to charge the battery charge the mobile every day. So, here is a PSK now we will see why we draw such a constellation. We will do it just the way we did it for PAM.

For PAM we took 1 one vector 1 one signal such that all the transmitted signals are scaled versions of that particular signal. Now, here we cannot express all the points as a scaled version of 1 point because it is the the phase it is rotating is increasing and decreasing. So, you have 2 dimensional signal set. Now, we will see how to express it nicely. So, suppose now we take we have seen that this PSK modulator signal can be expressed in this form. So, here already we can see that is something like this was our phi 1 t some scaling times this we have a separate scaling here.

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- constant energy modulation

$$\phi_1(t) = \sqrt{\frac{2}{E_p}} p(t) \cos 2\pi f_c t$$

$$\phi_2(t) = \sqrt{\frac{2}{E_p}} p(t) \sin 2\pi f_c t$$

- unit energy, orthogonal

$$\int \phi_1(t) \phi_2(t) dt = 0$$

$$x_m(t) = A_m \phi_1(t) + B_m \phi_2(t)$$

$$A_m = \cos\left(\frac{2\pi}{M} m\right) \cdot \sqrt{\frac{E_p}{2}}$$

$$B_m = \sin\left(\frac{2\pi}{M} m\right) \cdot \sqrt{\frac{E_p}{2}}$$

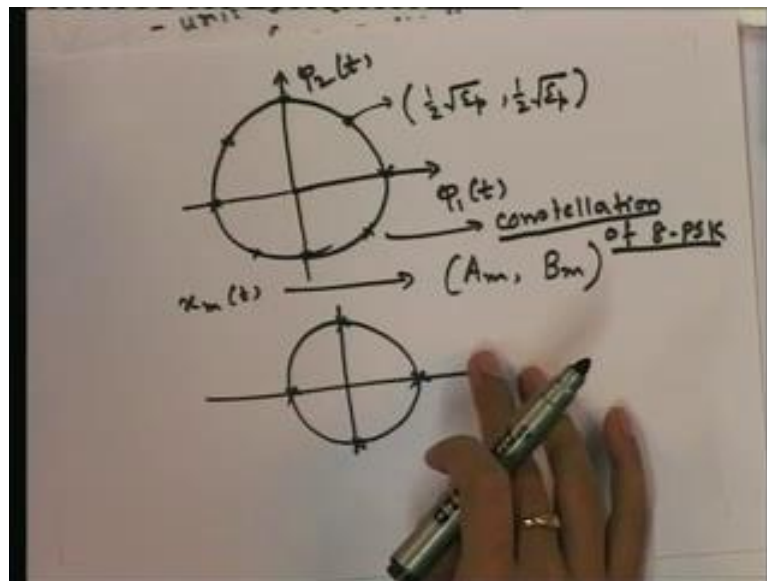
So, suppose we take 2 different signals now, $\phi_1(t)$ as $\sqrt{\frac{2}{E_p}} p(t) \cos 2\pi f_c t$ that is this part but, normalized that is it is scaled in such a way that the energy of the signal is 1. Then $\phi_2(t)$ is $p(t) \sin 2\pi f_c t$. And these 2 signals are equal unit energy each 1 is of energy 1 and they have some more nice structure that is they are orthogonal to

each other. What is meant by orthogonal? It means $\int \phi_1(t) \phi_2(t) dt = 0$. For complex signals we have to take conjugate of 1 of them.

So, now in terms of these 2 signals we can express every signal $x_m(t)$ simply by multiplying some constant to each and then adding. So, our $x_m(t)$ will be $A_m \phi_1(t) + B_m \phi_2(t)$. Where A_m from here you can see we have to multiply by \cos , but we have to divide by $\sqrt{E_p}$ by $\sqrt{2}$ by $\sqrt{E_p}$. So, we have $\frac{1}{\sqrt{2}} \sqrt{E_p} \cos(2\pi f_c t + \theta_m)$ and B_m similarly, is $\frac{1}{\sqrt{2}} \sqrt{E_p} \sin(2\pi f_c t + \theta_m)$.

Now, so we can say now that instead of having 1 signal $\phi_1(t)$ and then all the signals being able to express in terms of $\phi_1(t)$. We have now, 2 signals each unit energy and both are orthogonal to each other such that any signal can be expressed as a linear combination of those 2 signals.

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So, it is like we have 2 orthogonal vectors, these 2 are orthogonal vectors. We have like 2 orthogonal vectors we have 2 orthogonal vectors. So, this is $\phi_1(t)$ this is $\phi_2(t)$ and then any vector can be any transmitted signal is a linear combination of these 2 something times this plus something times this; A_m times this plus B_m times this. So, $A_m B_m$ is a point here we can say that $x_m(t)$ is denoted by $A_m B_m$. So, it is like this is this signal is denoted by 1 0 and this is 0 1 and this is a vector in 2 dimensional plane as

Am here. Then in terms of now we can write we can see that all the points only basically in the circle for PSK. And what will be the point?

Am Bm let us say for eight PSK if we draw all the points for m equal to 0 we will get 1 0, not 1 0, but Am is we can see the result for in equal to 0 cos of 0 that is times this. So, 1 times this so this point is root Ep by 2 and this point is root Ep by 2 root Ep by 2 and each 1 times 1 by root 2. So, this point for example, is half root Ep half root Ep basically, this is 45degree this is 45 degree. So, cos of that is 1 by root 2 1 by root 2 times this is this 2 comes out root over Ep half root over Ep that is same for both.

So, if you draw these Am Bm points this Am Bm these as points on the 2 dimensional plane then, we get these points. And this set of points is called the constellation this is called the constellation diagram; constellation of 8 PSK, this is the constellation of eight PSK. Similarly, if you have four PSK constellation of four PSK is this. And again from here we can see that the energy of each point is same because they are all on the same circle. So, length or from distance from 0 to that point is same for all the points. And as a result the energy which is the square of the distance from 0 is also same for all the points.

Now, we have seen in this class that PA modulation PA modulated signal can be expressed in every symbol interval as a point on the on 1 dimensional line. There will be points on the line and each point is basically a scaled version of 1 point. You can say it is the point 1 and that is a unit energy point that is ϕt . Then if you draw all the other signals as scaled versions of that signal then, those points on the line is called the constellation of the signals that we are using.

And similarly for PSK signal set we have seen for PSK modulation we can express all the signals that, can possibly be transmitted can be expressed as a linear combination of 2 fundamental signals. That is $\phi_1 t$ and $\phi_2 t$ such that, $\phi_1 t$ and $\phi_2 t$ are both unique energy signals and they are orthogonal to each other. So, in terms of those 2 signals all the other signals; all the signals that are transmitted can be expressed.

So, once we so if you have Am Bm as the; so $A_m \phi_1 t$ plus $B_m \phi_2 t$, if that is the transmitted signal. And that signal can be expressed as a point Am Bm on 2 dimensional plane because we have 2 signals $\phi_1 t$ $\phi_2 t$.

We have to have 2 dimensions to write the points draw the points. And set of all such points $A_m B_m$ for different m that set of points when drawn on 2 dimensional plane is called the constellation diagram for that signal set. So, for PSK signal set the constellation will be some points on a circle and the radius of circle actually tells us, what is the energy of the signals. If the radius is more the energy is basically more and that is energy of each symbol is that radius square.

The distance between the consecutive points actually keeps us a measure of what will be the probability of error. We have not seen how to compute the probability of error for PSK signal set for any signal set we have not done that. But, we can get an intuition intuitively we can see that we have close points closer to each other then, the probability of error will be more. And if you have now all the points distant from each other if the distance is more than the probability of error will be less. Because, even after the noise corrupts the signal we will still be able to detect which signal was transmitted reliably. So, we can detect that more reliably.

So, as a result the probability of error will be less. So, in this class we have seen a PSK modulated PSK modulation and we have summarized PA modulation. And we have especially introduced constellation diagram in this class and this particular concept will be useful even later on, when we discuss other modulation techniques.

Thank you.