Digital Communication
Prof. Bikash Kumar Dey
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 12
Digital Modulation Techniques (Part - 1)

In this class, we will start digital modulation. In the last two classes, we have discussed bandpass signal representation and bandpass systems representation in terms of lowpass signals; that is, if you are given a bandpass signal, we can represent the signal as a lowpass equivalent signal. And, we have seen how to get, how to derive the lowpass equivalent signal of a bandpass signal and also how to get the bandpass signal back from the lowpass equivalent signal. Also, we have the same thing for bandpass systems; that is, for any bandpass system, we can construct a lowpass equivalent system.

Now, also, if we have a bandpass system and we want to give one bandpass signal as input to that and we will receive one bandpass output, then we can do this whole operation in lowpass in low frequency. How? We can take the lowpass equivalent system of the bandpass system. The lowpass equivalent signal of the bandpass signal – that we are going to give as input. Then, we give this lowpass equivalent signal as input to the lowpass equivalent system. Then, we will receive one lowpass signal as output. We will have one output, which will be lowpass.

Now, if we now up convert, that is, take the bandpass signal corresponding to that lowpass signal, then we will get the output, which we would have got if we get the original bandpass signal as input to the bandpass system. So, this is something interesting because this way we can avoid high frequency implementation of bandpass systems. We can do down conversion of the signal and then pass it through to give it as input to the lowpass equivalent system. And then, at the end, convert the output to bandpass, that is, do up conversion.

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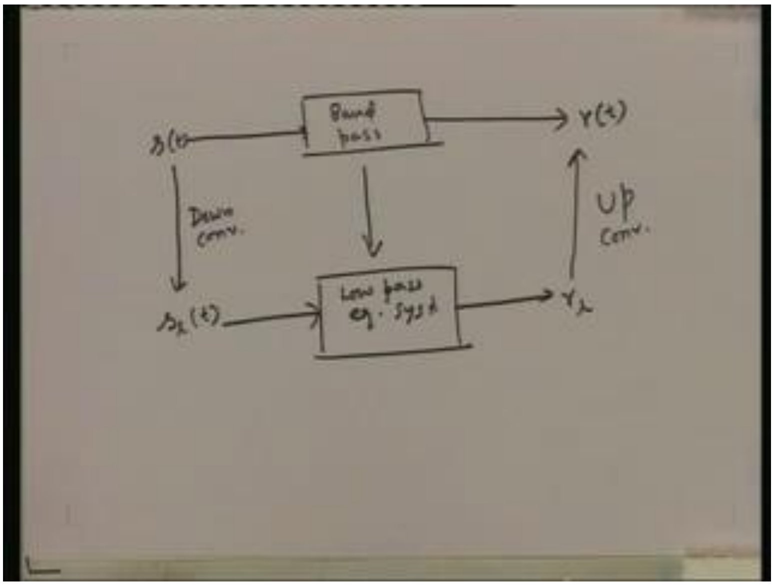

So, if you have a system – bandpass system, you give one bandpass signal as input to get this output. Instead of implementing this system, we can do down conversion to get s l t – the lowpass equivalent signal of s t; pass it through the lowpass equivalent system of the original system; then, receive some output, which will be actually the lowpass equivalent signal of this output. So, we can now do up conversion here to get this output. Instead of doing this way, we can do this way. And also, even if we do not implement things like this, we can at least for analysis purpose assume that, the system is like this. This is one way of representing the system also. Instead of saying there is a bandpass system, we can say that, we can deal with the lowpass equivalent signal, lowpass equivalent system. You can assume that, we have a lowpass equivalent signal, we have lowpass equivalent system, we have a lowpass output – lowpass signal as an output. So, we can treat this, analyze this system, because the original system is equivalent to this. It is only a frequency shift.

So, in this class, now, we will start digital modulation techniques. You must have done another basic communication course; where, there is you have studied analog modulation, amplitude modulation, frequency modulation; such analog modulation schemes. But, here as we said before that, digital communication will mean that, even if the signal – original signal that is to be transmitted is analog, we will convert it into digital signal and then transmit in digital means. So, when we are doing, we are discussing in this course about communication; we will assume that, the signal that is to be transmitted is digital signal. So, we have a digital signal – meaning where you have samples, may be binary; or, it may not be binary, but digital. So… But, any other digital signal can also be converted into a binary signal. So, we can without loss of generality, assume that, the signal we have to transmit is digital – is binary.

So, in this class, we will start discussion on how to transmit digital signal through analog channel. Remember the channel is still analog. Most of the channels in practice are analog channels. So, previously, we discussed that, there are several reasons why one should do modulation like antenna dimension to shift the signal in a higher frequency, so that the required antenna dimension is small; then, for purpose of multiplexing and such reasons. But, there is another reason for modulation in digital communication. When the signal that is to be transmitted is digital and the channel through which we have to transmit is analog; obviously, we have to do some conversion from digital to analog, because the ultimate transmitter signal should be continuous time analog signal. So, this also requires certainly some kind of conversion. So, that is also modulation. So, that is another purpose of doing modulation. That is quite natural in digital communication.

So, just now, we said that, the channel through which we are going to transmit is continuous time. That is why we need to convert the digital signal into analog signal and then transmit. So, that is one purpose. The other purpose may be of course, again to shift the frequency also like we said before. So, there are two kinds of modulation immediately we can see. One is that, the channel – maybe we do not really need to transmit in passband; we still want to transmit the signal in lowpass through lowpass channel. So, the signal – transmitted signal bandwidth signal will be still lowpass signal, not bandpass signal. But, still we need to convert the digital signal into continuous time signal. So, that kind of modulation is called baseband modulation; where, baseband means basically lowpass for starting from 0. So, there is baseband modulation, there is passband modulation. Purpose of passband modulation is all the objectives we said before for modulation like antenna – reducing antenna dimension, then multiplexing and other reasons. But, baseband modulation is just to convert the digital signal to analog signal and then transmit through continuous time channel.

So, let us first take the simplest example of a baseband modulation. You must know that, in computers for example, we transmit from one chip to another, from monitor to… from the CPU to the monitor, mouse, keyboard – all. So, these… There is some communication between all these devices. So, there is some communication going on and this communication is essentially digital. But, the channel, that is, the cable or the connections inside the circuit board – they are all analog; they can carry continuous time signals. So, how are they transmitted?

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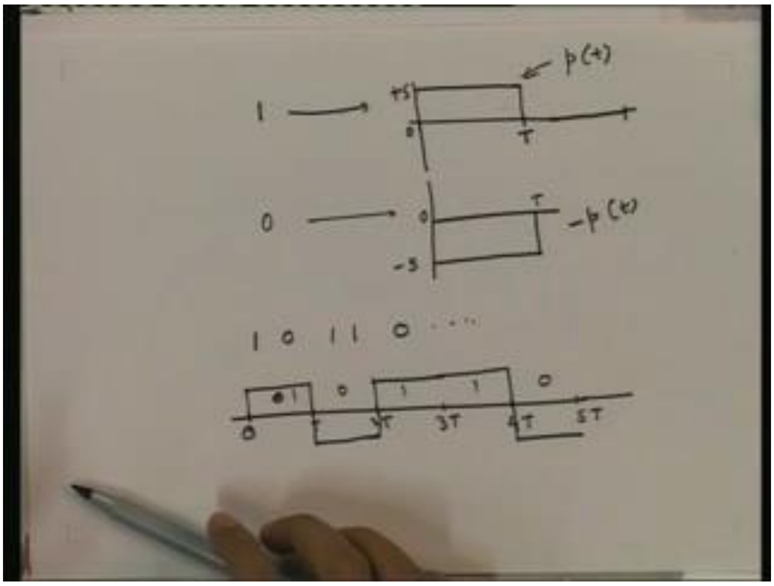

They are through this simple mean that, if one is to be transmitted; we assume that, the transmitter signal is binary – the binary message is to be transmitted; then, 1 is usually represented by one voltage and 0 is represented by another voltage level. So, one example is 1 for plus something – say plus 5 volt and 0, minus 5 volt. Also of course, this is limited to time. Probably, this is transmitted. For transmitting 1 bit, this is transmitted for maybe few nanoseconds or microsecond. So… And then, another bit will be transmitted in another same time – this time. So, this is one symbol interval. We will denote the symbol interval by T in which one symbol will be transmitted. In this case, the symbol is a bit actually. So, this is the simplest way to transmit through a cable.

Now, if we do this way, then if we have say these bits to be transmitted – 1 0 1 1 0 and so on; then, what will the transmitted signal look like? It will look like this. 1 means say this is 0, this is T, this is 2T, this is 3T, this is 4T, this is 5T and so on. So, here in this interval, we want to transmit 1. So, we will transmit say plus 5 volt here. Then, we want to transmit 0. So, we will transmit minus 5 volt here. Then, again 1, then again 1, then 0 and so on. So, this is 1 0 1 1 0 we are transmitting. So, this is a typical baseband modulation. This is the simplest baseband modulation. And, this whole transmission scheme can be represented in a neat analytical form. And, that is, suppose this pulse – we denote this pulse as p of T; we denote this pulse as p of t. Then, this is nothing but minus p of t. So, we say that, if we want to transmit 1, we transmit the waveform p t. And then, if we want to transmit 0, we basically transmit minus of p t. So, we have a fixed template signal – a fixed signal; we will transmit that signal. If we want to transmit 1, we will transmit negative of that if you want to transmit 0; that is the idea.

So, suppose that, we have now… We said that, we actually do not have 1 bit to transmit; we have many bits to transmit like we said 1 0 1 1 0. So, what will be the resulting waveform we are going to transmit? Can we express this in a neat way? So, for 1 bit, we know that, suppose this is the 0-th bit, this will transmitted in… We call this say… We call this b naught, then b 1, b 2 and so on. And of course, it might have started before b minus 1, b minus 2 and so on. So, this is a stream we are going to transmit. It starts from somewhere and goes on. So, we do not know where it is starts from. So, we can say that it starts from minus infinity in general. So, we are going to transmit this stream.

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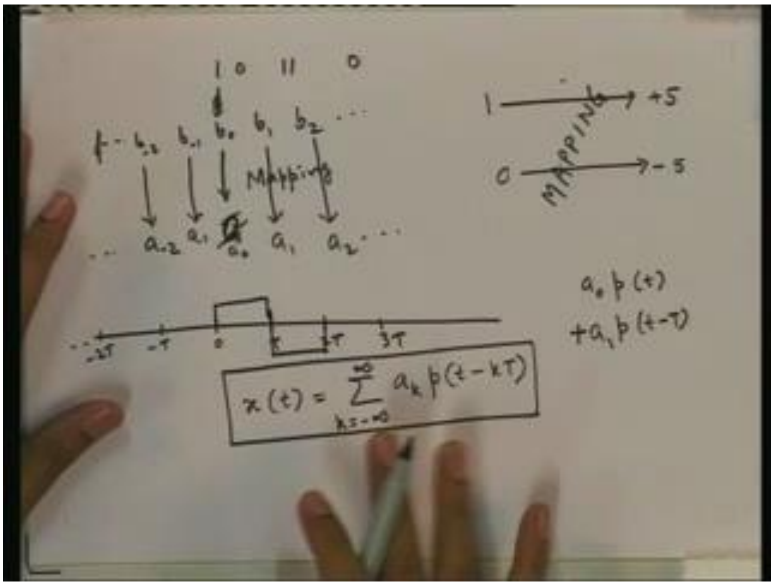

So, what is the way we have just now explained? We will now… We will rephrase it in a different way. So, we say that, each bit is first converted into a level – some voltage level or some real number. So, b naught – if it is 1, it will transmitted; it will be mapped to plus 5 or minus 5 – one of them. So, 1 is mapped to plus 5, 0 is mapped to minus 5. This operation we call mapping; this conversion from the bit value to a level. And then, what do we do? So, after this mapping, we then… So, first step is mapping. Suppose we get…

We will get some level for each bit. So, for this, we will get some levels. We call that a minus 2; the level is called a. So, the value of a is 5 for 1 and minus 5 for 0 for this example. So, from here we will get a minus 1, this a 0. From here we will get a 1; from here we will get a 2, and so on.

First, we have converted the bits into levels. And then, for generating the signal, we will basically multiply. So, in this 0 to T, we are going to transmit b naught, then 2T, then 3T, minus T, minus 2T, etcetera. So, we are going to transmit, construct some signal here; that is, a naught times p t we are going to put here. If this is 1, we are going to put 1 times p t; that is, it will come here. So, what have we got here? This signal if we do not put any other bits here, this signal is nothing but a naught times p t. Then, we will multiply a 1 by p t and put here. So, we will shift p t here and then multiply by a 1. So, what is the shift operation? p t is shifted by T. So, that waveform will have the expression p of t minus capital T. So, p of – we take this signal – we shift the pulse by T and then multiply by a 1. And then, add with this signal. Then, we will get the signal for transmission of b naught and b 1. So, if a 1 is 0, we will have this signal if we add these two. If we add these two, we will get this transmitted signal. But, we want to transmit all these signals.

So, the ultimate expression for the transmitted signal is nothing but x t is equal to… for all these bits. So, k equal to minus infinity to infinity; we are going to transmit b k. So, b k is the k-th bit we are going to transmit. So, that will be transmitted from k t to k plus 1 t in that interval. So, first, we have to shift the pulse to k t. It should start from k t; multiply by a k; and, we have to add all such signals for all the k's. So, we have to add a k p t minus k times T, because we are shifting the pulse by k times T. So, this is our ultimate transmitted signal. This is the expression for the transmitted signal, where a k's are obtained from these original bits to be transmitted by mapping. So, this mapping has to be decided. This is… If we have… For this example, we have taken plus 5 and 5; but, this could be anything. So, this is a typical… This is the simplest modulation we can have – the baseband modulation scheme.

Now, we have discussed here for this example that, the p t is a rectangular pulse; we have taken p t to be a rectangular pulse. But, there is no reason why this should be so. This should be so because… Why not have this pulse?

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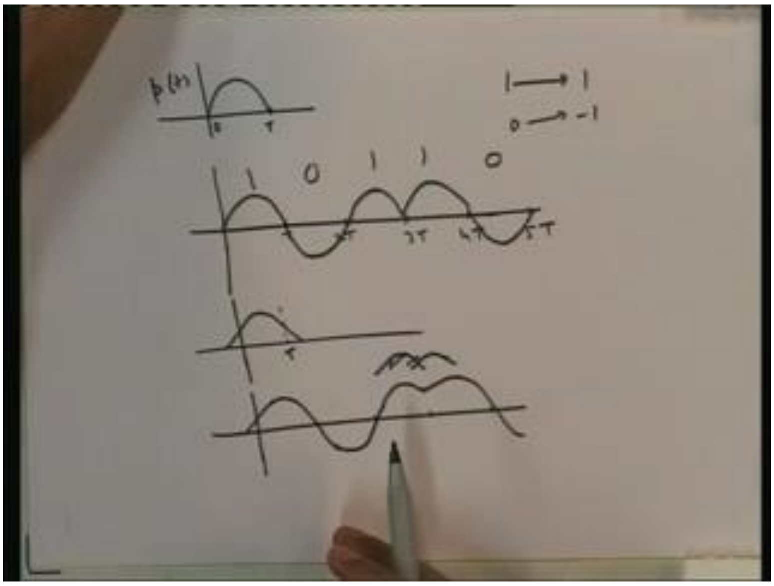

Instead of taking a rectangle like this, if we have like this, what is the harm? So, this is also one possibility. So, we can say this is p t. And then, how will our transmitted signal look? Again binary we will transmit plus p t or minus p t. Let us say 1 is mapped to 1, 0 is mapped to minus 1. Instead of saying 5 minus 5, we can also map to 1 or minus 1. So, the transmitter signal for the same bits as you transmitted before will look like – say this is T, this is 2T, this is 3T, 4T, 5T. Then, the transmitted signal for 1 0 1 1 0 will be 1 0 1 1 0. If you have this pulse, we will be transmitting this for transmitting 1 0 1 1 0.

Now, the pulse also need not be restricted to 0 to T; it could be for example, this kind of pulse. Then, the signals will overlap and they will be added. So, it will look like… They will be added. So, we want to transmit 1. So, this is T, 2T, 3T, 4T. So, when it goes to minus, it will cross through this; it will be something like this. There will have overlaps. This part for example, is sum of two such wave forms like this. Sum of this is this. So, the pulse can be anything. We can choose the pulse and then we can still have the same modulation scheme. So, this is a modulation scheme, because we have a bit stream, this is digital signal from which we are getting a continuous time signal, which is to be transmitted. So, this is the modulation scheme.

Now, instead of having these pulses, if you observe here the rectangular pulse, which we took before has the spectrum. What is the Fourier transform of the rectangular pulse? It will have the magnitude, which is sync function.

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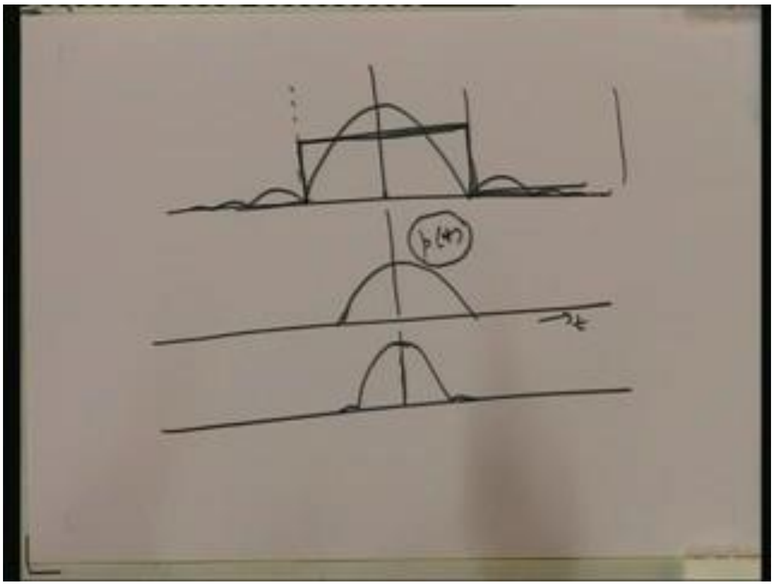

So, the magnitude of the Fourier transform of the pulse will have something like this shape. So, this is really a passband model. This is not ideally… If the spectrum was like this… So, it would have been best. See if we are transmitting this signal, the each individual one is actually a pulse – a rectangular pulse. So, Fourier transform of each one is a sink like this. But, when we add them, it will also be similar to this – magnitudes… The spectrum of the transmitted signal will be similar to this. And… So, it is a lowpass signal. As you can see, it is concentrated to 0 and it is dying off at higher frequency. So, because this is a baseband modulation, it should have been best if the spectrum of the transmitted signal was like this and then 0 afterwards, because probably, we are going to transmit something else here. Like multiplexing, some other signal will be transmitted here. So, we do not want them to interfere with each other.

So… But, we need not necessarily use the rectangular pulse as we said before. So, we can have instead of rectangular pulse, if we have this… This is in time domain if we have this for example, then its spectrum will be not missing function; it will be something else. And, it may be a better spectrum. It may be… It will have probably smaller side lobes. So, that is better. So, this is in baseband modulation also, there is an incentive for taking a different pulse shape such that we can… The purpose is basically to get a better lowpass transmitted signal. It should have better spectral containment it should be contained in a smaller frequency band; it should not spill out in higher frequency. That is the idea. So, it really may be useful to have different pulse shapes to transmit. Now, the pulse shapes need not be this also. Suppose we do not want to transmit in this band, it is not lowpass. We want to transmit in passband.

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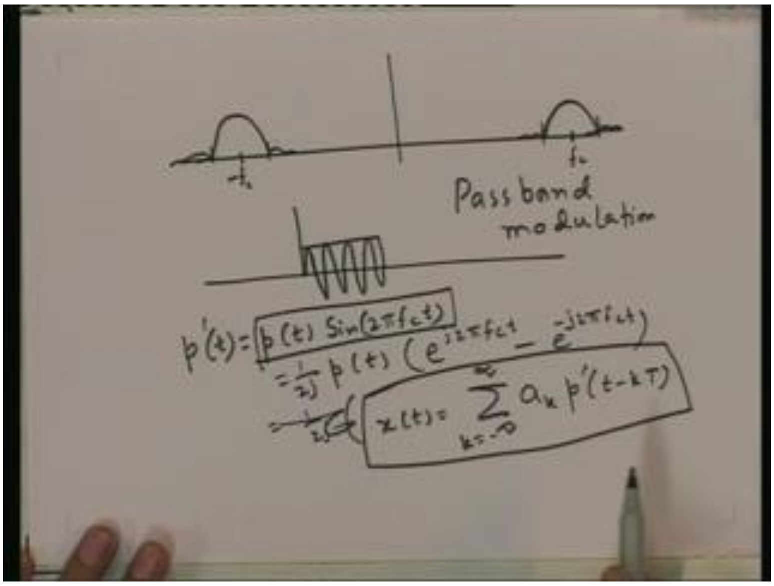

So, our channel is actually… Our channel – we want to transmit in this band let us say. So, it is simple. We can simply do this by… Suppose this is f c; we call this f c minus f c. Then, we simply do this. We instead of taking… Say we take the rectangular pulse shape. We can take any other pulse shape; also, we can do the same thing. But, let us say we take the rectangular pulse shape. This is the pulse shape we are going to take. But, now, we do not want to transmit this pulse as it is, because then the spectrum of the transmitted signal will be lowpass. We want the spectrum to lie here. So, what do we do? We basically shift this spectrum in two sides. So, we can then multiply this with the sinusoidal signal of frequency f c. We will get a signal like this. Then, what will happen? This signal is basically p t times sin 2 pi f c t.

Now, we can also write this as p t 1 by 2 j e to the power of j 2 pi f c t minus e to the power of minus j 2 pi f c t. And, if you can see that, p t times… So, 1 by 2j is a constant. p t times this and p t times that. So, what is the Fourier transform of p t times this? Fourier transform of p t shifted by f c. That is the Fourier transform of p t times e – the exponential of this. So, this Fourier transform will be here. This time this Fourier transform will be here. So, the resulting transmitted signal – if you take this pulse, the resulting transmitted signal will have spectrum like the sink here – shifted here. Similarly, this. So, this is fine with us, because we have now constructed a signal that is passband. This is lying here, where we wanted to transmit. So, this is now a passband modulation. So, now, the pulse is not a baseband pulse. Actually what we are going to transmit taking as pulse is this whole thing, not p t alone. p t is this pulse, but now we can say this itself is p t. This is rectangular and this p prime t – this is actually the pulse that we are going to transmit. So, the transmitted signal can still be written as x t summation k equal to minus infinity to infinity a k times p prime t minus k t.

Now, the pulse is p prime; that is all. And, p prime is not a simple rectangle, but a sinusoidal signal multiplied by that rectangle. So, it is a modulated pulse; it is a modulated pulse. And… But, this expression remains same for the transmitted signal whether it is a passband modulation or baseband modulation; only the pulse will change. The pulse itself for baseband modulation will be lowpass. So, it will be a rectangle or like this. But, for passband modulation, it will be a passband signal – bandpass signal. That is the difference between passband modulation and baseband modulation for this kind of modulation at least. Now… So, we have seen that for baseband, the pulse for baseband and passband modulations, only the pulse shape will be different; otherwise, the expression is same and the way you do the modulation is also same.

So, we will take now a particular example now for… So far, we have seen only binary modulation. All these are binary modulation. And, this kind of modulation is called pulse amplitude modulation, because what we are basically doing is that, depending on the bit that is to be transmitted… We have first taken a pulse; and, depending on the bit that is to be transmitted, we multiply the pulse by something and then transmit. For example, we can multiply by plus 5 or minus 5 for 1 and 0; or, 1 and minus 1 for 1 and 0 and then transmit. So, that is what we are doing before. Now, it is not necessary that, it has to be plus 5 minus 5, plus 1 minus 1 like this. It can be some constant like 2 times the p t for 1; 3 times p t for 0. Similarly, if you have… You can have different other ways also; we will see later.

So, for binary modulation, we have seen so far that, we have been actually changing the amplitude of the pulse by multiplying something and then transmitting. So, the amplitude of the pulse is transmitted depending on the bit that is to be transmitted. If it is 1, the amplitude is something; if it is 0, amplitude is something; we are doing it negative. So, why just binary? We can in fact combine two bits together, three bits together and transmit more bits in 1 symbol interval; that can also be done.

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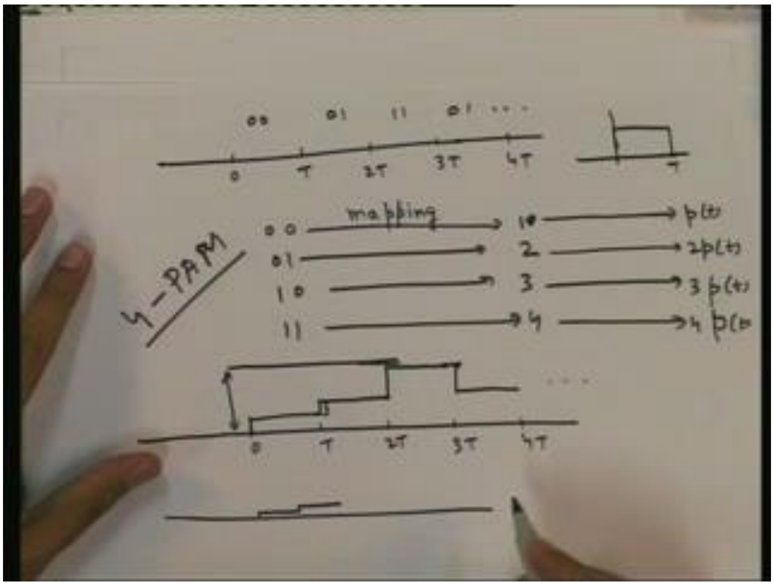

For example, let us say we want to transmit two bits at a time in 1 symbol interval. So, you have 0, T, 2T, 3T and so on. And, we want to transmit 00 here; 01 here; say 11 here; and, again 01 here and so on. So, we want to transmit two bits at a time; that means if you want to do the same thing as before, we need four values of amplitude; we cannot just take plus 5, minus 5, like that, because that is two levels. Using two levels, we can transmit only a binary signal. To transmit 2 bits at a time, we need four levels, because there are four possible values for the 2 bits; it can be 00, it can 01, it can 10 or it can be 11. So, we need four levels.

So, let us say that… For this example, let us say for 00; so, this is the mapping we are doing; map it to some level, some amplitude. So, this is 1 volt let us say – 1. 01 goes to 2; 10 goes to 3; 11 goes to 4. Then, modulation… In the modulation, we will multiply by p t. So, this will be p t; this will be 2 p t; this will be 3 p t; this will be 4 p t. This is the way we will do the modulation. So, if we take p t to be again rectangle for… because that is the simplest case easy to draw. If we take that, then what will be the transmitted pulse? 0, T, 2T, 3T, 4T. So, here we want to transmit 00. So, the level is 1. So, 1 times p t. So, we will transmit 1 here. Then, 01; we want to transmit 2. So, 2 times p t. Then, we want to transmit 11. 11 is mapped to 4. So, 4 times p t. We want to transmit 4 times p t here and then we want to transmit 01. So, 01 is again 2. So… And, so on. So, for this mapping and for this rectangular pulse, this will be the transmitted signal for this bit stream – for this bit stream.

So, here this modulation scheme – now, we have seen that, we are transmitting 2 bits at a time. What is the advantage? The advantage is that; obviously, we are transmitting more bits per symbol. So, the data rate we are transmitting is more now. Previously, we are transmitting 1 bit per symbol; now we are transmitting 2 bits per symbol. So, now, what is the compromise we are making here? Can we really stack more and more bits in 1 symbol and lose nothing? That is certainly… That must be impossible; otherwise, there will not be any problem in communication. So, there are different ways of looking at it. Suppose that we have some battery power restriction. Our mobile for example, if the battery lasts for 1 hour, then the mobile is of no use, because we have to charge every 1 hour; we have to recharge the mobile every 1 hour. So, there will be power restriction; how much power you can transmit that – there is some restriction on that. So, then, the pulse amplitude that you are going to send – that there must be some restriction on the power. So, you should not transmit too much average power.

So, now, what will that mean? If you now want to put more and more levels in that restricted interval, then you have to have… So, if you have different levels here and all the levels have to be now congested in the same interval, then we need to have these differences between these levels as smaller than what we have now. Suppose you want to put 8 levels in this same interval, then we will have very small difference between the levels. And, maybe when it is corrupted by noise, we cannot distinguish between the levels. So, that will be the problem. We will analyze this case more later. But, for time being, we will remember that, there is some restriction in practice in putting more and more bits in 1 symbol interval. So, this is 4 PAM we have considered. And, in general, this same technique can be used for transmitting multiple number of bits, any number of bits. This is for… We have two bits. So, the modulation scheme is 4 PAM.

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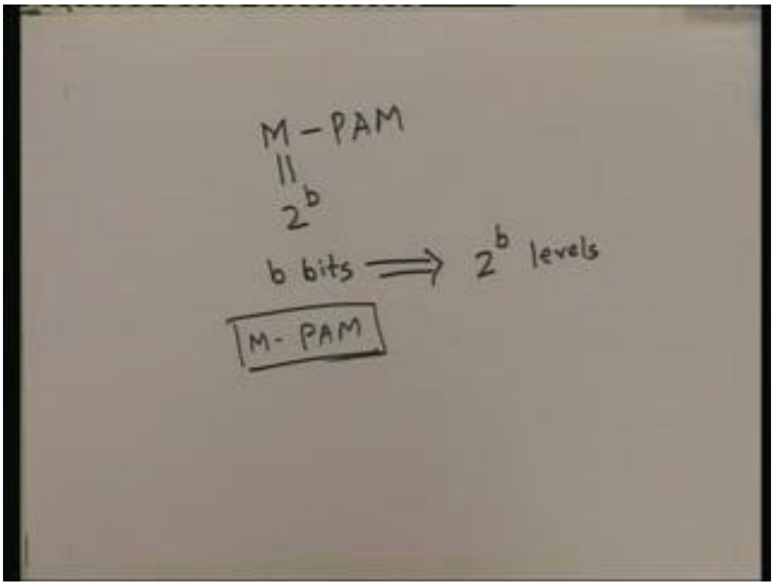

So, in general, we can have M-PAM; where, M is 2 power b. When we want to transmit b bits, that will mean 2 power b levels. And, that… So, if we now have so many levels and we multiply by that level to the pulse amplitude and then transmit, then that is the M-PAM. So far we have seen how binary PAM is done. We have seen how passband modulation is done and how baseband modulation is done; what is the difference. The type of the transmitted signal is basically same; the only difference is in the pulse shape. The pulse for the passband modulation will have a lowpass spectrum; whereas, the pulse for passband modulation will have bandpass spectrum. So, that is the difference.

And, we have seen that, instead of transmitting only 1 bit per symbol interval, we can in general transmit any number of bits per symbol interval though there will be other things we need to compromise for that. For example, if we do not want to compromise on probability of error, then we have to increase the battery power, the transmitted power to transmit more bits, because then we do not want to compromise on probability of error; that means that, the difference between the levels should not be reduced very much. If they are reduced, then they are corrupted by noise; when the signal is corrupted by noise, we will not be able to distinguish the different levels. So, difference between the consecutive levels should be kept high to have sufficient minimal low enough probability of error. So, then what is the way of increasing number of steps? Only way is to increase the range of values, range of levels. So, that will mean transmitting more power through the channel. And, that means again battery power if it is a provided by battery or whatever. So, we have seen in general that, we can transmit any number of bits per symbol using pulse amplitude modulation. But, we have to compromise on something. We have to transmit either more power or we have to compromise on probability of error.

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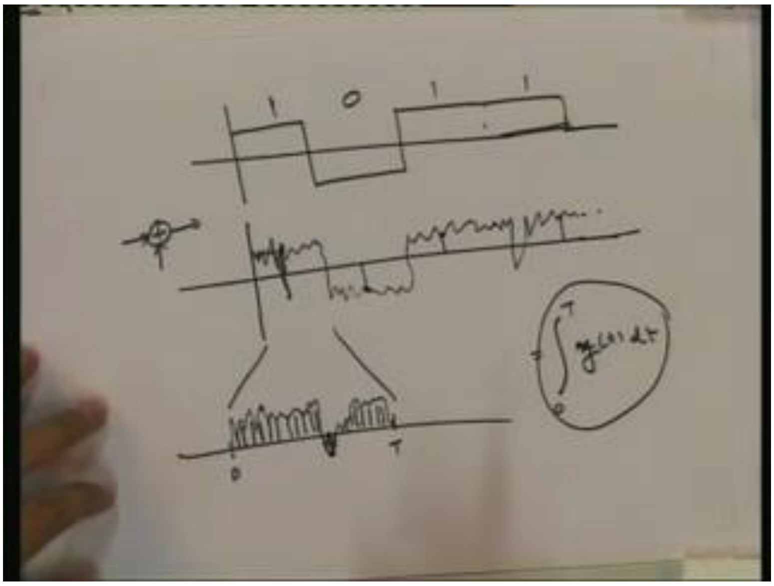

So, now, what is the way to demodulate this signal? We have… Let us say using a rectangular pulse, we have transmitted some bits. We still consider binary modulation because this is the simplest. Suppose we have transmitted this 1 0 1 1; then, how do we demodulate at the receiver? Suppose the channel is just AWGM; that means only noise is added – white Gaussian noise is added; then, this signal will be corrupted by noise and it will look like something like this. And then, at the receiver, we want to estimate these bits we have transmitted from this received signal.

Now, one obvious way that comes to our mind is that, we just sample at one point here – this sample value; and, see whether it is positive or negative. So, that is one way of doing it. But, that is not the best way, because the noise will create lot of problem with this kind of detection scheme – demodulation scheme, because the noise is I think white Gaussian noise. So, it can be from minus… The value of the noise can be minus infinity to infinity; any value. So, at this particular point, the value may be really negative. So, though the signal really looks very nice; here it is positive everywhere, but it may be negative at a very small range. So, if you amplify this part, the signal may look like this.

But, it is fairly clear that, 1 is transmitted in this interval. But, if you just happen to sample at this moment, then we are in trouble, because then we have detected 0 instead of 1, because this value is negative and we will think at the receiver that, probably the transmitter transmitted 0.

But, we can see that, the other parts of the pulse, is fairly nice. So, we should also take care… We should take into account all these values instead of sampling only at one place. So, one way to do it is just add all these sample values. So, most of the values will be positive. So, we will detect better. But, most of the… Add more samples means what? We are still leaving out these samples. So, we should in fact take all the values – take into account all the values and then decide. So, one way to do it is just take the integral of this signal in this interval 0 to T x t – the received signal; this will be y t. So, then what will happen is that, because the noise is white Gaussian noise with mean 0, the parts where this is negative will be fairly small. So, this will still have positive area if one was transmitted most of the times. So, we can take this value. This is the area of this curve. So, this part will contribute negative; these parts will contribute towards the positive. So, this value will be hopefully positive. So, this is a better way of demodulating the signal. And, we will analyze this in a different way and see how to implement such a system.

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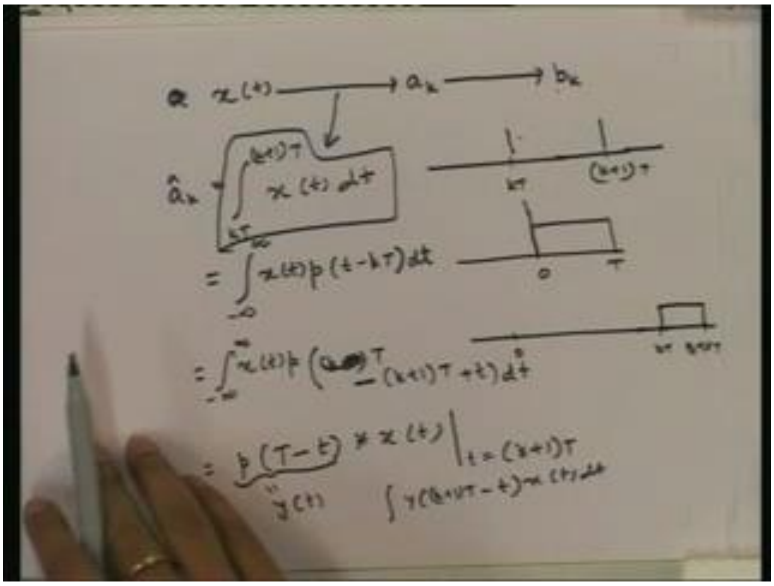

So, what do we want to do? We want to do integration of the signal. We want to integrate the signal from for detecting… First of all from x t, just like at the modulator – at the transmitter, we had first mapper and then modulator; at the demodulator, we will try to do exactly the reverse sequence. From x t, we will try to extract a k for different k's. For first, k equal to 0; then, k equal to 1, k equal to 2, so on. And then, from a k, we will try to estimate b k – the k-th bit or k-th bits; it may be many bits in the same symbol. So, then we will try to estimate all the bits in that symbol. So, in this, we are saying this operation we will do k T to k plus 1 T x t dt. This is what we will do. So, this is KT; this is k plus 1 T. We transmitted b k in this interval. So, we will integrate x t in this interval and then that will be the estimate of the value a k. So, this is the estimate of a k. This is the integration of x t from k T to k plus 1 T.

Now, this also can be expressed as… Remember that this is… p t is… This is the p t we have taken for this case; this is the p t. So, this can be written as the… This can be written as this; because what is the product of x t and p t – p t minus k T? p t minus k T is basically the shifted version of the pulse; it is the pulse shifted by k T. So, now, p t minus k T is basically… So, this is somewhere 0 here and this is k T and this is k plus 1 T. So, p t minus k T is this – signal. And then, we are multiplying that with x t. So, what we will have is that, this part of x t; other parts will be multiplied by 0. So, those values will be 0. So, this is same as this. It is same as integrating in this interval – integrating x t in this interval, because only this product will be nonzero only in this interval. And, in that interval, the value will be equal to x t.

So, this we can write as… This can be written as… We can verify that, it can be written as star x t. What is the formula for this? This is the convolution of x t and this at… We want to evaluate this at k plus 1 T. So, what is the convolution here? Suppose this we call… This is y t; y t equal to T minus – capital T minus small t. So, we want to evaluate it at k plus 1 T. So, in place of T, we will put this. So, the convolution formula is x t. Then, k plus 1 T minus t. So, if this is y t, then the convolution is y k plus 1 T minus t x t dt. So, y t is this. So, in place of t, we will put this. So, it is basically T minus – in place of small t, we will put this whole thing. So, k plus 1 T plus t dt. So, this is equal to this. We can see that, one T cancels. So, then t minus k times T; we have this. So, this is equal to this. But, then this is equal to this. To see this, suppose consider that, this signal is y t; then, convolution of y t with x t is – at this time is y of this time minus t; then, x t dt.

Now, in place of t, put this whole thing here. Then, we will get… We will evaluate this function at this point. We get this. So, what we have seen here is that, this is what we wanted to have actually. This turns out to be same as p capital T minus small t convolution x t evaluated at this point. So, what we have here is that, a filter… because convolution means filtering.

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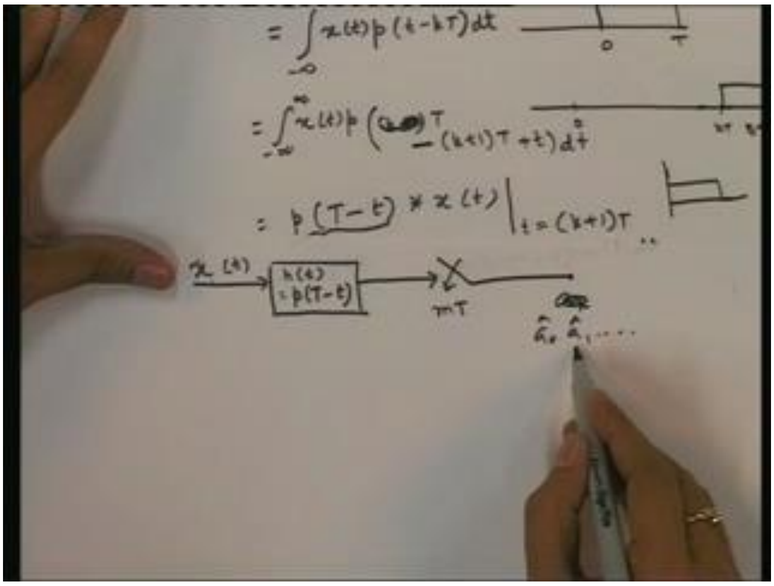

So, what we want to do is pass x t through a filter with impulse response this; this is x t. When you pass it through this, we get this convolution. Then, we want to have the value at this point. So, we sample it at different… Every t seconds, we sample. So, for m equal to k plus 1… At k plus 1-th sample, will be the value of a k. So, here we will get discrete values. The values will be a k. First we will get a 0, a 1 and so on. So, here we will have a 0, a 1, and so on. So, the k plus 1-th number – this will be estimates of course; k plus 1-th number will be the a k hat. So, we have seen here that, to do demodulation of the PAM signal, where we have used the pulse – a rectangular pulse, we have seen that, this is one way to do it and this is equivalent to doing convolution with x t. This convolution of this p… This is again the pulse of course. It is same as p t in this case. But, later we will see the significance of writing it this way.

So, we have seen that, this can be implemented in terms of a filter this way; first, filtering x t and then sampling at different points. Those will give us a t. And, from these of course, we can do demapping. If a 1 is positive; we say it was 1; and, if it is negative, we say it was 0. So, in this class, we have seen binary modulations first. We have seen that, there are different ways of taking the pulse shape. We can have rectangular, that is the simplest; but, we can have also different other pulse shapes to shape the spectrum appropriately. And then, we have seen that, instead of transmitting binary signal by using PAM, you can transmit b number of bits at a time in 1 symbol interval. And, that will be 2 power BPM. That is called MPM or M equal to 2 power b. And, we have seen just now how to demodulate such a signal. Instead of sampling at one time and then deciding of that sample, we have seen that, we will do better by integrating the signal in that interval to decide what was transmitted in that interval. And later finally, we have seen that, that can also be done in terms of first passing of… That can also be done by passing the signal through a filter with impulse response – rectangular impulse response and then sampling the output at multiples of T.

So, in the next class, we will see more on this demodulation; we will generalize this situation for different pulse shape; what should we do. For rectangle, this is the case; what is best way to do for other pulse shapes; we will do that in the next class. And, that will be mass filtering. So, in the next class, we will start with mass filtering and go ahead.

Thank you.