Digital Communication

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Lecture - 10
Bandpass Signal Representation (Part – 1)

In this class we will start digital modulation techniques; this is a very central topic of this course. And, first of all you must have already learned some analog modulation techniques like amplitude modulation, frequency modulation and their demodulation. But in this topic we will learn how to do modulation in digital communication. So, first of all why do we need modulation? This is a fundamental question; one should ask even before discussing analog modulation. So, why we need modulation is first of all if you want transmit the original signal just as itself then, it may be a baseband signal like if it is speech then it is frequency will be between minus 4 kilohertz to 4 kilohertz.

So, this is centered at 0 frequency; to transmit this signal let us say wirelessly through space we will need a very huge antenna to transmit first of all. Because antenna dimension is usually proportional to the wave length of the signal. So, it will be a very huge antenna; if you want to transmit the signal as itself in the baseband. So, on the other hand if you modulate it and place the signal in the some way in a higher frequency. So, if you shift the frequency at a higher frequency shift the band in a higher frequency and then transmit we will need a smaller antenna.

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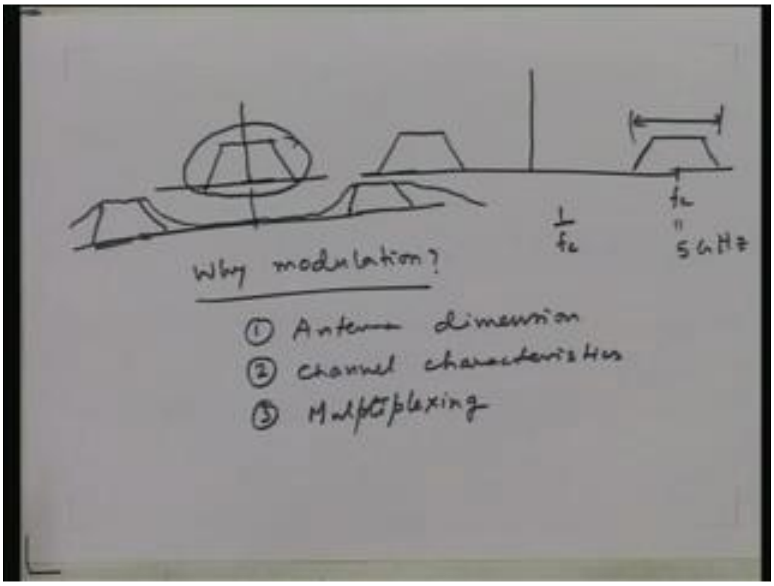

For example, so if you have the original signal like this the spectrum is like this then to transmit this signal you will need a higher a longer antenna; because the it has lower frequency the wave length of this signal will be very high. So, if you on the other hand translate this signal in higher frequency like this is a real signal. So, it will have both sides with centered f c; then, we will need antenna dimension proportional to 1 by f c because lambda is proportional to 1 by f c. So, first of all antenna dimension, so why modulation? Antenna dimension first. Second the channel to which you are going to transmit the signal may not have good response in the band in which the signal is lying.

For example if we are trying to transmit this signal the channel may have very high attenuation in this band channel may have response like say if something like this on both sides. So, if you want to transmit this signal through this channel we will have high attenuation to this signal. So, if you translate this signal on the other hand at a higher frequency, to a higher frequency then you say here signal is here then it will have low attenuation; because the channel is good in that band. So, channel characteristic may be such that the signal will undergo high attenuation; if it is transmitted through the channel as it is.

And, third is multiplexing what is this? Now, all of you know about TV broadcasting, radio broadcasting. There are multiple channels which are transmitted through the same channel the multiple signals from different stations, radio stations are transmitted through the same channel that is the space. Now, how are they transmitted through the same channel? They are put in different frequency band. So, the channel in the radio station in Delhi might be transmitting from through 1 particular band, frequency band; the radio station in Calcutta may be transmitting from transmitting in another frequency band and, so on. So, as a result they do not get mixed up at any receiver, at any radio receiver we can detect you can you can pick up any signal from any of the radio stations; because they are put in different frequency band.

If you wanted to transmit all the signals through the baseband the same band then they will all get mixed up and 1 will not be able to distinguish them in a radio receiver. So, that is another advantage of modulation. Using modulation you can modulate the signals by different, you can modulate different carrier frequency signals by different signals from different sub stations, different radio stations. So, in that way you can transmit many different signals through the same channel. So, these are the reasons why one should do modulation and we will discuss in this course some digital modulation techniques.

And, but before going to digital modulation techniques we will first discuss some basic signal representation fundamentals in this class. So, we have seen that by modulating a signal we actually put a baseband signal in a higher frequency. So, we have put this signal in these this band. Now, suppose this f c is 5 gigahertz; then, at the receiver or even at the transmitter to design such a signal to modulate. And, then the equipment you need to transmit this process, this signal is will become very costly and at the receiver also you will receive this signal. And, if you want to do all the processing required in this pass band itself in this band; then, all the electronics you will need will be very costly because they need to operate in this band.

All the electronics that will process this signal has to be have to be designed for 5 gigahertz band. So, designing devices electronic devices with very good frequency response which will also work for at very high frequency is difficult; first of all and they will be very costly as a result. So, we need to see whether we can process this signal first of all we can bring down this signal back here and then process. The channel will certainly distort this signal; and then we can bring down it bring it down to the baseband and then probably process it.

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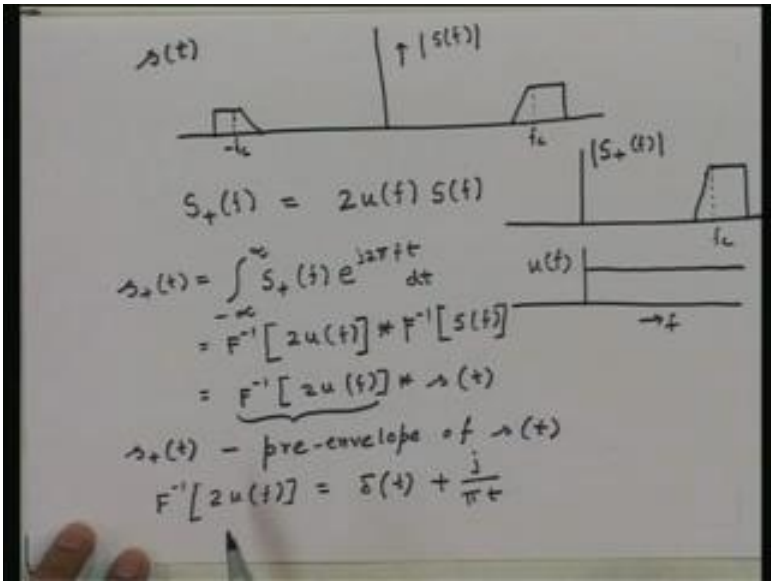

So, we will see how to do that in this class. So, suppose we have a band pass signal; so, you have a band pass signal whose spectrum is like this. Some arbitrary shape I am drawing here this is f c, this is minus f c. Now, this may be the received signal from the channel this is the channel, this is the signal we have received from the channel. So, the frequency response of the signal we are receiving, the signal in time domain is s t. Now, what we can do is just after receiving this signal we can do some operations. So, before we go into to that let us suppose that we define this new signal now; this is only the plus S, plus f means the positive side of s f.

So, it is basically we are multiplying by 2 to keep the energy same in the signal times S of (f). So, we keep only the positive side of the spectrum. So, S plus f will look like on the negative side there will be nothing, on the positive side you will have 2 times this okay. So, we have removed the negative half of this signal to get this signal. So, this is S plus f and we are plotting only the magnitude okay. And, this u f is the unit state function; it is looks like this so obviously if you multiply this u of (f) with s f you get S plus f basically you are suppressing the negative side by multiplying by 0.So, we have this signal which has only the positive half of S f.

Now, similarly what is the corresponding time domain signal of S plus f; the corresponding time domain signal is s plus t. And, this can be expressed as inverse Fourier transform of S plus f, this is inverse Fourier transform of 2 u times f times S of (f). But we know that inverse Fourier transform of the product of these 2, 2 u f and S f is the convolution of the individual Fourier transforms, inverse Fourier transforms. So, in the time domain convolution; if you do convolution in the time domain in the frequency domain the spectrums are multiplied. So, if you have a product of 2 spectrums the corresponding inverse Fourier transform will be the convolution of the individual inverse Fourier transforms.

So, using that principle we have here convolution of these 2.So, this is nothing but F inverse 2 u f; because inverse Fourier transform of S f is s of (t), small s of (t) okay. Now, this is plus of t which has this spectrum is called the pre envelope of s t, this s plus t this signal is called the pre envelope of s t. We will see why it is called pre envelope? Actually, we can get the envelope of s t from this. So, this is the pre envelope which has this spectrum. So, the pre envelope of this signal is has this spectrum. Now, if inverse let us see what this term is if inverse 2 u f this we know to be delta of (t) plus j by pi t. So, we have taken this part, this part this is a well known inverse Fourier transform that is delta of (t) plus j by pi t okay.

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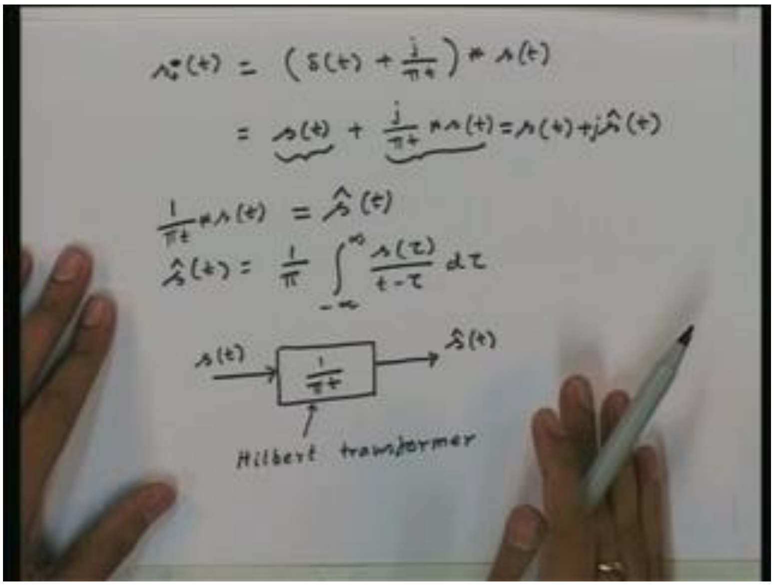

So, now put this here using that we get s plus t, s plus t is this delta t plus j by pi t convolution s t. So, delta t convolution s t is nothing but s t and then we have j by pi t convolution s t. So, this is the s t and this an extra signal we have. So, we have this s plus t they in the inverse Fourier transform of this signal is this signal plus another signal we have expressed it that way; so, s t plus this signal. Now, this signal let us denote it by not this; but j times this. So, j 1by pi t, convolution s t is denoted by s cap t; what is s cap t and how do we get it? So, we can actually get s plus t from s t and then adding with it j times this signal s cap t, s hat t. So, what is s hat t?

S hat t is the convolution of 1by pi t and s t. So, this is s hat t is we can write 1 by pi times; the convolution formula of convolution of 1 by t and s t. So, we have s of tau by t minus tau then d tau okay. So, this is what s hat t is; so, we can write this as s t plus s j times s hat t. So, we have seen that so we have seen that this s hat t is the convolution of 1 by pi t and s t. So, this conceptually this can be obtained from s t by passing it through a filter there is convolution actually gives you the filter output. So, filtered impulse response is 1 by pi t, this is the impulse response of the filter and if we pass s t through that filter we will get s hat t as output.

So, we can get s hat t from s t by passing it through this filter and this filter is called the inverse the Hilbert transformer filter. So, this filter is called the Hilbert transformer. So, and this operation is called Hilbert transform, the Hilbert transform of s t is s hat t. So, what is s plus t now? s plus t is the sum of s t and j times it is Hilbert transform. So, s plus t has 2 components; 1 is real part and imaginary part, real part is s t itself and the imaginary part is the Hilbert transform of s t. So, far we have seen this signal; we have designed this signal, we know how to get in practice, how to get this signal the inverse Fourier transform of this signal s plus t. Take s t that is the real part and to get the imaginary part; pass it through the Hilbert transformer that way we will get s plus t.

Now, from this signal now we can design another signal. So, before doing that let us also see; because we have seen Hilbert transformer what it means in the time domain? In the frequency domain what is the frequency response of Hilbert transformer?

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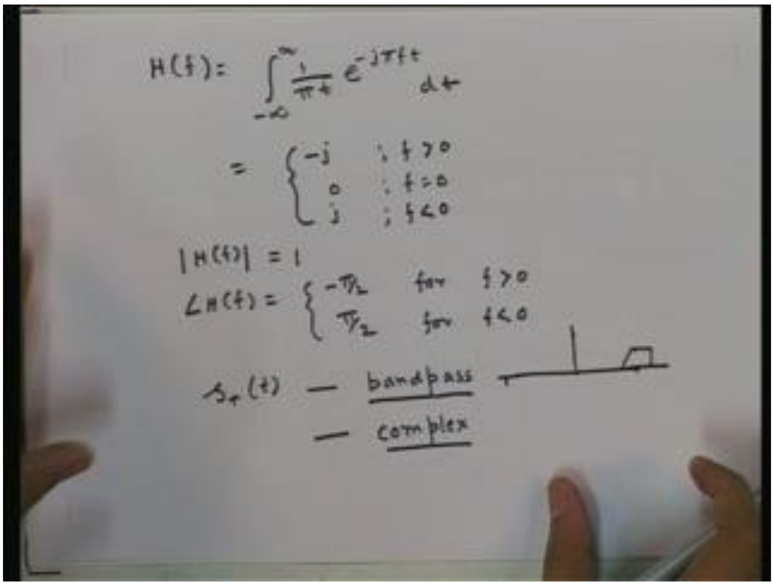

Now, from this signal now we can design another signal. So, before doing that let us also see; because we have seen Hilbert transformer what it means in the time domain? In the frequency domain what is the frequency response of Hilbert transformer? The frequency response is H (f) Fourier transform of h t 1 by pi t e to the power of minus j pi f t d t. So, this if you compute we will see that this is minus j for f greater than 0; 0 for f equal to 0 and j for f less than 0. And, so what do we see here is that magnitude of H (f) is1 except for f is equal to 0; which is already single point. So, we can say except for f equal to 0, H (f) is 1.

So, all positive and negative frequency nonzero frequency you have H (f) is 1, H (f) equal to 1 and what is the phase of H (f)? This is minus pi by 2 for f greater than 0 and pi by 2 for f less than 0. So, what does it do? What does Hilbert transformer do in the frequency domain? If you look at the signal in the frequency domain what it does is it takes every positive frequency and shifts it is phase by minus 90 degree; because an angle of H (f) in the positive frequency is minus 90 degree. So, if you take any positive frequency e to the power of j 2 pi f t it will shift it is phase only; not the it will not change the magnitude it will shift it is it will shift it is phase by minus 90 degree. And, if you take a signal of sinusoidal signal of negative frequency e to the power of minus j 2 pi f t then it will shift it is phase by 90 degree.

So, and if you take sin, say sin omega t what will it do? It will shift the shift the phase by minus 90 degree. So, this is basically Hilbert transformer is basically the 90 degree phase shifter; it changes the phase of the signal by 90 degree that is what it does okay. So, we have seen what Hilbert transformer means in frequency domain. So, we have seen that Hilbert transformer basically is 90 degree phase shifter. And, so we will summarize by saying that s plus t what which we have got s plus t is we will note that this is a band pass signal. But it has only positive side it is nonzero, it has nonzero values only in the positive side that is the difference between s t and s plus t.

So, this is the band pass and because it has only in the positive side this has value only in the positive side it must be complex. Because any real signal has Fourier transform this satisfies conjugative constant. So, whatever is the value here will be the conjugate of the value here that is for real signals; but here the value is nonzero, but here it is 0. So, it cannot be conjugate of this. So, the so this signal must be complex it cannot be real signal. Because for it to be a real signal it must satisfy conjugative constant that is this value must be conjugate of this value negative frequency value. So, this is a complex signal.

So, s t was band pass; but real signal from s t we have received we have constructed s plus t which is band pass still. But which has only positive frequency components and it is complex as a result. Now, from this signal we can construct another signal which will be low pass that is our interest actually, final interest; because we want to bring the band pass signal down to baseband. So that we can process in the base band with low complexity electronics. So, from this spectrum how do we do bring it back to baseband if you have this spectrum; simple it is we simply shift this signal here. So, how do we shift the spectrum here we know how to do it in the time domain we have to multiply by exponential.

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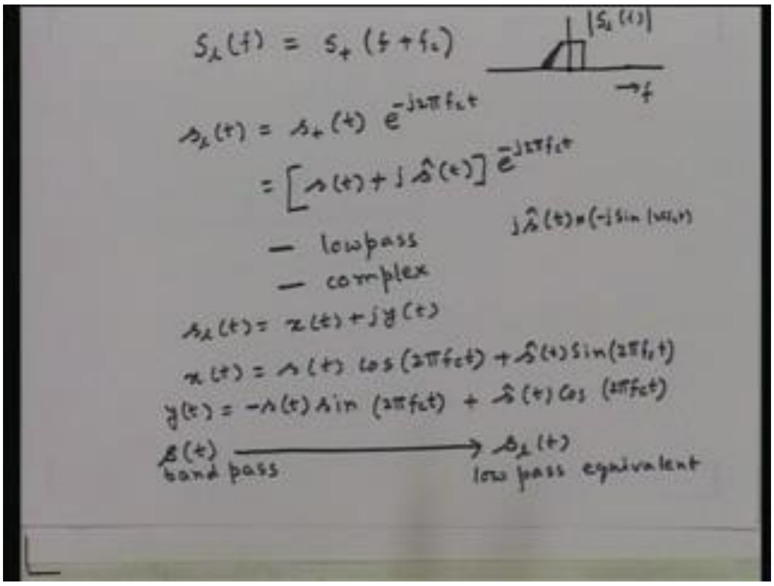

So, what we want to do in the frequency domain is that this new signal S l of (f) l to denote low pass S l of (f) should be S plus of (f) plus f c. Because we want to shift it by in the negative direction by f c; if you wanted to shift it by positive direction it will be minus f c. So, we have this if we take s plus of (f) plus f c we will have this spectrum it will come here. So, this has this spectrum f c has come here to 0; this is S l f and we are drawing only the magnitude always. So, we have shifted it to low frequency at baseband. So, in the time domain how does S l f look? It is inverse Fourier transform S l of (t) is s plus t as you said to shift the frequency in the frequency domain, in the time domain we have to multiply it by e to the power minus j 2 pi by f c t.

So, by f c we want to shift; so, this is what s l t is in time domain. And, what is s plus t in terms of (s t)? How do we get s plus t from s t? It is basically s t plus j times the Hilbert transform of (s t) okay. So, we have got S l t this signal we know how to construct in time domain. So, how do we do? We take s l s t pass it through s hat t and then we can in fact find out the real part and imaginary part of this explicitly and we will do that immediately. So, before doing that what kind of signal is s l t, it has this spectrum. So, it is baseband signal; so, low pass signal and then it is still complex signal. Because it may not have symmetry as you see we have drawn in that way this side is this way and this side is straight down.

So, this is not, this does not have the conjugate symmetry; if it was conjugate symmetry the magnitude will be symmetric so this is still possibly a complex signal. So, this is a low pass complex signal we will have at least got a low pass signal. And, it has all the information of s t we have not lost any information by getting s l t from s t; from s l t we can get s t back simply by shifting this. And, taking also on the other side on in the negative side it is image; in appropriate way magnitude same phase minus of the positive side that way. So, s l t is a low pass complex signal which has all the information of s t. And, we will see how to construct s l t from how to construct s l t from s t we have seen; how to construct s t back from s l t also we will see.

So, this is s l t in time domain and we will compute the real and imaginary part of s l t explicitly now. So, s l t suppose it is x t plus j y t this is the real part, this is the imaginary part, how do you compute x t and y t? We can compute clearly from here what is the real part of this? s t times cos 2 pi f c t. So, x t is s t times cos 2 pi f c t then s hat t times we have here; from here we will get minus j sin 2 pi f c t. So, minus j times, j is minus j square which is 1. So, we have s l t; so, real part will have j times, this is j s hat t times minus j sin this. So, this we will get plus, minus j times, j is minus, j square that is 1; so, we have s hat t sin 2 pi f c t. And, similarly y t can be obtained as s t minus s t sin 2 pi f c t plus s hat t cos 2 pi f c t.

So, we have seen how to get s l t from s t you take the Fourier, you take the Hilbert transform first you compute this. And, then do these operations multiply s t by cos of this, multiply s hat t by sin of this, take the sum of them you get the real part of s l t. Similarly, construct the imaginary part of s l t by combining s t time s sin 2 pi f c t and s hat t times cos 2 pi f c. So, we have seen how to compute the low pass equivalent signal from the; so, this signal is actually called low pass equivalent signal s l t, low pass equivalent signal of the band pass signal s t. So, s t we have s of t from this is the band pass signal band pass from here we have got s l t which is low pass; but what is the difference now low pass equivalent.

So, we have s t band pass signal from there we have a got a complex low pass equivalent signal s t was real signal it was band pass but real. So, from there we have got a baseband signal centered at 0; but it is complex signal. So, we have got a complex low pass equivalent signal of a band pass signal okay. So, now we will see, we have seen how to get the low pass equivalent signal from the band pass signal.

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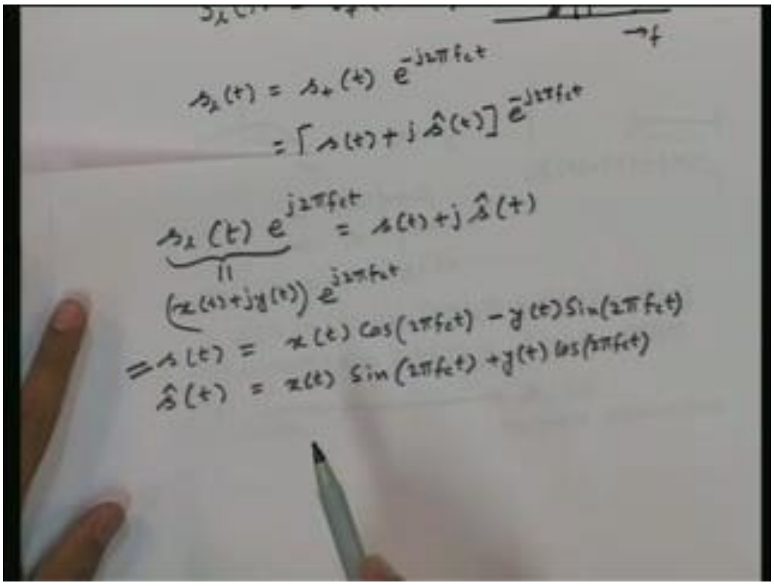

Now, we will we have to see how to get the band pass signal back from the low pass equivalent signal. So, that is also simple very similar from what we saw before; we have seen that s t, s l t, we have basically seen this part s l t is this from. Here we can see that from here we can see that s l t time s e to the power j 2 pi f c t taking this on the left hand side we have s t plus j times s hat t. So, from here we can see that s t is the real part of this and s l t is nothing but; so, this side is x t plus j y t times e to the power j 2 pi f c t. So, real part of this is s t real part of this we have to compute then we will get s t, St is x t cos 2 pi f c t; because e to the power j 2 pi f c t is cos 2 pi f c t plus j times sin 2 pi f c t.

So, this is multiplied this way and then minus y t sin 2 pi f c t okay. And, similarly s hat t is the imaginary part of the right hand side. So, we compute the imaginary part of the left hand side s t times sin 2 pi f c t x t times sin 2 pi f c t then j times y t times cos 2 pi f c t.

So, this is how we can get s t back this s t back from x t and y t. So, x t and y t we can get from s l t there are the real and imaginary part of s l t so from s l t we have got s t back. So, previously we have seen how to get from s t the low pass equivalent signal s l t. We can compute the real part and imaginary part of s l t from s t. Now, we have seen how to get from the low pass equivalent signal, how to get the band pass signal back.

So, this will be useful because in the transmitter we will usually design the low pass equivalent, we will not design the band pass signal. Initially we will design first the low pass equivalent signal and then translate it to the band pass 2 sides. So, that is constructing s t from s l t and then we will transmit through the channel and we will receive the band pass signal; then, the received band pass signal will be brought to the baseband. So, in the receiver we will need this operation from band pass signal to low pass equivalent signal. So, that is called down conversion; because we are bringing down the frequency, bringing down to the baseband. So, we have seen these 2 operations; in terms of signals now we have seen how to get from s t, how to from s t, how to get s l t this is actually down conversion.

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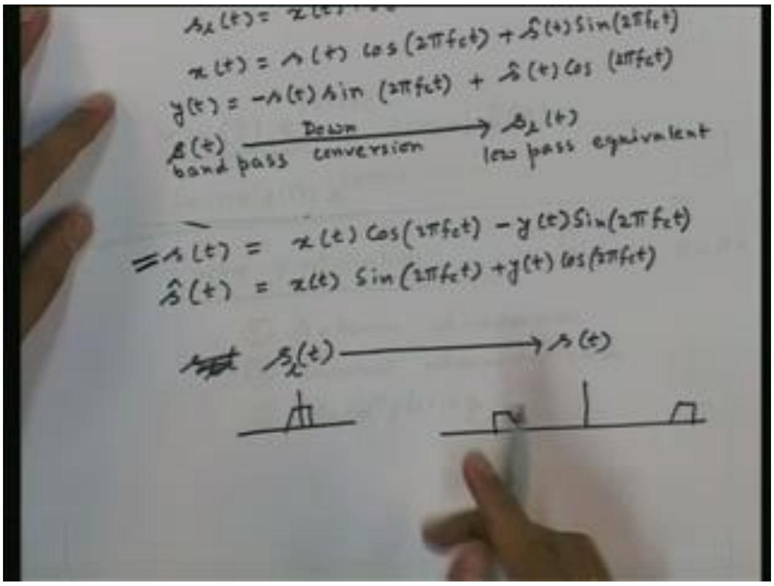

And, now we have seen also how to get s t back from s l t this way this equation. So, this is s from s l t to s t; this is from the baseband signal how to get the band pass signal back this is the spectrum of this, is the spectrum of this. So, this up conversion taking it up to the correct frequency; so, this is up conversion. So, we have seen these 2 operations we can if we have the band pass signal or if we want to design a band pass signal first we can design the low pass equivalent signal then we can do up conversion. So that we can do it at the transmitter; and then at the receiver we will receive a band pass signal and then you can do down conversion to bring it down to the baseband.

So, that will be the low pass equivalent signal of the received band pass signal. So that at the transmitter as well as at the receiver we can do these operations in the baseband with low cost electronics low cost and easy to design electronics we can be used to do this processing.

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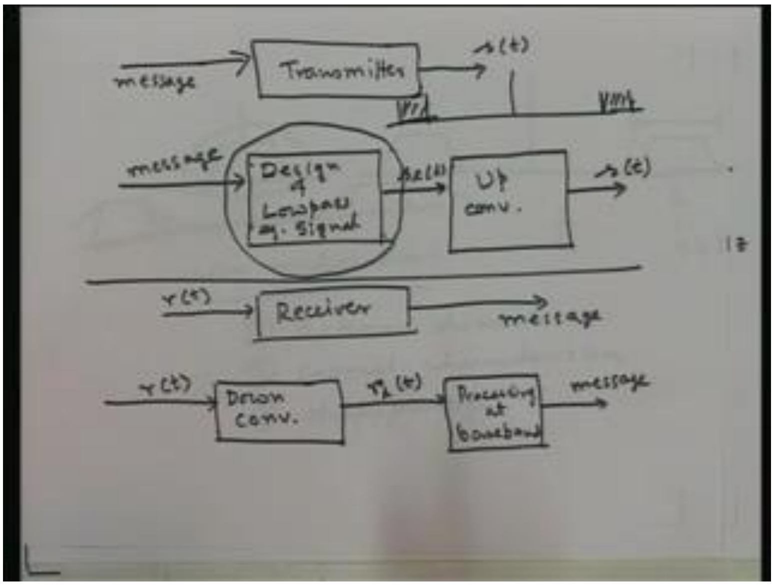

So, now if you summarize at the transmitter what we can do is first take the so transmitter has wants to transmit some message. So, transmitter wants to transmit some signal s t this signal we have probably already fixed that we should transmit in this band is given to us suppose Delhi radio station is given this band they have some message, some audio signal. So, that is the message that needs to be transmitted in this band that is the band given to them by the spectrum regulatory body. And, so they want to transmit in this band; so, we know that s t will lie in this band. But instead of designing that signal directly in this band they will first they can first design a low pass equivalent signal.

So, this design of low pass equivalent signal; so, they will design s l t and then do up conversion. So, this part now is simple; because at least this part can be implemented easily by low cost electronics comparatively, low cost electronics. Because this is a low pass signal; it is frequency is between minus 20 kilo hertz to 20 kilo hertz at most. So, you do not need to design it for in gigahertz band. For example, if you want to design this directly you would have to design in the high frequency. So, this is avoided by designing first the s l t and then doing the up conversion. Similarly, at the receiver actually wants do so this is transmitter.

Now, receiver want to actually do this it will receive some signal r t from channel then it wants to extract message from it; instead of doing directly processing this signal directly to get this message what it can do is it can first bring it down to. So, this will be very complex if it wants to get this message directly from r t the processing will be costly; because it has do all the processing in the high frequency. So, instead of that it can first bring it down to low frequency baseband by doing down conversion. And, then process this signal processing at baseband. So, this is r t this will be the low pass equivalent signal of r t; because the from this signal itself we can get message.

Because we have not lost any information by doing this we have seen that you can get r t back from r l t by doing up conversion. So that means there is no loss of information due to down conversion. So, you can get the message back as faithfully as you could have done from the band pass signal r t itself, there is no loss of information. So, we have seen that in the transmitter if we do first, if we design the baseband signal first. And, then up conversion and then transmit and if we do at the receiver if you first do down conversion and then process to get the message back; equivalently, we can say that the channel is a baseband channel.

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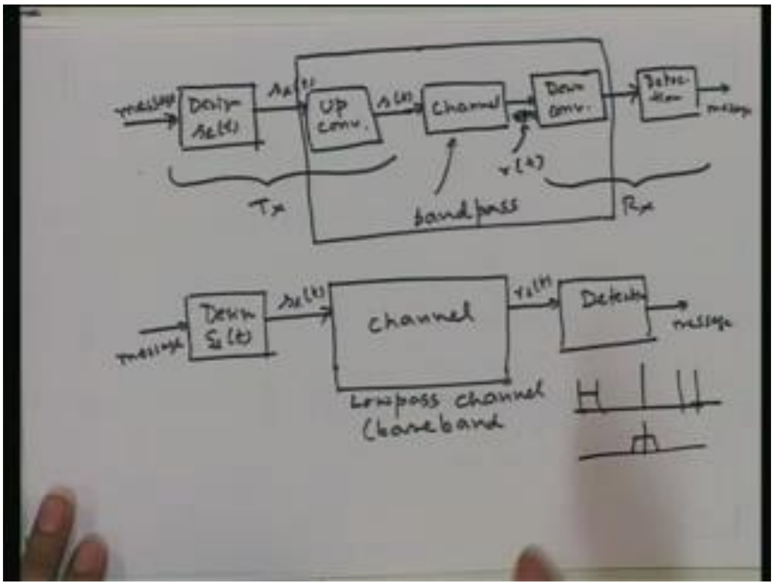

Now, let us see that here suppose you have so, transmitter and receiver if we combine in a single diagram we see that transmitter gets message from here it does some processing it designs the low pass signal from s l t it does up conversion. Then, s t is transmitted through the channel then the receiver first it receives r t another signal r t; it first does down conversion and then it does the processing or detection to get the message. So, from this message first this is a transmitter and this is the transmitter and this is the receiver and this is the channel. So, now if we consider this part combined way includes the up conversion and down conversion of the transmitter.

And, receiver inside the channel then this if you see draw this part alone. So, design of s l t from message from there is these block this box, we will see what it is; then detection this is s l t, this is r l t from here message. Now, if you consider this as a channel instead of this is a band pass channel this is band pass; but this channel let us see how this block behaves. What is input to this block? It is a low pass signal. What is output to this block? It is also a low pass signal. So, we are giving a low pass signal to that block which is the combined a block of combination of up conversion channel and down conversion.

So that block is taking s l t a low pass equivalent signal as input and giving again a low pass equivalent signal low pass signal as output. So, this is a kind of low pass signal, low pass channel or baseband channel. So, this is a baseband channel; so, we can say this is again a channel which is baseband. So, instead of saying that we are actually trying to transmit through this channel you know this channel in this band we were designing s t. And, then receiving r t transmitting s t receiving r t; instead of saying that we can say we have a different channel, we have this channel instead. So, take up conversion we know we will do up conversion; so, down conversion also we know we will do.

So, these 2 combined with channel. So, we can consider this whole block as a channel. So, if we consider that we know that this channel has not this; but we want to transmit in this. So, we will design a signal in the baseband transmit it through this channel we will get this. So, this is equivalent to a channel this whole block is equivalent to a baseband channel. So, we have converted now how do designs this signal so that I can decode this detect this message efficiently; with low probability of error is the question. So, we originally had a problem which is we have to transmit message through a pass band channel. But we have converted that into a problem of transmitting a message through a baseband channel.

But the channel is now complex; because we are transmitting a complex signal, we are receiving also a complex signal. So, originally we had a band pass channel; but real band pass real channel through which we have to transmit a message. Now, we have the problem as we have a baseband complex channel through which we have to transmit a message. So, we have to design this signal from the message in such a way that we can get this message back here in some way efficiently the channel is complex; so, these signals can be complex.

So, the purpose of this derivation is that to show that; if we have we may have a band pass channel or we have baseband channel. If you have a baseband real channel fine there is no complexity we can do with low cost electronics things can be done nicely. But if you have a band pass channel also in the high frequency say 5 gigahertz or 100 hertz, then also we can consider it as a baseband channel. Because we will design the scheme everything from the transmitter and receiver for band channel, corresponding baseband channel. And, then after designing the signal at the transmitter we will do up conversion and transmit through that channel through that baseband channel.

So, the design problem for the transmitter and receiver design problem for band pass channels can be translated into a problem of transmitting message through a baseband channel and the channel is now complex. So, from the next classical onwards we will assume that the channels are complex and the signals through which through which we can transmit and which we will receive are also in general complex. If they are real then we will the case is that the actual channel is also real. But if there are complex even then the actual channel is real; but band pass we can do up conversion and do actually translate this to a solution for this channel. So, we will consider signal design and how to transmit message efficiently for this kind of channel in general.

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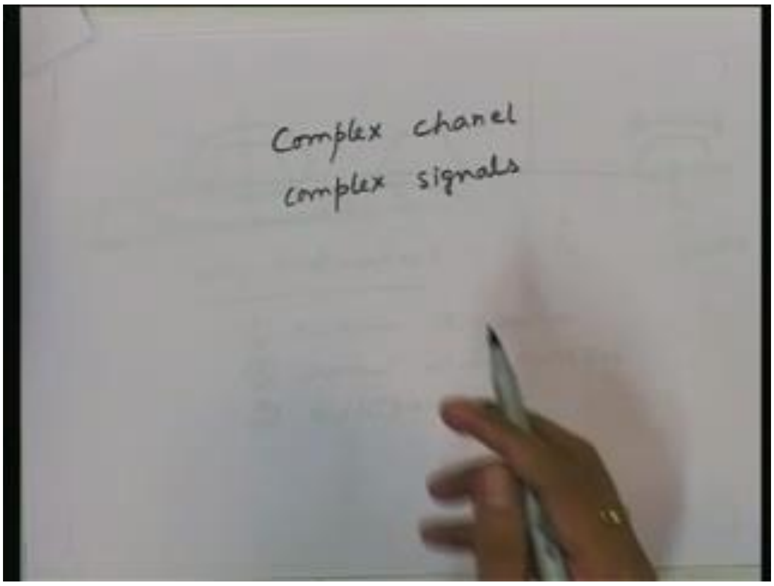

So, we will have complex channel from the next class we will have in general complex channel and complex signals. As transmitted as well as received signals are complex the channel is complex like this these 2 signals are complex and this channel is complex. So, this will be our assumption. And, we will see how to design schemes transmitter and receiver how to design for such a channel. So, in this class we have seen how to represent a baseband, band pass signal in baseband. So, how to do basically down conversion; so that there is no loss of information and also how to get the band pass signal back from the baseband signal.

So that these 2 can be done in the transmitter and receiver up conversion in transmitter, down conversion at receiver. And, then we have seen that this up conversion and down conversion; if they are the combined with the channel this can be considered as a different channel. Now, which is baseband channel? Which takes complex low pass signal as input and complex low pass signal as output? So, we have converted the problem of designing communication systems for band pass channels into a problem of designing communication systems for complex baseband channels. So, that is the fundamental idea and this is a very important idea. Because from now on we will consider such a channel that we will assume that the channel is baseband and the channel is complex; we will see you again in the next class.

Thank you.