Advanced Optical Communications Prof. R. K. Shevgaonkar Department of Electrical Engineering Indian Institute of Technology, Bombay

Lecture No. # 08 Signal Distortion – I

Up till now we discussed propagation of light inside the optical fiber. Initially, we investigated, what is called the ray model for propagation of light, we saw its limitations and then we shifted to more advanced model what is called the wave model. Then, in this wave model we saw that light propagates inside the fiber in the form of modes, and in general there are transverse electric, transverse magnetic and hybrid modes. However, if we consider the practical fiber where the difference in refractive index between core and cladding is small.

Then, we have what are called the linearly polarized modes. So, essentially for a practical fiber the model representation could be in the form of linearly polarized modes. And then, we saw that the first mode which propagates on the optical fiber is HE 11 mode or in terms of linearly polarized modes that is LP 01 mode. The second mode which can propagate on the optical fiber is LP 11 mode, which is the combination of T 01 mode, TM 01 mode, and HE 21 mode. The whole analysis we have been doing essentially to understand, if the light is sent in the form of pulses or if you have a light which is not continuous, but if it is modulated then how do the signal will get distorted when signal propagates on the optical fiber.

So, the next important issue which we want to investigate now from the understanding of propagation of light in the fiber is the signal distortion. So, now we discuss the very important topic, what is called the signal distortion. So, essentially the question we are asking is if the light is modulated and typically the light will be modulated in the form of pulses. So, you have a some kind of an amplitude modulation on this optical carrier. We are asking if the pulses are sent on the optical fiber, how this pulses will get distorted as they propagate on the optical fiber. Now, before we address this question we are asked a

fundamental question, what do we mean by distortion? And in the last lecture we essentially introduced the concept of distortion less system.

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 $= A \times C$
 $= A \times C \omega$

So, we said if we had a system to which the signal x t is a prime you get output y t. Then we say signal is not distorted, if y t is a replica of the input signal within the delay and with an amplitude factor. That means if the signal after passing through the system as a whole is reduced in the amplitude or increase in the amplitude and if the signal is delayed in time then we do not say the signal is distorted. And then you ask what is the condition to realize this in the spectral domain and then we found that if the amplitude response is constant and if the phase response is linear for the system then the signal does not get distorted while passing through the system. With this understanding that we had simply asked the question would these two characteristics be readily available for optical carrier.

So, we said suppose we had an optical carrier and if you modulate that by simple amplitude modulation what kind of spectrum will be generated and would these spectral characteristics satisfy the condition now which we are looking for distortion less signal.

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Radio frequency. $covrier \rightarrow f_c$ modulating frequency AM

So firstly, we saw that if you had a radio frequency carrier f c and if you modulate this carrier by signal f m then in the spectral domain we get what is called the carrier and the side bands which are red f c plus f m, and f c minus f m you are considering here the simplest possible modulation which is the amplitude modulation. And at radio frequency the carrier is very pure and what we mean by that is a spectral width of carrier intrinsically is much smaller compared to f m and as a result we get the side bands very clearly seen in the spectrum of the amplitude modulated signal. The question you wanted to ask is this true for optical carriers also? So, we said that if we take a typical optical source even like a laser, this source has a spectrum width typically if the order of about 1 nanometer. This thing will become clear later when we discuss the sources what causes to have the spectral width.

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But at this point of time let us accept that if you take a typical optical source like a semiconductor laser, it will have a spectral width of about 1 or 2 nanometers. Then around 1550 nanometer one can ask a question how much this 1 nanometer spectral would correspond to in terms of frequency and that I mention as a rule of thumb you can say approximately 1 nanometer is about 100 gigahertz. What that means, is that even without modulation if I take the optical carrier and put it on the spectrum analyzer, it will not look like a sharp delta function as we saw at radio frequencies.

But we will have a spectrum which is distributed something like this, with the center frequency which is the carrier frequency centroid with a width which is typically of the order of about few 100 gigahertz. Say even if we take a spectral width of about 1 nanometer, we will have a width of this carrier typically of the order of about 100 gigahertz. Then one can ask a question if I take this carrier and modulate this carrier by a signal which is the modulating signal f m.

Obviously now in the spectral width of this is very large, the side bands which are separated by the frequency difference of f m is much smaller compared to the spectral width and as a result you see that the side bands are completely masked by the intrinsic spectral width of the carrier. So, obviously whatever conclusions we could draw for the distortion less signal from the amplitude modulated system in radio frequencies though the arguments are no more valid because now the spectrum side bands are not clearly visible and if that is the case then our understanding now has to be completely modified in analyzing the distortion of the signal with optical carriers.

So, now essentially we take this fact into consideration that the optical carrier has a spectral width which is very large compared to the modulating signal frequency and as a result the side band will not be clearly visible. Let us come back to this point little later, but at this point it is very clear that when the signal is transmitted and the optical fiber a simple understanding which we have for radio frequencies will not be applicable in this case. So, in general now we can say that when the signal is transmitted on optical fiber the signal is distorted because the two phenomena and we will try to understand this two phenomena's in little more detail.

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The first phenomena is what is called the dispersion. Now, we have seen a dispersion phenomena when we are discussing the ray model that if you put a pulse energy inside an optical fiber, the pulse energy travels by different paths in the form of different rays and as a result the rays do not reach other end of the fiber at the same time and you have a broadening of the pulse. This phenomena we call as a dispersion because of the multipath propagation of light inside the optical fiber. If you take a single mode fiber then, obviously, this phenomena is not there.

However because of the finite bandwidth of the signal as we saw even the carrier itself has a finite bandwidth there would be dispersion and we will understand this little more as we proceed further. So, in general inside an optical fiber we have three types of dispersion the material dispersion, the intramodal dispersion and the intermodal dispersion which is same as the multi mode propagation or multiple ray propagation. So, inside a single mode optical fiber intermodal dispersion that is because of propagation of different modes is not present.

So, this dispersion is present only in a multi mode fiber whereas, if you take a single mode fiber then you have two dispersions one is material dispersion, other one is the intramodal dispersion. And then as the signal travels on the optical fiber there are losses which take place. So, the energy can be lost because of various mechanisms. Inside a material the energy would be lost due to intrinsic absorption of light inside the material, other possibility is that when you make a fiber the fiber has some kind of a microcenters created because of the manufacturing process or whatever it is.

And this microcenters have a scale size typically of the order of wavelength or smaller than that. So, you get the scattering of light which is a very strong variation as a function of wavelength. It goes as lambda to the power minus 4. So, even if you take a good fiber without putting this fiber in to a system then the fiber itself will have losses because of absorption and because of scattering of light. In addition to that when the fiber is put in to the system the fiber can be never kept in absolutely straight form, there are always some form deformations on the optical fiber which is what is called the micro bending and there would be losses because of this micro bending.

Further if the fiber is laid in the form of large arcs then we call that as the macro bending or that loss is what is called the radiation loss. So, when the signal travels on the optical fiber essentially it gets attenuated because of the loss which takes places over here and because of what is called dispersion. Now, if you take a typical system as we have seen earlier the optical carrier is around 10 to the power 14 hertz and the modulating signal has a frequency even if you consider the highest possible data rate which is transmitted today it could be the order of about few gigahertz.

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fc ~ 10^{14} Hz

fm ~ 10^{10} Hz (10 GHz)
 $Q = \frac{10^{14}}{10^{10}} = 10^{4}$

Let us take either rule of some 10 gigahertz which is essentially 10 to the power 10 hertz. So, if I look at the center frequency the carrier frequency for optical signal f c that is of the order of 10 to the power 14 hertz. Whereas, if I consider f m which could be the highest possible data rate which I want to send, this could be of the order of 10 to the power 10 hertz which is about 10 gigahertz. So, if I ask what is the fractional bandwidth this amplitude modulated signal would have or what will be the quality factor needed for the system typically you will get the quality factor which is 10 to the power 14 divided by 10 to the power 10. So, typically of the order of about 10 to the power 4. That means the fractional bandwidth which the system has is 1 10000 times of the carrier frequency.

So, if a consider now the loss which is there in the optical fiber because of various mechanisms one can ask is the loss going to vary substantially over the small bandwidth, small fractional bandwidth which is 1 upon 10000 of the carrier frequency. And one would notice that if I look at the loss performance as a function of frequency or as a function of wavelength, this is a reasonably smooth variation as in a very slowly varying function of wavelength.

So, if I consider a very small narrow band around the carrier frequency you will realize that there is no differential laws over the bandwidth of about 10 gigahertz. That means, the first thing which we wanted for distortion less behavior of a system that the amplitude response should be constant over the band that is reasonably satisfied by optical communication system. So, what the laws does as a whole the signal goes down as the signal travels on the optical fiber. Question is, is the phase condition satisfied as the signal propagates on the optical fiber?

So, we find that the first condition which you wanted the amplitude performance, the amplitude performance is good for optical fibers because of the amplitude variation there will not be any distortion of the signal on the optical fiber. So, the next thing is which is the phase performance and this far essentially is rated to this quantity what is called dispersion. So, we will see the dispersion in more detail to see whether this is going to affect the signal propagation or signal distortion.

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So, if we send a signal in the form of pulses what dispersion means is as the signal travels on the optical fiber the pulses get broadened and here we are not even talking about the shape of the pulses, you are not even saying the shape of the pulses retained or anything. We are saying if this pulse was representing a presence or absence of a bit. Then as the signal travels essentially this function broadens because of the phenomena of dispersion and as it travels more and more the broadening becomes more and more.

So, here the pulses were very clearly identified, when we travel a small distance the pulses will partly overlap, when we go further and the pulses will further overlap, if it travel further more on the fiber the pulses will become like that. So, what we find is that as the pulses travel on the optical fiber slowly they starts merging into each other or they start losing their identity. So, if they are travel still further you will see they will overflow so much that you will not be able to tell whether there are two pulses here.

So, before that thing happens essentially you have to stop and regenerate your pulses. So, essentially this pulse broadening phenomena puts a restriction on the maximum distance over which you can send this pulses so, that they can be recovered on the other side. If you send the pulses beyond that distance then signal will be lost hopelessly and you will not be able to recover the pulses from the signal. Now, since the energy in the pulse is fixed even if we assume that there are no losses in the optical fiber as the pulse broadens the peak amplitude of the pulse comes down. Since the total energy inside the pulse has to be conserved in the lossless system, the broaden pulse will have a lower amplitude.

So, as the signal travels on the optical fiber because of the dispersion two things happen one is the pulse gets broaden and other one is the peak amplitude of the pulse comes down and invariably since, we detect the signal without by integrating the pulse over the entire duration we sample it at a given instant of time, the signal to noise ratio gets affected because of the dispersion also. So, this phenomena the pulses gets broaden as it propagates on the optical fiber takes place because of as I mentioned because of three phenomena's one is the what is called the material dispersion, second one is what is called the intramodal dispersion.

That means, the dispersion within a mode because of the finite bandwidth of the signal and third one is the intermodal dispersion which is because of multiple propagation of modes and this one takes place inside the multi mode optical fiber. Now, let us come back to our original argument that if I consider a optical carrier then my spectral width of the carrier is very large. So, now, I have two situations at radio frequency, if I draw both spectrum of carrier and the modulating signal.

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So, this is the radio system. If I draw the spectrum, my carrier frequency is very small. Let us say this is my carrier frequency which is very narrow and my modulating signal frequency was large. So, this is my carrier frequency whose spectral width is much smaller compared to the modulating signal. So, my modulating signal spectrum was like that. So, this is modulating signal.

Whereas if I go to the optical system then I have a situation which is exactly opposite, that is my optical carrier is very large and my modulating signal is very small. So, this is my modulating signal which could be of the order of about 100 gigahertz or 10 gigahertz bandwidth whereas, you have the optical carrier which will be like that. So, this is the carrier. So, as we mentioned earlier when we analyze the radio systems since the side bands are very clearly visible, essentially you can detect the signal or recover signal by using either the spectral domain techniques or by using time domain technique because the spectrum is very clearly visible we can see the side bands.

Question one can ask is if I now go to the optical system where the carrier frequency is much larger or carrier spectrum width is much larger compared to the modulating signal then the total spectrum will be nothing, but combination of these two. So, total spectrum will be more or less identical to the spectrum of the carrier. If we get modified, but this modification will be very margin. So, whether we modulate the carrier or do not modulate carrier you see the spectrum which will be same as the carrier spectrum.

Or in other words what that means, is that there is no clear signature of modulation in the spectral domain for an optical system. Whereas, in radio system we had a very clear cut signature of the modulation because we could see the side bands very clearly and that is the reason we could employ either the spectral domain technique or the time domain technique, but now what we see for optical system there is no signature of modulation process in the spectral domain because the spectrum is more or less decided by the carrier spectral width.

What that means is then, that you cannot employ now the spectral domain technique for demodulation process, for recovering the signal. So, since the spectrum is hopelessly lost in this process we can employ only the time domain technique and a time domain technique which does not employ any spectral information is the amplitude modulation not even suppress carrier. So, we have various modulation processes as we know it could be simple amplitude modulation, it could be single side band modulation, it could be amplitude modulation suppress carrier and so on.

But single side band system or suppress carrier system, all of them require the spectral information. So, only modulation process which does not require spectral information is the amplitude modulation because for that signal we can recover the signal in the time domain by simply envelope detection. So, what we find is that because of this situation that the carrier is much broader compared to the modulating signal, intrinsically for optical communication the modulation process which is possible is the simple amplitude modulation process.

Or in other words as far as the modulation techniques are concerned the optical communication system is the most primitive system because simple amplitude modulation in for radio communication you should take place 100 years back then many sophisticated techniques came for modulation process, but in optical domain still the most prevalent modulation would be simple amplitude modulation. So, if I want to now visualize this and see how the signal will get distorted in the system.

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Only possibility is that I can consider now this carrier as a combination of many carriers which are simultaneously transmitted on the optical fiber. So, what we can say is that we have now intrinsic carrier which is having a spectrum like this and that is the carrier we want to modulate by amplitude modulation possibly we want to change the envelope of this total light which is having a spectral width which is about 100 few 100 gigahertz. So, this can be visualized as if I am transmitting multiple carriers simultaneously all up to this.

Each carrier is modulated by your modulating signal f m and all the carriers are simultaneously transmitted with that modulation. So, this system can be visualized as if there is a parallel transmission of multiple carriers taking place. Each one is modulated in a amplitude modulated fashion with the modulating signal. So, one can say that each of these carriers would have its own side bands and that is the reason the side band information is completely masked because you have a distribution and side bands are completely submerged inside this.

However, since we are having a spectral width which is large, typically of the order of about few 100 gigahertz. The pulse which rides on this carrier would travel with different velocity than the pulse which rides on this carrier because either we take the material intrinsically. The refractive index is a function of wavelength, say every wavelength or every frequency does not travel with same velocity. Also we know from our moral

propagation that since the b v diagram is not a linear diagram, the velocity is a function of frequency, the velocity is a function of v number and v number is proportional to frequency. So, we know that the velocity is a function of frequency.

So, essentially what we are saying is that the pulse which rides on this carrier travels with certain speed, the pulse which rides on this travels with different speed, this with different speeds and so on. And because of this different speed the pulses riding on different carrier, they reach at different time or they take different time to propagate a given length of fiber and therefore, essentially gives rise to what is called the material dispersion or intramodal dispersion.

So, one thing is now very clear that since now the broadening is going to be because of the suppression of different carriers within this spectral width of the source, more is the difference between this carriers, more is going to be the pulse broadening. So, larger the spectral width of the carrier more will be the pulse broadening. So, pulse broadening is now related to the spectral width of the carrier. Now if I try to see this thing in terms of time domain and that will become immediately clear that how the pulse will get broaden. So, this was in terms of frequency.

If I modulate now this light with let us say a pulse and see in terms of time how the pulse would look like. So, initially when we transmit all the pulses riding on different carriers they will be at, located at some location here on the fiber and all of them will be aligned in time. However, since the carrier amplitude is different at different frequencies and if I assume that all of them get modulated by the same modulation index, you will see that the pulse amplitude in time will be scaled exactly by this function.

So, if I see now. So, if I consider the frequency which is perpendicular to the plain of the paper and time is the time is the thing which is like this then at the launching point the pulse corresponding to this would have an amplitude which is like this, corresponding to this would be little more than this, corresponding to this will be little more than that and this will be maximum like that. So, you will have a pulse in time which will have a profile in frequency which is perpendicular to the plain of the paper it will be like this. But in time they will all be aligned so, you will see a pulse which will be a pulse which you want to transmit.

Now, let this pulses move around the optical fiber and let us say the high frequency travels faster compared to the low frequencies. Then the pulse riding on this carrier would go further or go ahead of the pulse riding on this one. So, let us say if I number this frequencies from f 1, f 2 up to let us say f n, the pulse riding on f n will travel more distance because it is travelling faster compared to the pulse riding on frequency f 1. So, if I go a certain distance you will see that the pulse now will be separated little bit like this where this f n has reached here and this is f 1.

Since, we have this frequency though I have drawn here this discrete kind of spectral lines here basically these are continuous distribution of carrier frequencies. So, we will not see this separate pulses, what we see is the envelope which will be like that. If it troubles further then there will be further separation of this so, f 1 goes here. So, you will get a distribution of the pulse which will look like that where again this is your f n frequency and this your f 1 frequency.

So, as we have argued earlier as this pulses travel more and more on the fiber the pulse gets more and more broad. One can ask what is the shape of this function? In fact, the shape of the function now is decided by this profile, where initially the spectrum was like this perpendicular to the plain of the vapor and as they travel this function slowly tilts like this to the projection of this function taken on time because of this differential velocities. That is what will get reflected in to this profile.

So, what we find something interesting in this case now is that a spectral width if I ask which is required for optical communication is more or less same as the spectral width of the source. Because even if you have side bands after modulation, hardly spectral width is going to change by a very small fraction, more or less will be decided by the spectral width of the source. You can ask what is the shape of the pulse as it travels on the optical fiber, initially the shape would resemble to the intrinsic shape of the pulse which you are transmitting, but as you proceed further you will see that this shape slowly will be dominated by this profile.

So, it is very interesting to see that in optical communication the bandwidth required is decided by carrier. You recall that if you take a radio system then the bandwidth required is decided by the modulating signal because for amplitude modulation you require a bandwidth of two times f m. So, the bandwidth has nothing to do with the carrier

frequency if I send a modulating signal of let us say 1 megahertz we will require a bandwidth of 2 megahertz whether I use a carrier of 1 gigahertz or 2 gigahertz it does not make a difference bandwidth will be always 2 megahertz.

Whereas if I going to optical frequency what we find is that for a 1 nanometer spectral width source the bandwidth required will be 1 nanometer which is about 100 gigahertz whether I modulate the signal with 1 megahertz or 2 megahertz makes hardly a difference because it will be 100 gigahertz plus 1 or 2 megahertz. So, the bandwidth is purely determined by the carrier in the optical system whereas, the bandwidth is purely determined by the modulating signal in the radial system.

Secondly more interestingly also we see that the shape of the signal or modulating signal is has a relation to the modulating signal which you originally sent in the radio system where I have, when you come to optical system we find that this shape at least after certain distance of propagation has nothing to do with the original pulse which you transmitted, but is decided by again the carrier profile, carrier spectral profile. So, when a signal propagates on the optical fiber it is not a small distortion which takes place, but in fact, the signal loses its complete identity.

Because even the shape of this pulse does not resemble to the original pulse which you transmitted and that is the reason we cannot really talk about distortions or that kind of thing, we talk about the pulse and if you are talking about a digital system then essentially absence of presence of a pulse gives you the information about signal. So, whether the pulse was like this or it is like this, it does not make a difference as long as you can identify there is a energy there, you can get the pulse.

So, the shape of the pulse and the spectral width required both of them are decided by the carrier in the optical communication system and this is conceptually different than what happens in the radio system. Once we understand this then the next thing is straight forward that if I transmit now more and more pulses then the, this broaden function will start overlapping with each other and because of that the signal will slowly start losing its identity. So, two things now are important to observe here, that if I talk about dispersion then the pulse broadening is directly related to the intrinsic spectral width of the carrier, more the spectral width more will be the pulse broadening.

Also the pulse broadening is directly proportional to the distance travelled because more the travel, more will be the difference in the distance travelled by the pulses because of difference in velocity. So, more will be the pulse broadening. So, they have we have this pulse broadening which now depends upon the distance travelled on the optical fiber and it also depends upon the spectral width of the carrier. With this understanding then one can go to more detailed discussion of the dispersion on the optical fiber.

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Dispersion Group Velocity $v_g = \frac{\partial \omega}{\partial \beta} = 2\pi c \frac{\partial}{\partial \beta} (1/\lambda)$ Group Delay per unit length $t_a = 1/v_a$ Pulse Broadening $\tau_z = \frac{dt_z}{d\lambda} \sigma_{\lambda} = -\frac{\sigma_z}{2\pi r} \left\{ 2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^{2\beta}}{d\lambda^2} \right\}$ $D = \frac{dt_e}{d\lambda}$ = Pulse broadening per unit
distance per unit spectral width $(ns/Km/nm)$

So, let us now give a framework to the dispersion. Here I have developed some simple mathematics to get the quantitative thing for a dispersion on optical fiber. So, what we are saying here is that different carriers which are modulated, they travel with different velocity and because of that you get a pulse broadening. So, since we are talking about propagation of pulse here we are now interested in what is called group velocity on the optical fiber. And as we have seen earlier the group velocity v g is given by d omega by d beta where beta is the phase constant of a particular mode and if I consider a single mode optical fiber this beta will be the phase constant of $1 \text{ p } 01$ mode or h e 11 mode.

We can write here for omega which is 2 pi in the frequency and frequency can be written as velocity divided by wavelength. So, we can write here 2 pi into c velocity of light d by d beta of 1 upon lambda. So, we can write down what is called a group delay per unit length which is nothing, but the group velocity and then we can write down the pulse broadening which is d t g by d lambda.

So, we are saying what is the difference in the delay, group delay per unit length over 1 unit spectral width? That quantity will be d t g by d lambda. If we multiply by what is called the spectral width of the source sigma lambda then we get total pulse broadening. See if I take this v g from here find out what is the group velocity take 1 upon that, derivative of this quantity with respect to lambda if we do that you get the pulse broadening which essentially is given by this. So, now we can define formally the dispersion. So, dispersion we define as the pulse broadening per unit distance travelled on the fiber per unit spectral width of the source.

See if I bring this quantity sigma lambda down here that will be the pulse broadening per unit spectral width of the source. So, essentially d t g by d lambda where t g is the group delay, that quantity is what is called the dispersion on the optical fiber. See you can see from here the unit for dispersion would be pulse broadening which normally is measured in terms of picoseconds per unit distance see your per kilometer and per unit spectral width of the source which is nanometer.

So, for the dispersion we have a unit picoseconds per kilometer per nanometer of the spectral width of the carrier, with this understanding now with this basic formulation now we can go and try to quantitatively find out the dispersion coming because of different components, because of intrinsically, because of medium properties and because of the modal properties on the optical fiber. However, you will note that the dispersion phenomena is rather a weak phenomena.

What that means, is that unless we want a very accurate value of dispersion one can assume that each of this dispersion can be calculated independent of each other. What I mean by that is that when I calculate material dispersion I assume that the waveguide dispersion is not present or negligibly small. That means, when I analyze the material dispersion I assume that the light is propagating in a infinitely large medium, there are no guided modes. So, light propagates in the form of a transverse electromagnetic wave, but just because of the material properties the velocity changes as the function of wavelength and that gives you dispersion.

This phenomena is same as the dispersion phenomena which you see in the prisms and lenses and so on. When we go to intramodal dispersion then we assume that the material dispersion is practically small. So, we assume that now the refractive index is not varying as a function of wavelength and only because of the modal nature of light we have a difference in velocities and because of that we have a pulse broadening. So, at a time we simply calculate a dispersion because of each of this and then finally, we simply add the dispersion to get the total dispersion on the optical fiber.

Intramodal dispersion we will discuss later. So, the argument which we are putting forward are essentially for these two dispersions, the material dispersion and the intramodal dispersion. So, let us take the fourth dispersion which is the material dispersion and what we find for material dispersion that now with the refractive index is the function of wavelength, but there is no guiding of light inside the fiber. That means, now if I ask what is the phase constant? The phase constant will be of a phase constant of the transverse electromagnetic wave.

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Phase constant
 $\beta = \frac{2\pi}{\lambda} n(\lambda)$ \rightarrow tg = $\frac{\partial P}{\partial \omega}$ $D = \frac{dt_g}{d\lambda}$
 $D_{mat} = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2}$

So, now the phase constant for this wave beta will be 2 pi by lambda where lambda is the wavelength which you measure in the free space multiplied by the refractive index of the medium. So, this is n, but now n is a function of wavelength. So, n is a function of lambda. So, now, I have a variation of this phase constant which is going to vary as a function of lambda. So, from here I can calculate the quantity d beta by d omega which gives me the group velocity or group delay. So, I have this quantity t g that is equal to d beta by d omega I am also I get this group delay then I can calculate the dispersion D which is d t g by d lambda.

See if I do this then we get the dispersion because of material which will be essentially given by this or it may write down there this quantity will get as D material that will be equal to minus lambda by c d 2 n by d lambda square. So, the material dispersion is first of all proportional to the wavelength, but interestingly it is also proportional to the second derivative of this refractive index function as the function of wavelength. So, at this point without looking at what variation the refractive index has as a function of wavelength we cannot really say much about what kind of material dispersion we have on the optical fiber.

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So, what we do is we take the material glass and plot its refractive index as a function of wavelength. So, you see if I vary the wavelength lambda from 0.2 micrometer 200 nanometer to about 3.5 micrometer we get a refractive index variation which will be typically like that. So, a refractive index decreases, but decrease in refractive index is not linear it has a variation something like this, that is the measured fact say for glass that is the variation of refractive index.

Now, if you look at this expression which you have got here the material dispersion is proportional to second derivative of n with respect to lambda and what does second derivative represent? The second derivative represent the curvature of this function n with respect to lambda. So, if I look at now this variation you see this function has a curvature like this. Whereas, here if I look at the curvature is opposite, it is something like this. So, that curvature changes the sign from this to this at this wavelength which is 1.27 micron or 1270 nanometer.

So, this quantity delta d 3 by d n, d 2 n by d lambda square goes 0 at the wavelength which is 1270 nanometer and as a result the material dispersion becomes 0 at 1270 nanometer. So, these are natural property now of glass that even if I not made optical fiber out of glass, the glass would have a 0 dispersion at a wavelength which is 1270 nanometer. Then this table show you now the dispersion which will get a different frequencies. So, the first window of optical communication which was 850 nanometer the dispersion is as high as 85 picoseconds per kilometer per nanometer.

This window 1310 nanometer which is very close to this. So, here dispersion is very small and now you will appreciate why the 1310 window was preferred against 850 because in this dispersion is very small. Again when you go to the wavelength which is 1550 nanometer the dispersion becomes negative 20 picoseconds per kilometer nanometer. So, if I not had the fiber, if I had only the material glass and if light had propagated in this, we would operate at 1270 nanometer because that is where the dispersion becomes 0 or that is where you can send the highest possible data rate inside glass.

So, we will continue our discussion on the dispersion. We have first see what the negative sign here means for dispersion and then we will investigate the second dispersion what is the material dispersion when we met in the next lecture.