Advanced Optical Communications

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Module No. # 01

Lecture No. # 38

Cross Phase Modulation and four wave mixing

In earlier lectures, we saw the non-linear effect coming from the chi three non-linearity. In that of course, we considered only one signal propagating inside the optical fiber. Then we saw that the refractive index depends upon the mod of the electric field square that is the power density, inside the optical fiber. Today we are going to see the same third order susceptibility if we take, and if we do not have only one signal propagating inside the optical fiber, if you have multiple signal propagating simultaneously. Then what kind of interaction takes place between different signals.

So, in this lecture we are going to discuss two phenomena; one what is called the cross phase modulation and second one is what is called four wave mixing. Both this phenomena are related to the term which is due to third order susceptibility. So, let us look at these phenomena of what is called the cross phase modulation. So, let us say we have the electric filed which has two frequency components.

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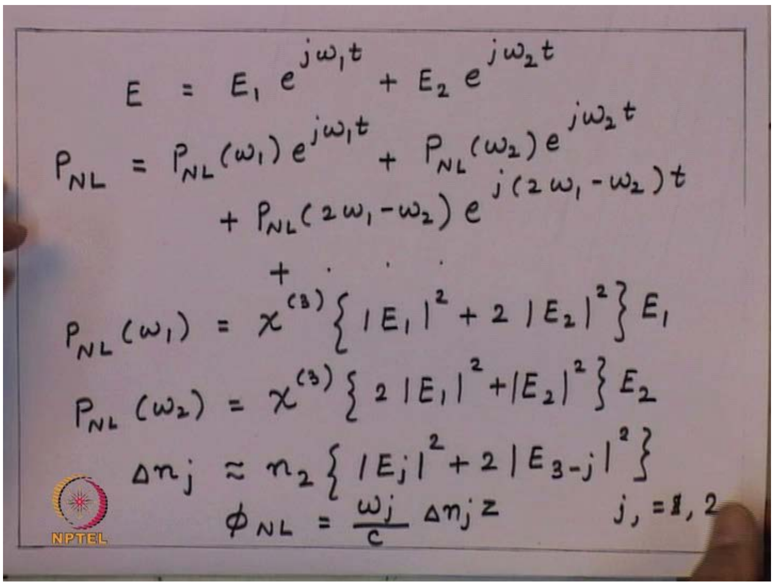

So, the electric field the two frequencies is amplitude, let us say E 1 and a frequency omega 1 and there are another signal with amplitude E 2 and frequency omega 2. Then if you substitute this into the polarization expression, we will get the no-linear polarization created inside the material that will have various components now. So, it will have component non-linear component at omega 1 E to the power j omega 1 t. It will have a component at frequency omega 2 E to the power j omega 2 t plus it will have other terms which law in the same frequency range. That is if we consider frequency let us say two times omega 1 minus omega 2 or two times omega 2 minus omega 1. These are the terms which will be approximately in the same frequency band.

So, you will have terms like 2 omega 1 minus omega 2 with the frequency 2 omega 1 minus omega 2 plus similar term with 2 omega 1 and so on. And this non-linear polarization which we get this at frequencies omega 1 will be ray to this chi 3 non-linearity. And the term mod E 1 square plus 2 times mod E 2 square E 1. Similarly, we can get the non-linearity at frequency omega 2 that is chi 3 2 times mod E 1 square plus E 2 square E 2. And then because of this non-linearity the change an refractive index delta n at jth frequency where j could be 1 or 2 that is given by this non-linear coefficient N 2 times mod of E j square plus 2 times E 3 minus j square. (No audio from 05:13 to 05:20).

Now, note here that when we had only one frequency that mean let us say E 2 was is equal to 0 in this only one frequency propagating that time. This quantity will be equal to 0 and then we get a change in refractive index which is the non-linear coefficient n 2 times this E 1 square. That is the case which is the Kerr non-linearity analysis which you have done, when we derived the non-linear Schrodinger equation considering only this term. What you are seeing now that when the other signal is also present here, the change in refractive index is going to be because of the signal itself plus there will be contribution coming from another signal which is co-existing along with this signal.

But, the most important thing to note here is that the change in refractive index due to this signal propagation has enhancement factor of 2 compare to this signal. And that is a very interesting thing to note what; that means, is that the non-linear effect of another signal is twice stronger compare to the signal itself. And as we seen earlier when the signal propagates inside the optical fiber, this change in refractive index gives me a phenomena what is called the self-phase modulation. What we are saying now is that since the refractive index change is going to take place, because of this frequency other signal which is this signal. The change in phase is not only due to because of the signal itself, but it also due to another signal which is propagating.

So, as we had a phenomena earlier when this was not present that phenomena we called the self-phase modulation, because the signal phase use to get modified by it-self. Now, we are saying that the phase is going to get modified by it-self plus another signal which is co-existent. That phenomena now than is called the cross phase modulation. So, it is possible that if the this signal is very week compare to the signal. The contribution coming because of this term in negligibly small, but this contribution could be very strong, because this intensity of the signal is much larger.

So, due to the third order non-linear susceptibility, we see this phenomena what is called the cross phase modulation. And when we are having a phenomena like this, then one can say that the phase change a non-linear phase change as we saw in terms of the self-phase modulation can be given as delta n j time set. So, here j could be 1 or 2. So, if I put j equal to 1 this is E 1 square plus 2 times 3 minus 1 which is E 2 square and if I put j equal to 2 this will be E 2 square plus 2 times E 1 square.

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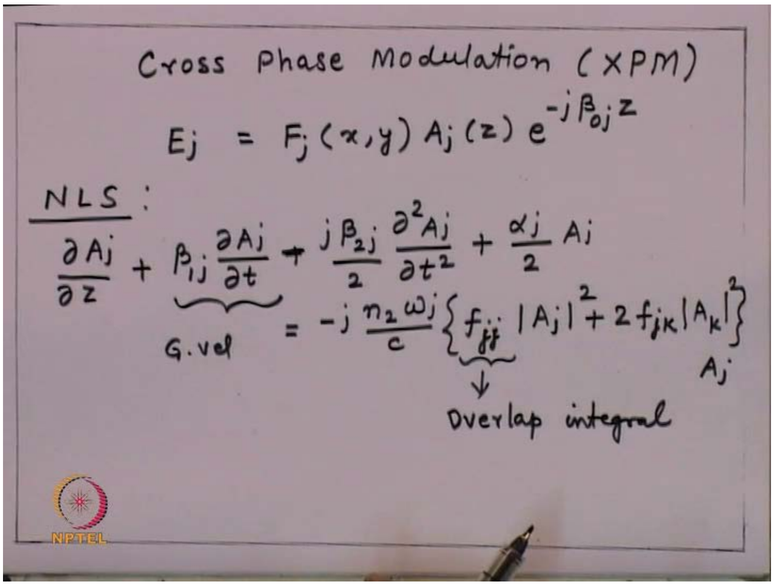

So, this phenomenon is what is called the cross phase modulation. So, we have these phenomena what is called cross phase modulation and in short it is called XPM. So, whenever we are going to have multiple channels transmit on inside the optical fiber, there would be these phenomena of cross phase modulation. Then as we know that for w d m system when large number of channels propagates if there is sufficient power in each channel then there would be change in the phase of E signal because of the presence of another signal. Also not here, that when we are having large number of channel transmitting let us say n channels are transmitting then each channel will be effected by the power present in remaining n minus 1 channel.

And since the effect of each of this channel is going to be two times stronger compare to the individual. So, you see the collective phenomena of self-phase modulation, cross phase modulation are much stronger compare to what you would see in the self-phase modulation. So, one can then say that if we want to do formulate a problem, essentially we require the same non-linear Schrodinger equation looking into the effect of the cross phase modulation. So, as we did earlier, also we can do the formulation similar way we say that the field distribution for the jth channel that is having a transfer function, field distribution which is in the x y. Which is the transfers plain for the fiber and envelope function which is going to evolve as the function of z.

And a phase constant which is minus j beta 0, but now this beta is going to be a function of the frequency so, this for the jth channel time z. So, if you do the same steps what we have done for the self-phase modulation. We again would get two equation one would govern this field is distribution and other one will give the evaluation of the envelope as a function of distance on the optical fiber. So, following the same steps what we have done earlier, one would then get the non-linear Schrodinger equation. That can be return as A j divided by d z plus beta 1 j d A j by d t plus (No audio from 12:23 to 12:28) minus j beta 2 j by 2 d 2 A j by d t square plus alpha j which is the attenuation term by 2 A j is equal to minus j N 2 omega j by c.

And the non-linear terms which are proportional to the mod of E square per since now the other channels are also present here along with this you will get contribution from other channels also. So, let us say along with jth channel even the kth channel is present. So, you will get here call f i i mod it is make j j square plus the cross phase modulation term 2 times f j k mod A k whole square. So, as we have seen earlier in the non-linear Schrodinger equation this is the change rate of change is the function of distance. This term gives me the group velocity, this term gives the group velocity dispersion, this term gives attenuation and this term gives the non-linearity.

And if you recall earlier we as to have a term which is simply mod is A square times A j. So, it has to be A j here. So, it use to have a term which is simply mod A j square multiplied by A j. However, now what we are seeing is we are having additional factor here which is this f j j or f j k. And this quantity is what is called the overlap integral. (No audio from 15:18 to 15:26) So, since there multiple signals propagating how much overlap of the field takes place between two signals that is capture by this quantity what is called an overlap integral.

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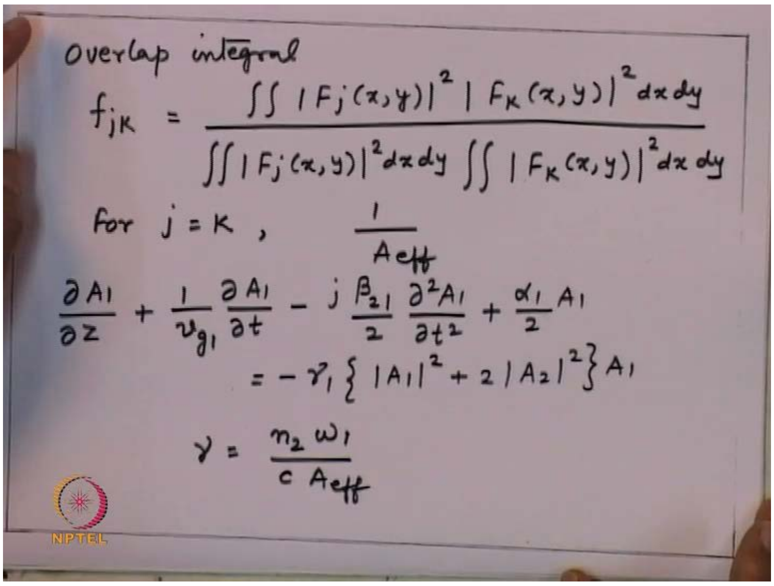

So, the overlap integral is define as (No audio from 15:43 to 15:50) f j k that is defined as over the cross section mod of F j x y square F k x y square d x d y divided by

(No audio from 16:24 to 16:30) integral mod of F j x y square d x d y multiplied by integral mod of F k x y square d x d y. So, when you are having same quantity here j is equal to k that time this quantity essentially will reduce to the effective area. So, you will see that this quantity will be for j equal to k that only one signal is propagating that time this quantity will be 1 upon A effect. That is what we got for the single signal propagation on the optical fiber.

However, when we are going to have multiple signal propagating, what this factor captures is it is sort of shows you how well the two phase one completely overlapping each other. Whereas, we know that this field distribution is going to be function of frequency. So, if we take widely different frequencies than the two fields will not have a good overlap and then this quantity will essentially reduce or the interaction between the two channels will reduce. So, this term essentially is capturing how effective the interaction between these two channels is going to take place having done this then the same equation now can be return in terms of that non-linear parameters and so on.

As we seen done earlier so, the equation will become a d A 1 by d z plus this quantity is beta 1 for the jth equation let us see we are writing for channel one. So, we know that this quantity will 1 upon group velocity or channel one. So, this is 1 upon v g 1 d A 1 by d t minus j you have beta 2 a for channel 1 by 2 d 2 A 1 by d t square plus alpha 1 by 2 A 1 is equal to minus gamma 1 mod A 1 square plus 2 times mod A 2 square A 1. And same thing we will have same equation you will have with for a 2 also. If I replace here a 2 A 1 by A 2 can I will get a equations similar equations for second signal also. Where this quantity gamma as we define in the earlier case this gamma is equal to this n 2 non-linearity coefficient j.

So, omega 1 divided by c times A effect. So, if the two frequency which I am considering are very close to each other than this quantity will be 1 upon a effective practically for both the things. So, the parameter which we have define a here which is f j j which is F 1 one or F 1 2 this two are equal and that is almost equal to 1 upon that a effective which we have define for the fiber. So, you see in the case of XPM, the non-linear Schrodinger equation has in addition to the original term which was this A 1 more time which is coming, because of this. And depending upon the amplitude of A 1 and A 2 this term and dominate or this term dominate.

So, one can then ask question if you are having this phenomena, what will happen, when the signal start propagating inside this? So, few things you here, first up all when two channels start propagating earlier what we have done is we are taken this term here and we are define a moving frame. So, that this term vanished, because we are got a frame which was moving with the pulse. So, in that of frame of reference than this velocity was equal to 0, but now since we are having two signals traveling, you cannot define now a frame of reference which can be synchronize with both of them.

So, essentially this term will always remain or even if you try to move a frame with one signal another frame will be moving with the difference of the two velocities. So, this term within general will always remain A with that. So, not the phenomenon is as follows. Imaginary situation, that you are having a pulse and there we know another pulse which is travelling, but these two pulses are not travelling with the same speed that travelling with different velocity. When the travel with different velocity, they essentially slide to each other as the propagate.

So, if you take this two pulses when the overlap, they interaction between this to take place and the cross phase modulation phenomena can be seen. When this slide through each other and the cross this then the selfish phenomena vanishes, because now this two pulses are not interacting with each other. This is what is called a walk of phenomena. So, this will be related to the difference in the two group velocities. So, if you consider a fiber where the group velocity is practically constant, then you will have a strong interaction taking place between these pulses.

But, if you consider the channels which are widely separated, then this two will travel with different velocity and only wherever there is overlap between the two the non-linear interaction will take place otherwise the interaction will not. So, the phenomena which we have seen earlier for the self-phase modulation, the similar kind of behavior we will expect in the situation also. Only thing is in this situation the non-linear phase will be effected by the additional channels also. So, if we want to these phenomena to be weak, then we should create situation. So, that the pulses do not travel together for long time; that means we should create the fiber which have a large group velocity change as the function of frequency.

That is saying that we have a large dispersion on the optical fiber. So, if you consider fiber where the dispersion is large, then there will be significant difference between the velocities of a two signals and then the walk of time will be very short. So, the non-linear interaction between these two pulses will be very small, on the other hand if you want to make use of these phenomena of self cross phase modulation. Then we should use a fiber which is more like a dispersion flat and fiber. So, that all the signals travel with more or less same velocity so, the walk of time is much longer and there would be strong interaction between the different signals.

So, if we use now a fiber which is dispersion flat and fiber in a w d m channel. You will see that the effect of cross phase modulation will be much more. So, the avoid the cross phase modulation effects in that is w d m signal, it is desirable to keep some dispersion into the distance, because you get the dispersion was made 0 then different frequencies would travel this same velocity. If the travel will be same velocity, then they will interact more and there will be more effect of cross phase modulation of the signal. Let us already mentioned that in a w d m system where the number of channels is very large, every channel is going to be effected by all the rest of the channels.

So, if you consider let us say one twenty eight channel w d m system, every channel will have the effect you to remaining one twenty seven channels. And each one even if is a carry power the order of about two three mill watts, the total power with one twenty seven channels would be easily of the order of about 500 mill watts. And then the non-linear phase which will created because of this will be significant.So, in a w d m system, one whose see that the cross phase modulation effects would probably we must stronger compare to the self-phase modulation. Because, the each channel is going to carry still a power which is very small few mill watts.

And at that power the self-phase modulation will be very weak, but collectively the cross phase-modulation would really have a stronger effect. So, it means that in w d m system, if you want that the channel should not get affected by the neighboring channels, because of this phenomena then one should use the fiber which has the dispersion. So, now you see originally when we talked about the optical communication we wanted to make the dispersion small. So, that the pulses broadening does not take place. Now, when we are talking about w d m channel, when we start looking into this additionally effects if find that making dispersion 0 over very wide wavelength range is not very good option. Because in that situation these different channels will start interacting and they will create some kind of cross stock between different channels. So, XPM effect the signals in different frequencies, it also give you the same phenomena like the spectral broadening as you saw in the self-phase modulation. Only thing is in this case the spectral broadening is not symmetric around this original frequency as was happening in the self-phase modulation. You may get a symmetric spectrum developed, because of these phenomena of course, phase modulation. So, this is one of the important aspect of the same third order susceptibility which was giving you the propagation self-phase modulation solution and so on.

When we are having a multichannel transmission, the same sort order susceptibility gives you these phenomena of course, talk to non-linear effects. The same phenomenon when I will look at it gives you one more effect and that is what is called the four view mixing phenomena. Now, what is the four wave mixing phenomenon? This phenomena if I compare with and the electronic system, this phenomena is identical to what we is to say as the inter channel mixing for inter modulation products. So, we know in an amplifier, when amplifier goes into saturation if you put two frequencies inside the amplifier, the third frequencies generated.

Because of the non-linearity, these two frequencies the gives a products which are some different products and so on. Exactly same phenomena we see in the optical fiber also, because of this certain order non-linear susceptibility.

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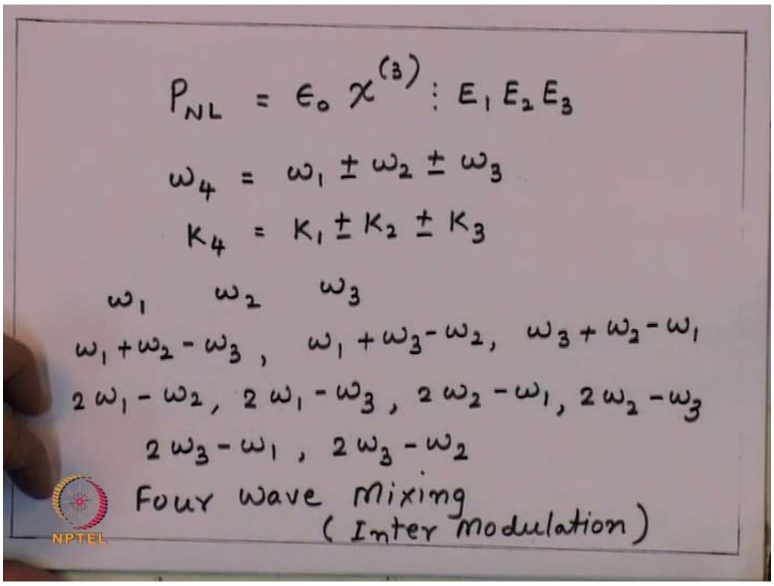

So, we are saying that if you consider now, a non-linear polarization which we have seen is third order susceptibility and let us say now I am putting three signals. So, I have three frequency simultaneous put inside the optical fiber. So, they will interact through this term and then, because of the intermediation they will produce the frequency force frequency. Which will be some and different frequency of the three signals. So, they will produce frequency which is omega 1 plus minus omega 2 plus minus omega 3. And they will be having the wave numbers which is k 1 plus minus k 2 plus minus k 3. So, when three frequencies go inside the material because of the non-linear measure of the material you will see there are additional frequency which would get generated.

In addition to the original frequencies which were omega 1 omega 2 omega 3. So, if i just take all possible combinations, you will see originally we had three frequencies omega 1 omega 2 omega three. But, now we are going to generate frequencies which is omega 1 plus omega 2 minus omega 3, omega 1 plus omega 3 minus omega 2, omega 3 plus omega 2 minus omega 1 2 times omega 1 minus omega 2 2 times omega 1 minus omega 3, 2 times omega 2 minus omega 1, 2 times omega 2 minus omega 3 2 times omega 3 minus omega 1 2 times omega 3 minus omega 2. So, you remember when we are talking about this term of non-linearity here.

We are said these are the different frequencies which are get generate and we are going to look at only those frequencies which lying the same frequency back. So, you see here that we had a original frequencies which we are omega 1 omega 2 omega 3 that is say the lying same band. You see all this quantities, all this frequencies they have the same range. So, this term has a frequency almost comparable to this, this one has almost comparable to this right and so on. We may have frequency where omega 1 plus omega 2 plus omega three, but this frequency will be approximately three times omega one. So, that will lying totally different frequency band.

So, this combination is going to create many more frequencies, but those frequencies which are double of any of these frequencies or three times. We are not looking into because they are going to be different frequency band, and very easily they can be eliminated by filtering process of whatever it is. But, these frequencies they all lying the same frequency band. So, essentially what we are saying is now that when different signals propagate, because of this non-linear process the some and different frequencies are created and this new frequency this are going to lie and the same frequency band. That is the phenomena what is called the four wave mixing (No audio from 34:35 to 34:41) and this phenomena electronic if you see this also what is called the inter modulation phenomena.

So, from the electronic communication terminology, this phenomena is exactly identical to what you see inside the amplifier and this frequencies water call the intermediation products. In optical communication this phenomena is what is called the four wave mixing. So, now we see that this phenomena four wave mixing which is coming, because of that term which is ASPM kind of term. This may affect the propagation of at WDM signal. So, let us consider a simple WDM system where there are three channels, and as we now in w d m the wave lengths are equispaced.

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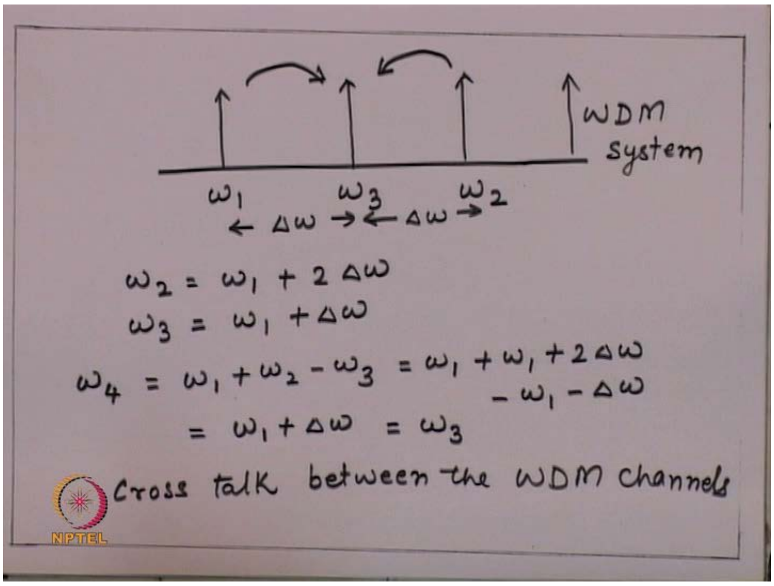

So, let us say we have three channels. Let us say this is channel here which is at omega 1, let us say I have a channel here which is omega three. So, this is a WDM system and here the delta omega is same for all this frequencies. So, this is also delta omega which is channel separation, this is also delta omega the channel separation and so on. Where in WDM system, the channels are equispaced. Now, let us say these two frequencies when this starts propagating. So, this frequency now omega 2 is nothing but omega 1 plus 2 times delta omega and this frequency omega 3 is omega 1 plus delta omega. And as we have seen earlier, now the frequency, which are going to be produced will be combination like this.

So, omega 1 plus 2 minus omega 3 and so on. One can now look at what kind of frequencies can be generated by these two. So, I will get the four frequency omega 4 which is omega 1 minus plus omega 2 minus omega 3. So, if I make omega 1 plus omega 2 minus omega 3 that will be equal to omega 1 plus omega 1 plus 2 times delta omega 2 minus omega 3 which is this so, minus omega 1 minus delta omega. So, this is equal to omega will cancel 1 delta omega will cancel. So, this is omega 1 plus delta omega which is same as omega 3. So, this is same omega 3.

So, now this phenomena look at this, when these two frequencies omega 1 and omega 2 soft propagating, they have a tendency 2 now, generate a frequency which is omega 3 and omega 3 frequency is the frequency corresponding to another WDM channel. But, what this means is that; that means, by this non-linear process these two channels are going to put the power into this channel. Or, you will see that there is to be going to some cross talk created between this channel, this channel and similarly, between this channel and in this channel.

Now, this we have seen only for the three channels when we are having large number of channels propagating. Each of this channel here would have tendency to get a power from this to and this to this. Or, in other words there will be going to be a cross talk created between all the WDM channels. So, you have a phenomena what is called cross talk between the WDM channels. What is the solution to this? If you want to avoid this phenomena, because very clear when we are looking and this phenomena it becomes very clear that when large number of channels are going to propagate. Because of this non-linearity the power is going to flow from one channel to another channel and that will create some kind of cross talk.

So, to avoid this phenomena then it looks that we should not transmit the channels which are equispaced. Because this phenomena can easily avoided if this frequency which is generated here omega four if we does not lie on this frequency here then there is no question of getting the power into this channel from this two. So, if the non-linear effects are present then one of the solution would be that we should put the channels such that they do not lie on this grid. So that the power from one channel to another channel cannot flow because of this kind of relations here. So, the equispace channels for the non-linearity point of view probably are a undesirable thing because that creates more cross talk because of the four wave mixing phenomena.

So, now, we see if you want to reduce the effect of the cross phase modulation and the interaction between different channels, we should not create channels between which are equispace. But, one more thing which can which can be done and that is again when the interaction between this thing take place here, it depends upon how this channels are traveling with each other. So, again the efficiency of this, four wave mixing depends upon what is called this, you know interaction factor or the walkover between different channels. So, again if the signals where traveling together, different frequencies were together, they will have a strong interaction and they would create more cross talk.

But, if they are not travelling together, then this phenomena will not be very efficient phenomena. So, again boils down to the same thing, that we should increase the dispersion on the optical fiber. So, that even this four wave mixing phenomena because weak and the cross stock into the system is reduced. So, it looks like that originally when we are considering a single channel kind of transmission, minimization of dispersion on optical fiber seem to be more desirable. But, now considering all this advance effects which we see, it looks that making dispersion zero on optical fiber is not really a good option. Probably, we should keep some dispersion on optical fiber.

So, that these effects do not build up and they are interaction and their effect on the signal propagation can be minimized. So, in modern high speed communication the requirement would be to not two make the dispersion zero, but me keep some residual dispersion. And by doing that this, non-linear effect can be reduced in a multichannel transmission. But, this same four wave mixing phenomena now can be used for something to something to achieve. For example, when we are talking about a wave length rooted network and if you want to crate sort of non-blocking situation, many times we require conversion of wave length. So, that wave length which is not present in to the next channel, the another wave length can be transmitted.

So, imagination situation that you want to do send some multiple wave length which are going on the network, and a lambda one can be transmitted of certain distance. But, further next path lambda one is not available because the lambda one was already allotted to somebody else. So, one possibilities that particular transmission is blocked that mean that communication channel the optical path is not available. Or other possibilities if you have a wave length conversion transfer the data from one wave length to another wave length and then the path is open.

So, wave length conversion may increase the through put inside an optical network, if the wave length conversion is there inside the optical network. Now, these phenomena of four wave mixing can be used for changing the frequency of a signal.

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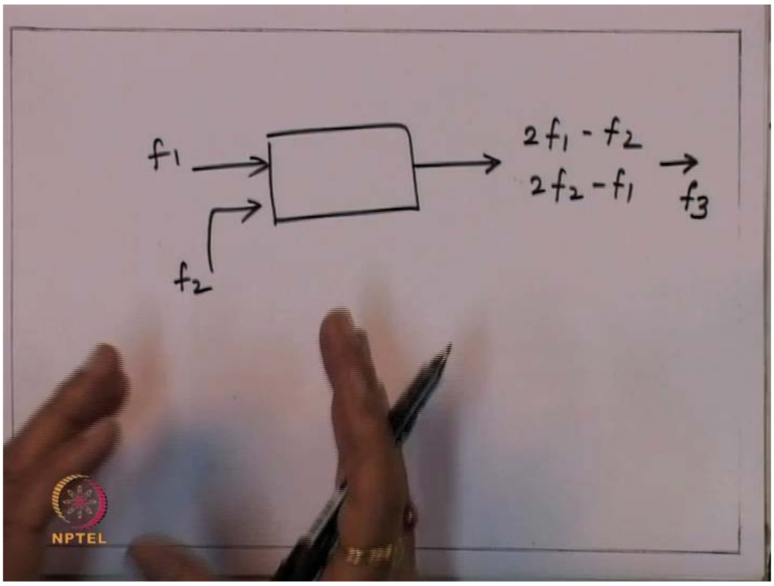

So, imagination a situation that if I have some device in which frequency comes in and I put another frequency is some frequency f 1, is some frequency f 2. And then the output I may get a frequency which is 2 times f 1 minus f 2 or 2 times f 2 minus f 1. So, if I can choose this frequency f 2 in a such a way, that any of these two frequency matches with the frequency in which you are interested for the next path. The signal data which is riding frequency essentially can get transfer to another frequency is f three frequency. So, these phenomena can be use for the frequency conversion.

So, four wave mixing phenomena if you really look at just for inter for interaction point of view. Yes it may create cross talk channels or the same phenomena when use intelligently this can be use as the frequency converter, which is use for wave length route and networks for creating non-blocking systems. So, like that you have sort of various system consideration when non-linear effects are present. So, let me summarize what we have done actually in this, we have to simply started with the non-linear effects saying that if the light intensity large, then the polarization does not have only the linear term which is the chi one term.

But, we also get contribution due to the higher order susceptibilities. Then we saw that for a molecule like glass, the second order susceptibility contribution is negligible. So, most of the contribution which you get non-linear contribution that is due to the third order susceptibility which is chi three, then we saw that for that non-linear contribution.

If we signal propagates, we have a phenomena what is call the self-phase modulation and then we saw that the self-phase modulation when combined together with a dispersion the group velocity dispersion.

You get a phenomenon what is called soliton. Then we saw that the same third order susceptibility where multiple signals propagate. The signal get effected by the other co-propagating signals, the phase as was happening in the self-phase modulation case. Now, the phase of signal case modified by the neighboring signal and that phenomenon then is what is called the cross phase modulation. The same phenomenon also creates new frequencies and that phenomena we saw what is called the four wave mixing phenomena and then we saw that the four wave phenomena mixing phenomena. Actually, creates cross talk between the neighboring channels and it effects can be neglected or sort of reduced by using the fiber which are having some dispersion.

So, that this signals can keep walking through each other. And because of this the efficiency of non-linear interaction weakens to source. You avoid the cross phase modulation as well as the four wave mixing phenomena. Probably, it is wiser to use the fibers which is a some dispersion and not a 0 dispersion fiber. Then we also saw that is the same four wave mixing phenomena can be used for creating new frequencies or for the frequency conversion which is required for wave length routed network for creating non-blocking situations.

So, in last couple of lectures essentially what we have seen is that inside the optical fiber we see a verity of non-linear phenomena. In fact, optical fiber is a very good laboratory of non-linear effects because you saw in the very first lecture of non-linear optics. That the non-linear effects inside the optical fibers get enhance by almost a factor of billion so, with very little power non-linear effects can be easily observed inside the optical fiber. So, we have seen just a cursory look at the non-linear effect which take inside the optical fiber and many of the effects are now use wisely to improve the quality of the signal or to getting certain personalities on the optical fiber network.

So, that concludes our discussion on the non-linear effects on optical fiber, the discussion was wave exhaustive. So, if one is interested having more in that understanding, there are wide range of books available very one can an develop in that understanding on each of the topic which were discuss. But, through the lectures a flavor has been provided, that what kind of non-linear effects can exists on a optical fiber and how they can be widely utilized for improving the transmission capacity and the quality of data on the optical communication channels.