Advanced Optical Communications

Prof. R. K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture No # 35

Self Phase Modulation (SPM)

We are discussing non-linear fiber optics, that is, when the pulse has high intensity, the refractive index of the material gets modified because of the pulse and that leads to the modification in the pulse propagation. So, in our last lecture we looked at the phenomena, what is called the group velocity dispersion and assumed, that the nonlinearity is not playing a role, but the dispersion is there. We solved the non-linear Schrodinger equation only for dispersion and then, we saw something interesting, that if you have a Gaussian pulse, then the shape of the pulse remains Gaussian as it travels on the optical fiber, but it goes on broadening.

We also saw, that internally the pulse gets frequency modulated and the chirp has a different nature depending upon whether you are in the normal dispersion regime or you are in anomalous dispersion regime. So, in the normal dispersion regime the frequency chirp is positive as a function of time, whereas if you are in an anomalous dispersion regime, which is for a wavelength greater than 1300 nanometer, then we have the negative chirp, that means, the frequency decreases as time inside the pulse.

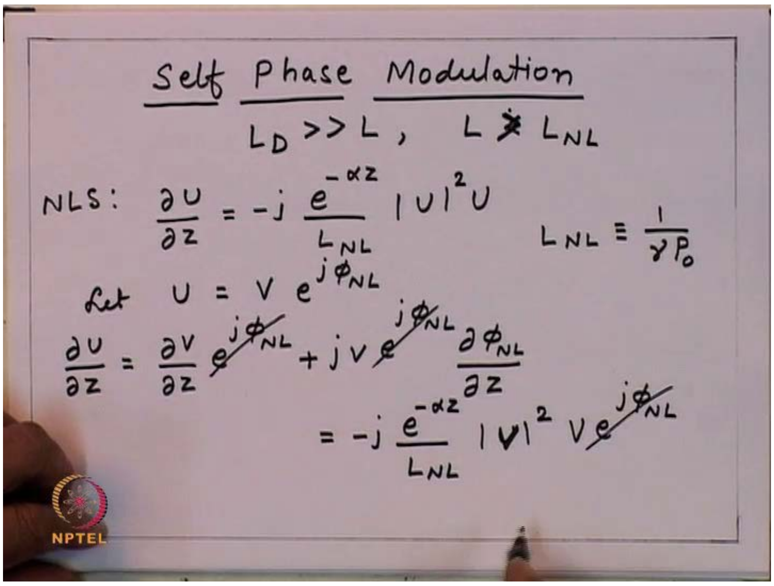
Today, we are going to consider the non-linear effect of pulse propagation, that means, we assume for time being, that the dispersion is not playing a role. We choose the parameter for the pulse in such a way, that the dispersion effects are negligible, but the non-linear effects are present and when that can happen?

It can happen if the pulse width is large, then the dispersion length is much larger. So, for physical length of optical fiber, the dispersion length could be much larger than the actual length of the fiber. So, the dispersion effects can be neglected, but the power in the optical pulse is large enough to have the nonlinearity length smaller than the optical fiber length. And in this situation, then what happens is that the refractive index, which the pulse sees is different at different location within the pulse because now you are having a refractive index, which is a function of intensity of light. So, at the center of the optical pulse, you see large refractive index. As I go on either side of the peak of the pulse, the refractive index decreases.

So, we have created some kind of a refractive index variation of them, material inside the pulse itself and since this pulse is moving inside the optical fiber, this refractive index profile also moves along the pulse inside the optical fiber. But since the refractive index is changing at different locations inside the pulse, so the velocity changes and because of that then you are getting the phase change inside the pulse. So, because of the nonlinearity there, what we see is that there is a phase change, which is going to be time dependent inside the pulse and this change in phase is created because of the pulse itself. Because of the pulse intensity changes the refractive index and that refractive index changes the velocity, which changes the phase. So, you are adding a phase change, which is produced by the pulse itself and that is the reason this phenomena is called the self phase modulation.

So, today we are going to discuss in detail the self phase modulation, which is coming because of the nonlinearity, the Kerr nonlinearity, inside the optical fiber.

(Refer Slide Time: 04:50)



So, we are assuming that the dispersion is not playing a role, but only nonlinearity is present in the optical fiber. So, this one we call as the self phase modulation. One can also show, though we are not going to go into the details of that, that because of the self phase modulation, a continuous beam of light is not a steady propagation. So, a small perturbation on the beam, essentially, breaks the beam into small, small pulses. So, in the presence of this nonlinearity the continuous propagation is unstable, so beam naturally gets split into the pulses. And therefore, we investigate the propagation the pulse propagation in the, in the optical fiber when we are having the nonlinearity present.

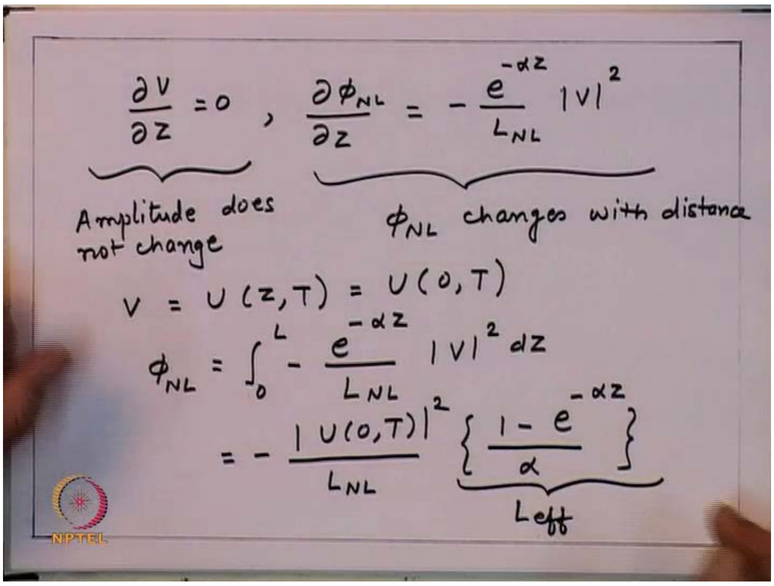
So, here, since we are interested only in the self phase modulation, we have the dispersion length, which is much, much greater than the physical length of the fiber, but the physical length is less a nonlinearity length, it is opposite, sorry. That means, the nonlinearity is present, now the dispersion is not present because this condition is satisfied.

So, in this case, now if we look at the Schrodinger equation, which we got last time with the normalized parameters, we all, what we have done is we have defined this normalized pulse function and then we got a non-linear Schrodinger equation, which was this we are saying now, that in this equation. Now, since the dispersion is not playing a role, this quantity is negligibly small, so we have only this term and this term. So, in this situation, then the NLS can be written as dU by dz, that is equal to minus j e to the power minus alpha z divided by L NL mod U square coming because of the nonlinearity term.

As you know, that this quantity here, L NL is defined as 1 upon gamma P naught, where P naught is the power inside the optical pulse. Now, this equation can be solved by assuming some, some solution to this equation. So, let the solution U be having some envelope V and some nonlinearity phase is given by phi N L. So, I can calculate dU by dz. So, from here I get dU by dz, which is if I differentiate this I will get dV by dz e to the power j phi N L plus differentiate this, so j V e to the power j phi N L d phi N L by dz.

So, this quantity now is equal to this quantity because this is equal to dU by dz. So, this is equal to minus j e to the power minus alpha z by L NL mod U square, which is V square times U, which is this quantity. So, V e to the power j phi, we can separate out real and imaginary part. So, this, first of all this term is common, e to the power j phi N L. So, this will be common. So, this is gone, this term is purely imaginary now. So, what that means is that this quantity, the real term here has to be equal to 0.

(Refer Slide Time: 10:02)

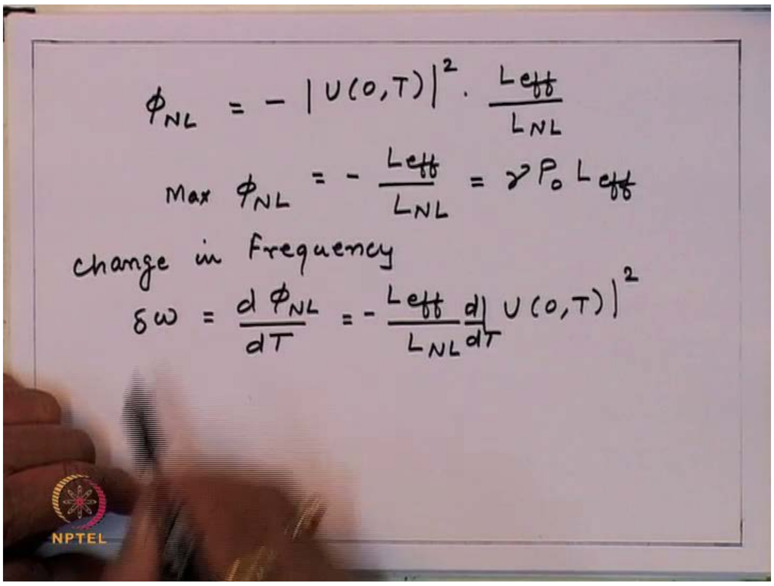


So, this gives, that two equations we get from here, one is that dV by dz equal to 0, that come from, this is real term, this term is imaginary, this term is imaginary, so there is no real term on this side. So, that means, dV by dz is equal to 0. From the imaginary part we get d phi by dz phi N L that is equal to minus e to the power minus alpha z by L N L mod V square. So, the important thing to note here is that now since dV by dz equal to constant, this means, that amplitude does not change as the function of distance when the pulse propagates. So, the amplitude does not change in the evolution of the pulse, only the phase changes as a function of distance.

So, this is saying, that phi N L changes with distance. So, that means, the V, which is U z, T that is same as what was initially at z equal to 0. So, the envelope of the pulse in time domain remains intact, but the phase inside the, inside the pulse changes as the function of time. So, from this equation, then one can get phi N L. I can integrate this as a function of distance, so this is 0 L minus e to the power minus alpha z upon L N L mod V square dz.

And since this quantity is not changing as a function of distance, that is equal to minus mod U 0, T square divided by L N L, then the integral symbol e to the power minus alpha z dz, which is a very simple integral to solve. So, you get 1 minus e to the power minus alpha z upon alpha. This quantity is nothing, but the effective length over which the power is, is propagating inside the optical fiber. Say, as we know, as the power propagates on the optical fiber, the power is continuously reducing because of the loss. So, that is the approximate length over which you will have substantial power. So, this is the one, which we can call as the L effective, that is, the effective length at which the non-linear effects can be seen inside the optical fiber.

(Refer Slide Time: 14:06)

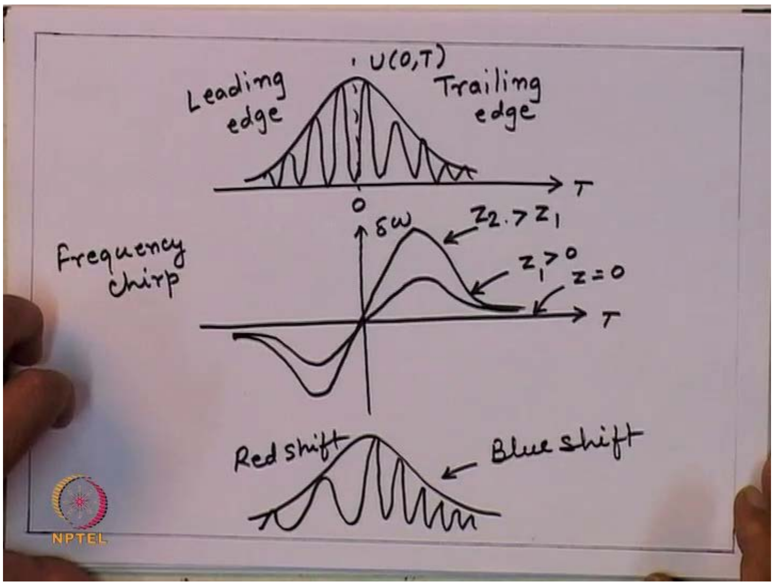


So, the non-linear phase, which you see in the pulse, then phi N L, that is equal to minus the pulse envelope square into L effective divided by L N L. So, one can calculate what the maximum value, the phase change you would see. So, the maximum value of the, of the phi N L, which you see on the fiber, that will be equal to minus L effective divided by L N L because the maximum value of this quantity is 1. We are considering the normalized pulse, so that is equal to gamma p naught to L effective, but the important thing to note here is that here also, now the phi, which is the non-linear phase change.

Since this function is not a linear function of time, the non-linear phase also is not a linear function of time. And as we saw earlier, that if the phase is not a linear function of time that leads to the frequency change. So, that means, even because of self modulation there is a change in frequency, which is a function of time because this phase is the function of time. So, because of self phase modulation also we see the change in frequency and this change in frequency, delta omega, is given as d phi N L by dt. So, that is equal to L effective minus divided by L N L mod of U 0, T square.

We can again consider the Gaussian pulse shape. So, this function will be square of that Gaussian function, like we have taken derivative of this quantity is d, d phi by dt. So, this, which has to be d by dt. So, whatever is this pulse shape, you take a time derivative of that and that quantity, essentially, will give you the change in the frequency as a function of time inside the pulse.

(Refer Slide Time: 17:16)



So, let us consider the Gaussian pulse as we have taken in the previous case. So, let us say, we have this Gaussian pulse as the function of time, this is T equal to 0, this is the leading edge, this is the trailing edge. So, first thing we will note here is that the change in frequency is the derivative of square of this envelope function. This is the quantity, which you have, which is U 0, T. See, if I consider this pulse and if I take square of this and take a derivative, then at this location the derivative is 0, that means, there is no change in frequency at the center of the pulse.

So, if I chirp turbo change in frequency, this is time, this is delta omega. The delta omega is 0 at the peak of the pulse, which is at this location. At this location the derivative of this function is positive, but I have a negative sign here. So, that means, the frequency, change in frequency is negative on the leading edge of the pulse as the same token, the slope here is negative. That means, the change in frequency is positive on the leading edge of the pulse.

Also, since it is going to be the derivative of this function, this quantity here, we will see, that it has a maximum, which goes somewhere here and as do you reach in this region, again we will see the derivative will be almost equal to 0. So, now, if I look at the, the change in frequency, the change in frequency would look something like that. So, you will see the center of the pulse, the frequency almost changes linearly; it goes from negative to positive, that means it has a positive chirp.

So, in this case also we have a frequency chirp because the frequency is changing inside the pulse. Only thing is now the frequency is always going to change from the negative to positive as the time increases because if I consider any pulse kind of shape like this, then on this edge the slope will be always positive, on this edge the slope will be always negative. So, you will always see the change in frequency, which will look typically like that. So, one can then see, that when z equal to 0, that time we do not have any change in frequency, I can take this function from here.

So, when I take z equal to 0, this quantity will be equal to 0, so this is equal to 0. So, I have no change in frequency. So, that is the change in frequency I will get at z equal to 0. As I move further, z increases. So, because of that, that this quantity, the phase will increase and I will see the change in phase, which will look like that for z greater than 0. If I go still further we will see, that the thing will become like that. So, this is for z, let us say z 2 greater than z 1, where this quantity let us say z 1.

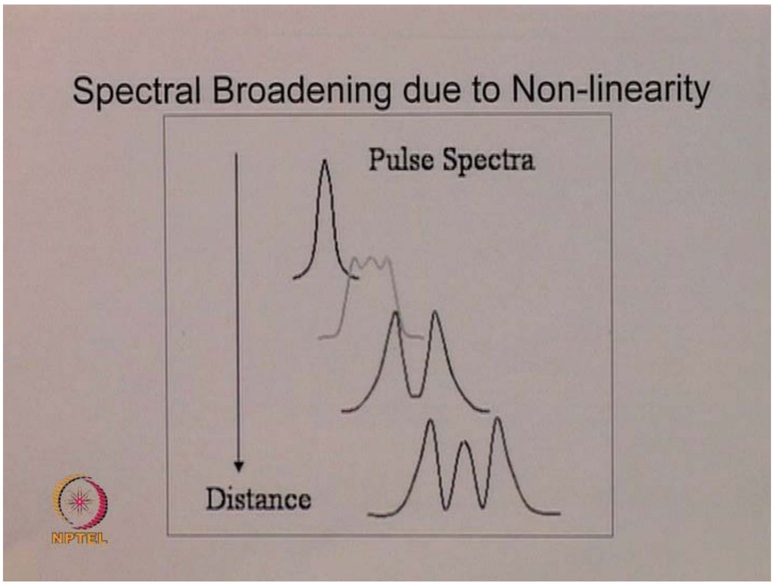
So, as you can see in this situation now, as the pulse propagates inside the optical fiber this chirp, which you say linear chirp, which is always a positive chirp, the slope of that goes on increasing as a function of distance, but the slope always remains positive. As we have seen when this is the kind of chirp, which we have, this means, that the high frequencies are accumulating on the trailing edge and the low frequencies are accumulating at the leading edge.

So, as you have seen in earlier case, if I consider the pulse shape like that and if I look at the carrier now you will see, that the frequency is little less compared to the center frequency here. So, originally, we had the carrier frequency, which was exactly same everywhere like that. Now, we will see, that the frequency is little lower on the leading edge. So, the frequency spectrum would look something like that. So, high frequencies are accumulating here and the low frequencies are accumulating on this edge. That means, here you are having some kind of a red shift and on this edge of the pulse, you have the blue shift.

So, the important thing which we note from this analysis is that when you are having the self phase modulation, the self phase modulation does not change the envelope of the pulse. However, what changes we have now? The change in the phase, so the time function, the envelope remains constant, but each frequency undergoes a phase change, which is non-linear as a function of time and that now modify the spectrum of the pulse.

So, this is exactly opposite of what we used to see for dispersion in the, dispersion, what was happening in the dispersion? The pulse spectrum was remaining constant, different frequencies are undergoing different phase change and as a consequence of that the pulse in time domain was getting modified. What we are saying for self phase modulation is that the time function remains same; different frequencies, there is an odd-even phase change across the pulse as a function of time and that gives a change in the frequency spectrum.

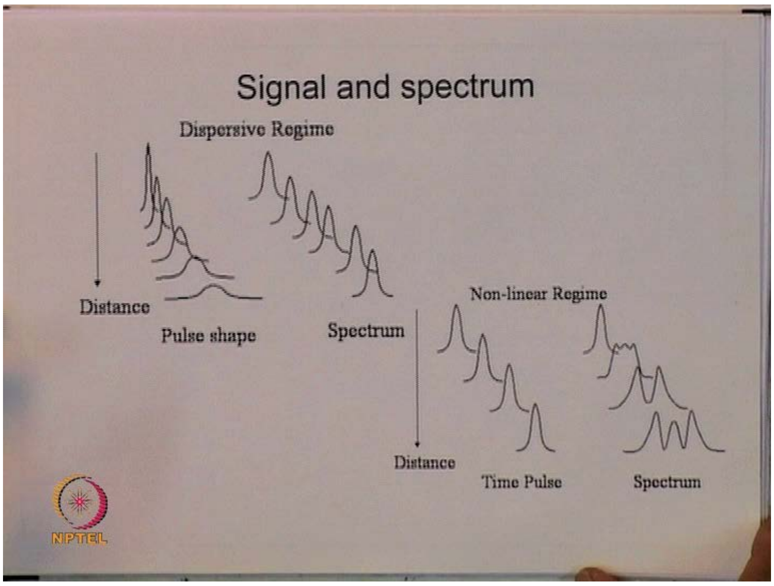
(Refer Slide Time: 25:00)



So, if I look at the self phase modulation, the phenomenon essentially looks like this. So, in the self phase modulation, because of nonlinearity we have the phenomena, what is called, spectral broadening. The time function is not changing, the pulse in time domain remains exactly intact, only thing it gets frequency modulated and because of that frequency modulation, you get the widening of the frequency spectrum.

So, initially, if you start with the spectrum like this, after certain distance the spectrum will become like this, after certain distance it will become like this, like this and so on. So, what it means is that in the presence of nonlinearity there are more new frequencies are getting generated into the signal. In the dispersion phenomena there was no frequencies getting generated, whatever frequencies were there, these frequencies were undergoing different phase change and because of that the time function was modifying in the presence of nonlinearity. We see that the new frequencies do get generated though the time function remains constant. The frequencies get generated because the nonlinearity phase, which is created inside the time pulse.

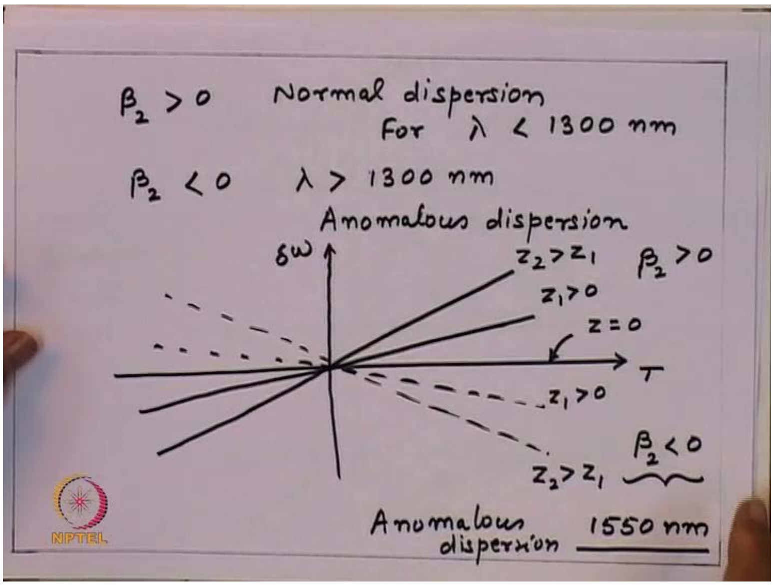
(Refer Slide Time: 26:28)



So, if I compare now these two phenomena, the self phase modulation and the dispersion the phenomena look exactly like this. So, this is the dispersion regime, so you have time pulse as a distance on the optical fiber. The pulse goes on expanding, it broadens, but the spectrum remains exactly same. On the contrary to this, when we are in non-linear regime, then the time function remains constant, the pulse shape does not change, but the spectrum goes on expanding or new frequencies go on getting generated.

Also, as we saw that accumulation of the frequencies in the dispersion regime and nonlinearity regime, the nonlinearity will always give you the frequency chirp, which is positive, whereas for the dispersion the frequency chirp could be positive or it could be negative.

(Refer Slide Time: 27:34)



So, we have seen, in case of the dispersion the frequency chirp could be either positive for the normal dispersion or it could be negative for the anomalous dispersion regime. That mean, if you are working at 1550, then the dispersion is anomalous and then the chirp frequency chirp is negative.

Now, imagine a situation, that the pulse broadening, which was taking place inside the optical fiber because of dispersion was primarily due to this frequency modulation, which is created because of the dispersion phenomena. So, the pulse modification, shape, pulse shape modification, which you used to see is essentially, because of this chirp, which is created. Since this chirp was not there, the pulse broadening also would not be there.

So, imagine a situation, that suppose, now we choose the parameters of the pulse such that the dispersion and nonlinearity, both play a role and they play role in such a way, that the frequency chirp, which is created by the dispersion in anomalous regime, which is negative, dispersion is negative, that negative slope is now balanced by the positive slope, created by the self phase modulation.

So, we have seen both the phenomena give the frequency modulation, they give frequency chirping. Non-linear phenomena always give frequency chirping, which is positive, that means, as a function of time the frequency always increases, is always positive chirp, whereas for dispersion the chirp would be positive or it could be negative.

So, if you are operating in the normal dispersion regime the chirp is positive. So, if the non-linear effect were present along with the dispersion, essentially what happens? The chirp further enhances, but if you are operating in the normal, not normal, dispersion regime, but in anomalous dispersion regime where the chirp because of dispersion is negative, then the two phenomena together can cancel this frequency chirp. And if this frequency chirp is cancelled, you have got the constant frequency all across the pulse, and if that happens, there is no pulse broadening also in the time domain, or in other words what we are saying is that the non-linear effect can be used to kill the dispersion on the optical fiber. It can compensate the pulse broadening, which is taking place because of dispersion and that is a very, very attractive proposition.

So, essentially, what we are saying is that when we are dealing in the normal regime without nonlinearity, as we transmitted data on the optical fiber the pulse was always broadened and then, there was a limit on the data rate, which you could transmit on the optical fiber because after distance, the pulse would have broadened so much, that you would not be able to distinguish between the pulses that keeps upper limit on the data rate, which one can transmit on a certain length of optical fiber.

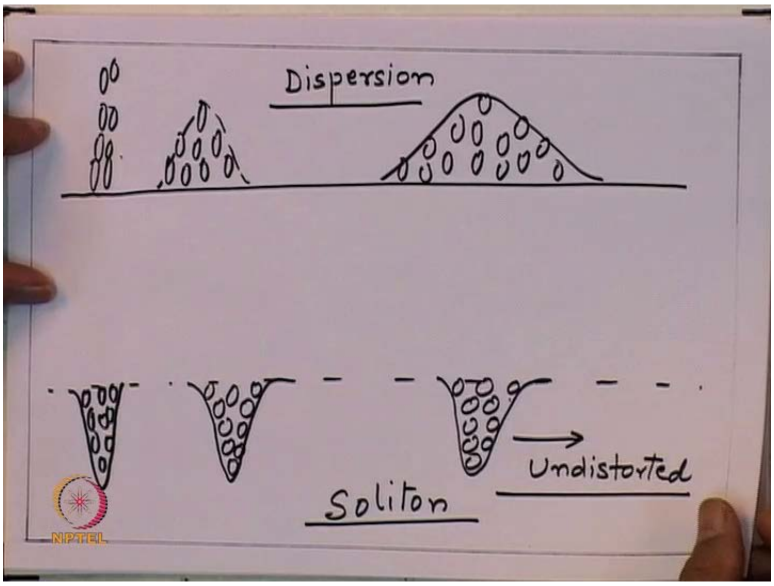
But now, what we are saying is that if we increase the power inside the optical pulse, so that the non-linear effects becomes observable and if we adjust the power in such a way, that the chirp created by the nonlinearity is balanced by the chirp created by the dispersion, then the pulse can propagate inside the optical fiber without any distortion in time domain. That means, the pulse can be transmitted over much longer distances because now the pulse is not broadening. So, there is no limit coming because of the merging of these pulses because of the broadening of the pulses. So, this balancing of these two phenomena together give you now a pulse, which can propagate without broadening, without contracting over long distances on optical fiber, provided the non-linear effects are maintained on the optical fiber.

We will see little later, that since there is a loss present on the optical fiber as the pulse propagates on the optical fiber, even if initially the non-linear effects were strong enough, the power was good enough to give you non-linear effects, and as the pulse starts propagating the non-linear effects become weaker and weaker and weaker. So, soon we will see that the non-linear effects are unable to compensate the effect created by the dispersion. So, if you create a situation where the power can be maintained inside the optical fiber, then the nonlinearity and the dispersion keep on balancing each other and then the pulse can travel on the optical fiber over much, much longer distances. This phenomenon, that these two are balanced by each other, so that the pulse does not go in a distortion, that is, what is called a soliton.

So, what we are going to now see is a phenomenon in which the nonlinearity and the dispersion, both are present. We adjust the power such a way, that the nonlinearity length and the dispersion length becomes comparable and then we look at how the pulse evolves. Actually, should not evolve because we are saying, now since these two are balanced, then you, you would actually get pulse without any distortion travelling on the optical fiber. So, that is the phenomena, essentially, we would like to, like to investigate and if you can create that kind of environment of the optical fiber, then one can imagine, then the data can be sent over long distances without any limitation coming because of dispersion as has been happening in the linear optical fiber.

So, just to give an analogy how these two would be would be balancing each other, let us look at the some, some mechanical analogy and see how this balancing would take place.

(Refer Slide Time: 34:50)



So, let us say consider that we are having a marathon, a marathon race. So, you are having a ground and there are lots of people are collected here to participate into the marathon race. So, everybody at some, sometime they all were accumulating here, all, everybody. Now, once the marathon race starts, you will see in this crowd there are some runners, which are going to be fast running, there are some runners, which are slow running runners. So, the fast runner would go ahead, the slow runner will be left behind. So, you will see, there are, somebody is here, somebody has gone here, somebody has gone here and so on.

So, the initially the whole crowd, which was, which was this; the crowd actually is spread into this move. Further, you will see that the slow runner will be further left behind and fast runner would have gone further ahead. So, if I do this I will see that phenomena, which will look something, like that. So, I will see, the crowd has become like this. This is the phenomena, which is exactly same as what we say is the dispersion phenomena, that you have a crowd here, which slowly get dispersed as it starts moving into this marathon race because each one does not run with the same speed. That is the phenomena, which gives you the pulse broadening on the optical fiber.

Imagine now a situation, that this marathon race, which you are going to hold is not on a hard terrain. Let us say, we, we have a huge mattress on which we want to conduct the marathon race. So, we have a thing, which is, which is like a soft material. Now, as soon as everybody assembles here for the marathon race, you will see, since this material is a soft material it does not remain like this, but it sort of sacks. So, now, material becomes like this people are standing they are, they are, they are different locations here because the material is sagging.

So, when everybody assembles there for the beginning of the marathon, they are essentially standing into this valley, which is created because of this soft material. Now, as people start running, as the marathon race starts, the person who is here, who is running has to essentially climb this slope, whereas the person who is standing on this side, he has to run down the slope and person who is standing here in the middle of the valley, for him it appears as if he is running on the flat terrain.

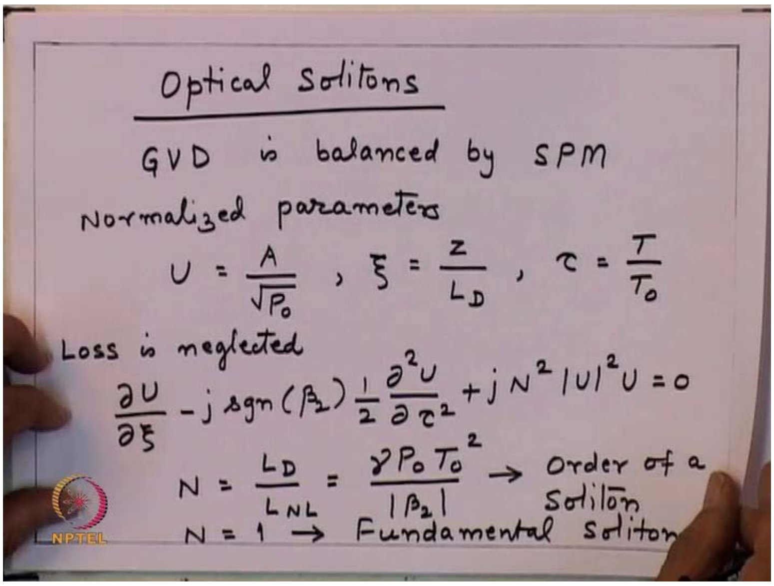
So, earlier what was happening was, that the fast runner was to run faster, slow runner was left behind and the pulse used to get broader. But in this situation, now the fast runner, since it is going to run up the hill because there is a slope is like that, his efficiency of running is reduced, now he cannot run as fast as he was running in this situation. Whereas, if I consider somebody here, then since he is going down the hill, he now can run much little faster than what he was otherwise running. So, as a result, what happens is that this person has to put more exertion, this person see little more easy way of running and this person sees as if nothing has changed as if he is running on the flat terrain.

So, everybody thinks that he is doing the best of his abilities, but this valley does not expand as it used to happen in this situation because everybody does his best and since the valley will be like this, so maybe some adjustment will take place to the valley like this. But this, once that adjustment has taken place, then everybody will keep doing to best of his ability, but even then this shape will remain same.

So, this valley go here, go here, exactly the same thing will remain, or in other words what we are saying is this change in the medium, which has taken place because of the mass, which you put inside this. This is the phenomena similar to what we are seeing because of nonlinearity because that is what nonlinearity was doing, that the intensity of the pulse used to modify the refractive index of the material; that is what precisely has happened here. And then, in this situation, the dispersion is unable to play a role as dominantly as it used to play in this situation.

So, a certain adjustment takes place for the valley you create. A shape, which is like this and this shape, now keeps on travelling undistorted. This is what is called a soliton. So, if we take this shape, which is readjusted shape and if it starts moving, then the nonlinearity and the dispersion, they will keep counteracting each other and then this pulse shape will travel on the optical fiber over long, long distances without a distortion So, as I said, this is a very attractive proposition, that if you can keep maintaining nonlinearity on the optical fiber, then one will be able to send the high speed data over much long distances, which otherwise was limited because of the dispersion or the pulse broadening on the optical fiber.

(Refer Slide Time: 41:09)



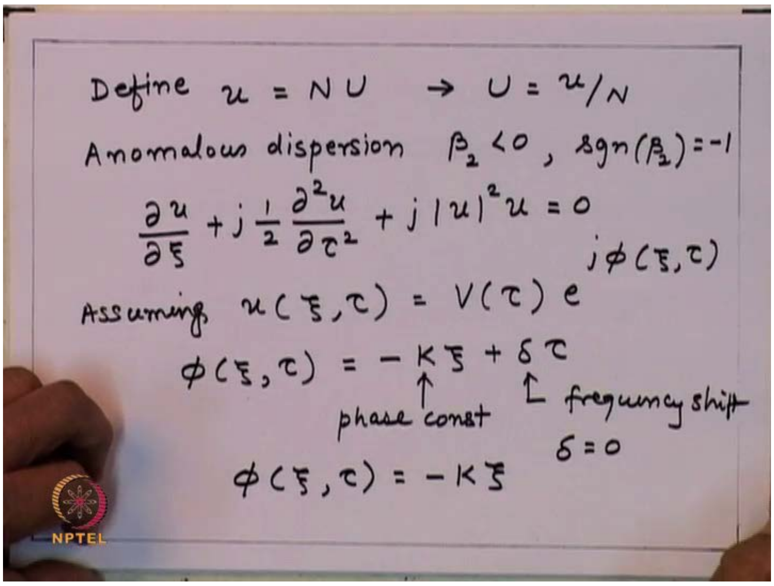
So, let us look at now the simple analysis of the solitonic propagation. So, let us consider here optical solitons. So, let me say here, what we have said, we said here the group velocity dispersion is balanced by SPM, the self phase modulation. So, we are saying here, that the dispersion length and the nonlinearity length are very comparable and in the regime, essentially we would like to look at how the non-linear Schrodinger equation would look like and what would be its solution.

So, let us define now certain normalized parameters as we have done earlier also. So, let us define normalized parameters, say as U define A by square root P naught. Let us define now all the distances by this zeta, which is the actual distance normalized with respect to dispersion length. And as we were defining the normalized time tau, which is now normalized with respect to the pulse width, now neglecting the loss. So, let us say the loss is neglected, that means, this P 0 , power inside the optical pulse remains constant as it travels on the, on the optical fiber.

Thus the non-linear Schrodinger equation, which we have seen earlier, that now can be written as dU by d zeta minus j sign of beta 2 1 upon 2 d 2 U by d tau square plus j N square mod U square U, that is equal to 0. Then, we have defined this parameter N here, that is, the ratio of the dispersion length to the nonlinearity length. So, by substitute for dispersion length and nonlinearity length, essentially this quantity will be gamma P naught T 0 square upon mod of beta 2.

This quantity N now is what is called the order of a soliton. And when L D is equal to L NL, that time this N is equal to 1 and we call that as the fundamental soliton. So, this quantity is what is called order of a soliton. So, N equal to 1 gives you fundamental soliton. Now, this we can do little more algebra to essentially eliminate N from, from this equation.

(Refer Slide Time: 45:12)



So, let us define another parameter, which is, so define some parameter U, that is, this order N times U that gives a U, which is normalized u by N.

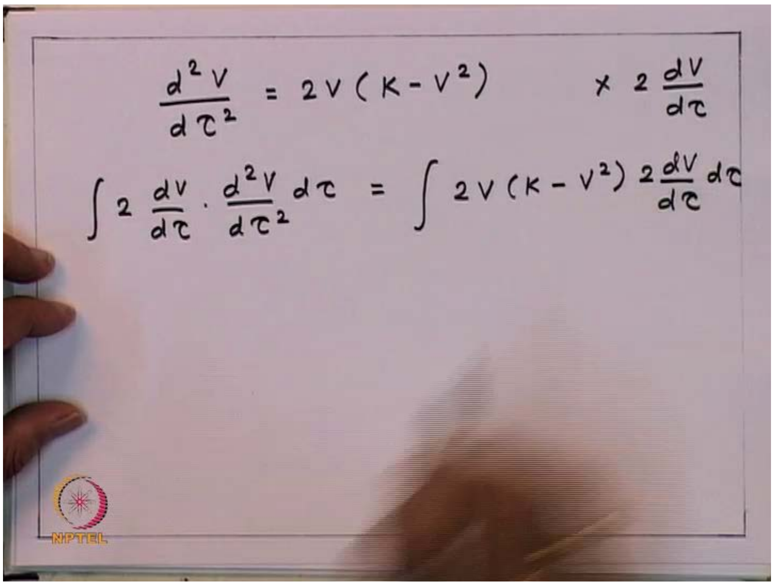
And now, since we are interested in the soliton and we know, that the soliton can be formed only in the non-linear dispersion regime in the anomalous domain because that is where the two chirps can be cancelled, so we are considering anomalous dispersion, that is beta 2 is less than 0. So, sign of beta 2 is minus 1; this equation sign of beta 2 is minus 1. So, this will become plus j and then substituting for this capital U in terms of this U and N, you get now the equation, which is dU by d zeta plus j 1 upon 2 d 2 U by d tau square plus j mod U square U that is equal to 0.

This equation actually has to be solved by what is called the inverse scattering method, but we can solve this in a different way. So, if you are having a general solution when the order of N is not 1, then one has to go to the inverse scattering method. However, for this case when N is equal to 1 we can solve this equation rather easily. So, since we are now looking for phenomena where the pulse shape remains intact, that is, the soliton, that is what we physically argued.

One can solve this equation by assuming this U, which is a function of zeta and tau, that is, some function of tau, that means, it is not changing as this phenomena moves on the, on the fiber and some phase function, which is a function of distance and time. So, if I do that, so I substitute this, I take this quantity, substitute into this and separate out real and imaginary parts, you will see, that you get two equations. One, you will get the equation for phi, other one will get equation for V and one can show, that the solution, which you get for phi as a function of g and tau, it is same as the wave propagation and a phenomena, which will give solution like that delta tau.

So, this quantity then would represent frequency and this quantity would then represent the phase constant. If we assume, that the frequency shift is not there when this phenomena is moving, one can take this delta is equal to 0. So, this phase as a function of zeta tau, that will be equal to minus k times zeta.

(Refer Slide Time: 49:34)



Once you get this, then you can substitute into the equation and the equation, which will get for, for, for V, that now can be obtained as this. This, the whole data of algebra you have to put in this because we are going to separate the real and imaginary part of this equation. So, from, from the, from the phase equation we get this expression; from the amplitude equation we will get the equation, which will be like this.

Now, this equation can be solved trickily. So, what one can do is, I can multiply both sides of this equation by this quantity, which is 2 times dV by d tau and then integrate this as the function of tau. So, what we will see here, that by multiplying this, this term here, this will become integral 2 dV by d tau d 2V by d tau square d tau, that is equal to integral 2 V k minus V square 2 dV by d tau to d tau. We will continue with this equation to see little more evolution of the pulse.

So, what we have done let me summarize. We started with the non-linear effects, we neglected the dispersion effects and then we saw, that in the presence of nonlinearity a phenomena, what is called self phase modulation takes place. The self phase modulation phenomena, it modifies the spectrum, the new frequencies are generated, but the time function is not modified. It is exactly opposite to what happens for the dispersion where the pulse shape is modified, but the spectrum is not modified.

We also saw, that these two phenomena give different frequency chirps. So, then, we said, that if we combine this two phenomena such that this frequency chirps are cancelled, that means, the dispersion is balanced by the nonlinearity, there will not be any frequency chirp inside the pulse. And if that happens, there will not be any change in the pulse shape also, and that is the phenomena, which we call as the soliton where the wave behavior becomes more like a particle. And that is the analysis, essentially we started doing, that if we consider a situation where the nonlinearity length and the dispersion lengths are equal. And if the fiber length is much larger than this, then we will see the evolution of a pulse, which will be in the balance of these two phenomena in solitonic equation.

So, by taking the simple, then functional form, we separated out real and imaginary parts and we are trying to get essentially solution for the solitonic wave. So, we will continue this analysis. So, in the next lecture when we meet, we are going to just take this equation, which you have got and then try to see what is the solution of this equation and then we see further, what is the behavior of the solitonic pulses in the optical fiber