Advanced Optical Communications
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Module No. # 01

Lecture No. # 34

Group Velocity Dispersion

(GVD)

We are discussing non-linear fiber optics. In last 2 lectures; we saw that when the light propagates inside the optical fiber, because of the intensity the refractive index of the material changes and because of that the pulse propagation gets modified.

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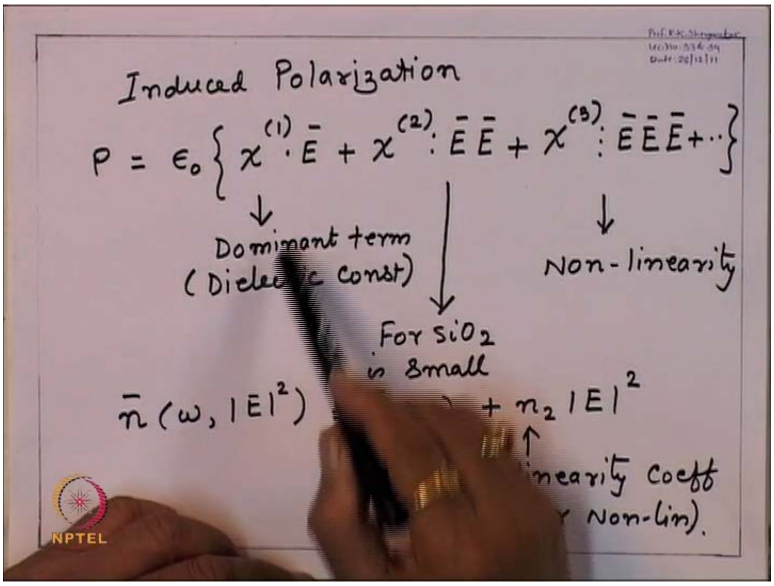

So, we show that in general the induce polarization inside any material has a component, which is this first water susceptibility, we also have component due to second order susceptibility and third order susceptibility and so on.

The first order susceptibility gives, what is call the dielectric constant and these are the non-linear terms, which you get in the dielectric constant. We also saw, that for the material glass, the second order susceptibility is negligibly small; and therefore, we see the effect of the third order susceptibility, which gives the non-linear contribution to the dielectric constant. And be to the case, what is call the Kerr non-linearity, where the refractive index of the material is related to the electric fields square that is the power density.

So, you are having the refractive index of the material, which is the linear index plus you are having this term, which is coming because of this non-linearity which we call as Kerr non-linearity. And then starting with the wave equation, we derived what is call the non-linear Schrodinger equation.

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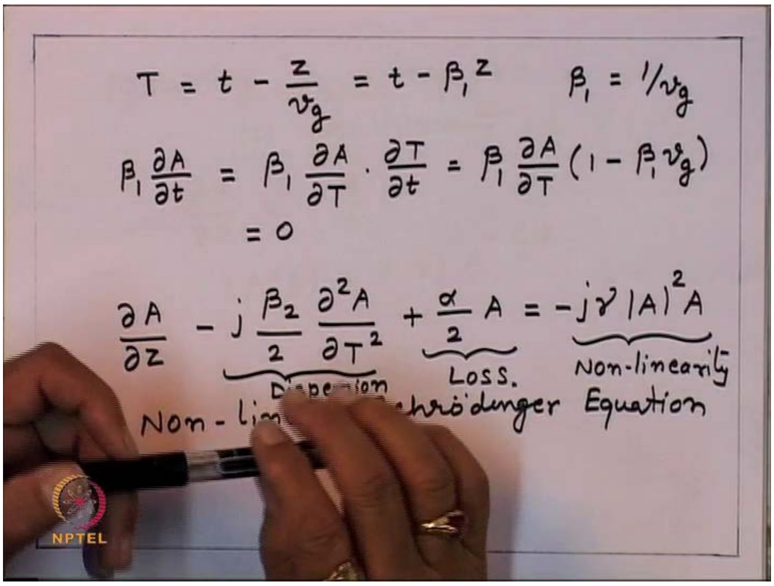

Which gives the evaluation of a pulse on an optical fiber you also, saw that on an optical fiber, the non-linear effect is enhanced by almost factor of billion, compare to the non-linear effect in bulk material. And that was for the simple reason, that once the power is confined to the core of the optical fiber; then this interaction the non-linear interaction keeps taking place over, the effective length inside the optical fiber, which due to very low loss on the fiber this lance turns out to be if you kilometers. And therefore, the overall effect of non-linearity observed inside the optical fiber is must stronger compare to what you see inside a bulk material.

So, we saw that the power levels which normally we deal in the optical communication, even for those power levels which are few mille watts of power, the non-linear effects become significant in the optical fibers. So, we that in mind then we wanted to understand when signal to travels on the optical fiber, what way it will get modified. And therefore, we derived this equation what is call the non-linear Schrodinger equation, after certain approximation on certain linearization. And we saw that if you consider a pulse with an envelope function, which is given by this A, then it evolves along the optical fiber by this equation.

So, here this term which is due to the dispersion on the optical fiber, this term is due to loss on the optical fiber and this is the term which is due to the non-linearity on the optical fiber. And then we saw that, we can consider different cases depending upon what is the pulse width, how high then power is in the pulse.

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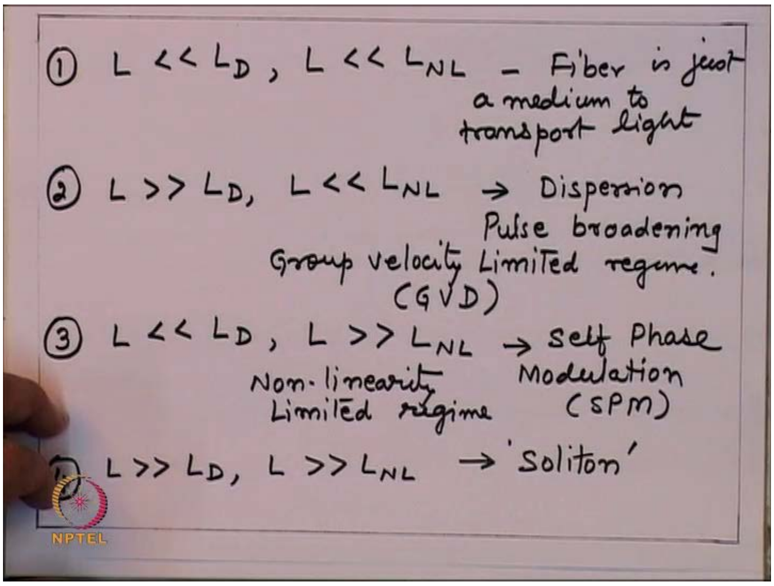

So, the different effects may dominant at different situations and therefore, we defined this 3 lengths or 2 lengths on the optical fiber, one we call as the dispersion length other one we call as the non-linearity length.

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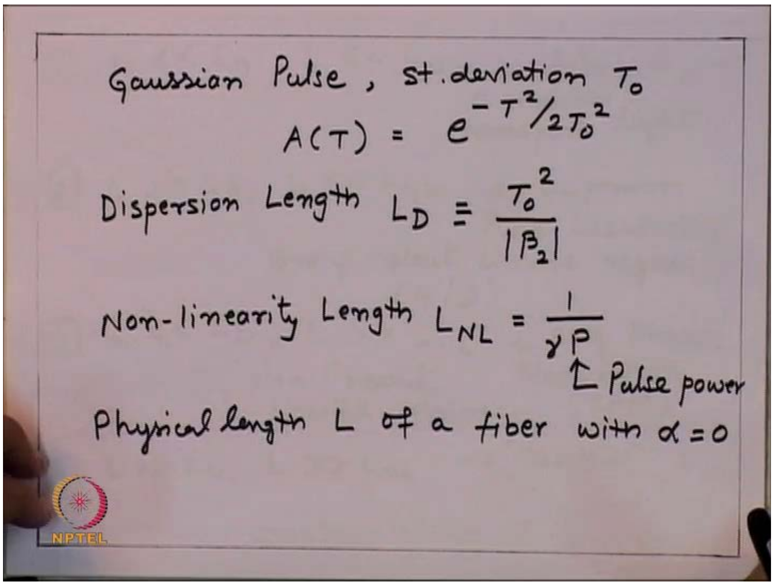

And then we saw that depending upon the physical length of the optical fiber, sometime the non-linearity length may be smaller than the physical length, so non-linear effects would dominate. Sometime the dispersion length would be less than the physical length of the optical fiber, so the dispersion effects would dominate. And when both the lengths are smaller than the effective length of the optical fiber, then the both effects would start contributing and that would give the evaluation of the pulse.

So, we saw that is three different situations (Refer Slide Time: 05:03), when L is much much less than L D and L is much much less than N L, the fiber becomes a bare medium of power transportation, the pulse does not get an evolution as it propagates in fiber. If you are having L which is much much greater than L D and L is much much less than L N L, then the dispersion effects dominate and we have seen already dispersion effect inside the optical fiber, that that gives you what is call the pulse broadening. Of course, we are going to see little more detail the analysis of of this today.

The third case was that the dispersion effects are not dominate but, the non-linearity effects are dominant, that will happen if you are having last power inside the optical pulse. Then we get a phenomena what is call self-phase modulation, that also will investigate and then when both the effects are present, then we get a pulse what is called soliton. So just to give the few numbers, let us put numbers in see, under what situation what effects would short showing it is presents.

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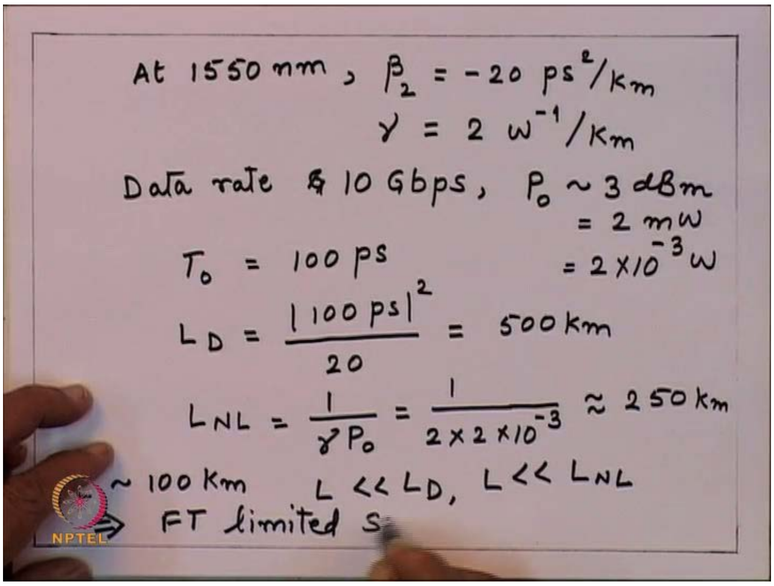

So, let us take it typical case of propagation, so let us say we consider the wave length of propagation, which is 15, 50 nanometer that is optical window which is used; the beta 2 at that wave length for a typical fiber is minus 20 picosecond square per kilometer. The gamma typically you see for for the optical fiber is the order of about 2 watt minus 1 divided by per kilometer. If you consider a data rate of 10 G b p s (No audio from 06:56 to 07:08) 10 g b p s and consider the power which were transmitting P 0 let us say about the order of about 3 d B m, so that is equal to 2 mille watts. So, data rate this would give the T 0, which is the pulse width it is 100 picoseconds, the P 0, which is the power in the pulse that is 2 into 10 to the power minus 3 watts.

So, if you calculate the dispersion length non-linearity length that will get as L D that is equal to T 0 square which is 100 picosecond square divided by mode beta 2, so which is 20; so that gives the dispersion length of the order of about 500 kilometers. Similarly, we can calculate the nonlinearity length or this power, the gamma is given which is 2 watt inverse per kilometer and P 0 is 2 mille watts.

So, we can calculate from here 1 upon gamma P naught that is equal to 1 upon 2 into 2 10 to the power minus 3, so that is of the order of about 250 kilometers. So, if you consider a typical optical communication link, let us say the L is of the order of about 100 kilometers, then for this one you see that L is much less than L D it is much less than non-linearity length also.

And therefore, in this situation neither we will see the dispersion effect nor the non-linearity effect on the pulse propagation. Of course, note here, that when we are talking about the pulse propagation, we are consider, that the pulses are Fourier transform limited. So, here we are assuming that the carrier is the (( )) carrier and the spectral width is coming just because of the modulation, which is this modulation; so we are intensically assuming that, we have Fourier transform limited spectrum.

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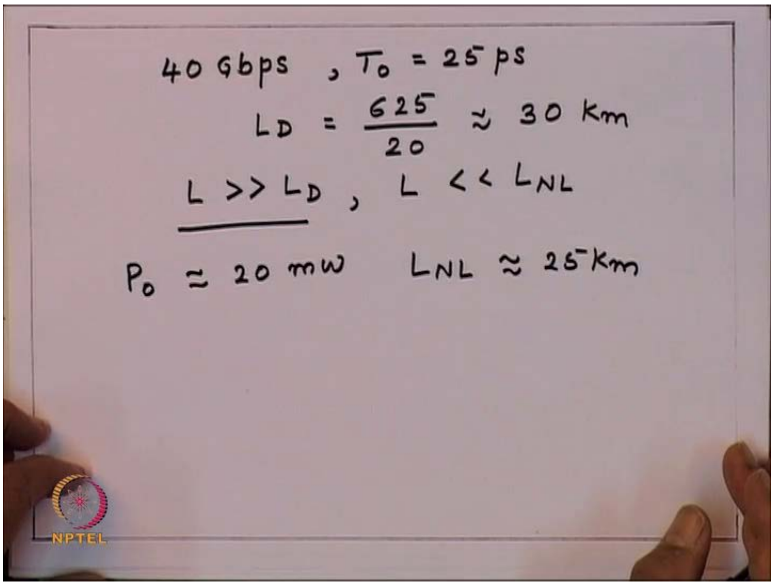

However if you narrow the pulse width further, then for the same 100 kilometer link, we will starts seeing the effect of the dispersion. So for example, if you consider 40 G b p s data, that will give the pulse width T 0, which is 25 a picosecond and that will lead to the dispersion length, which is 625 P 0 square divided by beta 2 which is 20, so the order of about 30 kilometers.

So, in this case then we will see that the L is much, much greater than L D but, L will still remain less than L N L, so for 40 G b p s pulses if you transmit on the optical fiber, you will see that the dispersion effect will starts showing up; we got that is the situation which will (( )) but, if you increase the power of the pulse from 2 mille watts, so let us say 20 mille watts.

So, if I make P naught let us say about 20 mille watts then the non-linearity length will reduce from 250 kilometers to 25 kilometers, then if you are having a link length of a 100 kilometers, we will see that the non-linearity length also will become much smaller compare to the physical length of the fiber. And in the situation than the dispersion and non-linearity both will start playing role in the pulse propagation.

So, these are the typical numbers which normally we see inside the optical communication, so we what we find is that for the typical power levels and for the typical data rate, which we are going to handle in the modern optical communication systems, the dispersion and non-linearity both effects can become visible. So, now we are going to take the one by one the case and then seen little more detail, what is the meaning of dispersion, what internally happens to the pulse, when pulse propagates in the presence of dispersion, what happens in the presence of non-linearity and when both are present, what would really happen?

So, today we are going to take one effect and try to see how the non-linear Schrodinger equation can be simplified in that situation; and then what way the pulse evaluation would take place. So, today we are going to discuss, what is the group velocity dispersion that means in the non-linear Schrodinger equation, we are considering the term, which is coming only because of the dispersion.

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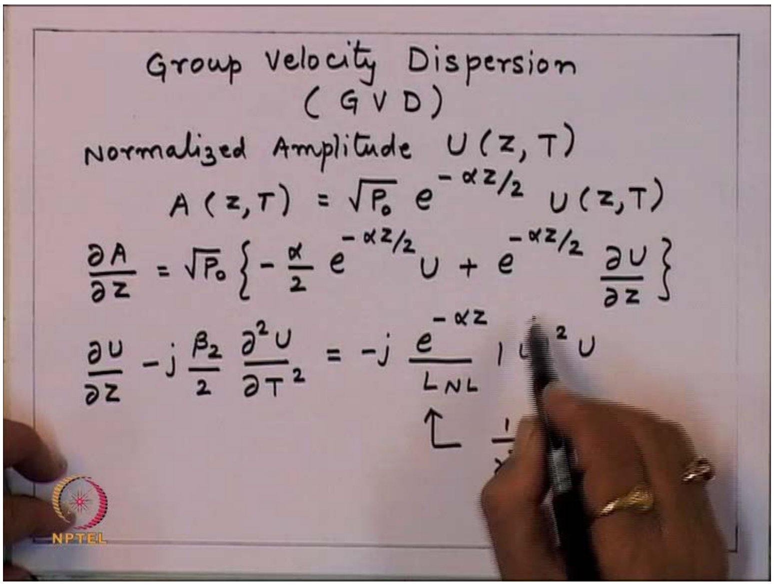

So, today we are going to talk about group velocity dispersion in short (()) GVD, no for a solving the non-linear Schrodinger equation, let us redefine certain parameters to taken to account the losses on the optical fiber. So, let us define and normalize amplitude (No audio from 13:52 to 14:03) U which is a function of Z distance and a time T; and you are define this normalize time T, if you recall this is the time in the moving frame with the pulse.

So, the annual up function, which we have in non-linear Schrodinger equation, this is related to this U at square root of P naught, which is the power in the pulse e to the power minus alpha z by 2, when alpha is the attenuation constant on the optical fiber U Z of T. So, from here we can get d A by d Z thus what we require in the non-linear Schrodinger equation, so that is equal to square root of P naught minus alpha by 2 e to the power minus alpha z by 2 U plus e to the power minus alpha z by 2 d U by d Z.

So, if I take this d A by d Z in substitute in do the non-linear Schrodinger equation, then we get the non-linear Schrodinger equation, then we get the non-linear Schrodinger equation as d U by d Z minus j beta 2 by 2 d 2 U by d T square is equal to minus j e to the power minus alpha z divided by L N L mod U square (()). So what we have done is we have taken the non-linear Schrodinger equation as we divide derived earlier, substitute into this equation (Refer Slide Time: 16:52) for A in terms of U; and that will get this equation. And now you see the significance of (()) define this quantity you in normalize terms, that after we define this the term which is to come, because of this loss thus term is now vanished.

So, in this equation in this equation in terms of this normalize parameter you have one term, which is coming because, of this dispersion and one term which is coming, because of non-linearity. And the non-linear length you already define, which is 1 upon gamma into P naught's, so this quantity is nothing but, 1 upon gamma P naught.

Now assuming that the power level inside the pulses is not very large, that means this term is negligible only the dispersion effects are present. So, since we are investigating here the group velocity dispersion, we are assuming that you have chosen the pulse parameter in such a way, that is the non-linear effects are negligible.

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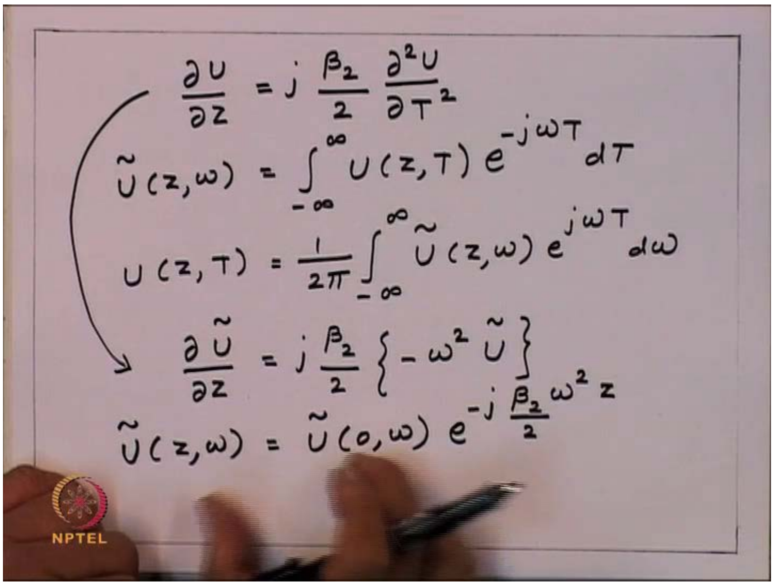

In this situation this term essentially can be neglected and then we have a the non-linear Schrodinger equation, which is d U by d Z is equal to j beta 2 by 2 d 2 U by d T square. Now, since you are having a second derivative which respect to time, this differentially equation can be solve, if you take the Fourier transform of this equation a in time, so the equation converted into frequency domain.

So, the second derivative will become simply multiplication of the square of the frequency and this then equation can be solve very easily in the Fourier domain. So, if we define the Fourier transform U tilde of z omega, which is (()) infinity U Z T e to the power minus j omega T d T. And the inverse Fourier transform as U Z T 1 upon 2 pi integral minus infinity to infinity U tilde z omega e to the power j omega T d omega.

So, I can take the Fourier transform of this equation now, with this definition, so I get this equation, the non-linear Schrodinger equation, which is d U tilde by d z equal to j beta 2 by 2 and this will become minus omega square U tilde. So, that is the secondary (( )) in time the frequency domain that will multiplication by the frequency square, (( )) equation can be solve very easily as a function of z.

So, now if you know the pulse in the Fourier domain, the spectrum of the pulse at distance z equal to 0, then the U tilde at any location z can be return as u tilde 0 omega at z equal to 0, time the phase function which is just coming because of this. So, e to the power minus j beta 2 omega square z upon. So, the solution in the Fourier domain if you see for the non-linear Schrodinger equation, when only the dispersion term is present is this, what does that mean, if look at this quantity here, you will see that this is the amplitude spectrum of the pulse and that is what is the phase, which were going to get.

So, initially whatever spectrum you had, which was this after a distance (()) the amplitude spectrum still remains unchanged, only thing what happens is each frequency component under goes the phase change, which is given by this. So, one thing first we note here is that, in the presence of GVD, the amplitude spectrum of the pulse remains unchanged, only different frequency component, they undergo the phase change. And that phase change is proportional to the frequency square; so the spectrum in the presence of GVD does not get modified.

However, since the phase is going to change for the different frequency components the time function will get modified. So, in the presence of dispersion the spectrum of a pulse remains intact only the phase spectrum changes but, as result of this the amplitude function of the pulse or the pulse shape gets modified.

So here we are seeing that, your having the phase of different frequency components that is proportional to omega square, it is also proportional distance (()). So, more the pulse travels on the optical fiber, more is going to be the phase change in the different frequency component, with this now one can take the Fourier inverse of this quantity to find out what the pulse shape would be; so one can get the time function, which is the pulse shape in time domain.

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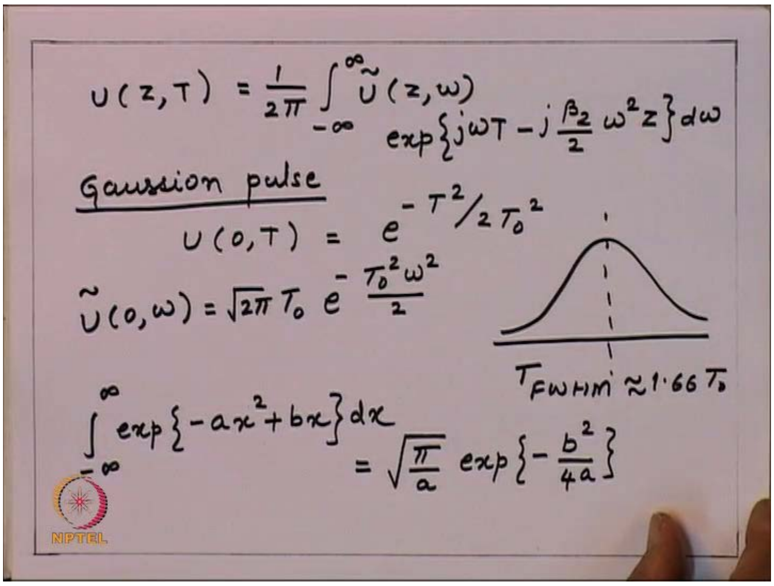

So U Z T that is equal to 1 upon 2 pi as we define the inverse Fourier transform minus infinity to infinity U z omega multiplied by e to the power j omega T, the Fourier inverse term plus the phase term, which is coming because of in the solution as we got here (Refer Slide Time: 24:25) this term; so, minus j beta 2 upon 2 omega square z d omega. So, this expression now with a general expression, we are not taken specific shape for the pulse only thing we started we some pulse shape which was U Z T, we took a Fourier transform to get spectrum.

And then we saw that the different frequency components undergo the phase change, so this term if add to the Fourier inverse and we can get the pulse shape at any distance on the optical fiber. However the analysis can be rather simple for a pulse shape, which is caution in nature, so what we are going to see here is the evaluation of pulse, which is caution in nature, because there we can get expression that which where visualization can be much better.

So, let us consider now that the pulse which we are launched inside the optical fiber is Gaussian. So, let us say, we are launching a pulse at z equal to 0, which is e to the power minus T square divided by 2 T naught square, where T naught is the the standard deviation of that Gaussian function, the half power width equity find for this pulse, so this pulse is caution in nature, so sigma for this pulse given by this T naught.

So, the half power width if I define for for this pulse, so T full width half maximum, that is approximately 1.66 times this quantity T 0. Since we have to take the Fourier transform the pulse, the Fourier transform this which is U tilde 0 omega that is equal to root 2 pi T 0 e to the power minus T 0 square omega square upon 2. That is just the Fourier transform of of this function, as we know the Fourier transform becomes in function is another Gaussian function. Now you have to substitute this into this equation and then take the inverse Fourier transform to see, what the pulse shape would be as the function of distance.

So, essentially this integral which we are seeing here it is of this form minus infinity to infinity minus a x square plus b x d x and this is a standard integra identity for which we know the integral; that is pi divided by exponential minus b square upon Fourier is we can use this standard integral identity, to solve this Fourier transform. So, once I get this pulse shape here, I substitute into this, identify what is the a and b and then I can use this standard identity to find out what the Fourier transform of this would be.

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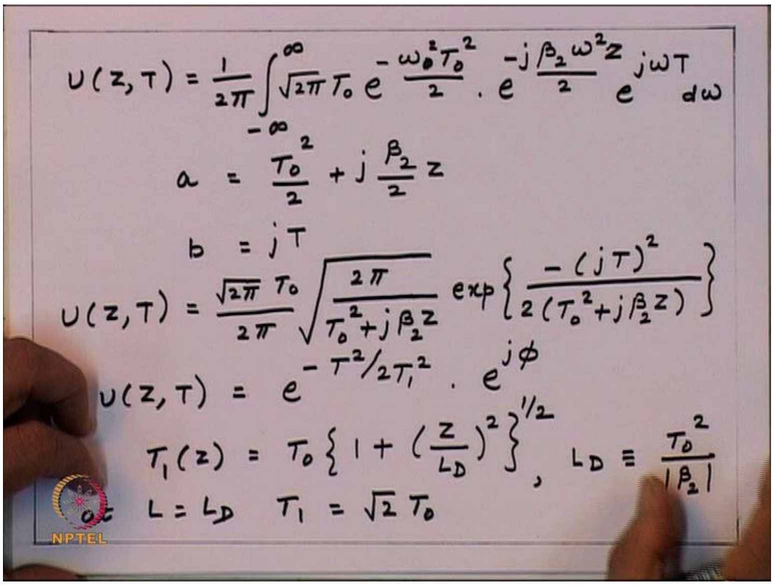

So, if I just substitute this, I get the Fourier transform as U Z T that is 1 upon 2 pi minus infinity to infinity root 2 pi T 0 e to the power minus omega 0 square omega square T 0 square upon 2. The phase function e to the power minus j beta 2 omega square z divided by 2 and the Fourier inverse term j omega T.

So, from here (Refer Slide Time: 29:47) then, we can identify what this a and b are, so if I compare this expression (Refer Slide Time: 29:55) with this expression here of the standard integral; we get a which is T 0 square upon 2 plus j beta 2 upon 2 z and b j times T. And that using now this standard integral identity, we can get the inverse Fourier transform of this pi get U Z T that is equal to 2 pi t 0.2 pi square root of 2 pi divided by T 0 square plus j beta 2 z exponential minus j T square divided by 2 times T 0 square plus j beta 2 z.

So, one thing one will note here is that, this function is also Gaussian function, so I can separate out really imaginary part for for this function; so you will see that this thing can be return in terms of the the phase function and the envelope function.

So, U Z T can be return as real part of this become minus T square upon 2 T 1 square where will be define T 1 plus the phase function some e to the power j some phi. And if you separate out really (( )) part of this you will not is that this quantity here T 1, which is the function of z that is equal to the initial pulse width T naught 1 plus Z by L D whole square half; and as we know the L D is defined as T naught square divided by mod of beta.

So, now the L D now, physically we can see meaning of L D, what the dispersion is really means, so what this is telling you this is telling now that the pulse width originally which was T naught. Now the pulse shape as till remain Gaussian but, the width of the pulse is change from T naught to T 1 and which is given by the expression.

So, if I take the distance z equal to L D, which is the dispersion length, this quantity will be equal to 1, so this quantity will be equal to root 2. So, what that means is now physically, the dispersion length is that length over which the pulse, (()) increase this by factor of root. So, you will see had L equal to L D, the pulse width T 1 is root 2 times T 0, we can take the imaginary part of this and then we can get the phase function of phi.

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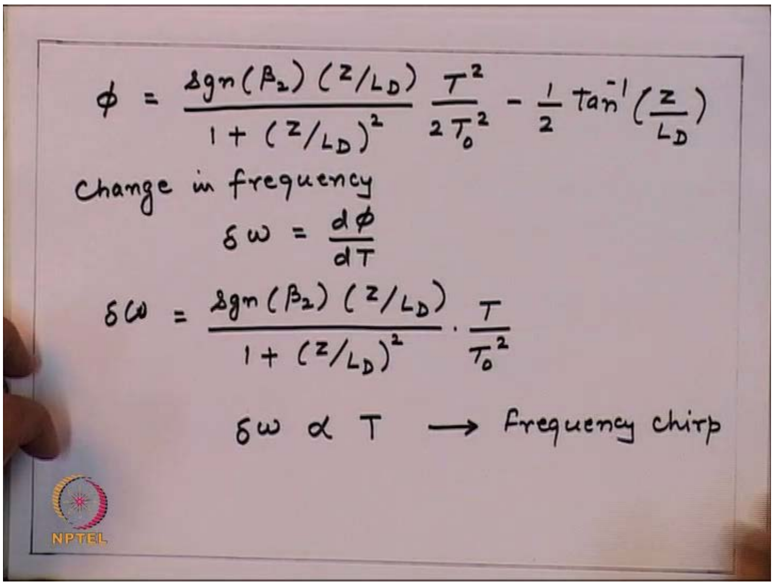

Which is given by phi which is equal to sign of beta 2 Z by L D divided by 1 plus Z by L D whole square T square divided by 2 t naught square minus 1 upon 2 tan inverse of Z by L D. Now not here (Refer Slide Time: 35:12) that when we are looking at the phase component of this function here, that is one phase term, which is going to come because, of this, another phase term which is going to become of this. So, the first term which was see here, this quantity that is coming because of the phase term here and this one is coming, because of the phase term of this quantity.

So, that is total phase you are going to see inside the pulse, so first thing what will not here is that, the phi the phase changes quadratically with pi a phase we are related to this quantities square. So, the phase with changing with linearly with time, then the rate of change of the phase with time that gives you a frequency that frequency remains constant. But, since the phase is not linear function of time, there is the frequency change as a function of time.

So, what we are essentially now saying is that initially when the Gaussian pulse or launched, there is a carrier which as the same frequency for the entire duration of of the Gaussian pulse. But, now since we are going to have a different phase change at different locations of the pulse, because the phase is a function of time and that is the non-linear function of time. there is the frequency change which will going to take place and this frequency change is going to be different different location inside the pulse.

So, if I look at Gaussian pulse here (Refer Slide Time: 36:53) I will see the phase change in the carrier at this location, it will be something we certain frequency here, if I go here the frequency would be different, if I go here the frequency would be different and so on.

That means initially the entire frequency was same inside the pulse but, know you see that the frequency is changing at different location inside the pulse, because of this non-linear phase function. So, since I have this expression here, we can you get the change in frequency, which is delta omega that is equal to d phi by d T, so I can differentiate this this quantity here with T.

So, will get change in frequency delta omega that is equal to sign of beta 2 Z by L D divided by 1 plus Z by L D whole square T upon T naught square, that gives the change in frequency delta omega proportional to that means inside the pulse (Refer Slide Time: 38:41) the frequency is going to linearly change from one end of the pulse to another and of the pulse. This phenomena of change in frequency is what is call the chirping phenomena, so this gives what is called the frequency chirp.

So, it is some kind of frequency modulation which is taken place inside the pulse, so initially, when we can consider a Gaussian function the Gaussian function had a same frequency inside the inside the pulse but, because of the dispersion there is going to be a frequency modulation inside the pulse. So, what have happened, because of the dispersion that the annual up inside the pulse if you really see, then whatever the constant carrier frequency or there inside the pulse, that is now frequency modulated.

And thus the frequency modulation gives you linear change in frequency across the pulse that is the effect of the group velocity dispersion thus what we see from here. So, now since the frequency is changing linearly the function of time and now not only that the change in frequency depends upon what is the sin of this beta 2. And as we know that inside the optical fiber, the fiber has characteristic that it has a inflection point, so if I plot the refractive index as the function of frequency for the optical fiber, then somewhere the beta 2 is positive, somewhere the beta 2 is negative. The region where beta 2 is positive is called the normal dispersion regime, when beta 2 is negative that regime is what is called anomalous dispersion.

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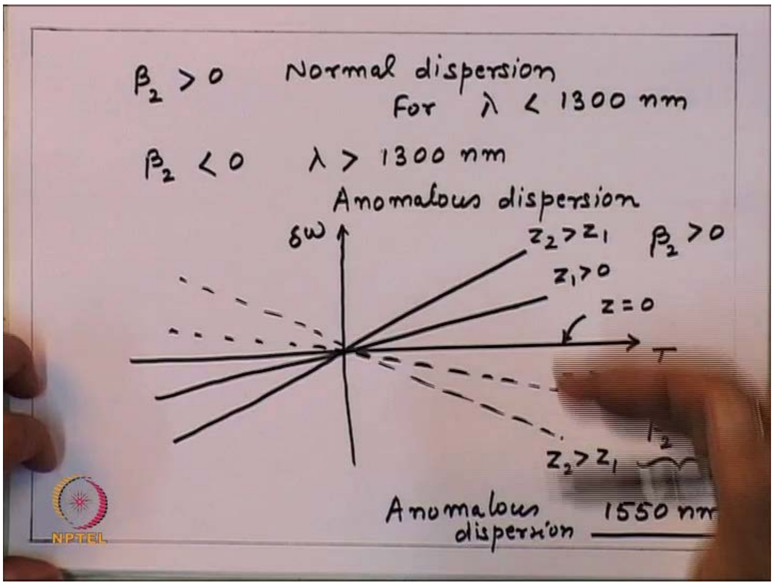

So, inside the optical fiber we see, beta 2 greater than 0, this we call as the normal dispersion and this happens if you look at the refractive index for optical fiber, this happens for wave length less than 1300 nanometer.

So, for lambda less than about 1300 nanometer, the beta 2 becomes less than 0 or become negative, this happens for lambda greater than about 1300 nanometer; and this regime is what is called the anomalous dispersion. Now, if I take this (Refer Slide Time: 41:58) change in frequency function and plot that as the function of time, I can see.

So, this is time, this is the change infrequency delta omega, so it z equal to 0 there is no change in frequency everywhere as function of time, the frequency is is same change infrequency 0, so this one gives you for z equal to 0. As the pulse starts moving as the z increases you have now, the frequency change in which is linear and if I consider (Refer Slide Time: 42:46) the sin of beta 2 positive, then the frequency increases as a function of T and it goes from negative to positive.

So, if we have situation, which is like this this is for some value of z 1 greater than 0, if I go to still further distance it remain linear chirp but, the slop of this change, so this is for z 2 greater than z 1 that so on. Now this happens if beta 2 is positive, that means I see the frequency chirp, which is positive like this for beta 2 greater than 0.

So, normal dispersion does the way frequency chirp is going to be, on the other hand if I consider beta 2 less than 0 there is anomalous dispersion, then the frequency chirp is negative, because now (Refer Slide Time: 43:58) this this quantity here is going to become negative, so frequency chirp is still linear but, now it will go like that.

So, this is the let us say some z 1 greater than 0, this is z 2 greater than z 1 and so on. So, you see two different types of chirp inside the optical fiber pulse, depending upon what wave length we are using. So, if we are below 1300 nanometer where the dispersion is normal we see the positive chirp of frequency, if I am at a wave length greater than 1300 nanometer, then we see the negative chirp of frequency. So, if I am operating at 1550 nanometer this quantity would be the one, so at 1550 nanometer we see a dispersion which is the anomalous dispersion.

So, we see here the anomalous dispersion but, note here whether dispersion is positive or negative you will see that the pulse width always increases (Refer Slide Time: 45:31) because the L,D which is defined as mode bit 2. So, whether dispersion is positive or negative you will see that this quantity is always increasing as the function of z, so whether you are in the normal dispersion regime or you are in the anomalous dispersion regime. The annual up always broadens and thus the phenomena, which is dispersion phenomena that when the pulse transfer propagating on the optical fiber, there is always broadening of the pulse thus what we are studied.

When we were studying the linear propagation in the in the optical fiber but, the chirping of the pulse inside the optical fiber or inside the pulse is going to we have opposite sense depending upon whether you are in the normal regime or you are in the anomalous regime.

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Now, what is the meaning of this positive chirp and negative chirp, so what this means is that, if I consider now a pulse which is like that and this is the time, this is the center which is capital T equal to 0.

So, this is the pulse edge, which is the leading edge of the pulse and this is the trailing edge of the pulse. Now, if now the change in frequency (Refer Slide Time: 47:17) is positive, what that means is that here the frequency is little lower compare to the center. And here the frequencies little higher, that means the low frequencies get accumulated at the leading edge and the high frequency get accumulated at the trailing edge but, if I consider the anomalous dispersion regime.

Then, the case will be exactly opposite because now the frequency will be decreasing, so here I will have as the function of T the frequencies decrease now, so here I will have low frequencies and here I will have high frequencies.

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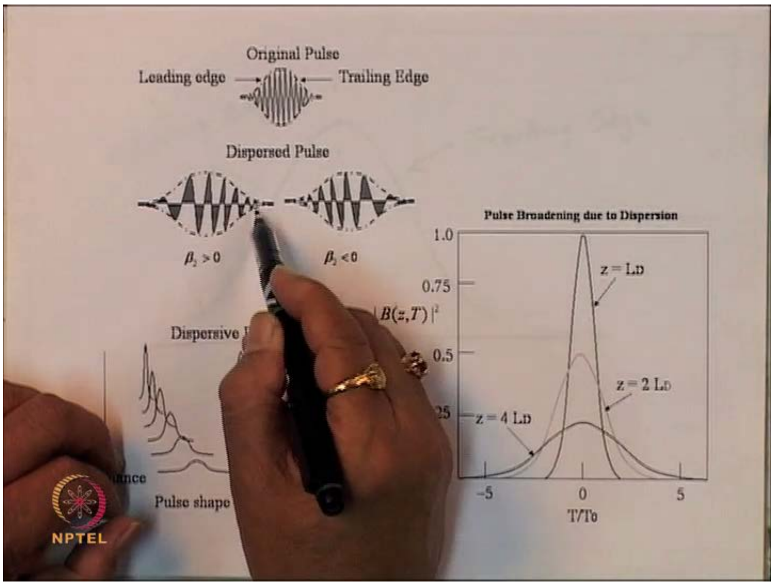

So, you see now that depending upon the situation, we may have situation like this you have the original pulse, which is like this as inside, the which is given as carrier. And you see there is a same frequency everywhere, when the pulse starts moving on the optical fiber and there is a dispersion.

So, when beta 2 is greater than 0 that means a normal dispersion you see the low frequency got accumulated here and high frequency got a accumulated here, when the dispersion is anomalous that time the high frequency got accumulated here and low frequency got accumulated here. So what are happened is that in this situation, the low frequencies I have going to head, because this is a leading edge; in this situation the low frequencies are left behind and the high frequencies are going to head.

So, if I consider the situation like this, then you will see high frequencies move faster then the low frequency in the anomalous dispersion (()). And exactly opposite happens the normal dispersion but, in both the cases the spectrum remain same, only the phase of different frequencies are getting modified.

So, here is the sort of pictorial representation of that, said this function of distance if you start with the pulse; the pulse was like that and does not we irrespective of whether you are having anomalous dispersion or normal dispersion. The pulse broadens but, the spectrum remains same, because the amplitude spectrum does not get modified only the phase gets modified. So here we are seeing the initial is pulse was like that after distance the pulse is broaden, I (()) get further broaden if is your further broaden and so on.

So, initially if you start with a Gaussian pulse, you will see as the pulse travels the shape of the pulse remains Gaussian but, its broadening continues as the pulse propagates on optical fiber. And there is the frequency modulation, linear frequency modulation inside the optical pulse. So, let us to summarize summarize, what is the effect of group velocity dispersion? The group velocity dispersion modifies the pulse shape in time domain, the amplitude spectrum of the pulse remains intact, what however modifies is the phase of different frequency components in the spectrum.

So, the amplitude spectrum is same but, the phase spectrum changes and as the result of that, the shape of the pulse in time domain changes. And irrespective whether you have anomalous dispersion or you have the normal dispersion, the pulse always broadens but, with different frequency modulation inside the pulse.