Advanced Optical Communications

Prof. R. K. Shevgaonkar

Department of Electrical Engineering

Indian Institute of Technology, Bombay

Lecture No. # 30

Tutorials

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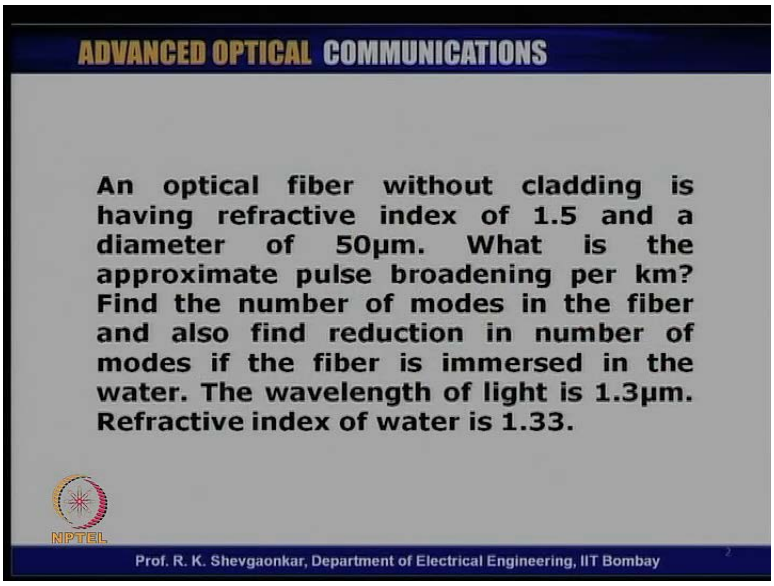

In this lecture, we are going to solve some problems related to optical fibers. Let us look at this first problem here. An optical fiber without cladding is having refractive index of 1.5, and a diameter of 50 micrometer. What is the approximate pulse broadening per kilometer on this fiber? Find the modes in the fiber and also find reduction in number of modes, if the fiber is immersed in water. The wavelength of light is given as 1.3 micrometer, and the refractive index of water is 1.33.

We all know that when the light propagates inside an optical fiber which is having a diameter much larger than the wavelength, then you can use the simple ray model for finding the estimate of the pulse broadening on the optical fiber.

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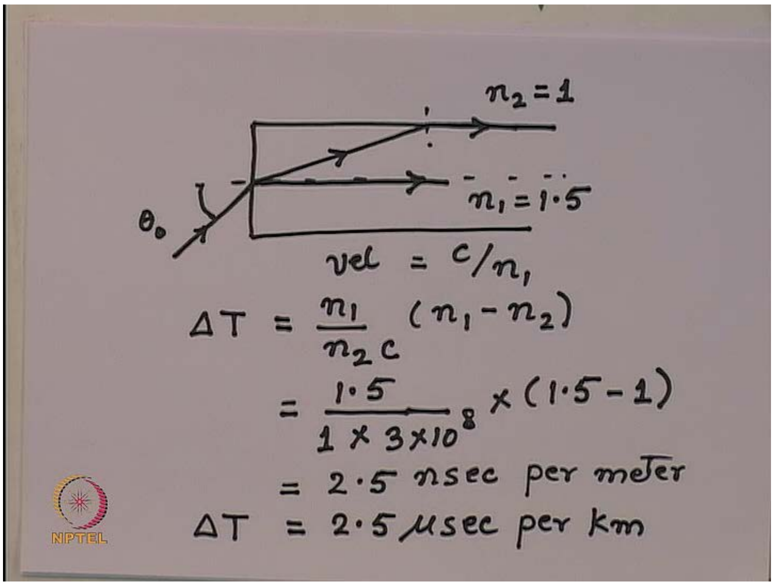

So, let us consider a simple optical fiber cross section like that; that is the axis of the optical fiber. When a rays launch at the maximum possible angle, the ray will be refracted inside this optical fiber and it comes here, and since this is the maximum possible launching (( )) here, ray actually goes along the edge of the optical fiber or the interface between the core and the cladding. Now, since there is no cladding for this fiber, the refractive index n 2 for this is equal to 1. The refractive index of fiber which is glass is n 1 that is equal to 1.5. This angle is theta 0 maximum that is the ray which is barely total internally reflected from the core cladding boundary.

Now, we know that when the light is launch inside this, there is one ray which can go along axis of the optical fiber which is like that and the extreme ray which will go in this direction. Since the refractive index is constant inside the optical fiber, the velocity for this ray and this ray is same which is c divided by n 1. So, the velocity of light for all the rays traveling inside the optical fiber, which is equal to c divided by n 1. Then as we have seen earlier in our discussion that the time difference of arrival between these two rays is given as delta T that is equal to n 1 upon n 2 c which is velocity of light multiplied by n 1 minus n 2. Now, since the cladding is not there for this fiber, n 2 is equal to 1.

So, n 1 minus n 2 is 0.5; n 1 is 1.5; n 2 is 1. So, that gives us the time difference is 1.5 divided by n 2 which is 1 multiplied by velocity of light c which is 3 into 10 to the power 8 meters per second multiplied by n 1 minus n 2 which is 1.5 minus 1 which is equal to 2.5 nanoseconds per meter. Since you are use the velocity of light in terms of meters here, the unit for this time is nanoseconds per meter. Normally, when we talk about time difference inside the optical fiber, it is given in terms of per kilometers. So, the time difference delta T will be this multiplied by 10 to the power 3, if you want convert the meters into kilometer.

So, this is 2.5 microsecond per kilometer. So, that is the difference in the arrival of these two rays and that is what essentially gives what is called the pulse broadening on n optical fiber. The second thing which is asked here is that how many modes are going to propagate on this optical fiber (Refer Slide Time: 00:33) and when the fiber is immersed inside the water how many modes will be there. So, what is the reduction in the number of modes? So, we can see here first of all as we know approximately since the fiber is multimode, the numbers of modes are approximately given by V square by 2. So, first we have to calculate what the V number of the optical fiber is.

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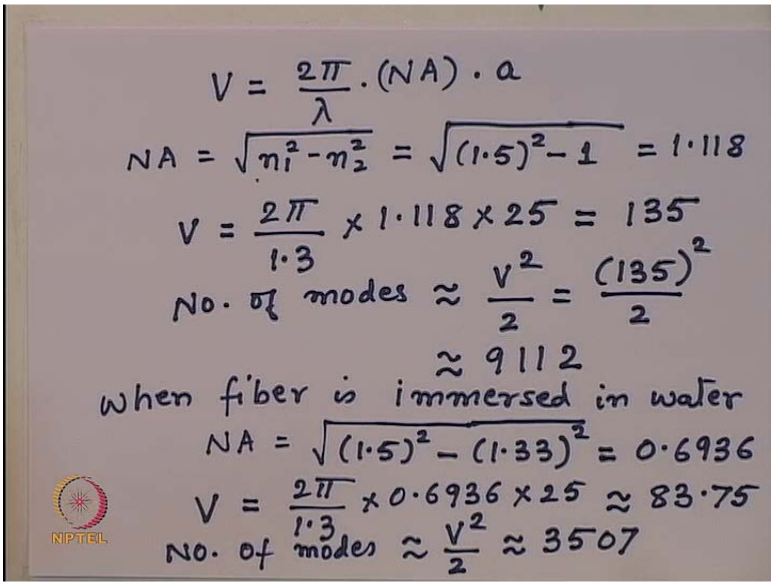

And as we know, the V number of an optical fiber is 2 pi by lambda in to numerical aperture in to the radius of the optical fiber. So, the numerical aperture for this optical fiber is square root of n 1 square minus n 2 square which is equal to square root of 1.5 whole square minus n 2 is 1; So, square of that is 1; So, this is one which is equal to 1.118. So, the V number for this optical fiber is 2 pi divided by lambda which is given is 1.3 micrometers. So, 1.3 multiplied by 1.118 multiplied by the radius of the optical fiber. It is given that the diameter of the optical fiber is 50 microns. So, a is equal to 25 micron.

So, we get the V number of the optical fiber is about 135 and the number of modes on this fiber then is approximately equal to V square by 2. So, that is 135 square divided by 2. So, that is approximately equal to 9112. So, as you can see without the claddings since the numerical aperture is very large, extremely large numbers of modes propagate on this optical fiber. So, this fiber is extremely multimode about 9000 modes are going to propagate in this optical fiber. Second thing which is asked is that when this fiber immersed inside the water, what is the reduction in number of modes inside the optical fiber?

So, first thing one should note is when the fiber is immersed (Refer Slide Time: 01:32) inside the optical inside the water, then the refractive index of surrounding now is increase from 1 to 1.33, where the refractive index of water is 1.33. That means now the n 1 minus n 2 is reduced and as a result, the numerical aperture of the optical fiber is reduced. So, now is the second case when fiber is immersed in the water in water, the numerical aperture will be square root of n 1 square minus n 2 square. But now n 2 is not 1; but n 2 is 1.33; that is the refractive index of water. So, this is 1.5 whole square minus 1.33 whole square and that is equal to 0.6936.

So, if I substitute now the numerical aperture which is 0.6936 inside this expression, the a is still 25 micron; lambda is 1.3 micron. The V number in this case now would be 2 pi divided by 1.3 multiplied by 0.6936 multiplied by 25 and that will be approximately 83.75. So, the numbers of modes in the situation are approximately V square by 2 and V is this now. So, this is approximately about 3507. So, when there was no cladding, that time approximately 9000 modes were propagating. When the same fiber is immersed inside the water, seen the refractive index of surrounding medium is increase; the V number of fiber is gone down and because of that, the numbers of modes are reduced to 3500.

So, now from approximately 9000 modes, the modes have reduced to about 6500. So, difference of this two that is reduction, which we have in number of modes. Naturally, one can ask a question at this point of time. What happens to these modes which were earlier propagating inside the optical fiber? The answer is very simple. The earlier the higher order modes which were confined inside the optical fiber; now when the fiber is immersed inside the water become leaky mode and because of that, the power corresponding to those modes essentially radiates out and as a result, the number of modes actually decrease inside the optical fiber. So, this is one of the simple problems; one can see related to the optical fiber.

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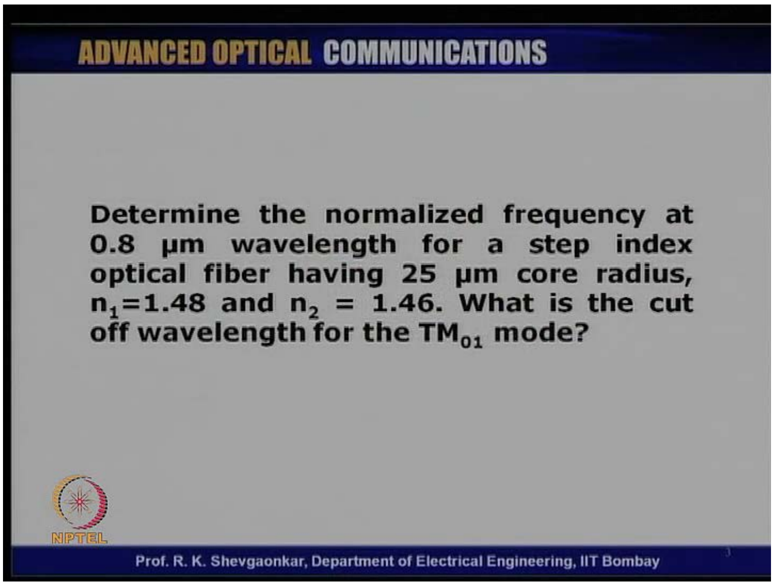

Let us look at the second problem. Now in this problem, it asks determine the normalized frequency at 0.8 micrometer wavelength for a step index optical fiber having 25 micrometer core radius; n 1, which is the refractive index of core is 1.48 and n 2, the refractive index of cladding is 1.46. It is also ask what is the cut off wavelength for the TM 01 mode? Now, this problem is a very simple problem to solve. First of all, as we know that first we have to solve find out what is the normalized frequency of this optical fiber and normalized frequency is nothing but the V number of the optical fiber.

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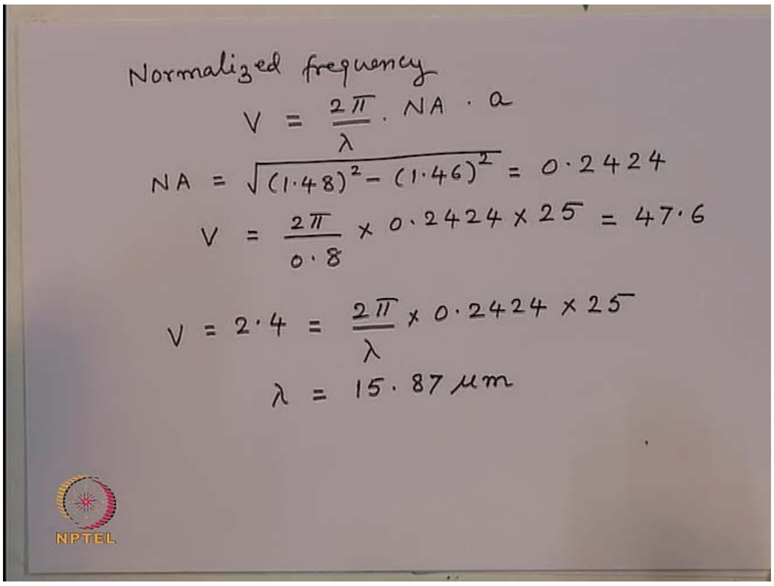

So, we have here normalized frequency which is the V number of the optical fiber, which we know is 2 pi by lambda in to the numerical aperture multiplied by the radius of the optical fiber. The numerical aperture as we have seen in the previous problem is square root of n 1 square minus n 2 square. So, this is 1.48 square minus 1.46 square and that is equal to 0.2424. So, the V number for this optical fiber that is 2 pi divided by lambda and in this problem, the lambda is given as 0.8 micrometers. So, this is 0.8 multiplied by the numerical aperture; it is 0.2424 multiplied by the radius of the optical fiber which is 25 micrometer. So, this gives the V number is 47.6.

So, the first part of the problem where asked to find out the normalized frequency of this optical fiber at 0.8 micrometer. The normalized frequency is nothing but the V number of the optical fiber and for this fiber at 0.8 micrometer, this normalized frequency turns out to be 47.6. The second question which is asked here is what is the cut off frequency for the TM 01 mode and as we all know, that on optical fiber the first mode which propagates that is the HE 11 mode. After that, you have bunch of modes which start propagating; T 01 modes are propagating; at the same time TM 01 modes are propagating and that happens, when the V number of the optical fiber is 2.4.

So, the frequency at which the V number becomes equal to 2.4, that is what is called the cut off frequency of that particular mode. So, in this case if we make V number equal to 2.4 and ask at what wavelength the V number will become equal to 2.4. We can work backwards and say that V number to be equal to 2.4. The wavelengths which want to find out which is lambda; substitute in to this expression here; the numerical aperture which is 2424 multiplied by the radius of the optical fiber which is 25 micrometer. So, if I just invert this, I will get the lambda which is 15.87 micrometer. So, as you can see that for most of the operating range of wavelengths which is used for optical communication, this fiber is going to remain the single mode optical fiber.

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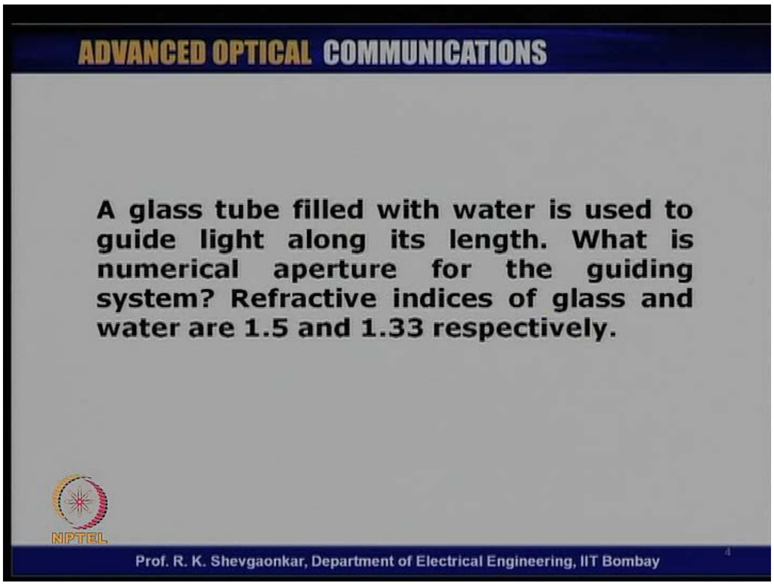

Let us look at the third problem. (No audio from 16:19 to 16:39) Now, this problem is a little interesting problem; not a very practical problem. Because this is not related to optical fiber in that sense though the principles involved; here are exactly same is what we have find inside the optical fiber. But this problem helps you in understanding how the rays actually are total internally reflected inside the optical fiber and the light is guided. So, what is problem says is problem says a glass tube filled with water is used to guide light along its length. What is the numerical aperture for the guiding system? Refractive index of glass and water are 1.5 and 1.33 respectively. So, look at the configuration which we are having here.

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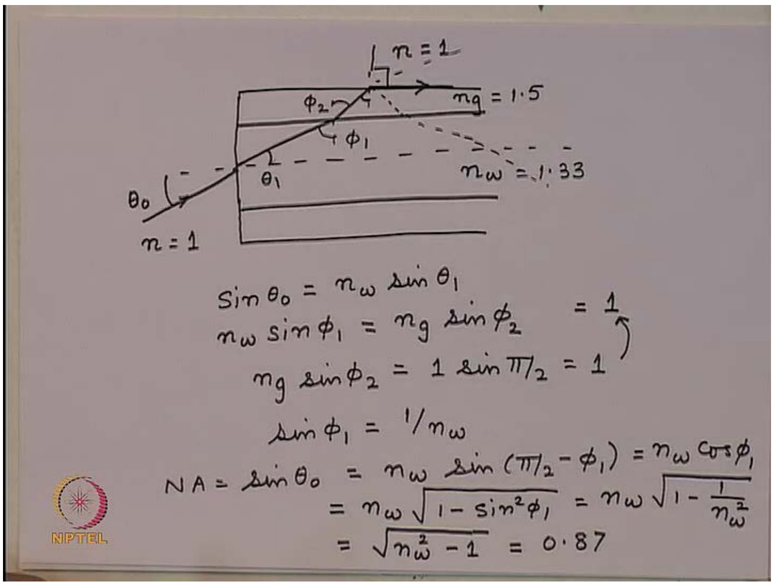

We are having a tube of glass and the tube wall have some thickness. So, we have this is a tube. So, if I take a glass tube and just cut it by plane; that is the way the cross section of the tube would look like. So, this is glass; this is glass and here the tube is filled with water. So, here the refractive index says this is the axis of the tube; refractive index here is n w, which is water. The refractive index here is of glass which is n g and here the refractive index is air, which is n is equal to 1. So, this this is equal to n equal to 1. Now in the first look, it may appear that if I try to launch a light inside this. Since this system will look as if you are having a core here which is water and you are having a cladding which is glass and then you are having a third medium, which is air.

Now, the water refractive index is 1.33; the glass refractive index is 1.5. Therefore in first look, it may appear that if I compare this system with optical fiber, the refractive index of the core which is 1.33 is lower than the refractive index of the cladding which is 1.5. So, in the first look, it might appear that this system will not be able to guide light along its length. However, if you really think little deeper, you will see that the question is not saying that the light will be guided inside the water. It is simply saying if you treat the whole tube as the system, then can the light be guided inside the system. So, let us look at how the light is going to propagate inside this. Again, we can consider a simple ray model.

Because the size of the tube is much larger compare to the wavelength. So, let us say I launch a ray from here inside inside the tube; inside this water. Now, seen the refractive index of water in is higher than this, this ray will bend like that. Now, at this point seen this angle cannot be greater than critical angle; because this refractive index is smaller than this. The rays going to be refracted and now this denser medium compared to this. So, the rays bend bending toward the normal at this point. So, the ray will go something like that. Now, if this angle at this location seen this is 1.5, this refractive index is 1 here. If this angle is less than critical angle, the ray will simply get refracted in the air.

If this angle is greater than critical angle at this interface which is the glass air interface, then the ray will get total internally reflected and then will will come something like that. Then again it will trace the path back exactly like the way it is going and then it will go on the other side, again go to this side of the glass again will get total internally reflected and so on. So, inside the water, the rays going to propagate get refracted inside this glass and if this angle happens to be greater than critical angle, then there will be total internal reflection at this interface. There is no possibility of total internal reflection at the water-glass interface.

But certainly there is a possibility of total internal reflection at the glass-air interface. So, now one can ask a question, what is this maximum angle possible; this theta 0. For which, the light will be guided inside the structure; that means this angle becomes equal to the critical angle or in that situation, this ray actually will travel along the interface between the glass and the air. Now, it is the simple Snell's law application on the different interfaces. So, first of all let us say if this angle is theta 1; this angle is phi 1; this angle is phi 2 and this angel since this ray is going along the interface, this angle is 90 degrees. So, if I apply the Snell's law at this interface and here the refractive index n is equal to 1.

I have sin of theta 0; that is equal to n w which is the refractive index of water multiplied by sin of theta 1. I can apply the Snell's law at this interface which is the water-glass interface, which is n w sin of phi 1; that is equal to n g which is refractive index of glass sin of phi 2 and this angel is phi 2. So, this angle is also is going to be phi 2. So, if I apply the Snell's law at this interface which is the glass-air interface, I will get n g sin of phi 2 is equal to this n which is equal to 1; sin of pi by 2. So, at this interface, this will be n g sin of phi 2 that is equal to n which is 1 sin of pi by 2, which is equal to 1. So, I see from here that the n g sin phi by 2 that is equal to 1 and n g sin phi by 2 is n w sin phi 1.

So, from here this (( )) this is also equal to 1. So, from here I can calculate sin of phi 1 that is 1 upon n w and phi 1 is nothing but pi by 2 minus theta 1 or theta 1 is equal to pi by 2 minus phi 1. So, now the numerical aperture for this system, which is nothing but sin of theta 0 when this angle is maximum; corresponding to the ray which goes on this interface that is equal to n w sin of theta 1 which is sin of pi by 2 minus phi 1 which is n w cos of phi 1; that is equal to n w square root of 1 minus sin square phi 1 and sin phi 1, we have got 1 upon n w. So, this is equal to n w square root of 1 minus 1 upon n w square which is equal to square root of n w square minus 1.

And since n w is given as 1.33, I can substitute the value of n w 1.33. So, the numerical aperture for this system would be equal to 0.87. So, what is interesting in this problem? The interesting thing in this problem is that when the light is trying to go inside this; though the refractive index of the inner region is smaller than the refractive index of the outer shell, in the conventional optical fiber sense we have cladding here; you have a core here and the core refractive index is smaller than the cladding. But if you are having a medium outside which is air, then the light is not guided inside the core. But the light can be guided at the cladding air boundary.

So, even if you are having a core refractive index smaller, there is the possibility of guidance of light at this interface which is the cladding-air interface; that is 1. Second thing to note here is the total internal reflection is going to take place at the glass-air interface. So, the for the guiding mechanism is concerned, that is the refractive index which is going to come in to picture; because that is where, total internal reflection is going to take place. But if you look at the numerical aperture, nowhere the refractive index of glass comes into picture; only the refractive index of water, where the light tries to get in; only that medium plays a role in deciding what the maximum angle possible is.

So, the interesting thing in this problem is that even if you are having refractive index smaller in the core; but if the clad is cladding is the good quality, then the light may get total internal reflected at the cladding-air interface. And there may be a guidance of light inside the core cladding structure together. Ofcourse, this is not a conventional way the light propagates inside the optical fiber. But just for sake of understanding, we can see here that even in this kind of system, the light can be guided inside the inside a glass tube or capillary tube or something like that; where the refractive index inside could be smaller than the than the tube material. So, this is the another interesting problem.

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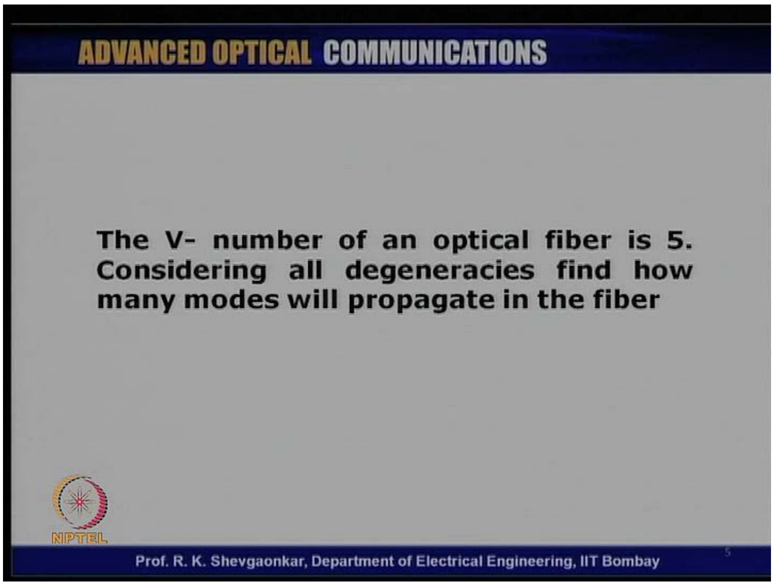

Let us look at another problem. (No audio from 28:20 to 28:29) Now in this problem, the V number of an optical fiber is 5. Considering all degeneracies find how many modes will propagate in the fiber. Now, we all know that when the optical fiber V number is given, we can very quickly calculate the number of modes propagating inside the optical fiber and that is V square by 2. However, you must remember that this expression that the total number of modes propagating inside the optical fiber is V square by 2 is an approximate expression. When the V number is large, this approximation is reasonably accurate. But, when the V number become small, then the accuracy of this expression is not very good.

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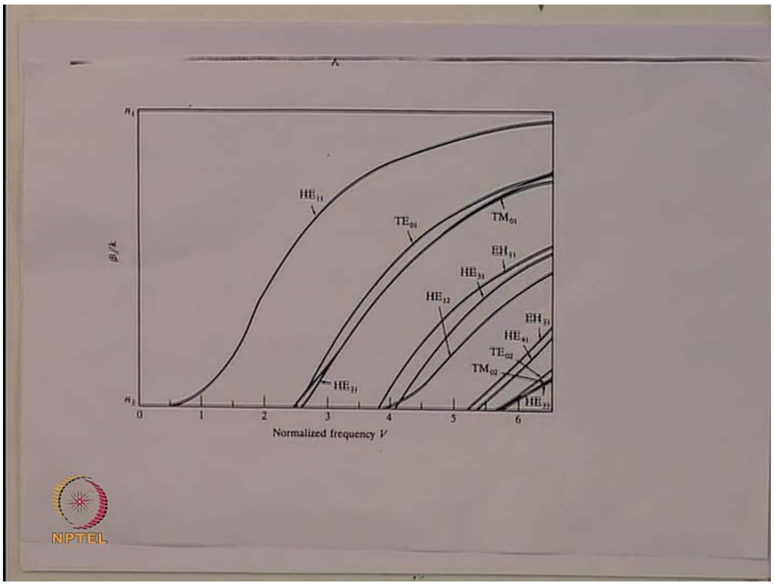

So, one can ask a simple question. If I have a V number which is let us say 2. Now, if I consider V number 2, then the V square by 2; this expression will give me simply V is equal to 2. So, V square is 4 divided by 2 will give me two modes. Whereas, now if I look at the b V diagram here; I can say very clearly that for this V number, there are no two modes propagating; there is only this module which which is propagating. If I consider degeneracies, ofcourse I will get two modes. But the point is that when I make the V number small, then that expression which is V square by 2 may not give you an accurate result for the small V numbers. And as V number becomes larger and larger, that time you may get a reasonably accurate a result for the number of modes propagating in a multimode fiber.

So, let us see now. It is said that the V number of this optical fiber is 5; that means firstly whatever modes are less than 5, V number 5 those modes are going to propagate. So, if I take this line here; this line here; this is the V number 5, then these modes are going to propagate inside this optical fiber. So, let us (( )) what are the numbers of modes which are going to propagate inside this optical fiber. So, as you can see here, this mode is HE 11 mode; then you are having T 01 mode; then you are having a TM 01 mode; then you are having a HE 21 mode; then you are having a HE 12 mode, HE 31 mode and EH 11 mode. So, let us write down that if I look at the b V diagram for V number 5, how many modes from the b V diagram will propagate.

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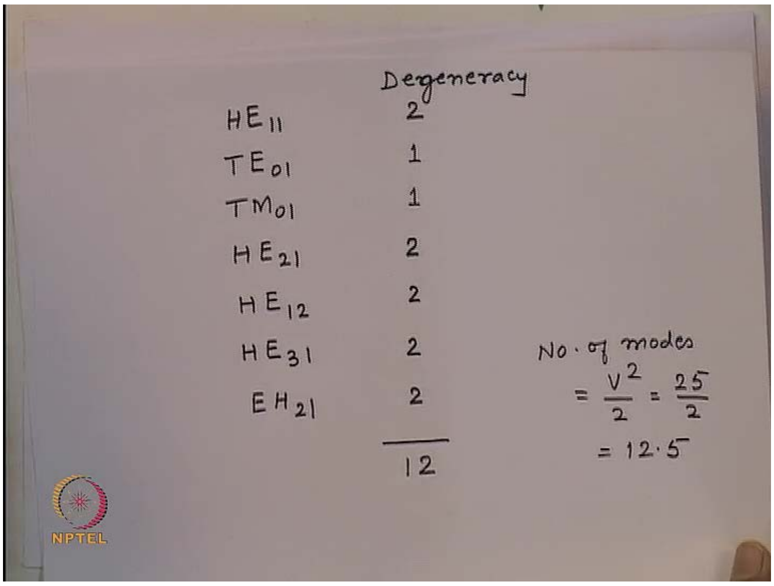

So, firstly we will get a mode which is HE 11 mode. Then I will get TE 01 mode; will get TM 01 mode; will get HE 21 mode and you will get HE 12 mode; you get HE 31 mode, EH 21 mode. So, you see here form the b V diagram which we have seen. (Refer Slide Time: 29:28) These are the three which are here; there are three modes in this and there is one mode which is propagating here. However, then the question is asked it is said very clearly that including all the degeneracies, how many modes are going to propagate. That means, though you are having this indices 11, it tells you the intensity distribution of the electrical magnetic fields inside the optical fiber. There are still the degeneracies because of polarization of the modes inside the optical fiber. So, let us ask a question what are the polarization degeneracies or the angular degeneracies corresponding to each of these modes.

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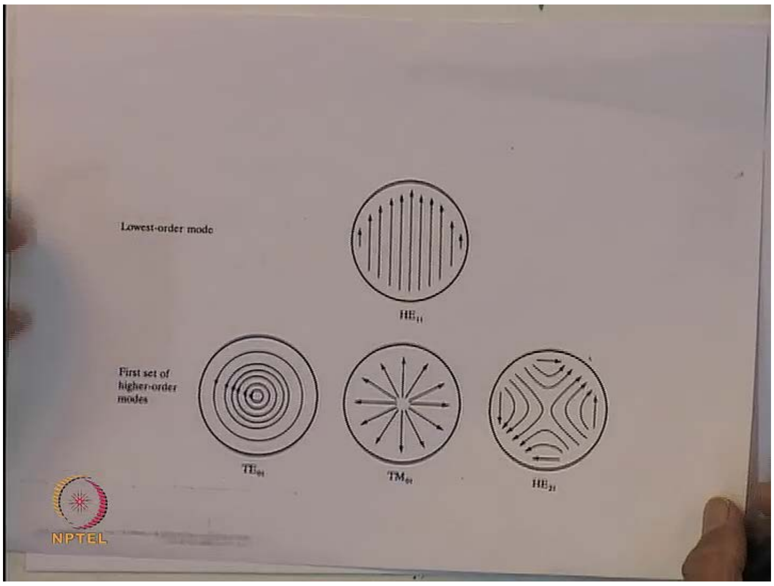

So, as you can see if I look at this diagram here; that is the field distribution which we are going to get for the HE 11 mode. Now, as you can see here the electric field is oriented this way, but there could be electric field which is perpendicular to this, which is orthogonal to this electric field; that will also have the exactly identically intensity distribution. Or in other words, it will also have the propagation characteristic identical to this direction of the electric field. That means for this mode, we are having two possible orientation of electric field, which will have a exactly identical propagation characteristics.

So, this mode has a degeneracy (Refer Slide Time: 31:16) degeneracy of 2. What about T 01 mode? For the T 01 mode, the field distribution is given like that. Now, since the field is oriented this way, no matter about orientation you have; you get actually the same field distribution. That means for this, the degeneracy is 1; because it is only one field distribution which we can get for T 01 mode. So, for this, degeneracy is 1. The same is true for T 1 is also; because if you orient this, you will get same filed distribution. So, you get degeneracy for this is 1. What about HE 21 mode? I have this is the field distribution for HE 21 mode.

But if I take this field, which is this way; if I changed to this direction where these are completely horizontal; this is radial, I will get the orthogonal field distribution corresponding to this. So, I will get a degeneracy corresponding to this mode which is a factor of 2. (Refer Slide Time: 31:16) So, I will get factor of 2 degeneracy for this. Similarly, you will get factor of 2 degeneracy for this and all this thing which are going to have the variation, which are going to be similar to this once. You will get degeneracy which have factor of 2; factor of 2. So, now I see the total number of modes if I ask which are going to propagate (Refer Slide Time: 29:28) inside the optical fiber with a V number 5.

From the b V diagram, we see there are about 7 modes which will propagate. But there are different degeneracies for the different modes and if I put the degeneracies, (Refer Slide Time: 31:16) I will see the total number of mode which are 2 plus 1 plus 1 plus 2 plus 2 plus 2 plus 2. So, this is you will get 2 plus 2 4 6 8 10 12. So, you will get total number of modes which are going to propagate from this is 12. How does it compare this with the approximate relation? If you are calculated by using the approximate formula, the number of modes in a fiber will be equal to approximately V square by 2 and V is 5.

So, this is 25 divided by 2. So, that is equal to 12.5. So, you see the formula is reasonably accurate. But when the V number is very small, that time the formula may give you little error. But the results are reasonably accurate. The formula (( )) 12.5 modes may be could round it of to 13; but you can round it of to 12. But this formula gives you the 12, which is exactly same as what you get considering various degeneracies right of the modes corresponding to HE 11 mode, T 01 mode so on and so on. So, this is another problem which where one can calculate the number of modes which are propagating inside an optical fiber.

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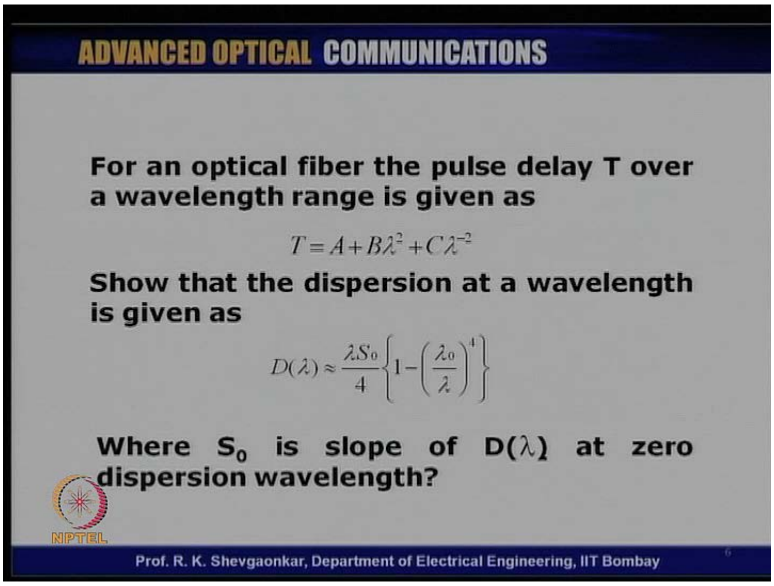

Let us consider another problem. (No audio from 36:58 to 37:10) Now, this problem is related to the dispersion on optical fiber. It is given that for an optical fiber, the pulse delay T over a wavelength range is given as T is equal to A plus B lambda lambda square plus C lambda to the power minus 2. Show that the dispersion at a wavelength is given as lambda times S 0 divided by 4 bracket 1 minus lambda 0 by lambda to the power 4, where S 0 is the slope of the dispersion as the function of wavelength and lambda 0 is the zero dispersion wavelength. So let us look at this problem.

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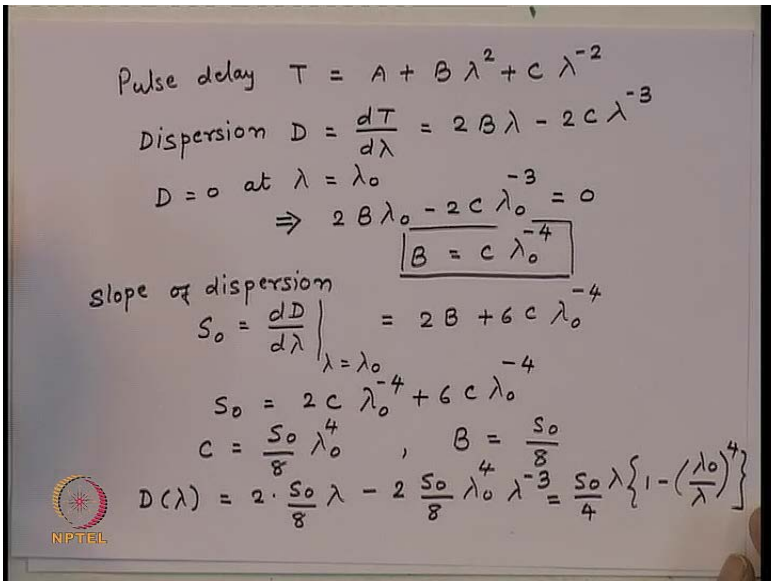

So, what is given here? Given is pulse delay T that is A plus B lambda square plus C lambda to the power minus 2. Now, you all know the dispersion is the derivative of the delay as a function of wavelength. So, we have dispersion D that is dT by d lambda which by differentiate this with respect to lambda; that will be equal to 2 B lambda minus 2 C lambda to the power minus 3. Now, it is given that a wavelength lambda 0, the dispersion is 0; that means D is equal to 0 at lambda equal to lambda 0. So, if I substitute this in to this, this gives us 2 B lambda 0 minus 2 C lambda 0 to the power minus 3; that is equal to 0 or from here, we can find the relationship between B and C.

So, this gives B is equal to C lambda 0 to the power minus 4. The second thing which is given there is that the slope of the dispersion at this wavelength lambda equal to lambda 0 is given by parameter S naught. So, we can calculate the slope of dispersion, which is d D by d lambda and this is given S naught at wavelength lambda equal to lambda naught. So, if I take the derivative of this as the function of lambda, we get 2 B plus 6 C lambda to the power minus 4 lambda naught to the power minus 4. So, from here I can solve; I can substitute for B in terms of C. So, from here I will get S naught is equal to 2 C lambda to the power minus 4 plus 6 C lambda naught to the power minus 4.

So, from here I can find out what is the value of C. So, the C is equal to S naught by 8 lambda 0 to the power 4. And if I take this value of C and substitute in to this, this will give me value of B which is S 0 upon 8. Then I can take this value of B and C and substitute in to the expression for the dispersion. So, we get the dispersion D as a function of lambda that is equal to 2 B into lambda. So, this is 2 S naught upon 8 in to lambda minus 2 C which is S naught upon 8 lambda 0 to the power 4 lambda to the power minus 3. So, if I simplify this, I will get the final expression which is S 0 upon 4 in to lambda 1 minus lambda 0 upon lambda to the power 4.

So, that is what is asked here that if the pulse delay was given by this expression, then the dispersion on the optical fiber can be written in terms of the zero dispersion wavelength, which is lambda 0 and the slope of the dispersion curve at the zero dispersion wavelength. Now, many time the dispersion characteristics of one optical fiber is approximated by this kind of relation and the dispersion then on the optical fiber can be calculated in the vicinity of the zero dispersion wavelength by using this simple simple formula. So, this is more like practical problem. Because that is the way many times, the dispersion characteristics are approximated and one can calculate the dispersion at on optical fiber at other wavelength in the vicinity of the zero dispersion wavelengths.

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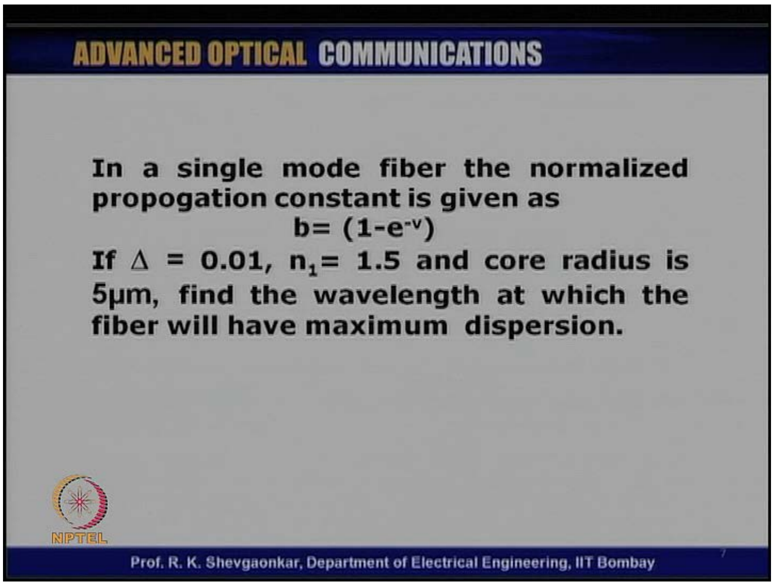

Let us look at another problem on the dispersion on the optical fiber. (No audio from 44:00 to 44:30) In a single mode fiber, the normalized propagation constant is given as b is equal to 1 minus e to the power minus v, where V is the V number of the optical fiber. If delta for the optical fiber is 0.01, the core refractive index is 1.5 and the core radius is 5 micrometer, find the wavelength at which the fiber will have maximum dispersion. Now, though the problem may not be may not give a number which are very realistic numbers, but this explains how one can approximately find the dispersion, if the dispersion curve can be expressed in some analytical form. So, as we know that the dispersion curves starts with 0 for the normalized propagation constant b and when the V number is very, very large, the normalized propagation constant b becomes equal to 1. So, the curve looks very similar to this kind of exponential curve.

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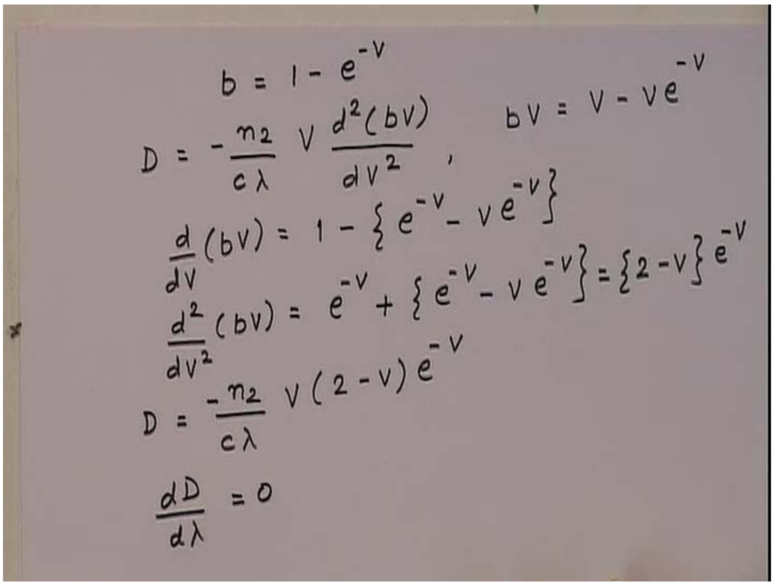

So, in this case it is given now that b is 1 minus e to the power minus v. Now, as we have derived earlier for the waveguide dispersion, the D is given as minus n 2 upon c lambda V d 2 b V by dv square. So, since the b is given as 1 minus e to the power minus v, we have b v; that is V minus V e to the power minus v. So, for this we can get d by dv of b V that is equal to 1 minus e to the power minus V minus V e to the power minus V and d 2 by dv square b V that is equal to e to the power minus V plus e to the power minus V minus V e to the power minus v. So, that is equal to 2 minus V e to the power minus v. We can substitute this in to the expression for the dispersion.

So, the D is minus n 2 upon c lambda V in to 2 minus V e to the power minus v. Now, we have to find out the wavelength at which the dispersion becomes maximum. Now form the expression, we can note few things here. Firstly, that when V is equal to 0, the dispersion is 0; because this quantity is 0. So, the dispersion goes to 0. When V is equal to 2, again the dispersion is 0; because this term will go to 0 and that therefore dispersion will be 0. Thirdly, when the V is equal to infinity; that means when the V becomes very very large, this quantity becomes goes to 0.

So, this again the dispersion will go to 0. So, this if you are having the normalized propagation constant given by this expression, the dispersion characteristics is fairly complicated. The dispersion is 0 at V equal to 0; the dispersion is 0 at V equal to 2 and the dispersion again is 0 at V equal to infinity. To find out where the dispersion will go maximum, first we have to find out the location where the derivative of the dispersion goes to 0 as a function of the the wavelength. So, we want to calculate when d by d lambda is equal to 0.

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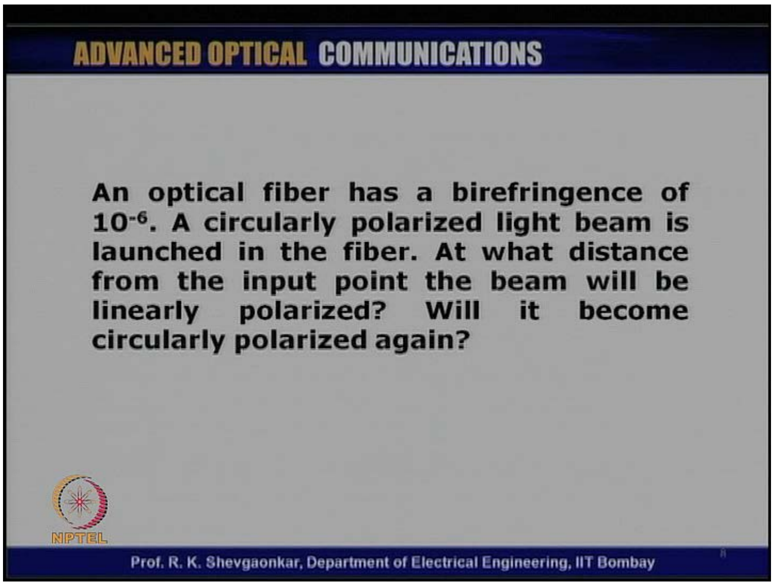

Let us look at this problem on birefringence on optical fiber. (No audio from 49:42 to 50:14) As we know that is the optical fiber is not absolutely circularly symmetric. The two orthogonal polarizations propagate with different propagation constants on the optical fiber. And because of this, the two polarizations undergo different phase change. Now, two orthogonal polarizations together can generate a state of polarization and if the phase change is the function of distance on the optical fiber, the state of polarization keeps on changing as the wave propagates on the optical fiber.

So, this problem essentially related to that; that an optical fiber has a birefringence of 10 to the power minus 6. Now, what is birefringence? Birefringence is the change in the refractive constant for the two orthogonally polarized modes on an optical fiber. So, that change in refractive index is 10 to the power minus 6. A circularly polarized light beam is launched in the optical fiber. At what distance from the input point, the beam will be linearly polarized? Will it become circularly polarized again? That is the question.

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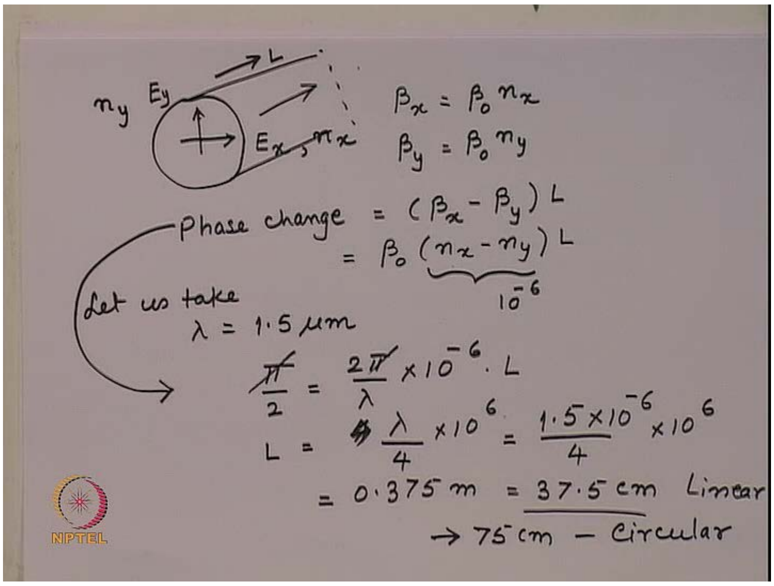

So, now as we said that if you are having the optical fiber core, the two fields which are perpendicularly polarized. Let us say this filed is E x and this filed is E y. They travel with different velocities or in other word, they see different refractive indices; effective indices. So, let us say this one looks a refractive index which is n x and this one see the effective index which is n y. So, the phase constant for this mode horizontally polarized mode is beta x which is beta 0; which is the propagation constant in the free space multiplied by the refractive index, which this polarization sees which is n x.

Similarly, the propagation constant for this polarization is beta y; that is equal to beta 0 multiplied by the effective index, which this polarizations sees which is n y. So, the phase change when the modes propagate on the optical fiber, then the phase change between these two polarizations is equal to beta x minus beta y multiplied by the length, which it travels along the optical fiber. Substituting for beta x and beta y, we get beta 0 n x minus n y in to length. Now, this quantity is nothing but the birefringence; that is given as 10 to the power minus 6. Though the wavelength is not given for calculation purpose, let us assume that we have a lambda which is equal to 1.5 micrometer. So, let us take lambda is equal to 1.5 micrometer.

Now, if the launch mode is circularly polarized that means these two polarization linear polarizations have equal amplitude and the initial phase difference between these two is equal to pi by 2; that is a condition we have for a circularly polarized wave. Now, if the wave has to become linearly polarized, then the phase difference between these two polarizations should be 0 or 180 degrees. Or in other words, if the phase change which these two polarizations are undergoing; difference in the phase change; if that is equal to pi by 2, then the initial phase change of pi by 2 is nullified by the phase change which is going to take place or added. So, either the either the phase change at that distance will be equal to 0 or 180 degrees.

That means, if this phase change when it becomes equal to pi by 2, the original circularly polarized wave will be converted in to a linearly polarized. So, we can ask question at what distance, this happens. So, if I put the phase change is equal to pi by 2; that is beta 0 which is 2 pi by lambda; birefringence which is given as 10 to the power minus 6 and you want to find out what is the length at which this is going to have. So, from here, we can calculate lambda equal to 1.5 micrometer. So, you get L is equal to 4 pi will cancel; so, you get lambda upon 4 in to 10 to the power 6; lambda as taken as 1.5 micrometer. So, you can substitute that; so this is 1.5 in to 10 to the power minus 6 divided by 4 in to 10 to the power 6.

So, that is equal to 0.375 meters or 37.5 centimeters. So, that means if a circularly polarized wave is launch inside this optical fiber at a distance of 37.5 centimeter from that location, the circularly polarized wave will be completely converted in to a linearly polarized wave. Now, if I keep moving further, the phase difference again will change and after the same distance, there will be accumulation of pi by 2 phase; that means, from this location where the wave was completely linearly polarized. If I move a 37.5 centimeter away, again the phase difference between these two polarizations will be equal to 90 degrees; that means the wave will become again circularly polarized.

So, at this distance, we get linear polarization. If I make the same distance again again; that means if I go to a distance of 75 centimeter, then I will get circular polarization again. We got the phase difference would be equal to pi by 2. If I again move 37.5 from that location, again I will see linear polarization and so on. That means, assuming that the two polarizations undergo the same amplitude change, we will see that the wave goes from linear polarization to circular polarization to linear polarization periodically and this period is 75 centimeter, where the initial polarization is again restored back.

So, the answer to the question, will the wave be converted to the circular polarization again? Answer is yes. It will be again converted to circular polarization and if you move further again, the wave will again converted will be converted to linear polarization and so on and in between these two locations, the wave will be elliptically polarized. So, what we see here is that because of birefringence on the optical fiber as the wave propagates, the state of polarization goes on changing continuously and the wave from right from the linear polarization can go up to circular polarization and so on.

So, in those optical communication systems which are polarization sensitive, one has to worry about the state of polarization of the outcoming light and that is where, one has to take the fibers which are what are called the polarization preserving fibers. Because those are the fiber in which the state of polarization will not change, as the light propagates on the optical fiber. But if you consider the normal optical fiber even with a small birefringence as small as 10 to the power minus 6, you will see a periodicity in the state of polarization. So, here are certain problems which we have seen today which are related to the basics of optical fiber.

We saw the problem related to V numbers, we saw the problem related to the cut of frequencies., We saw the problem related to dispersion calculations of the optical fiber. We also saw the problem how the rays are propagated in the core and the cladding, and we also found a situation where even if the core material refractive index is smaller than the cladding. But if the outside medium is air, again the light may be guided not at the core-cladding interface, but at the cladding-air interface. So, this problem essentially gives you more on hands on experience on solving the problems related to the basic light propagation inside the optical fiber.