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Lecture No. # 03 Ray Model – I

In the last lecture, we saw the basic structure for propagating light and that was the optical fiber. We also saw different windows in which the optical communication took place. So to start with the optical communication started in 800 nanometer window, later on it was shifted to 1300 nanometer window, and today the most of the optical communication takes place in 1550 nanometer window. We also saw the basic characteristics of light that means if we consider light as a energy source then it has parameters like intensity of light, the frequency of light or the wavelength of light and the spectral width of light.

However if we treat light as an electromagnetic wave then the vector nature is captured by a parameter, what is called the polarization. And we saw that an electromagnetic wave in general has elliptical polarization, which in limiting cases could be linear polarized or circularly polarized. We also saw that if you take a very wide band source then there is no very definite state of polarization defined, and that polarization we call as a random polarization.

So coming now to the structure that is the optical fiber; today we discussed the propagation of light inside the optical fiber.

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As we mentioned last time, the light can be considered like a ray in the simplest possible model. So, if we consider a source of light the phase fronts are moving out of the source and if we draw the line which are perpendicular to this phase fronts, then we will get those fictitious lines what we call as the rays. So in the simplest possible form, the light can be treated as a ray. Later on we will take the advanced version of the model and that will be the wave model, and finally, we have to take the model which is the quantum model where light would be the combination of photons.

So, today we take the simplest possible model which is the ray model and we ask that if you want to launch a ray inside the optical fiber what are the constraints on propagation of light in a bound structure like optical fiber. So today discussion is mostly going to be around the propagation of light inside the optical fiber in the form of rays, so as we discussed yesterday the optical fiber basically is a solid glass rod.

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Which consist of a inner structure which is called core and an outer shell which is called cladding, so the propagation of light essentially these two layers the core and the cladding are the important regions.

However to support the structure mechanically we have some other layers what are called the buffering layers but, they buffering layer does not have any role to play as far as propagation of light is concerned, so today we essentially we investigate if you have to launch a light inside the core under what condition the light will propagate inside this core over well long distance without much loss and then we find out some conditions the efficiency parameter or the data rates and the other things related to the propagation of the light inside the structure.

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So firstly let us say that if I had a structure consider a simple glass rod and this is the section of that, one thing you would note that if i put a ray of light from this side inside this ray would get inside this medium which is glass so is going from air to glass so they will go like that and then again it will reach to this interface it will again go away and this will come out so if the ray was sent on this optical fiber from the side walls then the ray will simply cross the structure and will never get guided inside this core.

So one thing is immediately clear that no matter how much light is there surrounding the optical fiber, if the light is impinging from the sides of the optical fiber there is no possibility of this light getting guided along the core of the optical fiber. That means only possibility if you have to send the light inside the core is that the light has to go from the tip of the optical fiber, so if the light goes from the tip of the optical fiber then it will get inside and if you have a reflection which is boundary it will get reflected like this, so through a multiple reflections the light ultimately will get guided inside the core of the optical fiber.

So what we notice that, if the light has to be guided inside the core it can be launched inside the structure only through the tip of the optical fiber. No matter how much light is present surrounding it, The light can never get inside structure though it can cross the structure, so if the tip is not exposed to the light, low light can get inside this core of the optical fiber and same is true otherwise also that if the light was propagating inside this optical fiber no light will come out from the sides of the optical fiber it can only come from the tip or the end of the optical fiber. That is what besides we talk about the security inside the optical fiber that if the tip is protected for the optical fiber form the sides of the optical fiber no light can get inside the core or no light can come out of the core. Now we essentially ask a question if you want to launch a light inside this core at what angle the light should be launched from the tip of the optical fiber, so that there is a total internal reflection at this interface which is the core cladding interface at this point we may ask would not a partial reflection suffice

Suppose reflection was not totally internal reflection but, suppose the reflection at the core cladding interface was partial reflection you will see that very quickly there will be leakage of energy where part of the energy will get transferred to the cladding which will leak out, so within a very short distance the power will be lost, so total internal reflection is extremely essential if you want to have a sustained propagation of light over very long distances, so even a smallest possible leakage of light because of partial reflection here would take away most of the power within a very short distance.

So that is the reason we say that the light has to be launched inside the core of the optical fiber such that the light get total internally reflected at the core cladding boundary and then through the multiple total internal reflections the light will be guided along the core of the optical fiber so since, we are talking about light rays now first we know that in the ray model the light obeys what are called the Snell's law.

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That means if we have a medium and 1 which is here and the medium in 2 which is above this line and if a ray is launched at an angle phi 1 which is script to the normal which is called the angle of incidence.

Then the light will be refracted in to the second medium and suppose the refracted ray makes an angle phi 2 with respect to the normal, then we have the Snell's law which says n 1 sine phi 1 is equal to n 2 sine phi2 note here the angle which you are measuring here are measured from the normal to the interface, so if this medium is rarer compared to this that means if n 2 is less than n 1 then phi 2 is greater than phi 1 so as we increase this angle phi 1 this angle becomes close to ninety degrees and that what is called the critical angle this angle will become equal to ninety degrees.

And beyond that if we launch a ray essentially the ray will be reflected in to this medium that is what the Snell's law says, so first thing is clear now that if you want to have a total internal reflection two conditions have to be satisfied firstly that this medium should have a refractive index higher than the refractive index of this medium, so in terms of the optical fiber what that means is that we must have the material chosen for core and cladding such that the refractive index of core is higher than the refractive index of cladding, this is the trivial condition.

Then you ask a question once you have chosen that refractive indices for core and cladding now if the angle phi 1 is less than critical angle than this ray will be refracted

and there will not be total internal reflection, so this angles phi 1 has to be greater than certain value then and then only it will have a total internal reflection at the core cladding boundary and the value of sustained propagation of light. Before we get in to this analysis and ask a question what is the angular zone from which the light is accepted by the optical fiber let us first try to physically see how the light ray can be launched inside an optical fiber.

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So one possibility is that, if I have the core of the optical fiber. I can launch a ray from the tip of the optical fiber such that it lies in a plane containing the access of the optical fiber a thing which is shown here, so if i take a plane which is passing through the access of the optical fiber which is the plain of the paper in this case and if i launch a ray which lies in this plain which is called a Meridional plain then you will notice that the ray will be refracted from here we will get total internally reflected but, this ray will always remain in the same plain.

So through multiple total internal reflections the light will get guided but, this ray will always remain in this plain or in other words if I now consider the set of rays which are going from the tip of the optical fiber any ray which is launched in a particular plain always remains in the same plain and since i have infinite plain which are passing through the axis of the optical fiber essentially i have a ensemble of rays which go like that again they will come and get totally internally reflected and come here and they will again join together and so on.

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So if i look at one possibility this is the axis of the optical fiber the ray is launched like that get totally internally reflected like that but, when I talking about one ray actually there are set of ray which are going which are making same angle with respect to axis i will have thing will be going like this also like this also and so on, so what we notice is that in this case all the rays start together travel a same distance again they come and meet at the axis of the optical fiber again they go like that again we will have the axis of the optical fiber and so on.

So in this case essentially you have the rays again and again meeting on the axis of the optical fiber. Now as a result you expect that you will have the maximum intensity at the axis of the optical fiber, so if i consider the rays which are launched in a plain containing the axis of the optical fiber then the ensemble of this rays would produce a light intensity distribution inside the optical fiber which would have a maximum intensity at the axis of the optical fiber, on the contrary you were say that we can launch a ray deliberately at an angle such that this ray does not lie in the plain containing the axis of the optical fiber.

So let us say I had a fiber like this instead of putting a ray which goes like this in the plain suppose you put a ray which goes like this now this ray you can work out that this ray will never now lie in the plain containing the axis of the optical fiber, so the ray will

go like this it will go like that it will go like that it will go like this and so on, so if the ray is launched at an angle with respect to the plain containing the axis of the optical fiber then the ray will never intercept at the axis of the optical fiber in fact it will go on spiraling around the axis of the optical fiber, so it comes from here it will come here it will come here it will come here and so on.

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These rays are called the skew rays so the previous case we have Meridional rays which always intercept the axis of the optical fiber and that is why i give you the highest intensity at the axis of the optical fiber whereas, the skew rays never meet the axis of the optical fiber and as a result they have low intensity at the center of the fiber, now when the light is launched inside the optical fiber there are two possibilities of intensity distribution and later on we will see what do they correspond to but, there are two possibilities and one possibility you have a intensity distribution which is maximum at the center of the optical fiber and other situation where at the center the light intensity is minimum.

Or in other words the light essentially is confined to the anural ring or the cross section of the core, so at the center we have very low intensity and the light essentially is confined to the anural ring, so light propagates in this form as if the core is not a solid rod it is like the hollow rod and most the energy is just confined to the rim of the core of the optical fiber, so there are two possibilities that means for launching of the light inside the optical fiber either the light can be launched in the form of the Meridional rays or the light can be launched in the form of skew rays.

For the simple analysis let us say that the light is launched inside the fiber in the form of Meridional ray, so let us take a cross section of the optical fiber, so this is the core of the optical fiber with a refractive index n one and this is the cladding surrounding the core of the optical fiber which has a refractive index n 2. Now let us say we launch a ray at an angle theta zero from air or in general you can see there is a medium which has a refractive index n this ray now goes inside the core of the optical fiber at an angle theta the ray reaches to the core cladding interface and then depending upon what is the value of theta naught either may get refracted in to the core cladding medium or will get total internally reflected inside the core.

So if the ray was launched at an angle which is beyond certain value then this ray will get refracted as i reduce the angle slowly i will reach to this angle corresponding to this red line for which the ray at this interface will be launched at the critical angle, so ray will travel parallel to the core cladding interface and if the angle is smaller than this then the ray will get total internally reflected what that means is that if i had a light source which could send the rays of light at all possible angles from the tip of the optical fiber the fiber accepts only those rays which have a launching angle less than the angle corresponding to this red line this quantity let us call this angle as theta zero max.

So what that means is that for a given light source which is capable of sending light from all directions the fiber is selecting in choosing only light coming from a certain cone or in other words there is some kind of a light launching efficiency associated with the optical fiber because only this cone corresponding to the two theta zero maximum that is the cone essentially is going to get launched inside the optical fiber which will be propagating in the core over long distance to that of total internal reflections all this rays which are beyond the red line they will get refracted and that is why that power will simply leak out.

With this underrating that one can simply apply the Snell's law at the two boundaries one can apply Snell's law here one can apply Snell's law here now ask what is the value of theta zero maximum in terms of the core cladding refractive indices see for apply a Snell's law at this point i have here and sine of theta zero that will be equal to n one sine of theta.

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$$n\sin\theta_{0} = n_{1}\sin\theta = n_{1}\sin(\pi_{2}-\phi) = n_{1}\cos\phi$$
$$n_{1}\sin\phi = n_{1}\sin\pi_{2} = n_{2}$$
$$\Rightarrow \sin\phi = \frac{n_{2}}{n_{1}} \Rightarrow \cos\phi = \sqrt{1-\frac{n_{2}^{2}}{n_{1}^{2}}}$$
$$\sin\theta_{0_{\text{max}}} = \frac{n_{1}\cos\phi}{n} = \sqrt{\frac{n_{1}^{2}-n_{2}^{2}}{n^{2}}}$$

So we can draw the snell's law that n sine of theta zero that will be equal to n 1 sine of theta.

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This angle phi which is the angle of incidence and the core cladding boundary which is nothing but, ninety minus this angle theta so we can write here n one sine of pi by two minus phi or equal to n one cos of phi. Now at the critical angle when this ray is launched we have here n one sine phi this angle now is pi by 2 angle of refraction in cladding so that will be equal to n 2 pi by two so that will be equal to n 2 so from here i can find out the value of the sine phi corresponding to maximum launching angle and that will be equal to n two upon none we can substitute and i can get a value of cos phi this is what is needed here so that you will be square root of one minus n 2 square one n 1 square you are substituted by this value of cos phi into this expression from here I get the maximum launching angle for optical fiber that will be n 1 cos phi divide by n that will be equal to square root of n 1 square minus n 2 square upon n square.

Invariably since we launched light inside the optical fiber from air and is equal to one so for air n equal to one, so we have a sine of theta zero maximum that is equal to square root of n 1 square minus n 2 square since this quantity is telling you the light collection efficiency or it has a effect very similar to if the wave was coming or if the energy is coming and you are having some kind of a aperture sitting in front of it the energy is stabbed by that aperture we call this parameter sine of theta zero maximum as the numerical aperture of an optical fiber.

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Numerical Aperture Sine of the maximum angle accepted by the fiber. It defines the light launching efficiency $NA = \sin \theta_{0\max} = (n_1^2 - n_2^2)^{1/2}$ For high launching efficiency NA should be as large as possible. Prof. R.K. Shevgaonkar, IIT Bombay

So numerical aperture is one of the most fundamental parameter of an optical fiber and that essentially defines the light launching efficiency of an optical fiber, so we have this parameter what is called numerical aperture which is equal to sine of theta zero maximum and this side you saw is square root of n 1 square minus n 2 square, so what we find from here that if you want to have a very high light launching efficiency then this quantity should be as large as possible.

Since we already identified the material for transmission of light that is glass, so i have to make a core of glass that means the refractive index of glass is more or less fixe d that is equal to one point five, so i cannot change the value of this quantity and one, so only possibility we have is reduce this parameter n 2 to as lower value as possible. Now since this quantity n 2 is always greater than or equal to one that is situation would be if i make n 2 equal to one and then i get the full launching efficiency by that time in that case this angle will be equal to pi by two.

But if i make n 2 equal to 1 essentially what we are saying is you have to move the cladding, so as far as the light launching efficiency is concerned the cladding is a undesirable feature because if we had a cladding then n 2 will be greater than one and then you will have a light launching efficiency which will be reduced, so in the forced look it appears that although the optical fiber structure is consisting of core and cladding the cladding is a undesirable feature because it reduces the light launching efficiency.

However if you think little deeper what you will realize is that light launching parameter is one of the aspects of optical fiber. One can ask a question was there a prime goal to put the light inside the optical fiber, if you take a light source and you can put that light will efficiently inside the optical fiber but, if it does not carry any information even with high efficiency that propagation is not a very used to communication, so one can ask the question that instead of continuous source of light if we put a light in the form which can carry information then what are the implications of the refractive indices of the core and cladding.

So numerical aperture which is a efficiency is one of the issues but, another issue is that.

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If you wanted to send the information on optical fiber, then the light cannot be of continuous nature you have to change the parameter of light, so let us say we have a light which is pulsed say you are sending a light pulses and now when you put the light inside this core since any ray which is launched within this cone can be total internally reflected and will get guided you can say that now the rays can go at different angles and that is what they travel different distances.

So if i consider a pulse of light here the pulse energy will get divided in to different rays one ray will go like this which were launched straight, the energy which group were launched at a angle slightly higher than this will go by this path, the energy which is launched at this angle which essentially go by this and so on, so what do we see is that the pulse of light which were launched here the pulse which has gone by this path has travelled a distance which is which is this the pulse which has gone by this path has

But still you having the range which are distributed continuously in this cone essentially you are having the pulse arriving some energy arriving at the this distance after sometime then after sometime here sometime here and so on, so the pulse which was very narrow at when at the launching point when it reaches through this location the pulse would be having a spread which will look something like that or in other words you have a pulse broadening phenomena because of large number of rays which are going inside this optical fiber and then these rays are effectively travelling different distance along the axis of the optical fiber because this is going by this path.

So then one can ask if i consider two rays one which is going around the axis and one which is going around the maximum possible angle theta zero maximum what is the time difference between these two and therefore, essentially will give me what is the pulse broadening which you are going to wait on the optical fiber, so the time different between the two extreme pulses that essentially would be given by this you can simply calculate you can find out if this distance is 1 this angle is given if you apply again the Snell's law and find out the difference in the time the light is travelling with the same velocity which is c by n 1 by this path, so i can take a projection of this on this and i can find out what is the time difference I get delta t that essentially will be given by this.

And as we know that within this time if i transmit another pulse the pulse will start overlapping with each other and essentially data will be lost, so for a given distance 1 we cannot transmit another pulse within this time and since the bandwidth or the data rate is of the order of one upon delta t essentially we find that the bandwidth now is related to this quantity n 1 minus n 2 and also n 2 which is the refractive index of the cladding (()), so ideally if you wanted to send a very high data rate this broadening of the pulse should be as narrow as possible.

Now since this delta t what we have here is related to the refractive indices of the medium you can look at two quantities here one is the ratio of n one upon n two and another quantity is n 1 minus n 2 l is the distance along the optical fiber c is the velocity of light so what we find is that the delta t is proportional to the ratio of n 1 upon n two and it is also proportional to the difference of n 1 and n 2 since the medium for core area identified is glass n 1 is 1 point 5 n 2 has to be less than n 1 so at most this value can range in between one and one point five.

So this ratio n 1 upon n 2 is very close to one it can range only between one and one point five, so essentially what we are saying is that this quantity delta t is depending on this quantity which is n 1 minus n 2 and if you have to make this quantity as small as possible that difference between the refractive indices of core and cladding has to be as small as possible now we have contradictory requirements as far as the numerical aperture is concerned or the light launching efficiency is concerned we want difference between n 1 and n 2 as large as possible and as far as the bandwidth is concerned we want this difference n 1 minus n 2 as small as possible.

So the question is whether we should be guided by this parameter or this constraint or we should be guided by the numerical aperture the answer essentially lies in the application for which you are using the optical fiber if we use the optical fiber for sensor kind of application where we want to measure very weak light then we cannot afford to lose any light or in other words we must have a very high launching efficiency inside the optical fiber so in those situation essentially we use the optical fiber which have a very high numerical aperture or we can use the fiber which do not have any cladding.

Whereas, if you go for communication where bandwidth is a rather important parameter launching efficiency of course, is but, bandwidth is of much higher importance where unless we have a bandwidth we will not be able to send the information on the optical fiber so for communication purposes we have to make the difference between n 1 and n 2 as small as possible or in other words for communication purposes the core is an integral part of the optical fiber if we remove the core you will have a bandwidth which is extremely small because this difference will become very large and then you will not be able to sense substantial high speed data on the optical fiber.

So most of the communication fiber this difference is made as small as the technology permits and that is the reason typically for communication fiber.

 $m_1 - m_2 \sim 10^{-3} - 10^{-4}$

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The difference n 1 minus n 2 is of the order of about tento the power minus three to ten to the power minus four so what that means is that the core and cladding although they will are two different regions the difference in the refractive indices of these two region is extremely small, so essentially we have the same glass which form the core and cladding only inner portion of this glass rod is doped with some material, so the refractive index is increases little bit or on the outer shell you leave the refractive index by very small amount and that rod essentially becomes an optical fiber

So this gives us very important now conclusion that for communication purpose whatever fiber we use that difference between the refractive indices of core and cladding has to be extremely small and then and then only you will be able to transmit a high bandwidth signal on the optical fiber you may do some calculation to find out that if n two was equal to one this quantity n one will be one point five n 1 minus n two will be point five and if you calculate this bandwidth this bandwidth will turn out to be as low as something like few hundred kilohertz or few hundred kilobits per second.

This bandwidth is much smaller than what twisted pair can afford so what that means is that unless we have a clanning in the optical fiber this structure will be a worthless structure because it will be able to support a bandwidth which will be even worst than what normal twisted pair or coaxial cable can support, so any useful optical fiber or communication has to have a cladding and the cladding refractive index should be as close to the refractive index of core as possible.

With this now let us now look at the total internal reflection phenomena in little more detail for the what we find here is that if we consider the light launch inside the structure any ray which is within the cone of this theta zero maximum will get guided because of total internal reflection inside the optical fiber say for as the ray model is concerned it appears there is a solid cone of rays enters the optical fiber and essentially zoom multiple reflection this cone properties.

Let us now try to put the phase fronts behind the rays well just we mentioned earlier the rays are only the fictitious lines which we have drawn essentially they are the phase fronts which are moving, so if you put a phase fronts behind this rays then ask a question that if this understanding correct that any ray which goes within the solid angle of this theta zero maximum would get guided inside the optical fiber, so let us look at the total internal reflection phenomena by putting the phase fronts behind this rays.



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So firstly let us say we have this core cladding boundary there is a ray of light which goes get totally internally reflected and let us at the moment consider only one interface will be another one here to multiple reflection the rays going and these are the phase fronts which are supporting this ray, so when the ray is reflected these are the phase fronts corresponding to this ray and so on, so let us say we have this green lines and red lines which tell you the phases of the phase fronts let us say the green shows you zero and this is red show you pi, so you got the zero pi then 2 pi and 3pi and so on similarly, I have here zero pi 2 pi 3 pi and so on.

So what we find in this region that the phase fronts corresponding to these two rays they overlap they intersect and whenever a red and green phase front intersects you have the two optical fields which are out of phase because the red line has a phase of all multiple and green line has phase of even multiple so whenever a red line and a green line intersect the light fields cancel each other, so we have a zero intensity whenever a green and green light intersect that time we have that what is called a constructive interference and you have a maximum intensity.

Similarly when a red and red line would intersect you have a constructive interference, so you have a maximum intensity, so whenever a red and green line intersect i have a zero intensity you got a cancellation where of destructive interference and whenever a red and red or green and green lines intersect that time we have constructive interference and we have a maximum intensity of light now if i consider a cross sectional line a line perpendicular to the core cladding interface and i ask what is the variation of light intensity in this direction.

So we get a light intensity which will typically look like that wherever green and green line is intersecting i have a maximum intensity similarly, red and red is intersecting i have a maximum intensity here this point will correspond to red and green intersecting i have a zero intensity and so on, so in this medium where total internal reflection has taken place the light intensity essentially varies from maximum to zero maximum to zero and so on, so you have a what is called the standing wave kind of behavior of light intensity pattern in a region where there is total internal reflection that is inside the core.

So inside the core we have the intensity distribution of light which would be like maxima, minima and so on if you go to the wave theory of light it tells us that a total internal reflection the light intensity is not zero in the second medium in fact if i go to the ray model this concept is completely missing because what the ray model says is that at total internal reflection light is completely reflected in this medium and it does not say anything about what is happening in this medium.

However if i take light as the electromagnetic wave it has electric and magnetic fields and if in this region if you have electric and magnetic fields which are finite then suddenly the electric and magnetic fields cannot go to zero in the second medium because you should have a continuity of after fields, so the ray model in fact does not tell you the correct picture of total internal reflection because it simply say the total power is reflected at the boundary a total internal reflection and it does not say anything what is happening on the other side of this boundary.

So if you use the wave analysis then you find that a total internal reflection the fields are present in the second medium and these fields exponentially decay as we go away from this interface and larger is the angle compared to the critical angle more sharper will be the decay of this fields from the core cladding interface nevertheless these fields are going to extend to infinite distance at least theoretically or in other words what we are saying is no matter how far away I go from the core cladding boundary I will always have these present fields present which are exponentially decaying and these fields are as important as the field which are this field because you have to maintain the boundary conditions at the core cladding interface

So that means we must protect the fields inside the cladding and unless those fields are protected the field which are inside the core also will get disturbed, so if i disturb this fields these field will disturb boundary field of the boundary this field will disturb these fields and essentially there will be a leakage of power which will be going from the cladding that is the reason we have to provide cladding so that at the outer edge of the cladding the field would have died down to substantially low value and these fields are not interfered by the external world

So as soon as I try to put the fields the phase fronts inside the rays I see something interesting happening of the propagation of light, so let me summarize what we said about total internal reflection we say that at total internal reflection there is a standing wave type of fields inside the core which is which is this and we have a decaying kind of fields inside the cladding in fact this understanding is required when we go to the more rigorous analysis which is a wave model unless we have this physical understanding we will not be able to get the solution to the equation which we get from the wave model

And third thing what is not explicitly seen here is that is when the ray undergoes a phase change at the reflecting boundary, so whenever there is a total internal reflection here suddenly there is a phase change between this ray and this ray at the reflecting point, so if i consider the two points which are very close to each other this point and this point the phase between them is not same and their phase change depends upon the angle of launching the refractive index of core and cladding and various other parameters

So a total internal reflection essentially three things happen you have a variation of intensity of light in the region where total internal reflection takes place which is core decaying fields which extend up to infinite distance in cladding and in a sudden phase change which takes place at the core cladding interface let us now consider with this understanding the two rays.

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Which are going like this and this dotted line here essentially the phase front which is common to this two rays now these two rays this way get total internal reflected which comes here this ray is still is going this way and when it comes here it now get reflected from this point and this is the phase front corresponding to this ray.

So now if i look at this phase front this phase front is common to this ray and this ray whereas, this phase front is common to this ray and this ray since this is the one ray is the common between these two essentially separation between this two phase front should be such that you have to have a phase condition satisfied that means what is called you have a sustained constructive interference, so the separation between these should be multiples of two pi then and then only this will sort a set of phase front which are moving this way and the phase front will satisfy the condition for this ray as well as for this ray and this ray.

So if you do a simple algebra for this what we find is that this distance s one you can get which is s one d upon sine theta where d is the thickness of this region s two which is this can be written like this. (Refer Slide Time: 47:24)

 $s_1 = d / \sin \theta$ $s_2 = AD\cos\theta = (\cos^2\theta - \sin^2\theta)d/\sin\theta$ $\frac{2\pi n_1}{\lambda}(s_1 - s_2) + 2\delta = 2\pi m$ $\frac{2\pi n_1 d\sin\theta}{2} + \pi = \pi m$

So the difference between s 1 and s 2 multiplied by the phase constant in the medium after refractive index n 1 that is the phase difference between the two rays or two phase fronts plus you have a phase difference at this total internal reflection and at this reflection so that is two times delta.

This total phase should be equal to multiples of two pi if you have a constructive a interference of light, so what you find now is that we require this condition to be satisfied if the light ray can propagate inside the structure there is something interesting because earlier when we talked about numerical aperture we find that any ray launched at an angle less than theta zero maximum would have a sustained propagation of light by total internal reflection now we say that is not enough even within the cone theta zero max if the light is launched at an angle which does not satisfy this condition then this ray cannot propagate inside this optical fiber

And since this m is an integer we have this expression which gives you discrete angles that means within the angle theta zero maximum also the ray can be launched only at very discrete angles then and then only there will be sustained propagation of light so what we have we have a departure from a solid cone of light to essentially the angular surfaces over which if the light ray is launched then and then only it can propagate inside the fiber if the ray is not launched at that angle then it will not satisfy the phase condition and it will not belong inside the optical fiber So now we have a departure from the continue theta to a discrete theta domain and therefore, essentially leads you to what are called the modes inside the optical fiber which are the discrete patterns of intensity of light inside the optical fiber so following this in the next lecture essentially we will develop the understanding of the model propagation of light and then we will go with the more rigorous analysis of light propagation inside the optical fiber.