Advanced Optical Communications

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Lecture No. # 24
Receiver Sensitivity Degradation

In the previous lectures, we investigated the noise characteristics of an optical receiver. We calculated a parameter, what is called the bit error rate or BER, which depends upon the signal to noise ratio of the optical signal. We also saw what is called the quantum limit of detection that means in the absence of thermal noise, just because of the statistical nature of the photons, how much minimum number of photons are required per bit to get a bit error rate of 10 to the power minus 9. We saw that this limit is very small.

So, in practice, we will not be able to go to that level. But in hypothetical situation, suppose we make the best possible receiver, where the thermal noise is negligibly small, and the short noise is the one which affects the signal to noise ratio performance. Then how much minimum power one should receive to get the required bit error rate? Now, when we are doing all this analysis, we essentially assume the ideal receiver. What that means is that when 0 level is transmitted, we assume that the 0 level corresponds to 0 power, also we assume that your clock is very, very stable. We assume that during 1 level, the amplitude of the light intensity remains constant.

Now, all these are very hypothetical or ideal situations. In practice, we always have deviation from this ideal situation; that means corresponding to 0 level, we do not have a condition of no light, but there always some light transmitted even during 0 level. Also because of the laser noise, we have fluctuation of the optical signal during level 1 and also we do not have a very stable clock. So now, one can ask a question that if the receiver had deviated from the ideal conditions, then what we have to do to achieve the same receiver performance that is the same bit error rate.

So, we are going to essentially discuss today the receiver sensitivity degradation that means, because of these practical constraints on the optical receiver, how the sensitivity of the receiver degrades. And to restore that sensitivity back, how much extra power has to be added into the system, so that we get the same receiver performance or same BER.

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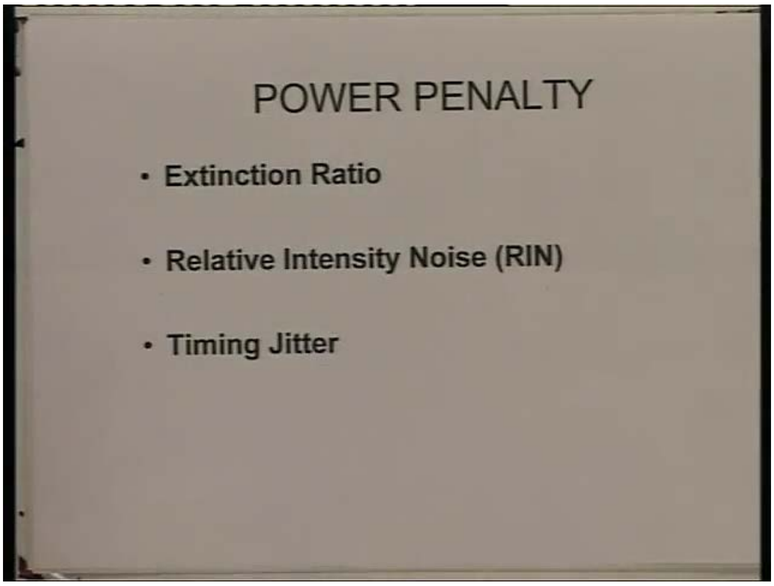

So, this is the quantity, which is called the power penalty; that power penalty essentially is the extra power, which has to be added in the optical system, so that we get the same BER performance and in this case, essentially we are going to discuss the power penalty due to the finite extinction ratio, which is related to non zero optical level corresponding to the 0 level of the data. Relative intensity noise what we call RIN that is related to the laser noise at level 1 and then because of the inaccuracy in the clock, there is always the jitter and because of that, we have statistical fluctuation in to the signal.

So, we are asking if the receiver had deviated from ideal conditions, because of this how much extra power has to be added and that is the quantity we call as the power penalty. So, in this lecture essentially we investigate each of these power penalties. Ofcourse, when one power penalty we are investigating, we assume that the other degradations are not there. So, at a time essentially we account for power penalty due to one effect and then we say total power penalty will be some of all the power penalties in terms of dB's. So, let us first look at the first power penalty, which is power penalty due to the extinction ratio.

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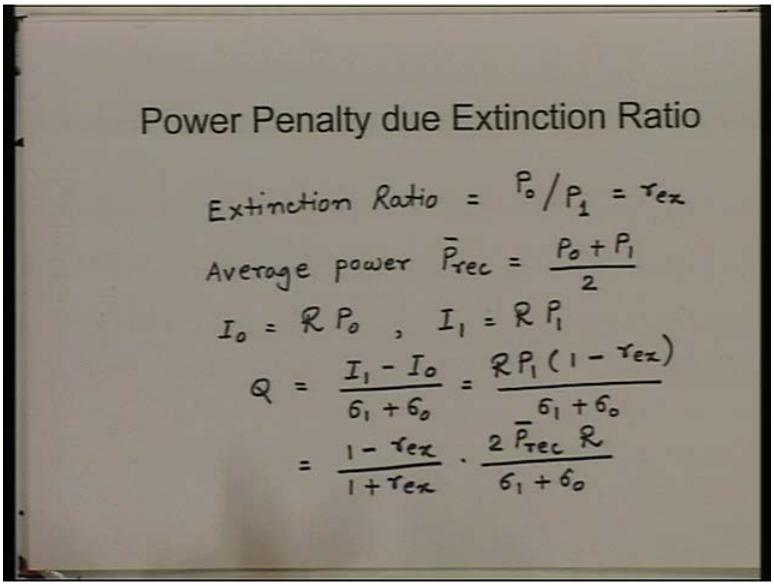

So, here what we are saying is that when the 0 level of the data is transmitted, the optical intensity is not 0. Now, there are certain reasons for that in a practical system. Firstly, if you recall when we talk about the optical detectors, we had something called the dark current. This is the current which is because of the ambient light, which is falling on the detector. So, even when no optical signal is transmitted, you will always have some light received and that essentially gives you a small DC current corresponding to the 0 level of the data. In addition to that, if you recall when we are discussing the laser characteristics the laser diode characteristics, we are seen that it has a threshold current.

And the speed of the device depends upon where the diode is biased with respect to the threshold current. So, if the bias is very close to 0 which is far away from the threshold current, then the device speed is less, because then it require certain time to build a population inversion and then the device goes into lasing action. So, to get a high speed, the laser diode is biased very close to the threshold current; but just below the threshold current. What that means is even in 0 data level conditions, you still have the threshold current flowing into the laser diode and therefore, there is a corresponding light emitted by the laser diode.

So, the two effects like the dark current and the biasing of the laser diode at the threshold current, they give you now non zero power corresponding to the 0 data level. So, then we define a parameter what is called the extinction ratio, which is ratio of the power in the 0 level divided by the power in the 1 level. So, we have a parameter what is called extinction ratio, which is the power received by the receiver corresponding to 0 data level divided by the power received by the receiver corresponding to the 1 data. In ideal situation, this quantity p 0 is 0; so the extinction ratio is 0.

Then we can define the average power as we did in the last lecture for finding out the minimum detectable power. So, we can say this is the average power say p rec; that is the mean of this. So, which is p 0 plus p 1 divided by 2 and the corresponding currents which will have corresponding to these two powers that will be the I 0; that will be responsivity into p 0 and I 1 the current corresponding to 1 level that will be responsivity into power p 1. And then we can define the parameter Q for the data, which is equal to I 1 minus I 0 divided by sigma 1 plus sigma 0. So, let us say this quantity is denoted by r extinction ratio, r ex. So, I can now take this quantity I 1 common.

So, this thing can be written as I 1 which is R into p 1. So, this responsivity into p 1 into 1 minus I 0 divide by I 1, which is proportional to p 0 divide by p 1. So, that is equal to r ex. So, we have this quantity r ex divided by sigma 1 plus sigma 0. If you substitute in terms of the average power, then this thing can be rewritten. So, we can get a quantity 1 minus r ex divided by 1 plus r ex; that corresponds to this mean power into 2 times p rec into responsivity divided by sigma 1 plus sigma 0. Now, if you assume that the receiver is thermal noise dominated, then essentially this quantity sigma 1 and sigma 0 there will be approximately equal to sigma T.

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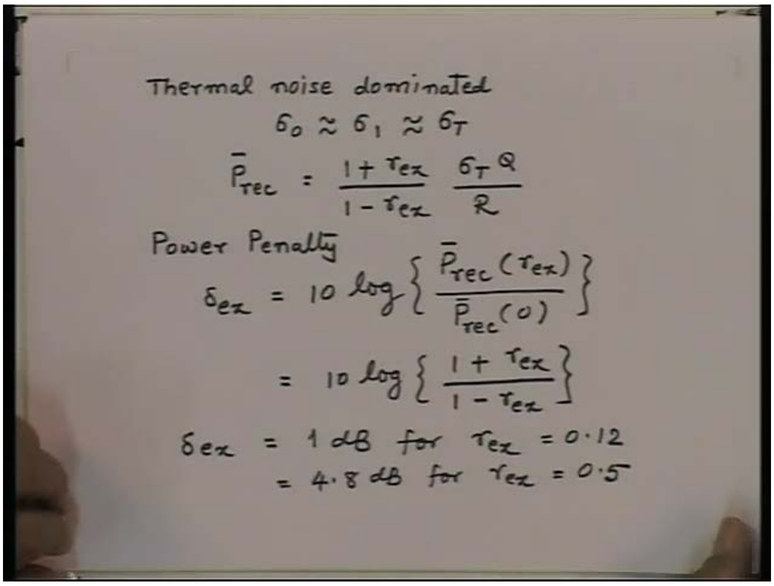

So, under the assumption that my receiver is thermal noise dominated. We can get sigma 0 approximately equal to sigma 1 approximately equal to sigma T and then in this situation, we can get the average power required. This we can get essentially by inverting (Refer Slide Time: 05:23) this expression. So, we can get average power received which is 1 plus r ex divided by 1 minus r ex sigma T Q divided by the responsivity. Now, we say that if the extinction ratio was 0 that is ideal receiver, then we would require the average power in the data which would corresponding to r ex equal to 0. So, that quantity will be simply sigma T Q divided by R; whereas, now when the extinction ratio is finite, you will require little higher power p rec compared to ideal situation.

So, the ratio of p rec for a given value of r ex compared to when r ex was 0; that we can call as the power penalty. So, we can define the power penalty. Let us say it is denoted by delta ex that is in terms of dB's will be 10 log of p rec for given value of extinction ratio divided by p rec for 0 value of extinction ratio. So, this quantity is a function of r ex that is what we are writing here. So, substituting now for this quantity, the p rec 0 will be nothing but this. So, we get the power penalty in terms of dB's that will be 10 log of 1 plus r ex divided by 1 minus r ex. So, one can essentially plot this function of the power penalty as a function of the extinction ratio.

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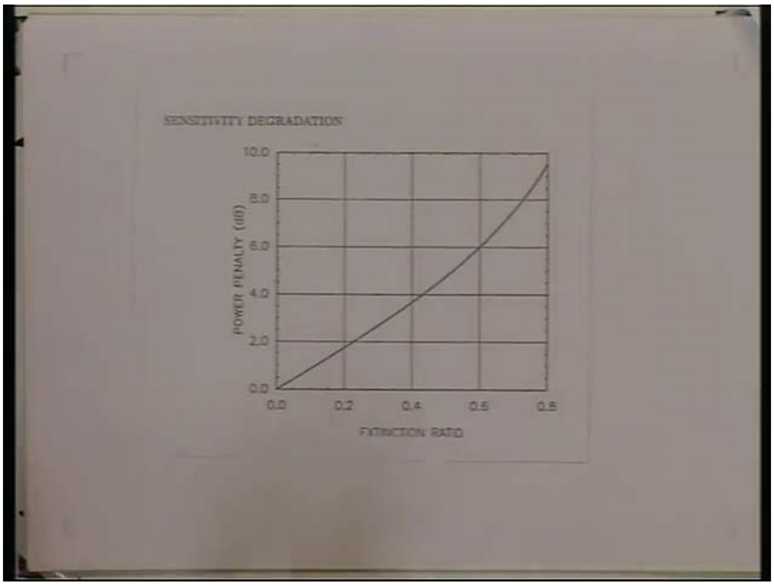

Say, if we plot this, typically the the plot will look like that. Here essentially we have plotted the extinction ratio starting from 0; that is the the ideal receiver to going up to 0.8 and this is the corresponding power penalty in dB. So, we can see here that for an extinction ratio up about 0.2, we have a power penalty typically of the order of about 2 dB. And if you have extinction ratio of 0.4, there is the power penalty about 4 dB and so on. So, essentially if you say that for a receiver the 1 dB power penalty is probably acceptable, then we can get corresponding to 1 dB extinction ratio that will be approximately 0.12.

So, from this plot, we can get (Refer Slide Time: 11:39) the numbers that this quantity delta extinction that is equal to 1 dB for r ex of 0.12 and this will be as high as 4.8 dB for r ex is equal to 0.5. Typically, the extinction ratio would lie in the range of this. So, you will have a power penalty corresponding to the extinction ratio, which would be of the order of about a dB in the in the system. So, this is one of the power penalties, when we design a practical receiver.

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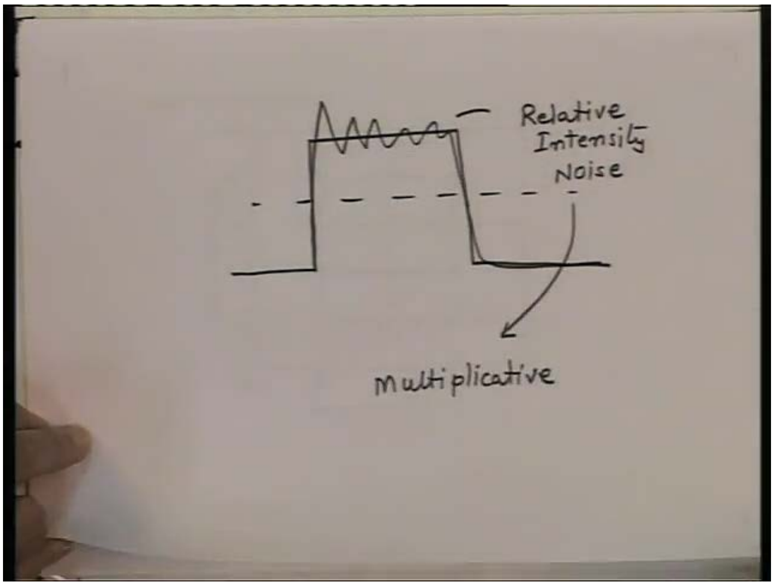

The second power penalty which we have is the power penalty due to relative intensity noise. And that is, if you recall that if the pulse has to be transmitted like this because of the relaxation oscillations and all other noise present in the laser, the signal does not go like that; the signal essentially goes something like this. So, we are having some fluctuations in the optical intensity corresponding to the 1 level. This is the fluctuation which is called the relative intensity noise. And one can see that this effect essentially the multiplicative effect because this fluctuation which we are going to get into this amplitude that will be proportional to the amplitude of the pulse.

So, this noise which is the relative intensity noise; this noise is the multiplicative noise; because this noise is proportional to the amplitude of the pulse. So, what we do is we define now the variance for the relative intensity noise and then this treat this noise in addition to the noise which is present because of the shot noise and the thermal noise.

And then we can ask a question because of the additional noise which is there because of the relative intensity fluctuations, what will be the extra power added to the system to get the same BER.

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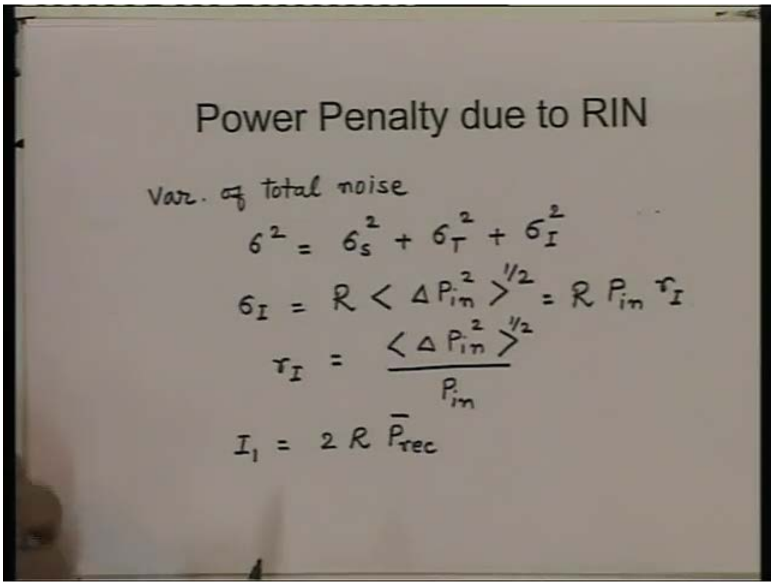

So, we define now the variance of the noise which is sigma square. So, let us say this is the variance of the total noise sigma square that will be equal to sigma shot noise square plus sigma thermal square plus the additional noise which is the relative intensity noise, which we denote by sigma I that square; where sigma I we are denoting by the responsivity and the average value of the change in the input power. So, this quantity is the root mean square value of the input optical power multiplied by the responsivity; that is this quantity, which is the sigma of the relative intensity noise.

So, this one we can say that this is responsivity multiplied by your p in into a parameter what is called the parameter for the relative intensity, where this r I is defined as delta p in square to the power half divided by p in. Then what we do is essentially find out now the I 1 and I 0. And now since we are considering only the relative intensity noise, we assume that the extinction ratio is 0; that mean there is no power which is coming corresponding to 0 level and in this situation, then we can find out what is the average value. So, in this case in the extinction ratio is 0, corresponding to 1 level we have the current I 1; that will be 2 times R into average value of the power which is average p received.

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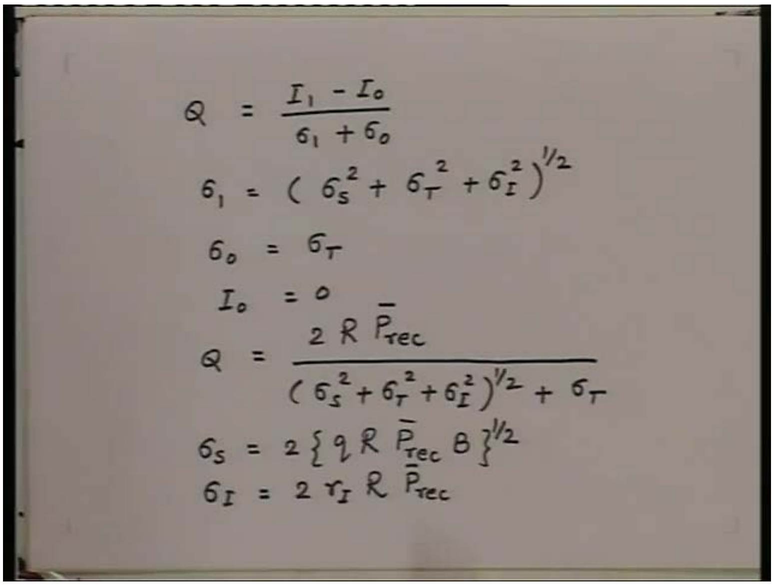

Then the Q factor which we have for us this case will be Q which will be equal to I 1 minus I 0 divided by sigma 1 plus sigma 0. Now, when 1 level is transmitted, we have all the noises present. (Refer Slide Time: 19:11) We have shot noise present; we have relative intensity noise present and we also have thermal noise present. Though may be thermal noise may be negligible compared to this, but for sake of completeness we can say that when 1 level is transmitted, all three noises are going to be present. Whereas, when 0 level is transmitted since the extinction ratio is 0, we do not have shot noise and we do not have relative intensity noise.

So, we have sigma 0 which will be only sigma T. So, we have now a situation where sigma 1 will be square root of sigma S square plus sigma T square plus sigma I square and sigma 0 will be only sigma T. And also since we are assuming that the extinction ratio is 0, I 0 is 0. So, we have this quantity is also 0. So, if we substitute now these parameters inside this, then you get this parameter Q; that will be 2 times R p average divided by quantity which is sigma S square plus sigma T square plus sigma I square to the power half plus sigma 0 which is sigma T; so this equal to sigma T.

And now if we write this value of sigma S, this is the shot noise. So, sigma S is equal to 2 times q responsivity average power to bandwidth to the power half. And we also have sigma I as we defined that is equal to 2 times r I is the relative intensity parameter responsivity and p average. So, we can take this thing and substitute into this and now note here that we have p rec which is here and also these quantities are now depending upon p rec. So, if we invert this thing and solve for p rec, we get the received power.

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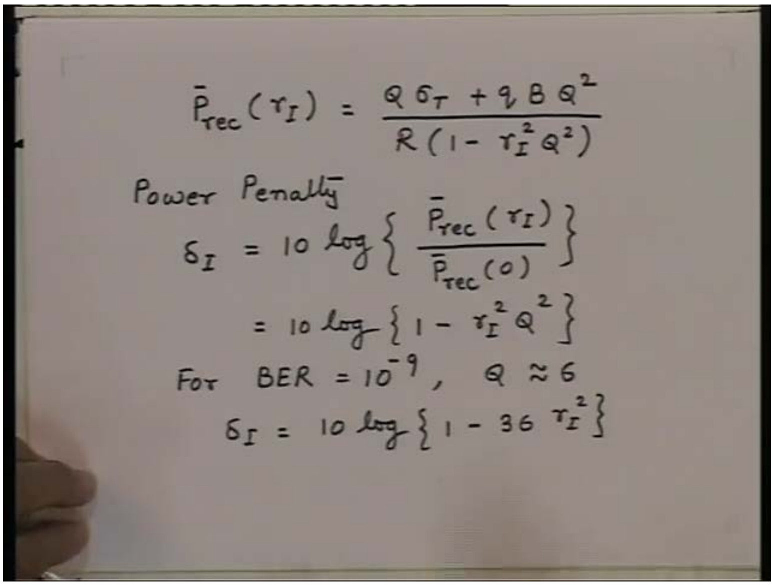

As the function of the parameter q that will be p rec average, as a function of the RIN intensity noise parameter r I that will be equal to Q into sigma T plus q B Q square divided by the responsivity into 1 minus r I square into Q square. Again from this expression if we substitute r I equal to 0, then we get the required received power in the ideal situation; that means when the relative intensity noise is not present and in the presence of this r I, this quantity will be non zero and as a result, we will require little more power.

So, we can define the power penalty in this case, which is the ratio of p received r I divided by p received when r I is equal to 0 and again in terms of dB's, if you want, then we can define 10 log of that. So, we have in this case the power penalty. Let us call it delta I that is equal to 10 log of p rec bar for r I divided by p rec for r I equal to 0. So, again this term is same. So, when I take a ratio, essentially this is the term which will cancel. So, we get the power penalty which will be 10 log of 1 minus r I square into Q square. Now, for the acceptable level for the BER, as we know this quantity Q has to be atleast 6.

Say, if we see that we are working with bit error rate of 10 to the power minus 9, this quantity Q will be about 6. So, for BER of 10 to the power minus 9, the Q is approximately 6. Say, if we substitute into this, you get delta I that will be 10 log of 1 minus about 36 into r I square. We can plot this function again as a function of this parameter relative intensity noise and the plot would look something like this. (Refer Slide Time: 14:58) So, we have got here texting the (( )) intensity noise plotted and here we are having power penalty in terms of dB's. So, we see that as a relative intensity noise increases, the power penalty increases very rapidly or in other words, the system degrades very rapidly.

Now, note here the noise which you are indicating here is essentially indicating a value of sigma of the relative intensity noise. The peak to peak deviation of the noise is about 5 to 6 times sigma; that means one sided deviation if you take on the data, then you are having (Refer Slide Time: 17:18) approximated two and half time sigma deviation of signal cum corresponding to mean level on one side. So, when sigma I is 0.1, essentially we have a peak to peak noise about 0.3 of this amplitude. If you make the sigma which is about 0.2, that time we will have the peak variation which is about 0.6.

And then you will see that there are large number of values, which will be lying below the threshold value which is half. And that is the reason as the sigma half for a relative intensity noise increases, very rapidly the system degrades and that is what essentially shown (Refer Slide Time: 14:58) by this figure. That by the time you go to a relative intensity noise of about 0.15, the power penalty is as high as about 8 dB and beyond that, the power penalty essentially becomes unmanageable. So, power penalty due to relative intensity noise is the very sharp function and that is the reason, we have to keep the relative intensity noise as small as possible right inside the data.

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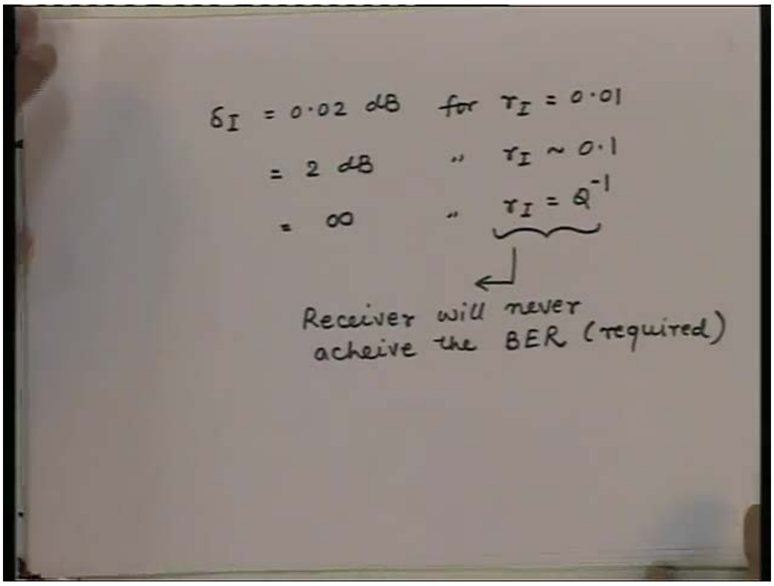

So, just to give few numbers if we write for this, we have say delta I that is equal to 0.02 dB for r I is 1 percent 0.01; that is equal to about 2 dB for r I about 10 percent or 0.1 and r I is equal to infinity for r I is equal to Q to the power minus 1. As you see here (Refer Slide Time: 25:30) when this quantity r I is Q 1 upon Q; that time this quantity will be 0 and then this will be infinity. So, that means if we are having the relative intensity which is 1 upon Q, then the power penalty is very high; that means your receiver can never perform satisfactorily.

Because now the power penalty in fine infinity; that means to get the bit error rate of 10 to the power minus 9, we have to essentially increase the power by infinity dB's. On in other words, if the r I becomes comparable to 1 upon Q, then we will never be able to achieve the bit error rate what we are desiring; which is 10 to the power minus 9; that is what essentially is shown in this. (Refer Slide Time: 14:58) By this plot, it is very steeply increases has a relative intensity noise increases. So, we say that for this value receiver will never achieve the BER required.

So, that is the reason when we talk about the noises, especially this going to come from the laser. So, this noise has to be kept low. So, that the power penalty is again within the specified limit. But in other words, if you say that may be about a dB of kind of power penalties, then the relative intensity noise should remain of only few percent levels. Otherwise, the power penalty will be excessively large. The third power penalty we are seeing now is what is called the power penalty due to the timing jitter. Firstly, one can ask a question why the timing jitter gives a noise, which will affect the bit error rate.

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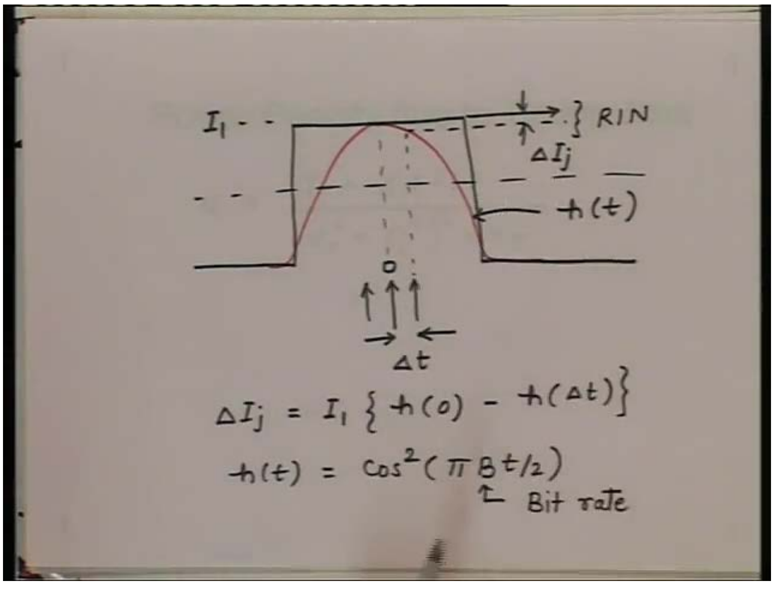

One can again go back to the data and one say that if we had a pulse, which is like this. After all when the pulse is detected, we have a sampling time at which essentially we ask whether signal is below threshold or above threshold and depending upon the decision, we assign the level either 0 or 1. So, if you are having a pulse which looks like that, even if you having a small jitter in the clock; that means if this pattern warbles little bit, it have not make any difference. Because you are going to always sample this amplitude. However what happens in practice because of the finite bandwidth of the system, we have now a pulse which is rather distorted. Also to reduce the inter symbol interference; we may not transmit a pulse which is rectangular in shape.

We may have some other form of the pulse. So, let us say the pulse which is transmitted or which has become after distortion look something like this something like this. And then over and above, the noise is superimposed on this. So, now if you are having now a timing jitter; that means on this function, the sampling point essentially is having a random fluctuation; that means sometime we sample here, sometime we may sample here, sometime we may sample here and so on. Now, since this function is not constant over this time, when I am sampling when I sample here, we get an amplitude, which is this. Whereas, if we sample here; that means if my clock had drifted little bit, then we sample the amplitude which is this.

What that means is now that we are having now a random fluctuation in the intensity or the amplitude of the signal because of the jitter in the clock or jitter in the sampling time. So, the sampling time jitter essentially gets converted into the corresponding amplitude fluctuations. So, basically what we are seeing here that the statistical fluctuation which we are having in the sampling time or in the clock, they essentially get converted into equivalent relative intensity noise, because here also how much fluctuation you are going to get will depend upon the amplitude of the pulse. So, we have got here equivalent some kind of relative intensity noise; that is because of the fluctuation in the sampling time. So, now we can define the the parameter.

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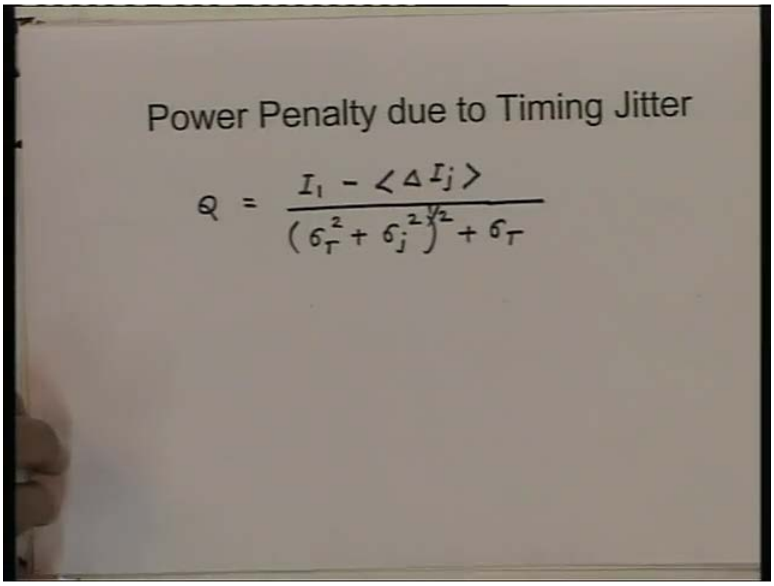

$$Q = \frac{I_1 - \langle \Delta I_j \rangle}{(\sigma_T^2 + \sigma_j^2)^{1/2} + \sigma_T}$$

Assuming that again the current is 0 corresponding to 0 level and thermal noise is also small, there is no thermal noise and all that. We can define this parameter now Q, which will be I 1 minus this quantity which we have got delta I to to jitter value divided by sigma T square plus sigma j square; where sigma j is the noise because of the jitter are converted into corresponding to relative intensity noise 1 by 2 plus sigma T. So, here we are having essentially this quantity sigma j is the rms value of the fluctuation of the current due to the sampling time fluctuation. Now, obviously what fluctuation we are going to get in this (Refer Slide Time: 35:30) and its statistical variation will depend upon the shape of this pulse. So, if we have a pulse shape which is rectangular, then obviously there is no fluctuation in this.

So, we do not have any jitter noise; whereas, if you have a function which looks like that, then we will have a corresponding jitter noise. So, essentially what we are saying is because of this fluctuation here, let us say this function is given by a function plus say which is h of t. And let us say fluctuation which we have in sampling is given by delta t and let us say, this is the peak is the time which corresponds to 0. So, I measure at the peak. This is the best sampling time and because of the jitter, I can sample on other side and time deviation could be delta t. So, then the amplitude change which we are going to get here will be essentially delta I j; that will be I 1 which corresponds to this level into h of 0 that corresponds to t equal to 0 minus h of delta t.

So, this is the function now; h of t that is the one which essentially is going to give me now the change in the amplitude. So, this quantity which we got here that quantity is nothing but delta I jitter. Now, we can choose different pulse shapes. So, either the pulse shape could be cosine square. But generally for the reduction of the inter symbol interference, we use this pulse shape. So, let us say I use this function which is the cosine square function. So, we get h of t is cos square of pi B t by 2, where this quantity is the bit rate. So, the pulse is no more rectangular pulse now. The pulse is having a shape which is a cosine (( )) shape and the corresponding transfer function you write which reduces essentially the inter symbol interference; we get the raised cosine filter for this pulse shape.

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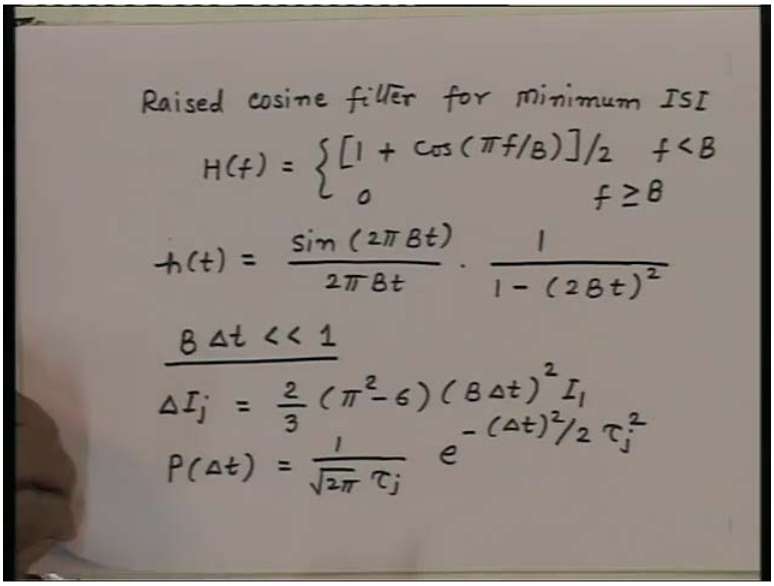

So, we get now raised cosine filter for minimum inter symbol interference, ISI. If you do that, then the transfer function for this receiver would be given as 1 plus cosine of pi f by B by 2 for frequency is less than B and equal to 0 for frequencies greater than all equal to B. So, this is the filter which is normally used in the receiver to reduce the inter symbol interference and for this, then we can get this function what is called the impulse response h(t) which is given by sin of 2 pi B t divided by 2 pi B t into 1 upon 1 minus 2 B t square. So, assuming that now this quantity here B into delta t, which is the time fluctuation is small compare to 1. What essentially we are saying is compared to the bit duration (Refer Slide Time: 35:30) compared to the bit duration here if the time fluctuation is small, so delta t is much smaller compared to this bit duration.

That essentially is given by when B delta t is much much less than 1 for this condition. We can get the change in the current for this pulse shape; that will be equal to 2 upon 3 pi square minus 6 into B delta t whole square into the mean current level I 1. So, you have this quantity here delta I j; that is what we are time to find out for this function and I 1 is the current corresponding to 1 level. So, that is now the fluctuation which we have. Assuming that the distribution of this fluctuation is Gaussian, then we can calculate the density function as the function of delta t; that will be even 1 upon root of 2 pi. Let us say tau j is the standard deviation of the fluctuation of the sampling time e to the power minus delta t square divided by 2 tau j square. And we can find out the corresponding now the density functions for the current for this.

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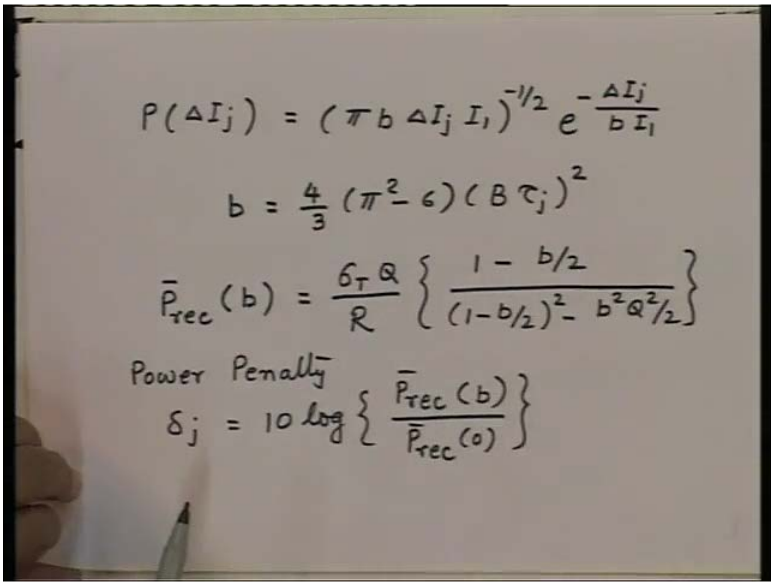

So, we essentially get the density function for delta I j; that will be equal to pi some parameter which we define b delta I j I 1 to the power minus half e to the power minus delta I j divided by b I 1; where this parameter b is 4 by 3 into pi square minus 6 into B tau j square. So, once I have this density function now for the current, we can now go back and substitution into the noise expression. We can find out what is the relative intensity noise produced because of the timing jitter. So, if we do that, then we can get now the received power p for a given Q and now this will depend upon this quantity here which depends upon this tau j, which is the standard deviation of the timing fluctuation or jitter in the clock.

So, this is now going to be a function of this parameter b that is equal to your sigma T into Q divided by responsivity R into 1 minus b by 2 divided by 1 minus b by 2 whole square minus b square Q square by 2. So, again by substituting this quantity b equal to 0, we get the power required in the ideal situation. So, we essentially have now the p rec corresponding to be equal to 0; that is the ideal condition and now because of the finite value of b which is related to the variance of the timing jitter, we get the corresponding power penalty. So, we can define now the power penalty delta j; that is 10 log of again the same thing which is p received for b divided by p received, when b equal to 0. So, essentially this is the quantity. If you take 10 log of that, that will give me the corresponding power penalty.

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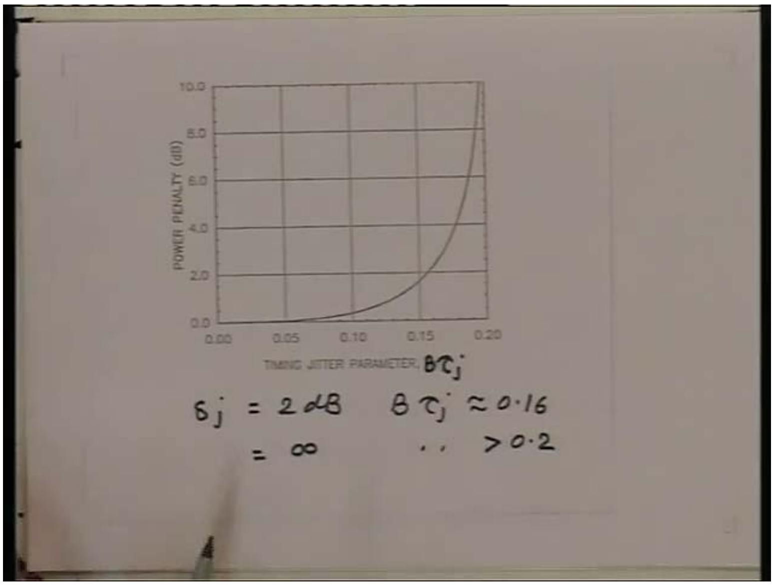

So, if you plot this function as the function of the timing jitter now or this product B into tau j. So, we have parameter here B into tau j and we have a corresponding power penalty, which is plotted here. Again we see here the power penalty increases very rapidly, and by the time we have this product about 0.2, the power penalty practically becomes infinity. And that we can see from essentially from the eye diagram that if the sigma of these quantities about 0.2 that means the clock peak to peak (( )) will be over the bit duration and that essentially the pulse will not be sample properly. So, we can see from here that for delta j of about 2 dB. This parameter B into tau j that should be approximately 0.16, and as we saw this will become infinity, when this parameter is greater than 0.2.

So, essentially what we are saying is timing jitter should be small and a value of about few percent timing jitter may be acceptable to give you a power penalty of between 1 and 2 dB. So, what we have seen in this lecture essentially is that because of the non-ideal situation of the optical receiver like you have a finite extinction ratio; you have the relative intensity noise, because of the laser fluctuations and you have a timing jitter, you have to essentially add extra power into your system to achieve the same bit error performance. So, we are seen here three major parameters or major three major effects, which contribute to the power penalty. So, total power penalty will be sum of the power penalties of this and that is the reason when we design an optical link, we have to take this thing into consideration, and essentially supply more power to the system to overcome these power penalties, so that we can get the satisfactory receiver performance.