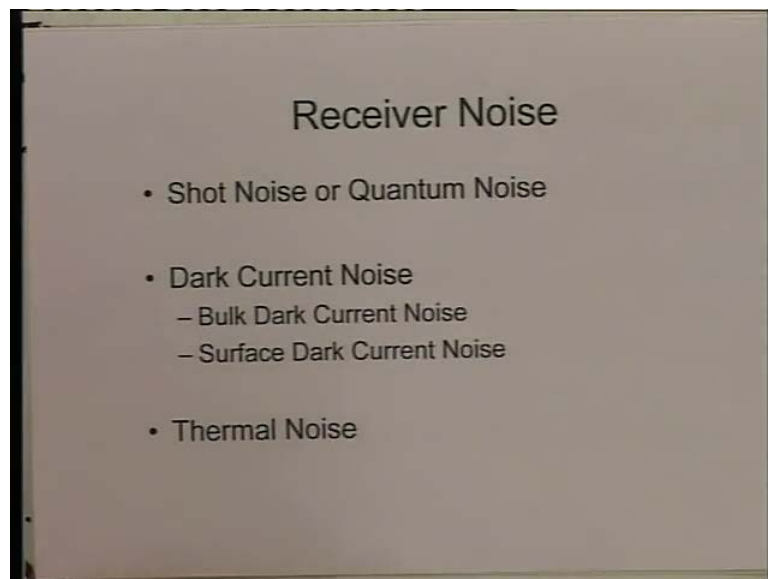


Advanced Optical Communications
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Lecture No. # 22
Optical Receivers - I

In the last lecture, we investigated the characteristics of a photo detector. Now, we see if the photo detector is used in optical receivers, what are the performance parameters of an optical receiver and in this case, essentially we are going to discuss the digital data that means, if the information is sent in the form of bits, how do we measure the performance of an optical receiver? So, in this lecture, essentially we are going to discuss the performance of the optical receivers.

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We have seen in the previous lecture that in a photo detector or in general, in an optical receiver, there are various types of noises which are present. So, we have a shot noise, which is because of the statistical nature of the photons and the statistical nature of the interaction of the photon with the matters. This we call also as the quantum noise. We saw that this noise has a poisson distribution, and this noise is multiplicative in nature. We also saw that there is a noise now, which is what is called the dark current noise that

in the presence of signal, we have this noise. But when the signal is not present, then there is some ambient light falling on the photo detector, and that gives some fluctuations in the photo current, which we call as the dark current noise.

So, dark current noise is divided into two categories, what is called the bulk dark current noise which is essentially inside the device; whereas, we have the current which is flowing almost on the surface of the device, which we call as the surface current. And then we have fluctuation in that current, which we call as the surface dark current noise. And then we saw that because of the resistances used in the electronic circuitry, we have the so called thermal noise, and this thermal noise is additive in nature, and also its distribution is Gaussian. So, in any optical signal, essentially we have combination of all three noises present and depending upon the optical intensity, either this noise will dominate or this noise will dominate and so on. So, last time we wrote down the expressions for these noises.

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Noise in photo Detector

1. Quantum or shot Noise

$$\langle i_Q^2 \rangle = 2q I_p B \overbrace{M^2 F(M)}^{\text{Avalanche detector}}$$

↑
Bandwidth
2. Dark current Noise
 Bulk: $\langle i_{DB}^2 \rangle = 2q I_B M^2 F(M) B$
 Surface: $\langle i_{DS}^2 \rangle = 2q I_S B$
- 3 Thermal Noise:

$$\langle i_T^2 \rangle = 4KT B / R_L$$

So, we have seen that the quantum noise or the shot noise has the variance or mean square value, which is proportional to the mean photo current and it is proportional to the bandwidth. Also if we use the photo detector which is avalanche detector, then there is internal gain which is given by this factor M and then we have a noise figure; because again the avalanche process also is a statistical process. Similarly, we had the expression for the dark current noise. So, this is the expression for the bulk dark current noise. We

see the amplification because of the avalanche process and the surface current does not see amplification because of the avalanche process. And then we have the thermal noise, which has a variance equal to 4 times the Boltzmann constant and the thermal temperature of the receiver, the bandwidth of the receiver and the load resistance and then we said that assuming that these noises are independent of each other.

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Signal power = $\langle i_p^2 \rangle M^2$

Signal to Noise Ratio

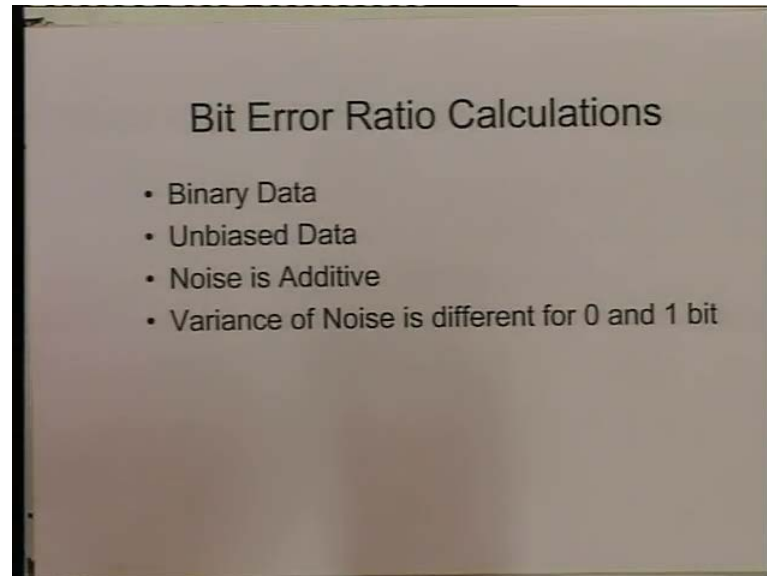
$$SNR = \frac{\langle i_p^2 \rangle M^2}{\langle i_q^2 \rangle + \langle i_{DB}^2 \rangle + \langle i_{DS}^2 \rangle + \langle i_T^2 \rangle}$$

So, total variance will be sum of their variances. So, we essentially got the noise power which is the quantum noise plus the dark current bulk noise plus the dark current surface noise plus the thermal noise. And then we got a signal power, which is proportional to i_p square and if you are using the avalanche photo detector, then this quantity will be having some value M . If you are using simple $p-i-n$ detector, then M is equal to 1 and then there is no internal amplification in this process. So, essentially now we are going to make use of this quantity to find out the system performance, which is what is called the bit error ratio.

I mentioned last time that if you are considering the analog communication system, then signal to noise ratio is the parameter which is of importance; whereas, if you are using the data which is digital in nature, then more useful quantity is what is called the bit error ratio; that means is the ratio of number of bits wrongly detected to the total number of bit transmitted or received by the receiver. So, today essentially we are going to do the

analysis of the bit error ratio. So, we want to now calculate what is called the bit error ratio and for that, we can make certain justifiable assumptions about the data.

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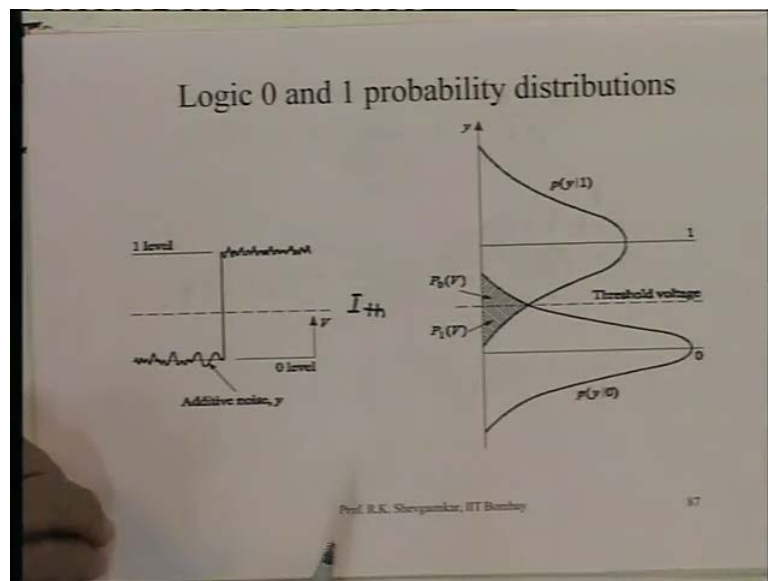
So, firstly we are going to do the analysis of this for the data, which is binary in nature; that means we are sending the information in the form of only two levels, 0 and 1. And let us say, 0 means no optical power and 1 means some optical power. While discussing the optical fibers, we have discussed that if you consider the normal semiconductor lasers, the spectral width of this laser is very large. And because of that, sophisticated communication modulation techniques cannot be employed; because the spectrum of the signal is completely washed out. So, for normal optical communication system, essentially we use the amplitude modulation; because that is the modulation, which can be recovered in time domain.

It does not require information about the spectral domain and that modulation if we convert in to corresponding digital form that is the one which we call as the amplitude shift keying; that means now the data 0 and 1 is transmitted in the form of optical pulses. So, if 0 bit is transmitted, there is no optical pulse; if 1 bit is transmitted, there is an optical pulse. So, we are assuming now the data is binary. So, data is send only either presence of a optical pulse or absence of a optical pulse. Also we are assuming that the data is unbiased; that means there is no preference given to 1 level or 0 level. There is

equal possibility of getting 0 bit or 1 bit. Also what you are assuming here is that the noise is additive in nature and it is Gaussian in nature.

So, recall when we talk about the shot noise, we said that actually the shot noise is poisson in nature; also it is multiplicative in nature. However to do the analysis simpler, we take the appropriate variance value for the noise in two levels 0 and 1. But beyond that point, we assume that this noise is distributed with Gaussian distribution and then we can do the analysis for the bit error ratio. So, to account for the shot noise appropriately into the analysis, essentially what we are doing now is we are saying that the noise is different for the 0 level and for the 1 level. So, appropriately we can provide these variances for the two levels for the data. If we do that, then essentially we have now the bits which are going in two levels.

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So, we have a 0 level here and we have 1 level here and the noise is superimposed on this. So, these are the mean levels which are denoted level 1 and level 0 and this is the photo current. And photo current is fluctuating around this mean value corresponding to 0 level and around this mean value, which corresponds to 1 level. And in general, as we said we are assuming that these quantities are variances of these two are not equal and that is what is shown here, if you look at this distribution here. This distribution looks narrower compared to this distribution. And there is for the simple fact that in this case,

we will have the shot noise present; whereas, in this case we will have only noise which is dark current noise or the thermal noise.

So, what is plotted here is the probability of the current lying around this mean position which is 0. So, the current will fluctuate around this value which is this and there is the density function for that current for the 0. So, that is the density function which we say is $p(y|0)$ corresponding to 0 level. Similarly, we can get here $p(y|1)$ corresponding to level 1, where y is the quantity which is representing current or corresponding voltage, if you are measuring the voltage across the load resistance. So, then we can write here the probability of the current for level 1 and level 0.

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$$p(I/0) = \frac{1}{\sqrt{2\pi} \sigma_0} e^{-\frac{(I - I_0)^2}{2\sigma_0^2}}$$

$$p(I/1) = \frac{1}{\sqrt{2\pi} \sigma_1} e^{-\frac{(I - I_1)^2}{2\sigma_1^2}}$$

So, we say that the density function; so p of let us say photocurrent I given 0. So, that is the density function for the 0 level; that is equal to $\frac{1}{\sqrt{2\pi} \sigma_0} e^{-\frac{(I - I_0)^2}{2\sigma_0^2}}$; where σ_0 is the standard deviation of this Gaussian noise or σ_0^2 is the variance of this Gaussian noise corresponding to 0 level. And I_0 is the mean value of the current which can be 0. If we assume that, (Refer Slide Time: 09:17) no light was transmitted in the corresponding to 0 bit. But due to some factors, there may be certain low value of light which will be transmitted even during the 0 bit.

So, in general let us leave it that this level is not 0; but this level is given by some I_0 . Similarly, we have a probability of the current given 1 level and that will be equal to 1

upon square root $2\pi\sigma_1 e^{-\frac{(I-I_1)^2}{2\sigma_1^2}}$, where σ_1 is the standard deviation of the noise for 1 level. Say you seen here if the (()) shot noise is present, then you will have a value of the standard deviation larger for this compared to the 0 level. So, σ_1^2 and σ_0^2 are the total variances of noise corresponding to 1 and 0 level. So, now you can say that (Refer Slide Time: 09:17) if the one 0 level is transmitted and seen the data is binary.

Essentially, what we do? We have a threshold level for decision. If the signal lies above the threshold value, then we detect the bit as 1. If the signal lies below the threshold value, then we detect the level at 0. So, we have this level here which we call as some I threshold and this compared at the instant, when the bit is expected. If the current is greater than this value which is I threshold, then we assign the bit as 1. If the current is less than I threshold, then we assign the bit as 0. Now, with this density function which is given here; that means, we can now calculate what is the probability that 0 level was transmitted.

But the signal would have excited this value, which is I threshold and that probability essentially is given by this area under this. Similarly, if 1 was transmitted and if the signal happens to lie below the threshold value, then will take 1 as 0 and then there will a bit error. So, for 0 level transmission, if the signal exceeds I threshold, there is bit error; whereas for 1 level transmission, if signal lies below I threshold, then there is a bit error. Now, we can say that the total bit error probability is the probability of bit error for 1 level plus the probability of bit error for 0 level and ofcourse 0 then 1, the appearances also will have their own probabilities.

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The image shows a whiteboard with handwritten mathematical equations for the Bit Error Ratio (BER). The title is "Bit Error Ratio". The main equation is $BER = P(0/1)P(1) + P(1/0)P(0)$. Below it, it states $P(1) = P(0) = 1/2$. Then, $BER = \frac{1}{2} \{ P(0/1) + P(1/0) \}$. The next two equations are $P(0/1) = \frac{1}{\sqrt{2\pi}\sigma_1} \int_{-\infty}^{I_{th}} e^{-\frac{(I-I_1)^2}{2\sigma_1^2}} dI$ and $P(1/0) = \frac{1}{\sqrt{2\pi}\sigma_0} \int_{I_{th}}^{\infty} e^{-\frac{(I-I_0)^2}{2\sigma_0^2}} dI$.

So, we can write what is called the bit error ratio, which is nothing but the probability of any bit detected wrongly. So, we can write BER is equal to the error probability of detecting 0, when 1 was transmitted. So, that is probability of getting signal 0, when 1 was transmitted multiplied by the probability of transmission of 1. So, which is probability of 1 plus the probability of detecting 1, when 0 is transmitted multiplied by the probability of transmission of 0. Now, note here we were assume that the data is unbiased; that means, there is a probability of 1 level and 0 level transmission equal. So, we have here p of 1 is equal to p of 0 is equal to 1 by 2; that means in a data stream which we are receiving 0 and 1 levels are equi probable.

Therefore, we can write here the BER that is equal to 1 by 2 into this probability. So, that means probability of detecting 0 when actually 1 was transmitted plus probability of detecting 1 when actually 0 was transmitted. And as we have seen probability of this, when 1 was transmitted detecting 0 (Refer Slide Time: 09:17) is essentially given by this area. So, if I take this density function corresponding to 1 and if I integrate for the currents manage infinity up to this point I threshold; that is the area which gives me the probability that the current will lie below threshold value and therefore, 1 will be detected as 0.

So, we can write that then that this probability of detecting 0, when actually 1 is transmitted; that is $\frac{1}{\sqrt{2\pi}\sigma_1} \int_{-\infty}^{I_{th}} e^{-\frac{(I-I_1)^2}{2\sigma_1^2}} dI$

power minus I_1 minus I_1 whole square divided by $2\sigma_1$ square in to d I. Similarly, the probability of this which is probability of detecting 1, when actually 0 was transmitted (Refer Slide Time: 09:17) that will be given by this area. So, if I take a density function corresponding to 0 level and if I integrate this from the I threshold up to infinity, this area is the error probability which will be p_{10} .

So, we can write p_{10} that is equal to $\frac{1}{\sigma_0 \sqrt{2\pi}} \int_{I_{th}}^{\infty} e^{-\frac{(I-I_0)^2}{2\sigma_0^2}} dI$. So, once I get these two probabilities, there I can substitute into this expression and then we got the bit error probability or bit error ratio. Now, these integrals cannot be solved in the close form. So, essentially they are represented by functions what are called the error functions or complementary error functions. So, the complementary error function is defined as follows.

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Complementary Error Function

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy$$

$$P(0/1) = \frac{1}{2} \text{erfc}\left(\frac{I_1 - I_{th}}{\sigma_1 \sqrt{2}}\right)$$

$$P(1/0) = \frac{1}{2} \text{erfc}\left(\frac{I_{th} - I_0}{\sigma_0 \sqrt{2}}\right)$$

$$\text{BER} = \frac{1}{4} \left\{ \text{erfc}\left(\frac{I_1 - I_{th}}{\sigma_1 \sqrt{2}}\right) + \text{erfc}\left(\frac{I_{th} - I_0}{\sigma_0 \sqrt{2}}\right) \right\}$$

(No audio from 21:40 to 21:56) erfc as a function of x that is defined as $\frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy$. So, taking this definition for the complementary error function, these integrals now can be written as error functions. So, we get the probability of getting 0, when actually 1 is transmitted; that is nothing but half **half** error function I_1 minus I_{th} divided by $\sigma_1 \sqrt{2}$. Similarly, this quantity here p_{10} , we can write as half complementary error function of I_{th} minus I_0 divided by $\sigma_0 \sqrt{2}$. So, infact there are standard complementary

error function tables are available. So, if we know this quantity the threshold current and the level I_1 and I_0 , then we can find out this quantity here.

Then we can go to the tables for the complementary error function and from there, then we can find out this probability of getting zero or getting 1. And then we can find out the total probability of error in the data, which we call as the bit error rate. So, combining these two and substituting into this expression for the BER, we get now the BER for the data which is equal to $\frac{1}{4} \text{erfc} \left(\frac{I_1 - I_{\text{threshold}}}{\sigma \sqrt{2}} \right) + \text{erfc} \left(\frac{I_{\text{threshold}} - I_0}{\sigma \sqrt{2}} \right)$. So, this is the expression for the bit error ratio. Now, as one can see here the bit error ratio now is going to depend upon the threshold current and which make sense. Because (Refer Slide Time: 09:17) if you look at these plots here, if the threshold value is brought down, then there will be more a errors in detection of 0.

Because now there will be more probability of signal crossing the threshold level. So, more 0's will be detected wrongly (()) there will be lesser 1's detected wrongly; so more 1's will be detected correctly. Similarly, if I bring the threshold value higher, then more 1's will go wrong and more 0 will be detected correctly. So, there has to be some optimum value for which essentially we get the BER minimized. So, now we have questioned that if we are having the bit error rate given by this quantity which is the function of $I_{\text{threshold}}$, then what is this value of optimum value for $I_{\text{threshold}}$? For which, this bit error ratio would go minimum. One can show that this bit error ratio was minimum, when we have this condition satisfied.

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BER minimum when

$$\frac{I_1 - I_{th}}{\sigma_1} = \frac{I_{th} - I_0}{\sigma_0}$$

Optimum threshold current

$$I_{th} = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$$
$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

So, we get BER minimum, when we have $I_1 - I_{th}$ divided by σ_1 is equal to $I_{th} - I_0$ divided by σ_0 . So, essentially what we are saying is that (Refer Slide Time: 09:17) when these two areas this area which gives me the error probability of 0 bit and this area which gives me the error probability of 1 bit. When these two areas become equal; that means, when the bit error probability for 0 and 1 become equal, that is a time we will get the minimum bit error rate. And that is what essentially given by this condition here that when these two become equal, the two areas become equal and that is what gives you the minimum bit error rate.

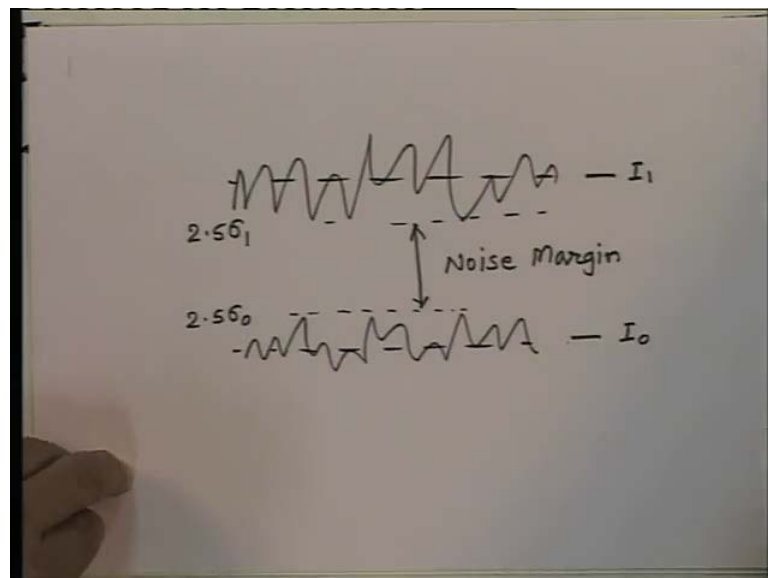
So, this is the optimum value of the threshold current. So, we can now say that then the optimum threshold current I_{th} that by solving this, we get essentially as $\sigma_0 I_1 + \sigma_1 I_0$ divided by $\sigma_0 + \sigma_1$. One can verify that if we have a situation that the noise is equal on the 0 bit and the 1 bit; that means when σ_0 is equal to σ_1 and that is what will happen if we consider the noise which is only thermal noise, then the σ_0 equal to σ_1 . So, I_{th} essentially will become $I_1 + I_0$ divided by 2. So, for a binary data (Refer Slide Time: 09:17) for which the noise is equal on both the levels, that time the threshold level has to be half way between these two levels.

So, it is $I_1 + I_0$ divided by 2; whereas, when the σ_1 and σ_0 are not equal, that time we have to adjust the threshold appropriately to minimize the bit error ratio.

And in the optical communication, it is more appropriate to treat the sigma 0's and sigma 1 different. Because there may be a possibility that in this level, you may have a shot noise which may be substantially larger compared to the noise which we you get here because of the essentially thermal origin. So, in general essentially then we are having the optimum value of threshold current, which essentially is given by this. Now, one can consider the case when sigma 1 is much **much** greater than sigma 0, there could be one extreme condition or other condition could be sigma 0 could be equal to sigma 1, which we call as the thermal noise condition.

So, firstly for optimum value of this thing if I substitute this I th into this now, we can get this quantity which we call as the Q parameter of the data. So, we define a quantity Q which is this quantity essentially. Say if I substitute this optimum value in this, these two will become equal **right** and that Q quantity will be given by $I_1 - I_0$ divided by $\sigma_1 + \sigma_0$. So, now we are saying this is the quantity Q, which is the parameter which is now deciding the bit error rate on the data. Now, if you look at this thing, what it **it** is telling you physically? If I look at the levels here, we have a 0 level some fluctuations are there; we have 1 level, on this some fluctuations are there.

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So, now if I say this is my 0 level, and this is my 1 level, and there is some noise which is present on this and there is some noise which is present on this. This quantity, which is the level difference between 0 and 1 that quantity is $I_1 - I_0$; I_1 this level is I_0 . So,

the difference between the mean value of this 1 and mean value of 0 level that is the swing, which we have in the two levels of the data and σ_1 and σ_0 are essentially deviations from this mean level. So, if you consider a Gaussian distribution, typically this would be of the order of about 2.5 σ_0 from the mean value. Similarly, from this mean value this deviation peak deviation would be something like 2.5 in to σ_1 .

So, now we are having encroachment in this swing, which is from I_0 to I_1 between the two levels and the encroachment is now from both sides. This side it is 2.5 time σ_1 and this is 2.5 time σ_2 or this encroachment is proportional to σ_0 plus σ_1 . So, we have this quantity which is the swing in the two levels, which is I_1 minus I_0 and then we are having encroachment because of the noise in this swing; that is proportional to σ_0 plus σ_1 . So, essentially this range which is available now clean. One can call this as noise margin; that is the quantity which essentially is this parameter Q. (Refer Slide Time: 26:25)

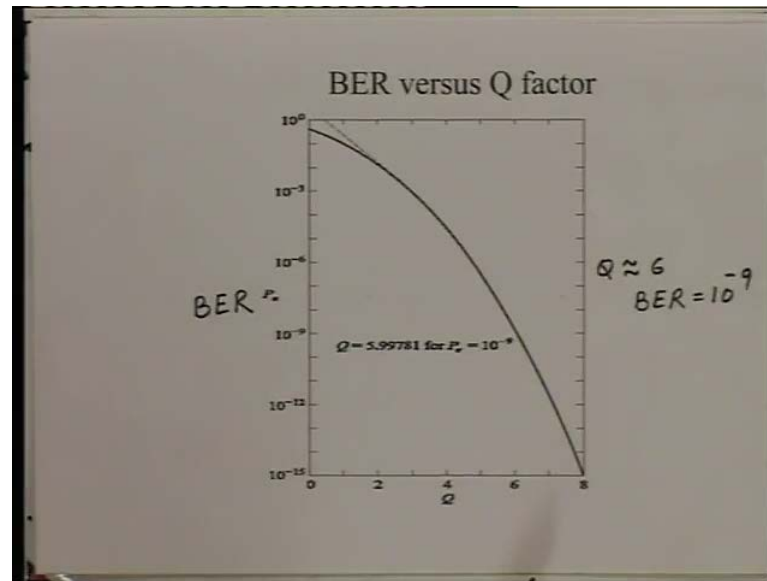
So, we are essentially talking this quantity Q which is saying that we have a total swing, which is available between two levels I_1 minus I_0 and then you are having this is the encroachment parameter. So, this is a quantity which is in some sense is a measure of what is the noise margin present in to your system and that is the quantity, which is now going to decide the bit error rate of the receiver. So, now if I substitute this Q seen these two quantities have become equal now for the minimum bit error ratio, the BER expression now can be simplified further. So, for this optimum condition (Refer Slide Time: 21:42) these two terms have become equal and these quantity here I_1 minus I_0 threshold divided by σ_1 ; that quantity is nothing but Q.

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$$\begin{aligned} \text{BER} &= \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \\ &\approx \frac{e^{-Q^2/2}}{Q\sqrt{2\pi}} \quad Q > 3 \end{aligned}$$

So, we can write down now the bit error rate which is BER; that is now equal to half erfc Q divided by root 2. This can be approximated to an exponential function for large values of Q . So, we can say this is approximately e to the power minus Q square by 2 divided by Q root of 2 pi and if Q is greater than about 3, then the approximation is quite accurate. So, within few percent, we get this quantity equal to the complementary error function. So, now essentially what we are saying is we have now derived the expression for the bit error ratio. And we have defined this parameter Q , which essentially the measure of the current swing between the two level 0 and 1 and the noise present at the two levels. So, for a data quality if you can measure this quantity Q than under the assumption that the noise is of Gaussian nature, we can calculate the bit error rate for the data. Now, as we can see here this quantity is a very rapidly decreasing function of Q . It is going is e to the power minus Q square.

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So, if you plot this function, the function essentially looks like this. So, what is plotted here is quantity Q and on the vertical axis, you have plotted this quantity BER. So, here we have BER. So, that one can see this vertical scale is logarithmic and here every division is 10 to the power minus 3. So, this is 10 to the power 0 which is 1; this is 10 to the power minus 3; 10 to the power 6; 10 to the power minus 9; 10 to the power minus 12; minus 15 and so on. So, we can note here that as the Q increases, very rapidly this functions drops; that means the bit error ratio or bit error rate drop very rapidly as a function of Q. And if we take some standard number now what bit error ratio will be acceptable value, then we can find out what is the corresponding value of k.

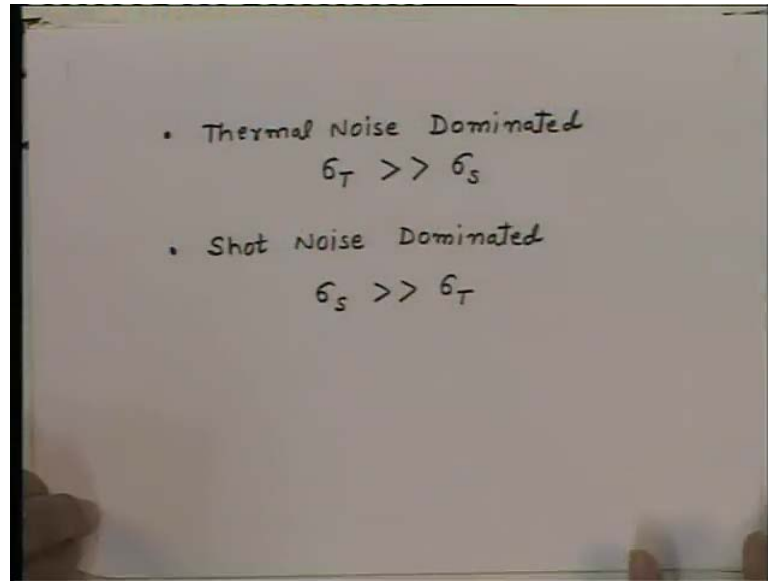
So, one can do one can say that I have a standard bit error rate acceptable for the data. And in optical communication, the acceptable bit error rate is 10 to the power minus 9 without any error codes, error corrections. So, on the raw data you must get a bit error rate of 10 to the power minus 9. So, if you go to this **this** quantity 10 to the power minus 9, you will get this value typically about Q equal to 6, where **BBR** will be 10 to the power minus 9. So, we get Q equal to approximately 6; for BER 10 to the power minus 9 and then for every one unit increase in Q, the BER practically dropped by three orders of magnitude. So, if we increase Q from 6 to 7, the BER would have dropped almost to 10 to the power minus 12. If we go from 6 to 8, the BER would drop to 10 to the power minus 15.

So, we see that just by increasing **the** this Q factor for the data, the BER can be very rapidly reduced. So, by even small increment in this quantity Q, we get substantially small value of the **the** quality fact. If we consider now a situation that the noise is let us say shot noise dominated, then the σ_0 could be negligibly small and one can make certain approximation to this data. But in general, as we have seen essentially if then σ_0 then σ_1 's are not equal, then essentially we have to deal in general case and then we can define this parameter Q, which will now decide the bit error rate of the data. So, let us see what we have done up till now. We have first said we are having various noises present in to the receiver.

We find out the total variance of the noise, which is sum of the variances of various noises. From that variance, then we say that we have the noise various corresponding to 0 level; have the noise variance corresponding to 1 level. And without worrying about what is the actual distribution of this fluctuation, we say that this fluctuation is of Gaussian nature. And under this assumption, then we say that we are having two distributions. One corresponding to 1 level; other corresponding to 0 level and these two distributions have different variances. So, 0 level, we have the variance which is σ_0^2 and for level 1, we have variance which is σ_1^2 . And then we general we derived the expression for the bit error ratio.

We find the optimum level for which the bit error rate is minimum and then for that, we get a parameter what is called the Q parameter of the data and then we get the bit error rate as a function of that Q parameter. As I mentioned for a standard data, the bit error rate should be less than 10^{-9} . So, that means for a data, we require Q to be of the order of 6 or more. With this understanding of BER, now one can go back to our signal to noise ratio expressions. And as I mentioned, we can make certain approximations two domains; that means one high power domain; other one is low power domain. So, essentially the operation we can divide into two categories.

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One is what we call as the thermal noise dominated and other one, we call as the shot noise dominated. In thermal noise dominated situation, essentially we are saying that the sigma T is much **much** greater compared to sigma S, the shot noise and this essentially happens for the low optical powers. So, when low optical powers when the shot noise is small, that time essentially system is limited by the thermal noise. And in that situation, we have sigma T much greater compared to sigma S. And this application probably more suited for the 0 level of the data; whereas, if we go to the 1 level of the data or if the data amplitude is large, then we have what is called shot noise dominated and in that case, we have exactly opposite.

So, we have sigma S which is much **much** greater than sigma T. Note also in an optical communication system, if the receiver is very close to transmitter where the power is not attenuated significantly in the fiber, then the situation probably would be the shot noise dominated; because optical power will be high; whereas, if you go to a distance significantly far away from the transmitter, the optical signal would have attenuated significantly. And in that case, essentially the shot noise contribution will be less and then the system will become thermal noise dominated. So, essentially we have depending upon the location of the transmitter and receiver and the distance between them, we may get a situation which is rather thermal noise dominated or shot noise dominated.

So, let us consider now the two limiting cases; thermal noise and shot noise dominated. So, what we do? Now, we consider this expression which we have got for the signal to noise ratio from here. And then we say that if we have a situation which is the thermal noise dominated, which is this. Then this is the term (Refer Slide Time: 04:25) which is present and all these quantities are negligibly small; whereas, when we are having a shot noise dominated situation, that time we have only this noise which is present. We can even assume that the dark noise also is negligibly small and thermal noise is also negligible. So, what we can do is in these two regimes, we can now get the signal to noise ratio.

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Thermal Noise Dominated

$$SNR = \frac{R_L R^2}{4KT B} P_{in}^2$$

$$SNR \propto R_L P_{in}^2$$

Noise Equivalent Power (NEP)

$$NEP = \frac{P_{in}}{\sqrt{B}} = \left(\frac{4KT B}{R_L R^2} \right)^{1/2}$$

1-10 pW/Hz^{1/2}

So, if I go for the thermal noise dominated in this situation, the signal to noise ratio would be equal to R_L responsivity square divided by $4KT B$ into p of the data; so we call an p in. So, note here this quantity we had the photo current square for the signal. Now, if the responsivity of the detector is known which is R , then the photo current will be R into p in optical power. So, that is your signal to noise ratio. So, for given receiver all these quantities are constant. We have responsivity constant, bandwidth constant, temperature constant. So, essentially we can say that this quantity is proportional to R_L and p in square.

So, in thermal noise dominated regime, the signal to noise ratio improves very rapidly as the optical power. It goes at the square of the optical power. Also the signal to noise ratio

improves with the load resistance and that is the reason as you have seen in the detector that we use a resistance, which is normally of a large value. Because that essentially helps you in giving you last signal to noise ratio. Then for a given receiver even if R_L is fixed, then we can define a quantity what is called the noise equivalent power, (No audio from 46:52 to 47:03) which is now a parameter of your receiver in the thermal noise dominated case. Let us say this is NEP; that is p in divided by square root of the bandwidth.

So, this quantity from this expression, essentially we can get $4 kTB$ into divided by R_L square responsivity square to the power half. So, for a given load resistance, we have this noise equivalent power and then this is given as watts per square root hertz and typical receiver as this quantity ranging between 1 to 10 picowatts per hertz power 1 by 2. So, the important in the note here is the when we are having a thermal noise dominated regime, that time the signal to noise ratio can be improved rapidly by increasing the optical power; because it is proportional to the square of the optical power. If I consider on the other hand the thermal noise not the shot noise dominated regime, then the situation is different.

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Shot Noise Dominated
 $\sigma_s \gg \sigma_T$
 $SNR = \frac{R P_{in}}{2qB}$
 $\propto P_{in}$

So, let us say if I consider now the situation, which is shot noise dominated; that means σ_S is much **much** greater than σ_T and in that case, essentially we get SNR, which will be equal to responsivity times p in divided by $2q$ into B . We can see from

here (Refer Slide Time: 04:25) these quantities negligible; this is negligible. So, only we have this quantity here and this one is proportional to the photo current. So, one photo current will cancel with this. So, that is how you got this quantity, which is $R \cdot p \cdot i \cdot n$. So, in this case, now the signal to noise ratio is proportional to the $p \cdot i \cdot n$. So, what is important to note here is that when I go to the low optical powers, at that time the system is dominated by thermal noise.

And then by small increase in the optical power, improve the signal to noise ratio significantly; whereas, as the power increases in the optical signal, then slowly the shot noise starts coming into picture. And for high optical power, the thermal noise becomes negligibly small and then the signal to noise ratio does not improve that rapidly as it was happening at the low optical powers. So, earlier the signal to noise ratio was going a square of the power, now it starts going only linearly as the optical power. This transition from the dominants of the thermal noise to the shot noise, somewhere it takes between minus 20 to 30 dbm of power.

So, if you have the optical power in the data, which is more than about minus 20 dbm, then system essentially tries to go towards the shot noise limited regime; whereas, if you are having optical powers lying in the range of about minus 40 minus 50 dbm, then the system essentially is thermal noise dominated regime. So, essentially what we have done in this lecture? We have analyzed the bit error rate performance of an optical receiver in the presence of various noises. We assume that all noises can be put together and there will be equivalent variance we can define for the total noise, which will be different for 0 and 1 level and then assuming that the noise is Gaussian.

We can calculate the bit error ratio for the optimum value of the threshold and we saw that the bit error rate or bit error probability drops very rapidly as a function of this parameter what is called q . So, a small change in this quality factor or data quality factor q that can improve the BER performance significantly. And then we saw that the system can be of two limiting cases, thermal noise dominated and shot noise dominated. And in thermal noise dominated case that means a low optical powers, signal to noise ratio improves as the square of the signal; whereas for the shot noise limited, the signal to noise ratio scales as linearly. So, it does not increase that rapidly.