Advanced Optical Communications

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Lecture No. # 16

Laser – II

We are discussing principles of laser. In the last lecture we introduce the concept of coherency; we saw there are two types of coherencies in light. One is the temporal coherency, another one is special coherency. And then, you ask a question from optical communication point of view, why do you require coherency in light? And then we found that the temporal coherency is related to the spectral width of the source whereas, the special coherency is related to the beaming effect, of the radiation coming from the source.

Since, for low dispersion on optical fibers the spectral width has to be very small, we require high temporal coherency and for high efficiency since, the radiation has to be launched inside an optical fiber, we require a very narrow beam of light we require special coherency.

So, we found that for optical communication directly the coherency is not required, but the parameters which are desirable for an optical source from communication point of view; they imply a high temporal coherency and high special coherency. And then, we started investigating the principal of laser. So we said that if you take a simple material consider a very simple system of two energy levels and then you ask a question what kind of process they electron transition can have?

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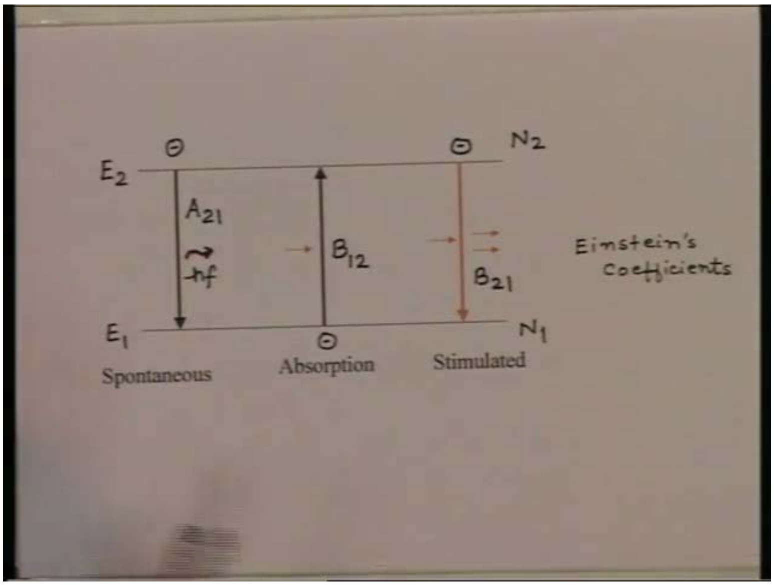

And then we identified three process which can take place inside the material one is what is called the spontaneous process, other one is the absorption process and third one is stimulated process. We saw that if the electron is an exited state, then naturally it has a tendency to relax to the lower energy state and in this process the energy difference is released in the form of photon.

And this transition probability is given by this parameter in one whereas, if the photon is incident on this system then there are two possibilities either the photon can be absorbed by the material, and the energy of the photon can be given to the electron and which makes the transition upwards, or the photon can force an electron to make a downward transition; and to meet another photon which is in complete coherency with the original photons. And this probabilities of transition upward transition and downward transition where given by this constants B 1 2 and B 2 1.

So, we saw that this constant a, is the transition probability without presents of a photon and a parameter B, essentially gives you transition probabilities which are related to the presence of a photon. These quantities A and B are calling the Einstein's coefficients, in fact this process which is the stimulated process that was one which was postulated by Einstein's.

And we find that even if there is no direct evidence for this process should take place inside the material, it was propose that if this process takes place then essentially that there are to downward process which are taking place here and there is one upward process. So, will thermally equilibrium these two processes must balance, so then we wrote the equations for these to processes.

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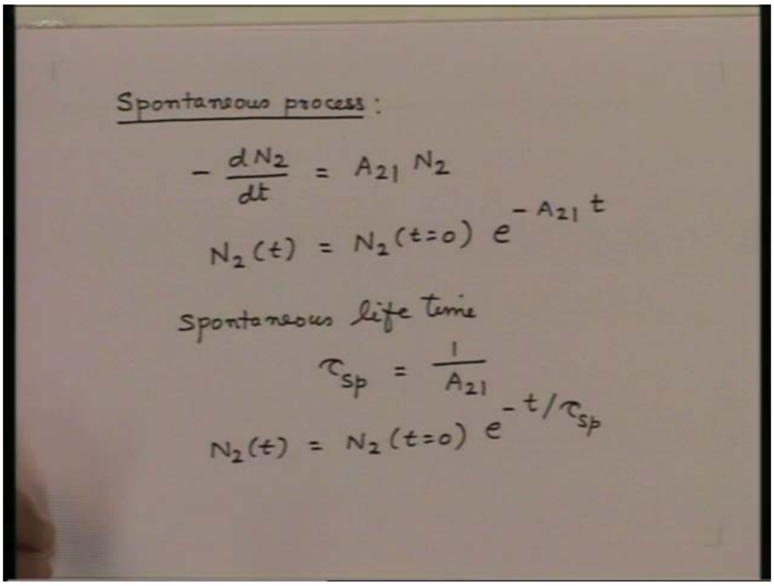

So, was stimulated process we said that the depletion rate of electrons at the higher energy level that is given by A 2 1 multiplied by N 2 because rate of change of electrons and higher energy level is proportional to the electron density at higher energy level. And then we got a solution of this equation, where the electron density as the function of time decreases as exponentially at the higher energy level and that time constant is what call is the spontaneous life time.

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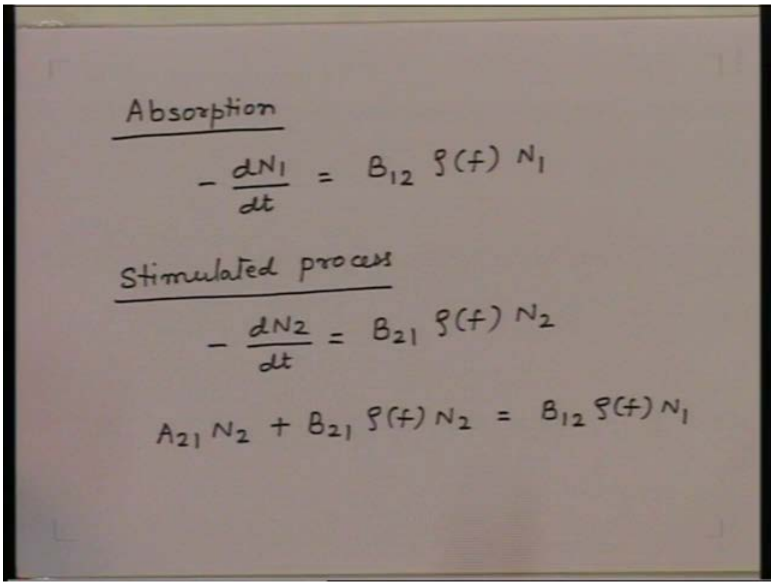

Similar exercise, we did for the other two processes the absorption process and stimulated process. And here he said the rate of depletion of electrons in the lower energy state will be equal to these transition probability which is given by this constant B 1 2. But this process also depends upon the photon flux, so this is the photon density row (f) and it is also proportional to the electron density at the lower energy state.

Same thing, we have return for the stimulated process thus minus d N 2 by d t is equal to this probability which is B 2 1 again photon density multiplied by N 2. And then we said at thermal equilibrium, since the number of electrons at lower energy state and higher energy state is same essentially the downward process must be completely balance by the upward processes.

So, we got this equation that these two processes are downward processes spontaneous emission, stimulated emission. And this is the upward process which is the absorption process just by rearranging the terms bringing all the rows in terms for onside and simplified, we got the photon flux density expression which was this.

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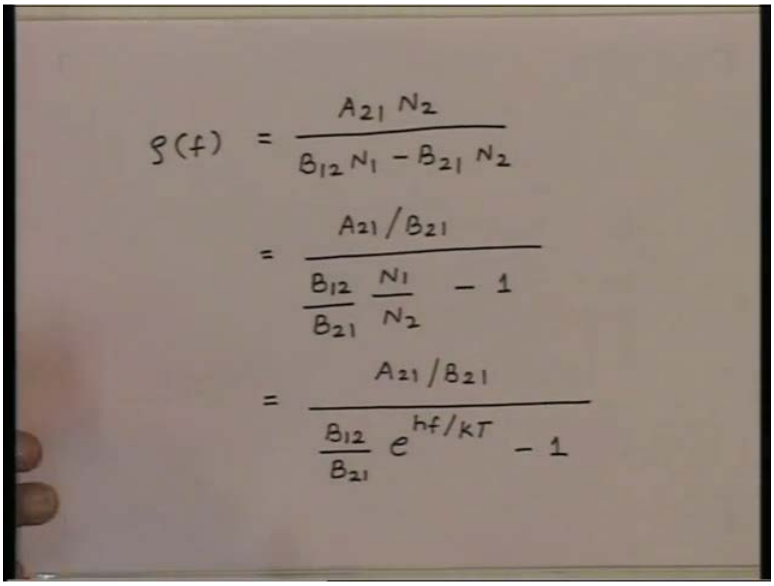

So, a some rearrangement of the terms essentially says that the photon flux density is given by this expression A 2 1 upon B 2 1 divided by B 1 2 upon B 2 1 N 1 upon N 2 minus 1. Now, as you seen that this ratio N 1 upon N 2 there is the ratio of electron densities at the lower energy level and higher energy level, they are given by the bolds one distribution, so N 1 upon N 2 is e to the power h f upon K T. So, we can substitute into this, so we got photon flux density at thermal equilibrium which is given by B 1 2 upon B 2 1 e to the power h f upon K T that is N 1 upon N 2 from bolds one distribution minus 1.

So, this simple exercise of balancing the three processes the spontaneous process, the absorption process and the stimulated process says that there is net photon flux from this material which is at temperature t and the photon flux density is given by this expression.

What one can do next now, that we know that if you have a body at temperature t then this body gives black body radiation. So, what we can say that whatever, the photon flux density which we have got from balancing this three processes this must be same as the photon flux density, what we get from black body radiation and that is what precisely done.

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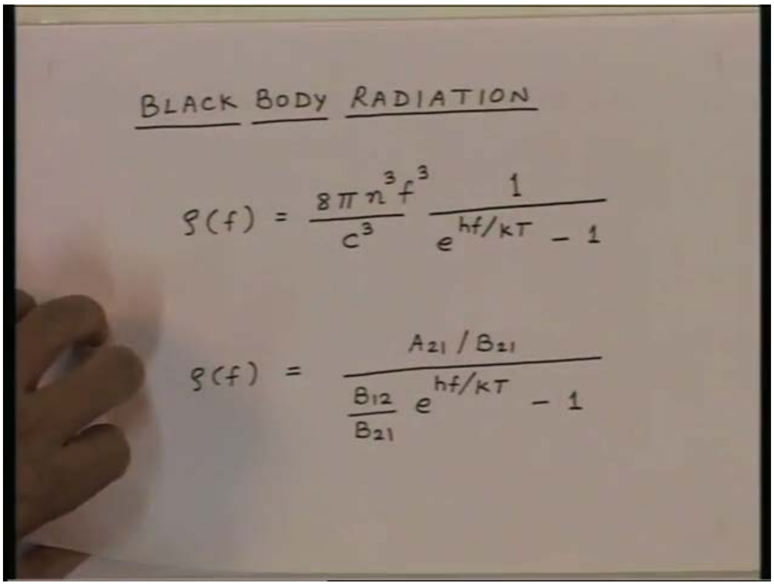

So, if we get the expression for the black body radiation the photon flux density which we get is given by this, so this your photon flux density as a function of frequency, that is related to this quantity N which is the refractive index of the medium, in which is the radiation is coming see is velocity of light f is the frequency of radiation and here h f is the photon energy and K T is the thermal energy.

So, when these three processes were proposed the black body radiation expression was while established. So, it was a straight forward to just compare these to expressions one which is like body radiation expression really jeans low and the expression which you derived by balancing this three processes. So, if we compare now these two you will see that this quantity here A 2 1 upon B 2 1 is this constant.

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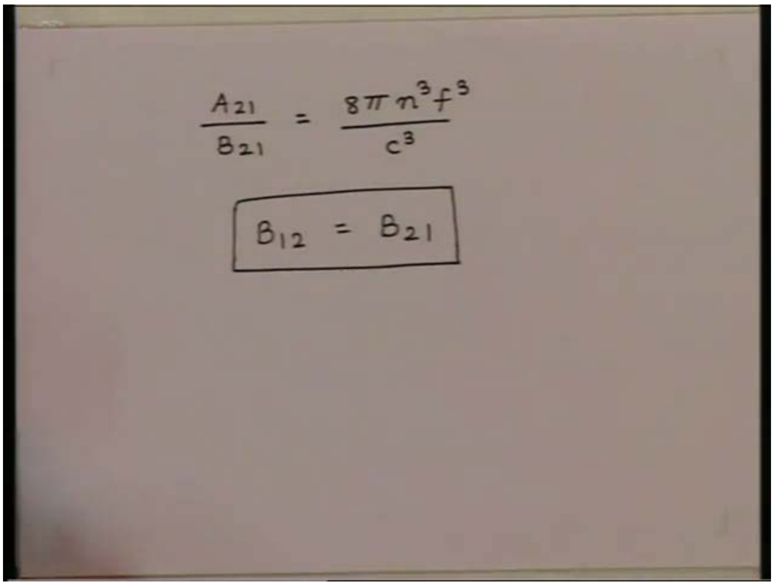

So, we can write down just by comparison, that we have A 2 1 upon B 2 1 that is equal to 8 pi n cube f cube upon c cube. So, essentially we get the relationship between A 2 1 which is related to the spontaneous life time and also this Einstein coefficient B 2 1 which is the transition probability for downward transition from energy level 2 to energy level 1.

The most important and striking thing which we see by comparison of these two expression either if I look at this quantity here, now which is 1 upon e to the power h f upon K T minus 1 and compare with this quantity what we find is that B 2 1 has to be equal to B 1 2. So, the second thing which we see by comparison is B 1 2 is equal to B 2 1. So, we see here that B 1 2 is equal to B 2 1, and what is B 1 2? B 1 2 is a transition probability of electron going up and B 2 1 is the transition probability of electron coming down in the presence of the photon.

And saying that B 1 2 is equal to B 2 1 say that these to transition probabilities are equal that means, the absorption process and the stimulated process are equiprobable. No, note here that the absorption process is always observed in life, if you go and search file the system where light beams is the incident it always observed there is light beam attenuates as a travels inside the medium, so the photons loose energy there is a absorption of photons.

We never find the stimulated process in nature we always find only the absorption process in nature. But this simple analysis tells that this two processes are equiprobable processes and that is a very perform conclusion, that one process which is always observed in nature and the other processes which we never observe in nature or two equiprobable processes. One can then ask a question, why do a find the absorption very often in a nature, and we never find the stimulated emission in nature?

And the answer is essentially lies in the Boltzmann distribution that all though the probability of transition is equal, it depends upon were the electrons are. In thermal equilibrium as we have seen there are more electrons on the ground state and less electrons on the higher state. Even if photon has equal probability of forcing a transition upward are downward there be always more transitions which will be going upward because there are more electrons which are resigning on the ground state. So, the net result will always see is that there will be always more upward transition and loss of photon which is the absorption process.

So, what you find from the simple analysis (Refer Slide Time: 14:22) is that the two process the absorption process and stimulated process, there are equiprobable processes; however, we always find the absorption process dominated in nature and that is, because we have more electrons in the lower energy state in thermal equilibrium their higher energy state.

Naturally their one can ask a question if you could create a situation where there are more electrons on the higher energy state compare to the lower energy state then what would happen. And the natural answer would be that now this stimulated process will dominant over the absorption process.

So, if we want the stimulated process to dominate and to become more visible, then essentially we have to change the natural distribution of electrons in different energy levels. So, we have by some mechanism, we have to transport more electrons to higher energy levels and if there are more electrons and higher energy level compare to lower energy level. Then the process downward process will be dominant over the upward process and then the net effect will be stimulated emission.

So far seeing a stimulated emission then essentially, now we have to create a situation which it against the nature, so we have to invert the population of the electrons and there process is what is called the population inversion. So, by creating a population inversion (Refer Slide Time: 16:04) if the photon is incident on the medium, then you will have the coherent emission from the material.

Though, we can say that that is no net effect which is coming because of the stimulated emission, one can ask question that radiation which is coming in the form of black body radiation even that radiation is not of core and nature it is predominantly incoherent nature. So, we are having two processes which are essentially giving you the radiation out one is the spontaneous process, other one is a stimulated process. The spontaneous process present requires presence of a photon whereas, stimulated process requires presents of a photon.

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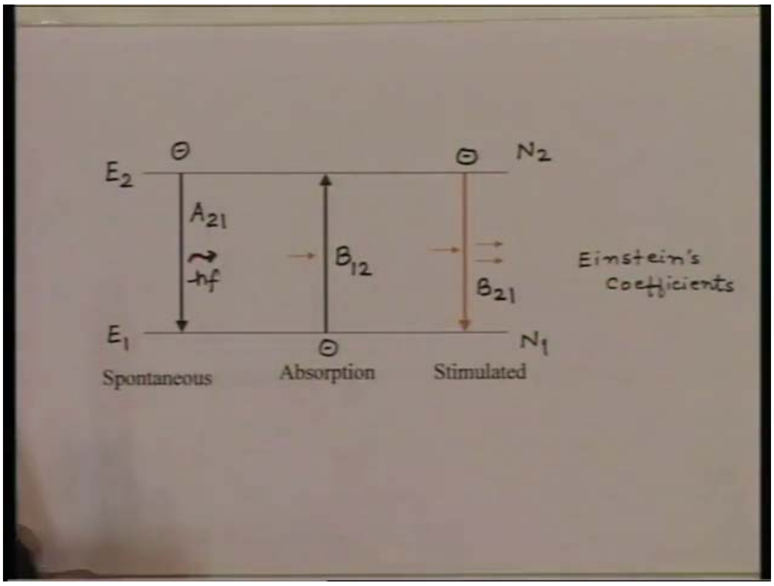

Going back to again the probability of these two being equal, one can now visualize this thing as follows, when we are now looking at the interaction of a photon with the system of energy levels where difference is exactly equal to the energy of a photon essentially we are talking about a resonant phenomena.

The photon as an energy that means, it has a definite frequency similarly, the system has to an energy levels and the energy difference is constant which again is equivalent to certain frequency. So, we are having a characteristic frequency of the material which is the energy difference divide by the Planck’s constant and we have a frequency of the photon.

So, we are having now to sustain one is photon which is having a frequency and then we are having another system which is having a resonant frequency or characteristic frequency which is equal to this photon frequency. And whenever, we are having to systems which are having is same natural frequencies, there is exchange of power between this two systems.

Precisely, that is happening here that since the frequencies of the photon energy level system is same their now exchange of energy takes place between the photon and the material. Of course, which were the energy flow will take place will depend upon where the more energies. So, if you are having more energy with the photon, then the energy will flow towards the system and if it is the other way around then the energy will go from the material to the photon.

So, in natural process thermal equilibrium process the energies with the photon it can imparted energy to material it is electron gets exited where as if you go to the stimulated process, then there is more energy now with the material. So, energies now important to photons you get more number of photons.

So, essentially this phenomena is more like a force vibration of phenomena and whenever, the force vibration takes place essentially, the photon which is getting emitted here (Refer Slide Time: 19:32) is now completely under the influence of the original photon and thus why the characteristics of these two photons are excited identical.

So, if by creating a mechanism by which those the stimulated process dominates in the material, then we will get an ambition which will be coherent in nature. One can do a very quick calculation to find out in a typical material all though this three process are taking place what is the contribution of thus spontaneous emission over thus stimulated emission.

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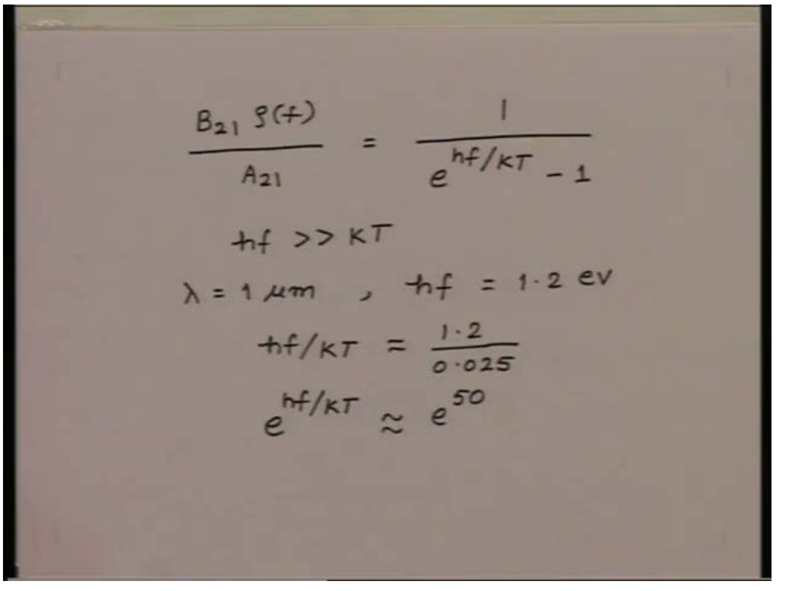

Now, as we seen that the spontaneous process is proportional to B 2 1 and the photon flux whereas, the stimulated sorry this is the stimulated process. The spontaneous process is proportional to this quantity at 1. So, from the expression which we have got here here (Refer Slide Time: 20:34) if you simplify you will get this B 2 1 row divided by A 2 1 that will be nothing, but 1 upon e to the power h f upon K T minus 1.

And if you consider typical materials the h f is much much greater than K T. So, if we consider let us say a radiation for lambda equal to 1 micrometer, for which the h f would be about 1.2 e v this quantity h f upon K T would be 1.2 divided by 0.025 that is the thermal energy at room temperature.

So, this quantity here e to the power h f upon K T, that will be of the order of e to the power 50 or if you ask at room temperature if you look at body whatever, radiation is coming out of it, the ratio of this stimulated emission to spontaneous emission is e to the power minus 50; that means, the simulative emission is practically nonexistent at room temperature under thermal equilibrium.

And that is the reason all though all the three processes are internally taking place inside the material that net effect what we see from the black body radiation is incoherent radiation. Because, for a typical energy which we are talking about this quantity here e to the power h f upon K T is very very large and because of that this ratio stimulated to spontaneous that will be negligible small having said that.

Now, let us concentrate only on this process which is the stimulated process (Refer Slide Time: 23:00) or let us concentrate on this two process which are taking place in the presence of the photon. Why we are saying that that is because when this process starts, this process is multiplicative process one photon incident on this will give twice to two photons which are have same characteristic as a original photon; if there are more electrons available here then this two photons will pull down two electrons and with create two more new photons. So, very quickly the photon number will increase and as the photon number increases more and more electrons will you pull down and more and more photons will be realized.

So, this process is rather a catastrophic process (Refer Slide Time: 23;49) is a multiplicative process where, as if you look at this process here which is the spontaneous process then this process naturally the electronics time to come down. So, we will see that the time constant of this is much much larger than the time constant of this process.

So, whenever this process starts and star dominating this process does not have much (( )) show because there is a number grows very quickly the electrons will you pull down there will be (( )) of these electrons to show the spontaneous emission. So, now onwards whenever, we discuss now the transitions let us ignore this spontaneous emission this we already seen while discussing LED's and we concentrate only on these two processes is a absorption process and the stimulation process.

So, now what we find is that if you want to have a stimulated emission then you must have population inversion; that means, there was more electrons on the upper level compare to the lower level. One can now, ask very basic question if I have to create population inversion since, naturally the electrons are to be going to be on the ground state here by some mechanism I must transport the electrons to the upper level by what is the mechanism by which I can transport the electrons again I have to supply energy to this system and a energy is given to the electrons and the electrons are transported to the higher level. But when again I am giving the energy to this system the energy will be again given in the form of photons.

And the energy has to be exactly equal to the difference in the energy levels; that means, for taking the electron from here to here (Refer Slide Time: 26:04) I must supplier energy for photon which is having a exactly the energy which is equal to the difference between these two energies.

Now, if the energies exactly same as difference of this that means, the frequency of the photon which is to be supplied for transporting electrons from this level to this level is same as what photon essentially we want to generate by the stimulated emission. What another words what it means is to get the stimulated emission out which require population inversion I have to provide the photons to this system precisely at the same frequency (Refer Slide Time: 26:40) that means, this photons are temporarily coherent. And this temporarily coherent photons can transport electrons from lower level to higher level which then are trigger by stimulated process will give the coherent emission out.

Obviously, since for generating the coherent emission I cannot have a coherent emission in advance simple to energy level system will not work. However, what cannot here is that when the energy supplied (Refer Slide Time: 27:13) in the form of this photon for transporting electrons from here for creating population inversion. These photons need not be especially coherent, so they must be coming from various directions only if they are temporarily coherent.

Then, they can supply the energy which is exactly equal to this difference and it (()) be transported to here to here. So, in practice when it appears that a simple two energy level system, what we have discussed up till now, cannot provide population inversion. Because it will require coherent emission to create that and therefore, a laser cannot be created. So, what is done is then that you create essentially the systems which are more energy level systems.

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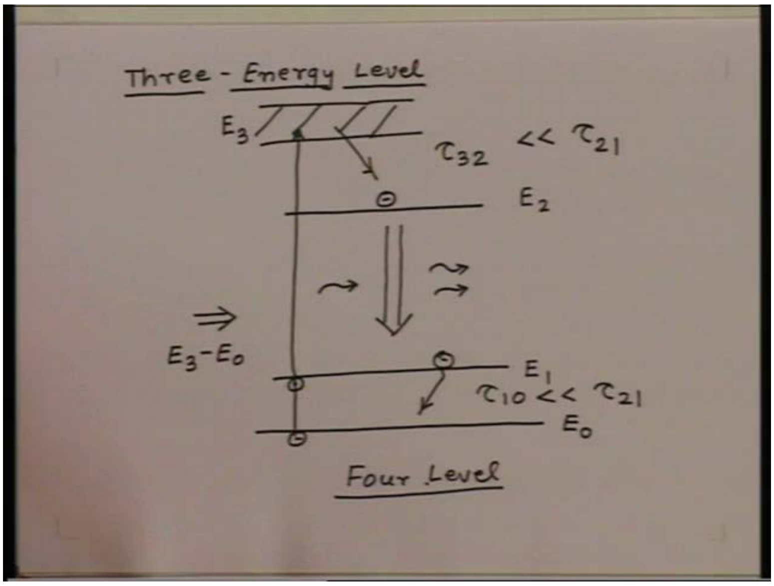

So, let us say we have energy level here, these energy level and let us say this is the energy level which is E 1, this is the energy level E 2. And then what we do is we identify the material which have other energy levels here which are not as discrete, as this energy levels and your having essentially a band of energy is where electron can be transported.

Now, since you are having a finite energy width here by providing temporally in coherent radiation corresponding to these energy width electrons can be transported from the ground state to this state. So, let us say this energy level is having energy denoted by E 3. So, now the process is follows the electron is transported from the ground state to this upper state, energy state.

And then if the life time of the electron to relax from this level to this level is much smaller than the electron will be transported to this level and very quickly, because of the continues process this electron will relax to this level. So, the electron will come here to here and it will come to second level.

So, if I say the energy level 1 2 3 they have different time constants, spontaneous time constant associated with them. So, let us say this time constant is tau 3 2 that means, the relaxation time from level 3 2 level 2. And if this time constant is much smaller compare to the time constant corresponding to energy level 2 and 1. So, if it is much much smaller compare to tau 2 1, then as the energy supplied to the system corresponding to this energy difference it remains the one.

The electrons will be transported to this level very quickly the electrons will come here to energy level E 2 and now since tau 2 1 is much larger compare to tau 3 2 it will weight their at energy level 2. Then if the photon is incident now, with an energy difference which is E 2 minus E 1 then this electron will be pull down and we get the stimulated emission out from this system corresponding to the energy reference E 2 and E 1.

So, what it appears now, that to create a laser we require a at least a three energy level systems, out of which one energy level has to be fairly brought. So, that electrons can be excited to that energy level by as using (Refer Slide Time: 31:20) incoherent photons.

And then these electrons will decade to another (Refer Slide Time: 31:26) energy level and then an actually the lasing action for stimulated action will take place between these two energy levels. So, the triggering photon will have a frequency corresponding to E 2 minus E 1.

So, in principle then it looks like in practice a three energy level system, will be the minimal system which will require for creating the population inversion. However, we will see that even the three level systems will not work very efficiently in practice, and the reason is very simple.

Even if the electrons are accumulated, now at this region here when this photon comes as we saw that the probability of interaction of this photon with this electrons or this electron is equal; that means, when this photon comes either they photon can pull down this electron to emit another stimulated photon or it can pull an electron up from the downstate to this state and the photon may get lost.

And as you seen to see the next stimulated process, taking place the number of electrons in the upper level have to be larger compare to the number of electrons here (Refer Slide Time: 32:56) which is what is call the population inversion. So, what that means is that total numbers of electrons which are there on ground state more than half of those electrons have to be transported to this level, then and then only one can see the next stimulated emission from the system.

If we do a quick calculation that very huge number of electron which are present on the ground state and if they would be transported to the upper level a huge amount of energy is to be supplied to the system to get a very tiny stimulated emission coming out of this (Refer Slide Time: 33:37). So, again you will see that in principle though the three level systems can create the stimulated emission it will not be very efficient, because it will require a huge amount of energy to be supplied, so that more than half the electrons are transported to the upper level.

So, what one can do one can essentially create one more level here (Refer Slide Time: 34:03) and now we call this is truly the ground state. So, let us say this is the energy level 0 and is slope transporting now electron from this two this electron is transported from here to there to this level. So, this is the ground state from here the energy supplied now corresponding to the difference which is E 3 minus E 0 and this band is brought.

So, this is the in coherent photon flux, which is incident on this exciting the electrons, the electrons go from here to here since, this time is much smaller compare to tau 2. It will relax with this it will rate, because this time is much larger here compare to this time. However, if I want to have a population inversion between these two levels, what we must guaranty is than after the electron as made a transition due to this process. It should not wait here in this level, because if it weights in this level then again we have similar problem that you have more number of electrons which are here.

So, what we should have the time constant between this two levels, now has to be much smaller again compare to this time constant. So, electron again will make a transition from here to this level by this time constant which is tau 1 0 and tau 1 0 has to be much much less again than tau 2. This is, now created essentially a four level system energy level system where E 1 a small number of electrons can be transported to this level and if this level is clear very quickly even the small number of electrons can create population inversion between these two levels.

So, if we want have efficient system which can work in practice at least of four energy level system probability is required with the appropriate time constant (Refer Slide Time: 36:25) of transitions between different levels and then and then only the stimulated emission can take place.

We will see that later on when we see it various laser systems that actually the material which are use for lasers, they have a fairly complicated energy level system and a appropriate energy bands are chosen. So, that you have appropriate time constants which will create population inversion and then we have a lazing action between those two energy levels.

So, a four level (Refer Slide Time: 37:11), four level system or more than four level system probably is more appropriate for creating the coherent emission from an atomic system with good efficiency. Let us now go back to our original thing will this understanding that by doing this mechanism we have create the population inversion and then we have certain number of electron which are in the upper level and there are certain number of electron which are at lower level.

And then, when the photon is incident now with equal probability either upward transition will take place or downward transition will take place and as we mention since this process is a very fast process, the spontaneous emission will not have any chance to show. So, a now our discussion let us assume that thus spontaneous emission is negligible and essentially this two process are only the one which are taking part.

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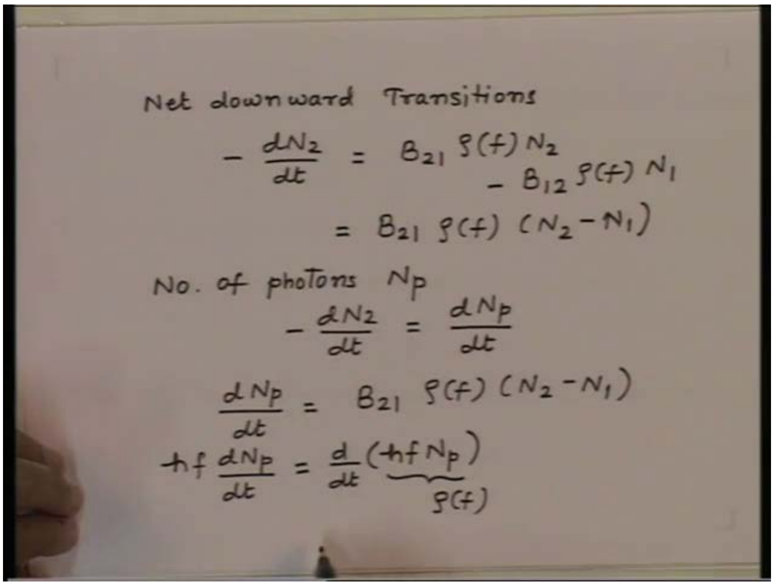

If we do this now, then our equation of the transitions of electrons the net downward transition, if you want to write we can get the net downward transitions, which will be net downward transitions which will be minus d N 2 by d t that will be equal to B 2 1 rho of f into N 2 this are downward transition minus the upward transition which will be B 1 2 rho of f into N 1 as we shown B 2 1 and B 1 2 r equal, so this thing essentially can be return as B 2 1 rho of f N 2 N 2 minus N 1.

So, if N 2 is greater than N 1, which is the population inversion, then there will be more downward transitions, if N 1 is greater than N 2 then there will be more upward transitions. So, a normal situation N 1 will be more than N 2, so you will have more upward transition which absorption process and for population inversion N 2 will be more than N 1. So, there will be net stimulated emission process.

Now, the each transition which is going to take place which is downwards is going to give one photon out. So, if I say that the number of photons are given by n p, then the downward transition rate of electrons that will be equal to the photon generation rate, so that means minus d N 2 by d t that will be equal to d N photons divided by d t.

So, this equation what we have here we can get d N P by d t that is equal to B 2 1 rho of f into N 2 minus N 1. This is the rate of photons, if I multiplied this quantity by the energy of photons which is h into f, and then this quantity essentially will tell us the energy per unit time in the form of photons. So, if I say that this quantity here multiplied by h f d N P by d t which we can write as d by d t half h of f into N P.

But, this quantity is nothing, but the photon flux density which is nothing, but this quantity rho of f, so you have this quantity is rho of f. So, now, you got very nice relationship that the rate of change of this photon flux from the system will be equal to B 2 1 rho f into N 2 minus N 1.

So, we got equation for the photon flux from the system which is (Refer Slide Time: 42:55) d rho by d t is function of frequency that is equal to B 2 1 rho of f into N 2 minus N 1 knowing the relation between B 2 1 and A 2 1. And A 2 1 is one upon the spontaneous life time.

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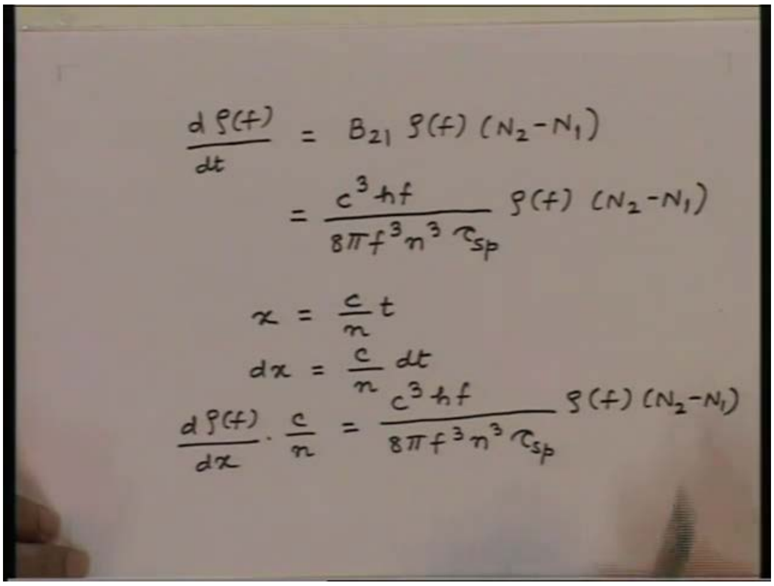

If you go back and get those expression as we derive from comparing a, the black body radiation and the plank's constant, then we can get for B 2 1 expression. And this expression essentially would become c cube h of f divided by 8 pi f cube n cube tau's spontaneous into rho of f into N 2 minus N 1.

So, now we are seeing that the photon flux varies as the function of time and that is given by differential equation. But, when we look at the photons, the photons essentially is the electromagnetic wave. So, photon is not going to be staying at the particular point as a function of time, it moves with the velocity of light that means, at the photon flux is going as function of time, the photon also moving in space.

And we know that (Refer Slide Time: 44:57) that the x distance traveled by the photon is equal to the velocity of light which is c by n into t. So, we can write from here the d x is equal to c by n into d t.

So, I can get here 1 by d t from here that is equal to c by N 2 1 by d x, so now we can change this equation which is a differentially equation as the function of time to as a function of space. So, we can write quantity here which is d rho f of d x into d x by d t which is c by n which is c by n that is equal to this quantity here, so c cube h of f divided by 8 phi f cube n cube tau spontaneous rho of f into N 2 minus N 1 once by will cancel with that.

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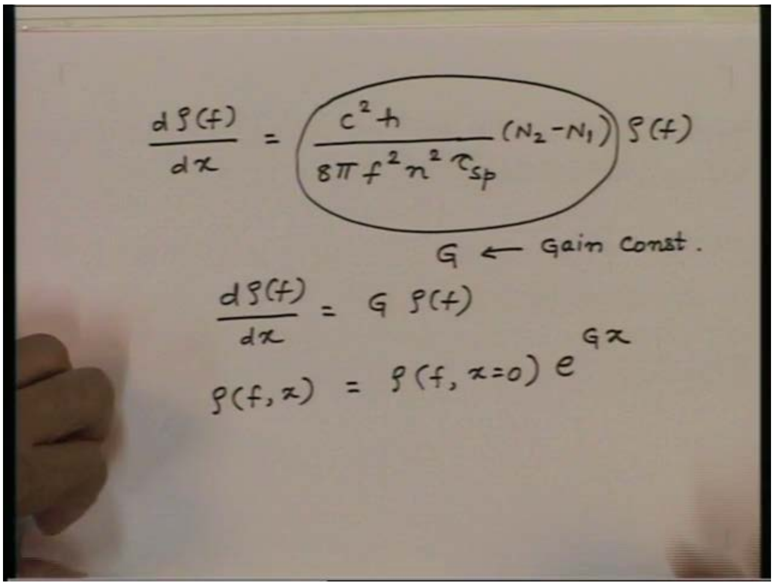

And then finally, we get the equation for the photon flux as a function of distance which will be d rho of f by d x that will be c square h upon 8 phi f square n square tau spontaneous into N 2 minus N 1 N 2 rho f. So, if we define this quantity is for here, a some quantity g which is the function of frequency, then this equation can be return as d rho of f by d x h is equal to g into rho of f and solution of this equation would be rho of f as a function of x that will be equal to rho of f at x equal to 0 into e to the power G x.

So, this quantity then we can call as the Gain constant of the system, and the photon flux will go exponentially as you travel inside the material with this gain constant. And as we see the gain constant depends upon various parameters, it depends upon the frequency, it depends upon the refractive index of the medium, it depends upon the spontaneous life time, and per more importantly this quantity is directly proportional to this difference N 2 minus N 1. And if N 2 is greater than N 1 then G is positive, if N 2 is less than N 1 then this quantity is negative and N 2 is equal to N 1 that time is quantity 0.

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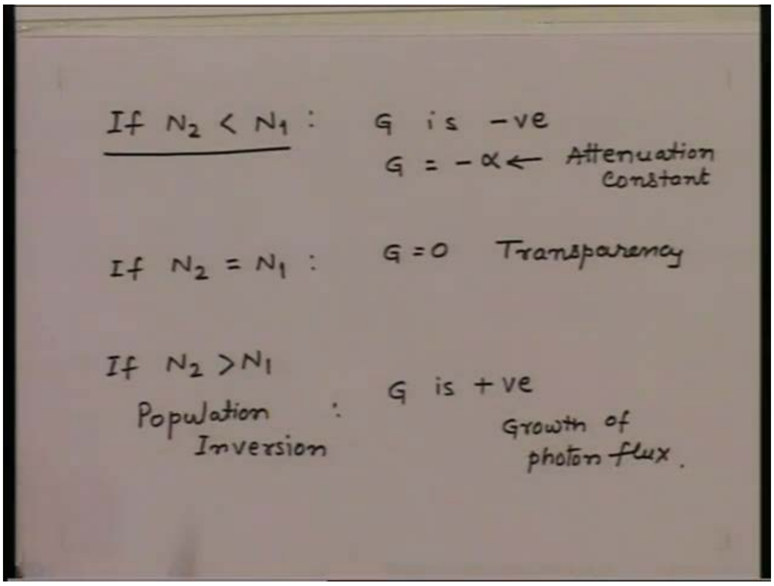

So, essentially we have now these three things possible one is if N 2 is less than N 1, which is the normal have a condition then, G is negative or it represents exponential d k of this function. So, g is equal to minus alpha where this quantity is the attenuation constant, so if I have normal situation then there is a exponential loss and energy will d k exponentially by the attenuation constant alpha.

If N 2 is equal to N 1, then G is equal to 0, and then we call this thing as the transparency condition that means, the photon flux neither goals no decays as a travels inside the material. Third possibility is if N 2 is greater than N 1 which is population inversion then G is positive and you have a growth or the photon flux.

So, we say that now quantitatively whenever we have a population inversion then the photon flux is grow into grow exponentially inside the material as a travels. So, now he want to really grow the stimulated emission inside the material if the radiation travels more and more inside the material and if the population inversion is there then there radiation is will keep growing.

So, now the idea as essentially to mate the radiation travel more and more (Refer Slide Time: 51:45) inside the material a certain mechanism maintain the population inversion and if we do that then we get the stimulated emission coming out of the system which is the coherent radiation.

So, this is the fundamental now equation for the laser, when we meet in the next lecture we will try to see this characteristics that is gain constant depends upon and what it means really a for the lasing action. And then we also go and ask the question are even if the stimulated emission is internally created inside the matter would it be externally visible and how do we make this radiation coherent which were essentially looking from.