

Advanced Optical Communications
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Lecture No. # 15

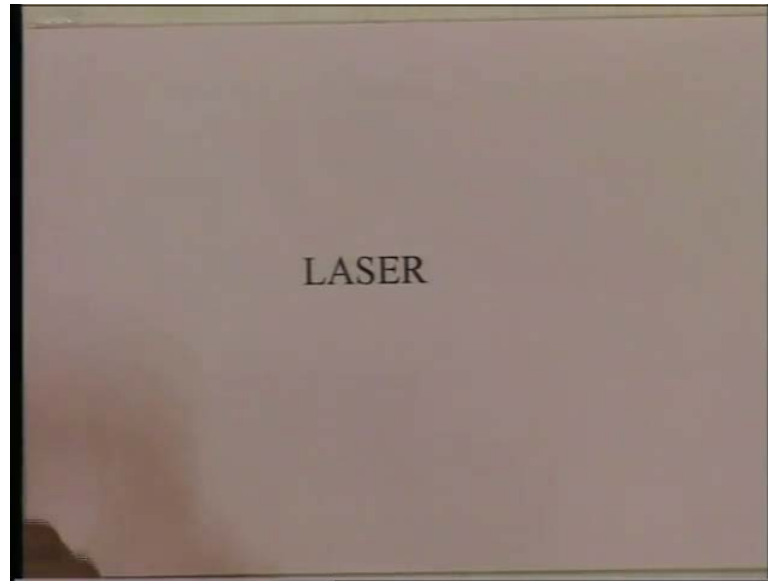
Laser - I

In the last lecture, we discussed various aspects of light emitting diode. We had seen that for optical communication, two sources could be possible. Both are semiconductor based, one is light emitting diode and other one is the injection laser diode. We saw the limitations of LED that LED have large spectral width and also it has a very low efficiency, because of that it cannot be used for long distance communication. So now, in the following lectures, we investigate the operation of a laser and also see, how laser is superior compared to LED from the view of optical communication.

As soon as we say word laser, we think of coherent radiation. However, up till now whenever we have discuss the optical communication aspects, we never made a case for coherent radiation. We have seen certain characteristics, which are required in the source for optical communication. However, no were explicitly we have seen that the coherency in the light is required for optical communication. One can ask then question why is the laser, which is supposed to be a coherent source is more suited for optical communication.

And this question one can understand better, if we understand properly the meaning of coherency. And then, we see what are the characteristics which are embedded inside the coherency, which are suitable for long distance optical communication and then we can make a case for laser as a source for long distance communication.

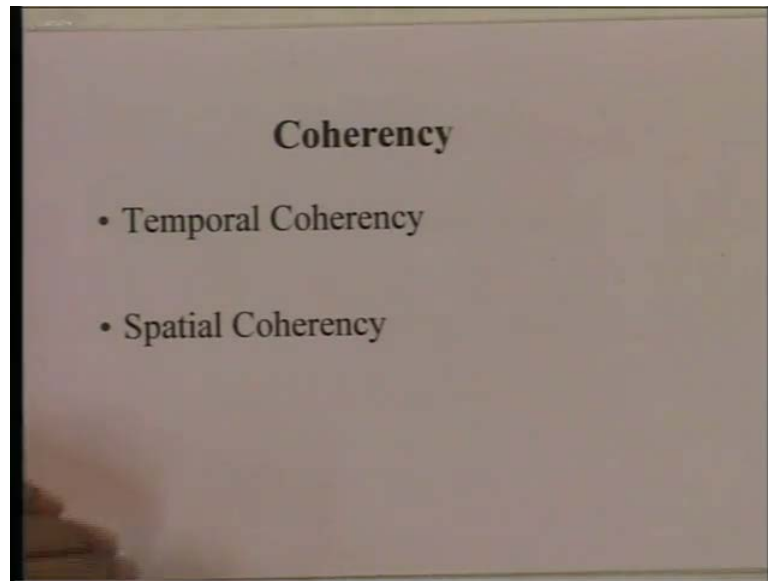
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So, in this case essentially we are going to talk about the basics of laser. First, we will talk in general the principles of laser and then, we will go specifically to the semiconductor lasers. As you all know the laser is a short form of light amplification by stimulated emission of radiation. So, firstly one should note that the laser is not an optical source, laser is an amplifier. When I compare this with LED the LED is an optical source, so you have a electrical energy you give to LED it gives you a light out; whereas, when we go to laser principally laser is an amplifier. And every amplifier essentially requires an input signal, which can be amplified by this device. So, in principle essentially laser requires some input signal, which can be amplified and you get a large optical output. However, when we are investigating laser here we are interested in optical source.

So, we will see later that first, we investigate the characteristics of laser as an amplifier, then by giving proper feedback an amplifier can be converted into an oscillator, which becomes an optical source. So, before we get in to various aspects of laser. First, let us understand the meaning of coherency and how the parameter, which are related to coherency, affect the communication of optical fiber.

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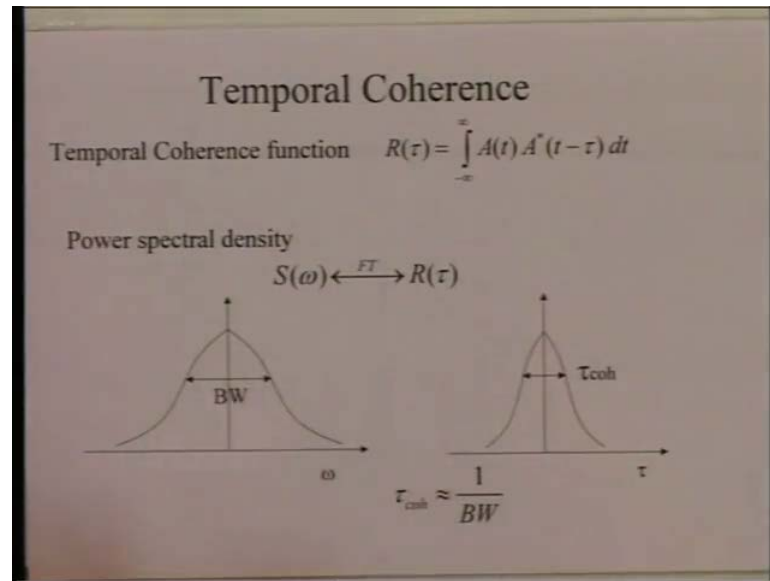


So, coherency there are two types of coherencies, which you see in light, one is what is called the temporal coherency; that means, coherency as a function of time. And other one is what is called the spatial coherency; that means, in space the radiation is coherent. Now, if we consider light as the function of time and space, so you are having a light as electromagnetic wave which is moving in space. See, if I look in the space as a function of time you will be having certain electric field, which will be varying in time it will be also varying in space.

What one can then do is one can ask, what is the behavior of this electric field or the amplitude of the signal as the function of time, and what is the behavior of the signal in space in a plain perpendicular to the direction of this light propagation. So, when I try to capture the characteristics of this as a function of time that is what is called the temporal coherency.

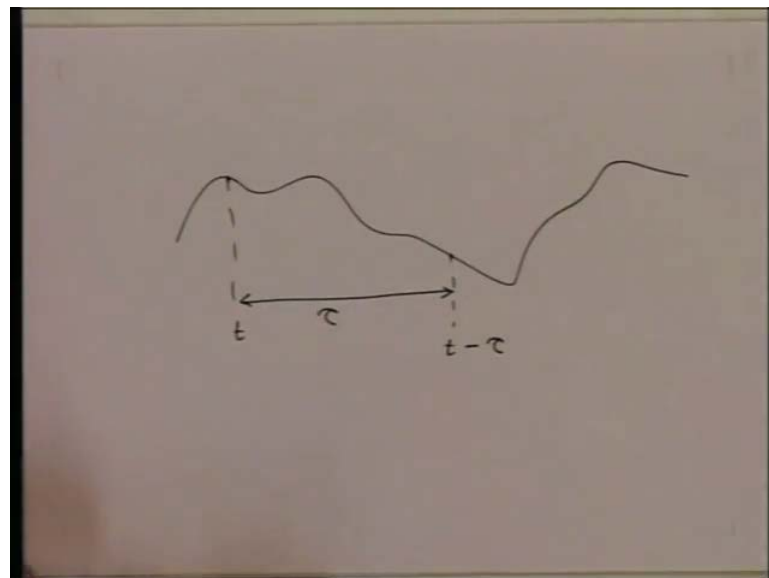
Whereas, when I see how the signal is behaving in a plain perpendicular to direction of propagation of this light that is captured by this quantity, what is called the spatial coherency. So, if I assume that the electric field let us say is varying as a function of time and it is having certain amplitude A .

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Then, I can define what is called the temporal coherence function is given by R of tau, where time is integrated from minus infinity to infinity A t, A conjugate t minus tau d; where tau is the parameter, which is the delay parameter. So, what; that means, is if we have a arbitrary time function.

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You just take the sample points, so I have a certain function which is like this I consider some point here, at some time t. Then I consider another point here which is at t minus tau as the difference in time is tau, I take the product of these 2 amplitude values. And I

take all possible pairs which are separated by distance τ from minus infinity plus infinity and if I take their sum that is a quantity which will be this quantity R of τ (Refer Slide Time: 07:15).

So, what R of τ essentially tells you is the how a particular point on this waveform is correlated with **another**, another point on the waveform, which is separated by a time τ . Say one can see very easily if this τ is very, very small which is somewhere here, then the 2 points are very well correlated. Because whatever is happening at this point more or less same thing will be happening in the neighboring point also whereas, if I consider a point which is very far away, then it is possible when the signal is positive this signal may be negative.

And if I consider all possible pairs you will see, that sometime you will have this positive this negative; whereas, this two goes to the positive as far same distance and so on. So, what we find is that, if we consider a typical signal, which is varying like this if τ is very, very small. Then the correlation is large; that means, the value of this R of τ is large. Whereas as the τ increases essentially this quantity the correlation decreases, so that is what essentially shown here, that if I take the plot of this quantity R of τ as a function of τ , then typically we get a behavior which will look like that.

So, when τ is equal to 0 you get the maximum and normalize this function is normalized to 1, so you say that the correlation is 1. And as the time increases then, this function drops; that means, the 2 points on the wave function, they become more and more uncorrelated. So, this function typically the correlation function which we have typically has a peak at τ equal to 0 and then it drops as τ increases. One can ask in this question, what is the use of this temporal coherence function.

And one can show that if we calculate the spectral power density of the signal this signal here, then the spectral power density is related to this (Refer Slide Time: 09:47) coherence function through a Fourier transform relationship. So, if I take this quantity R of τ and take it is Fourier transform, then I get the spectral distribution of the radiation.

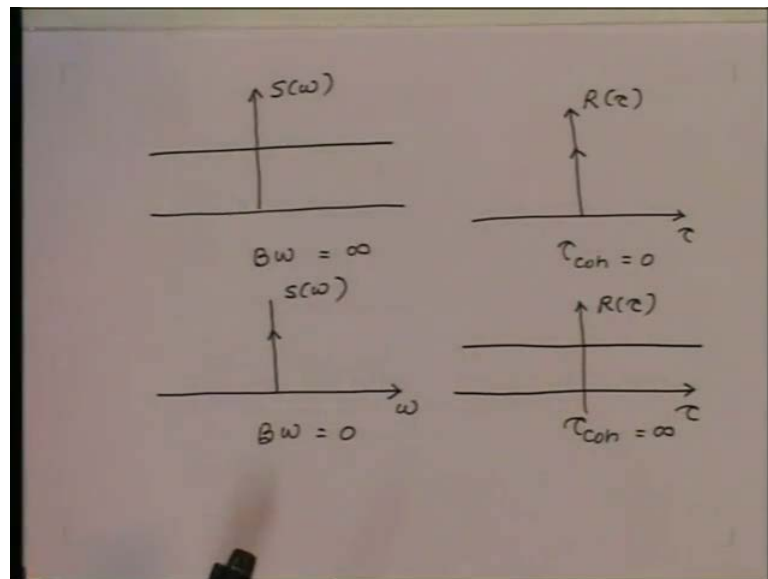
So, typically if I have the temporal coherence function, which is like this the Fourier transform of that would look something like this. So, here this is the plot a power as a function of frequency, for this radiation. So, we can measure the effective width of this function and that is what is called the bandwidth of the signal; whereas, if I measure the

effective width of this coherence function, we call that catch the coherence time which is denoted by T coherence.

So, in time domain if I capture the characteristic of this radiation, then it can be captured in the form of this tau coherence, which is the effective period or effective time over which the radiation will be correlated or will be coherent. If I go to the spectral domain then this is the frequency range (Refer Slide Time: 11:05) over which the power is more or less confined. And what one can show is that this tau coherence is approximately 1 upon bandwidth.

Say if bandwidth is small then we have a tau coherence which is very large and if the bandwidth is large, then the tau coherence is very, very small. So, if we consider the extreme cases of this two things.

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If I consider the tau coherence 0; that means, if I plot this R of tau, as a function of tau and if this quantity is a delta function. What that means, is that the signal is varying in such a way that every point is correlated with itself, but the neighboring point no matter how small, away it is the correlation is going to be 0. If this function becomes a delta function then it is spectral density which is the Fourier transform of this becomes a constant function.

So, this is S of ω , so this will become uniform function or in other words you will get a spectral width, which will be infinite; if the correlation function becomes a delta function, so in this case your τ coherence is 0 and your bandwidth is infinite. If we take exactly opposite case that the τ coherence is infinite spread like this, so this is R of τ what; that means, is that the radiation is now completely coherent in time. You know the value of the signal at a particular point and with 100 percent confidence you can predict the signal at any other time then the correlation function will be constant. And then the spectral density for this if you calculate that will be a Fourier transform of this which will be a delta function, so this is ω .

Now, a delta function frequency spectrum essentially represents a purely sinusoidal wave. So, what we find is, if we consider a sinusoidal wave then its correlation is perfect it is correlation function is constant and its coherence time is infinite, so in this case your τ coherence is infinite and your bandwidth is 0. In practice we neither of this situation nor of this situation, so we have a certain bandwidth and we have a certain coherence time for the radiation. But, the important thing to note is that they have inverse relationships.

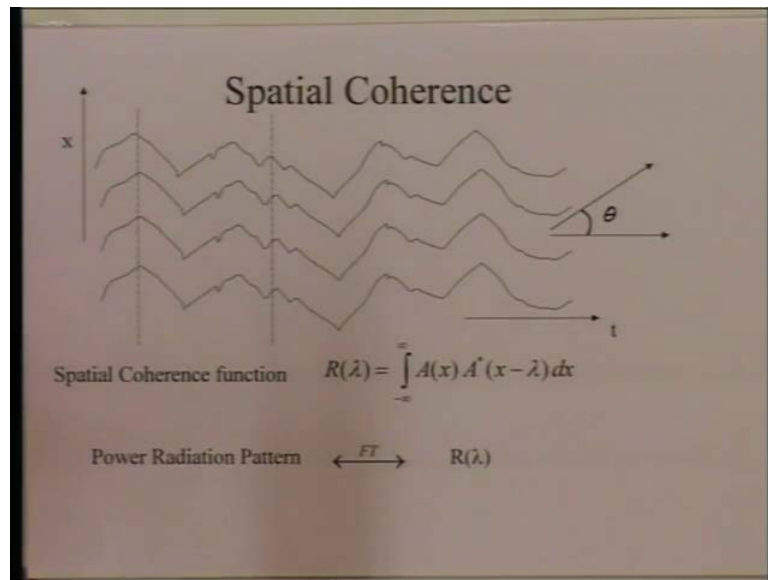
So, higher the correlation in the radiation means τ coherence is small that means, the bandwidth is large (Refer Slide Time: 14:45). So, if this quantity is small, then the bandwidth will be large if this quantity is large; that means, you will be able to predict the signal over very long distance the bandwidth will be very, very small. So, now, you can ask a question why you require actually the coherency in to your radiation. And what do we say is that since the dispersion is related to the spectral width of the source.

We have seen that the intramodal dispersion or chromatic dispersion is directly proportional to the spectral width, larger the spectral width worse will be the source (Refer Slide Time: 15:33) for optical communication. So, for a good source the spectral width should be as small as possible or in other words this bandwidth should be as small as possible, which in turn means that the τ coherence should be as large as possible or my radiation should be temporally coherent.

So, now, it makes sense why we are looking for coherency, so when we say that we are in need of temporal coherency, indirectly we are saying we are looking for an optical source, which has a very narrow spectral width. And that is the desirable feature, because

that has a direct bearing on the dispersion. So, here we conclude that we are looking for temporal coherency, because that is the one which guaranties you, the unless dispersion on the optical fibers. The next thing is the spatial coherency that do we require a spatial coherency in the radiation.

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So, let us ask what spatial coherency means, so let us say if I have a electromagnetic wave, which is travelling the wave is travelling in this direction. And this is the plain which is perpendicular to the direction of propagation, I have a variation as a function of time, which is not very systematic some randomly varying function is there. So, if I calculate the correlation between this point and this point, the temporal coherence in this signal is very small, because this function is varying more like randomly as a function of time.

But, if I ask what is the relationship between different points in the plain perpendicular to direction of propagation. And you will see that when this point is positive maximum this also is doing same thing, this also doing same thing, this also doing same thing (Refer Slide Time: 17:41) though this point is not doing same thing what this point is doing. So, these 2 points are not very well correlated, but these points are very well correlated.

So, if I define now the spatial coherence function, similar to what we have done is temporal coherence function that R of λ is again integral A of x a conjugate x minus λ $d x$. Where λ now is a separation in the plain perpendicular to

direction of propagation, I will get tau a coherence function, which will be similar to what we saw for the for the temporal variation. Only thing in this case you may have coherence function which would have two dimensional.

But, let us for simplicity assume that I am just plotting this function as a function of x . Now, the radiation is moving in this direction, so let us say if I consider any other direction at an angle θ with respect to this one. What one can show is that the power radiation pattern for this radiation has a Fourier transform relationship with the spatial coherence function. So, again we can use the same properties, which we saw earlier if this quantity is uniform; that means, all the points are perfectly correlated from minus infinity to infinity in this plain.

Then the Fourier transform of that would give me radiation pattern, which will be delta function. So, that will correspond to the propagation of radiation precisely in that direction, which is θ equal to zero. Whereas, if I have the this function which has a finite width we have also finite width for the radiation pattern. So, if I am having a function, which is partially coherent in a plain perpendicular to the direction of propagation. It essentially means that the radiation is not focused in this direction, but it goes over a **finite angle**, finite angle zone.

Now, if you recall we are seen when we are discussing LED, that the major loss of efficiency takes place, because the photons when they are born, they can go in any arbitrary direction. And only those photons which are within the critical angle cone can escape from the device. If by means of some mechanism, if you can create the photons only within the critical angle cone, then most of the photons will come out now, because they are lying within the critical angle cone.

So, if I make sure when the photon is born it travels only in the direction, which is within the critical angle cone, you will see that substantial improvement in the efficiency will take place. Similarly, if we have a very focus radiation or beaming radiation, then it can be launched very efficiently inside the optical fiber, because again this will lie within the numerical aperture cone of the optical fiber.

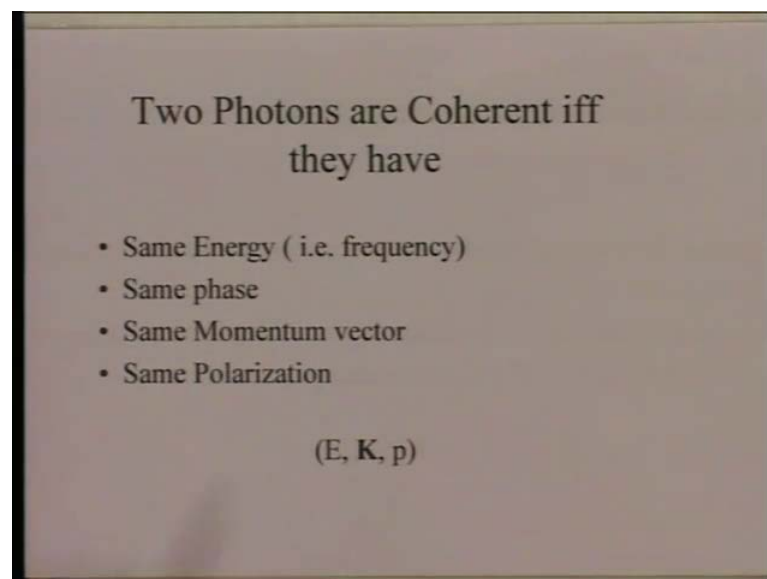
So, from the efficiency point of view essentially we are needing a highly directional radiation pattern we are looking for a **a** pattern, which will look like that. And if that happens then the efficiency of light generation will be high. And since the width of this

is related to the spatial coherency, small will be this width if the spatially coherent radiation is there, we want the spatial coherency in the source.

So, what we now find is that for increasing the bandwidth or for smaller dispersion, we require temporal coherency in the source, because that gives me the narrower bandwidth. On the other hand if I have a spatial coherency in the source, then it gives highly directional radiation and that in turn increases the efficiency of the device. So, now, it is very clear, that when we are looking for a laser, the laser gives a radiation which is coherent and it is coherent spatially as well as temporally.

And this coherency is needed because, indirectly it gives parameters which are desirable for high speed optical communication. So, now let us say if I do now define the coherency between the two photons what is the precise definition now is there for the coherency.

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So, we say that the two photons are coherent if and only if they have same energy and since the energy is equal to h into frequency and h is the planks constant it should have same frequency. They should have same phase the photon should be moving in the same direction as we saw for the spatial coherency all the photons moving in same direction, the mean is highly directional. So, this comes from the spatial coherency, this comes from the temporal coherency.

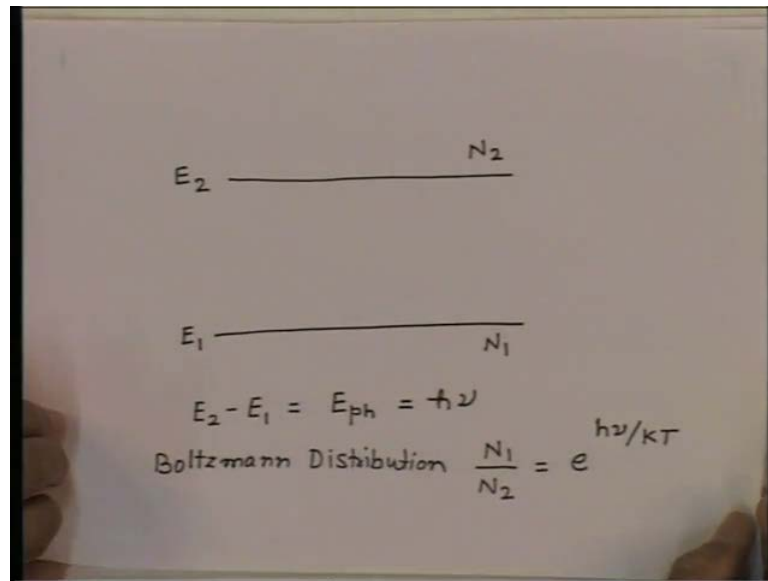
But now, if you look at the photon as an electromagnetic wave, then you have the vector nature of light; that means, the electromagnetic wave has electric and magnetic fields. And the electric field has different behavior as a function of time, which is characterized by this parameter, what is called the polarization. So, it is possible that the two photons are moving in the same direction, so they have the same momentum vector they have a same frequency. But, if the electric fields are oriented perpendicular to each other these two photon will not show any interference.

So, coherence **coherence** you are coherent behavior of light can be seen through the interference phenomena and for that to take place not only these quantity should be same, but the polarization of the photon also should be same. Now, how do we define the polarization of a photon, it is a polarization of that electromagnetic wave to which this photon belongs. So, if I take an electromagnetic wave or light source, I can find out what is the state of polarization for that light beam. The polarization state of a photon which is taken form that light beam is the same as that of the light beam.

So, then we say that two photons are coherent if these three quantities energy momentum and polarization are same. So, essentially coherence can be defined by a triplet, which is energy momentum vector and the state of polarization. So, we can say the two photons will be completely coherent, if the triplet is same for the two photons; with this understanding of coherency.

Now we can go to the very basic processes which take place inside the material and then from there we try to establish the coherent radiation. Conditions under which the coherent radiation would take place which will essentially give the foundation of the laser.

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So, let us consider any material and let us consider only a simple system of two energy levels. So, let us say we have this two energy levels. And let us say this energy level is given by E_1 this energy level is given by E_2 . This is the lower energy level and this is the higher energy level, difference of these two energy level E_2 minus E_1 is let us say is denoted by E_{ph} . Now, because of thermal excitation the electrons occupy different energy levels. So, you have a density of electrons in this energy level and you have a density of electrons in this energy level.

So, if I consider a general material, then the density of electrons at different energy levels are related and that is given by the Boltzmann distribution. So, if I say that the electron density in this level is given by N_1 and at this level is given by N_2 . Then by Boltzmann distribution (No audio from 27:49 to 28:00) N_1 upon N_2 will be $e^{-h\nu/KT}$ where $h\nu$ is the energy difference in this two levels. So, $h\nu$ in this case is E_{ph} .

So, this quantity is $h\nu$, where h is the planks constant and ν is the frequency of the photon, K is the Boltzmann constant and T is the absolute temperature. So, firstly, what we note here is that the density exponentially decreases as we go to the higher energy levels, but N_2 upon N_1 will be $e^{h\nu/KT}$. So, one can just take typical numbers and try to see, if I calculate at room temperature some typical

energy levels, what is the ratio of the electron density at energy level E 1 and energy level E 2.

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$$T = 300 \text{ K}, \quad \lambda = 0.7 \mu\text{m}$$

$$E_{ph} = E_2 - E_1 = hf = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.7 \times 10^{-6}} \approx 3 \times 10^{-19} \text{ J}$$

$$KT = 1.38 \times 10^{-23} \times 300 = 4 \times 10^{-21} \text{ J}$$

$$E_{ph}/KT \sim 100$$

$$\frac{N_2}{N_1} = e^{-E_{ph}/KT} \sim e^{-100} \approx 10^{-32}$$

So, let us say I take the room temperature, so T is 300 k and let us say this quantity here nu (Refer Slide Time: 29:35) that corresponds to a wavelength, which is about 0.7 micrometer, so let us say we have lambda is 0.7 micrometer. Now, this energy difference E 2 minus E 1 that is h into f or h into nu, which is h into velocity of light divide by lambda. I can substitute now the values here, so for h which is 6.6 into 10 to the power minus 34 into velocity of light, which is 3 into 10 to the power 8 divided by a wavelength lambda which is 0.7 into 10 to the power minus 6.

So, approximately this number would be 3 into 10 to the power minus 19 watts second, we can calculate the thermal energy which is K T. The Boltzmann constant is 1.38 into 10 to the power minus 23 multiplied by temperature, which is 300. So, that is approximately equal to 4 into 10 to the power minus 21 watts second. So, if I get this quantity E 2 minus E 1 by K T, so this is which is nothing, but E p h, see if I get this quantity E p h by K T. This will be typically of the order of about 100 or the N 2 by N 1, which is equal I am taking N 2 by N 1 here, see e to the power minus E photon divided by K T.

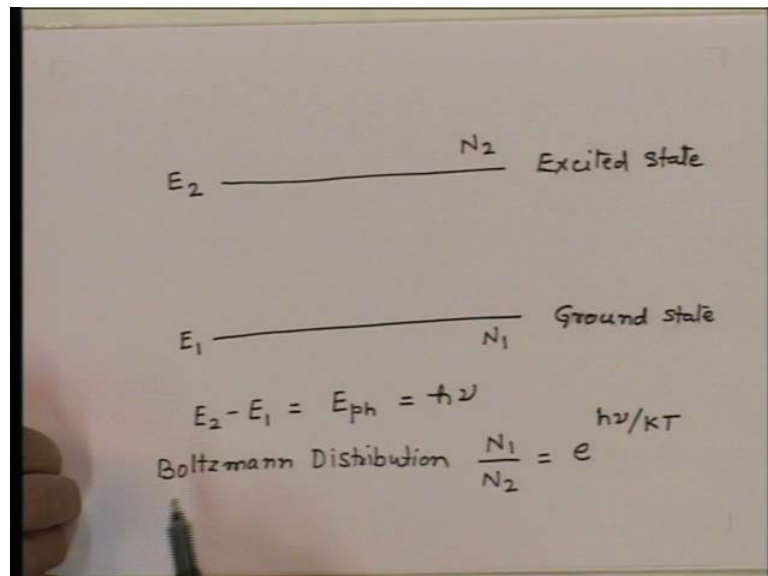
So, this will be e to the power minus E photon by K T, which is of the order of e to the power minus 100 (No audio from 32:29 to 32:40). So, this number is typically of the

order of about 10^{-32} . So, what; that means, is that if we consider the energy, levels which have energy difference correspond to about 0.7 micrometer then the ratio of the electron density in the upper level to the lower level is 10^{-32} .

That means, this level is practically empty compared to the electron density in this level, most of the electrons are going to be in this level. And there are hardly any electrons will find in the upper level if the energy density or energy difference is of the order of this quantity which corresponds to about 0.7 micrometer, this is the natural situation. So, what we find is that if I consider a system at thermal equilibrium, which has a temperature t , then the electron density at the upper energy level will be negligibly small compared to the electron density at the lower energy level.

Now, let us say in general we are having now the electrons, which are lying at the upper energy level and they are lying in the lower energy level. Now if the electron is lying in the upper energy level it has a tendency to relax to the lowest energy level state. So, if the electron was in the upper energy level, we say the electron is in excited state, when it comes down to the lower energy level, we say the electron is relax to the ground state.

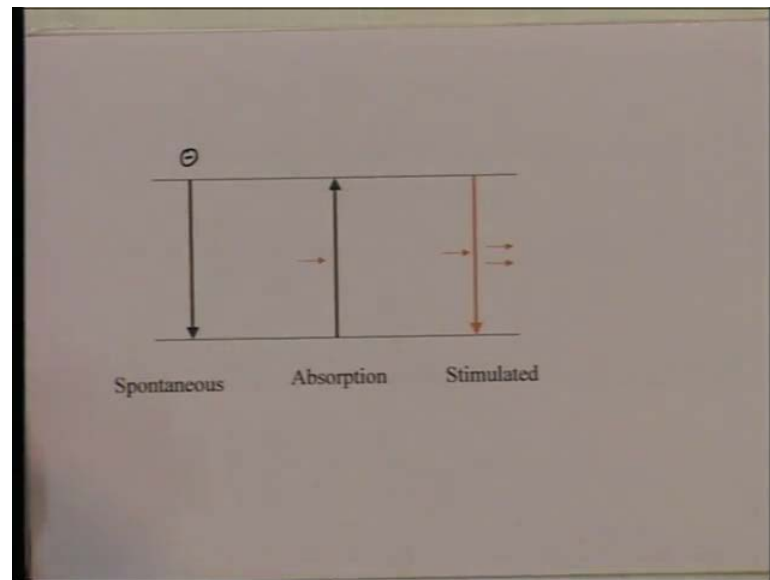
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So, the two states which we have shown here we can say this is the ground state and this is the excited state. So, since you are having now the distribution of electrons there will be some electron, which will be in the excited state in the thermal excitation; and large

number of electrons will be there on the ground state. However, what we are saying is if the electron is in the excited state it has a tendency to relax to the ground state and when that happens essentially the energy is shed away. And this energy will be emitted in the form of photon of energy which is $E_p = h\nu$ which is $E_2 - E_1$.

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So, what we say is that, if the electron is in the excited state which is here then naturally it has a tendency to come down to the ground state or relaxed to the ground state. And when this transition takes place a photon is released in this process, so let us say this photon is released in this process. Now, since this process does not require anything else it is just that since the electron is there in excited state and it has a natural tendency to relax to the lower energy state this process is what is called the spontaneous process.

And whenever spontaneous process takes place a photon is released of a frequency which is $h\nu$, where $h\nu$ is nothing but, the energy difference between these two levels. So, as we have call this is E_2 and this is E_1 , this quantity $h\nu$ will be $E_2 - E_1$, this is one process which takes place inside the material. The other process which you observe is the suppose we take a material and shine light on the material, when the light propagates through the material the intensity of the light reduces.

What that means, is that the part of the energy of the light beam has been absorbed inside the material, what does this energy do these energy essentially excites the electrons from the ground state to the excited state. So, if the photon was incident on the system and if

the electron was there on the ground state. Then this process, what is they call the absorption process, will take away the energy from the photon give the energy to the electron. And electron will excite it get excited to the higher energy state and the photon will be lost.

This process we always see in nature because we have seen always when the light beam passes through a material it attenuates. So, number of photons are lost the energy of these photons is given to the electrons and electrons are excited. So, internally the material is excited, because these electrons have gone to the higher energy state and the photons are lost. Also when we consider a normal material, we also see that it gives you the black body radiation.

So, you always see some light or electromagnetic waves coming out of a body. So, even this phenomena in some sense has been observed, you can see that this phenomena would exist. What is proposed now is that in addition to these two phenomena which one can observe very easily in nature there is also a process which is the force process. And that process is if you are having a electron in the excited state here and if the photon is incident on to the system.

Then this photon may force this electron to jump down and when this jumping takes place it will release a photon. So, this original photon will force this electron to jump and emit another photon. So, this process which we are talking about is not taking place on it is own by electron it is a force process by photon and this process we call as the stimulated process.

And the postulate was that when this forced process takes place, the time the photon which is released now additional photon. Since, it is now completely under the influence of the original photon, the properties of the new photons are exactly identical to the original photon. Or same in terms of coherency the two photons now which we have they are completely coherent photons. And we have seen what complete coherency means they are spatially coherent and they are temporally coherent and they have the same polarization.

So, the two photons now will be travelling together in the same direction, they have a same frequency and they will have the same polarization. So, this process now is with the presence of a photon, **this process is with the presence of photon**. So, when the

photon or light beam is incident on a material two processes might take place, one is the photon may get lost and the electron can get excited. Or the photon may force the electron if it is in the excited states to jump down and release another photon which is completely coherent with the original photon.

Now, this process is rather a multiplicative process, because if you are having one photon and given 2, this 2 if you are having more electrons present here they will pull down more photons and so on. So, you have a process which grows very rapidly the stimulated process, what one can do is now one can define some quantitative parameters to define this processes. And these are defined by the transition probabilities which are called the Einstein coefficients. So, the process which does not depend upon the photon that is given by the coefficient which is called A_{21} and the process, which takes place in the presence of photon that coefficient is denoted by B .

So, this process which is taking electron up from energy level 1 to 2 that is coefficient is given by B_{12} and the stimulated process the coefficient is given by B_{21} . So, now, we can say that if you are having a electron density here. The rate of electron transition downwards by it is own nature by spontaneous nature will be proportional to the number of electrons present in the upper energy level.

So, now, we can write down essentially the equations for this processes, so we say if the electron density in the upper level is N_2 . And the electron density in the lower level is N_1 then we can write down the equations for three processes as follows.

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Spontaneous process:

$$-\frac{dN_2}{dt} = A_{21} N_2$$
$$N_2(t) = N_2(t=0) e^{-A_{21} t}$$

Spontaneous life time

$$\tau_{sp} = \frac{1}{A_{21}}$$
$$N_2(t) = N_2(t=0) e^{-t/\tau_{sp}}$$

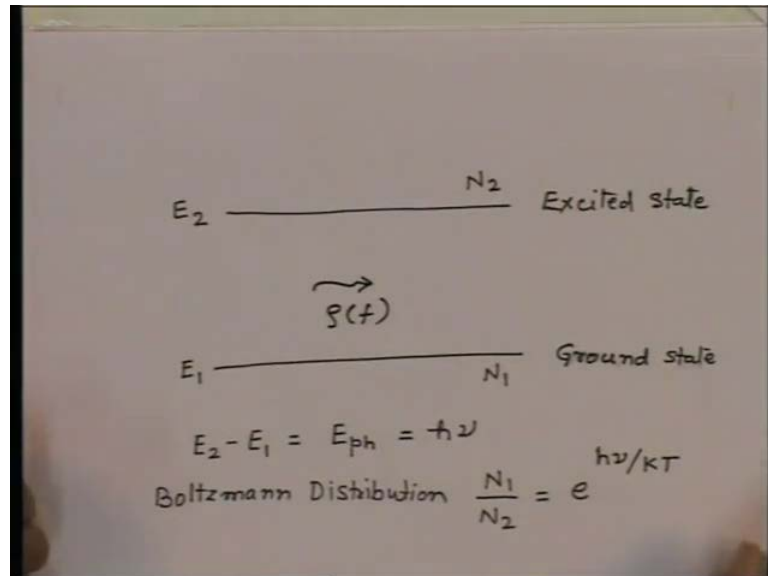
So, first take spontaneous process (No audio from 43:13 to 43:26), so here the rate of electron transition downwards or rate of depletion of electrons in the upper level, which will be given by minus dN_2 by dt (Refer Slide Time: 43:41). That will be proportional to N_2 the electron density in the upper level and the proportionality constant will be this quantity A_{21} . So, we can write the equation minus dN_2 by dt rate of change of electrons in the upper level or decrease of electrons per unit time in the upper level that will be equal to A_{21} into N_2 .

We have solved this equation in a simple solution for this will be N_2 , as a function of time that will be N_2 at t equal to 0, e to the power minus A_{21} into t . If we define a parameter what is called the lifetime of the electron in the upper level. And call that lifetime as the spontaneous lifetime then this quantity A_{21} will be nothing but, 1 upon the spontaneous lifetime. So, you can define the quantity what is called spontaneous lifetime τ_{sp} and that is equal to 1 upon A_{21} .

So, this quantity here density of electrons in the upper state will be N_2 at t equal to 0 to e to the power minus t upon τ_{sp} . So, what that means, is that if the electrons are transported to the upper energy level, then naturally the electrons will have a tendency to decay. And this decay will be exponential with a characteristic lifetime which will be τ_{sp} , so this process is a very straight forward process.

Similarly, we can write down the equations now for the other two process and since the other two processes depend upon the photon present there.

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So, let us say there is a photon flux density which is given by rho of nu, so what we are saying here having a material like this you are having a photon flux with here, which is as a function of rho of f where f is the frequency.

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Absorption
 $-\frac{dN_1}{dt} = B_{12} \rho(f) N_1$
Stimulated process
 $-\frac{dN_2}{dt} = B_{21} \rho(f) N_2$
 $A_{21} N_2 + B_{21} \rho(f) N_2 = B_{12} \rho(f) N_1$

So, now we can write down the absorption process, where electrons are taken from the ground state to the upper state (Refer Slide Time: 47:04). And now the rate of change of

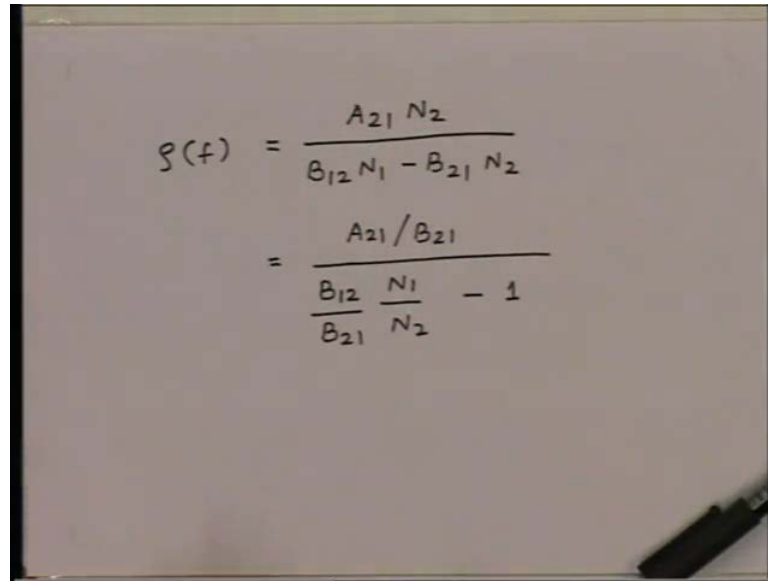
the electron density in this level will be proportional to this quantity, will be proportional to what is this quantity or density here in the lower state. And is also proportional to the photon flux which is ρ of f . So, we can write here this is $-\frac{dN_1}{dt}$ that will be equal to $B_{12} \rho$ of f into N_1 .

Similarly, we can write down the stimulated process where a transition is going to be downwards (Refer Slide Time: 48:13) again the rate of change of electrons in the upper level will be a proportional to N_2 , will be proportional to the photon flux density. And the proportionality constant will be B_{21} . So, we can get here $-\frac{dN_2}{dt}$ that will be equal to $B_{21} \rho$ of f into N_2 . So, now we have got 3 processes simultaneously operating inside the material that 2 processes, which are bringing the electrons down from the excited state to the ground state. And then the one process which is upward process which is taking electrons up from the ground state to the excited state.

So, in thermal equilibrium since the number of electrons are not changing either at a ground state or in the excited state, the total downward transition must be completely balanced by the upward transition. So, we can essentially write down the balance equation from here, which says that the upward transitions are equal to downward transitions. So, we can have these two things equal to this, so we can get from here essentially $A_{21} N_2$ plus $B_{21} \rho$ of f N_2 that is equal to $B_{12} \rho$ of f into N_1 .

So, these two are downward transition this is the upward transition and in the equilibrium these transitions are completely balanced. So, neither electron density is changing in the upper energy level nor it is changing in the lower energy level. If you rearrange this we can find out what is the net photon flux coming out of the material.

(Refer Slide Time: 50:28)


$$\begin{aligned} \rho(f) &= \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \\ &= \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \end{aligned}$$

So, if I simply rearrange this and substitute in to this, we can get the net photon flux which will be coming out of this rho of f that will be equal to $A_{21} N_2$ divided by $B_{12} N_1 - B_{21} N_2$. By taking this $B_{21} N_2$ common from here that will be equal to A_{21} / B_{21} divided by $B_{12} / B_{21} \times N_1 / N_2 - 1$.

So, what this expression essentially is giving you that if all the three processes spontaneous emission absorption and stimulated emission take place simultaneously inside the material. Then there is a net photon flux from the system and that photon flux density is given by this; what one can do now is that since the system is a thermal equilibrium. This photon flux must be identical to the black body radiation, because if the body is at some temperature t it gives you radiation.

So, if I now take the black body radiation expression and compare this expression with that, then we can reduce some relationship between these quantities A_{21} / B_{21} , A_{12} / B_{12} and B_{12} / B_{21} . These quantities now A_{21} / B_{21} and B_{12} / B_{21} are called the Einstein's coefficients, because you are the one first we investigated this problem. And then try to establish the relationship between these quantities.

So, in the following lecture what we will do is we will take this expression of the photon flux density, compare this expression for the photon flux density from a black body radiation and then deduce some very important conclusions.