Advanced Digital Signal Processing - Wavelets and Multirate
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Lecture No. # 46
Zoom In and Zoom Out using Wavelet Transform

A warm welcome to this session; continuing the theme of wavelets and multiresolution signal processing and multirate filter banks. It is my proud privilege today to introduce a guest speaker that we have in this series. And I would like first to mention some of his biographical details, before bringing him before you. And also to stress and emphasis the role - the very important role that he has played in the construction of this course, in the construction of this series of sessions in lectures. Professor Aditya Abhyankar is going to come before you today, and the most important qualification that he has is, that he is very well known in the area of wavelets and filter banks in the country. It has been my proud privilege to have him as a reviewer for this series, and he has carefully and meticulously given suggestions on the content, and the manner in which the lectures have been constructed. And therefore, it is both out of gratitude in pride, that I have invited him today to speak on the theme and to present some of his own experience, and some of his own thoughts, in this broad field of wavelets multiresolution signal processing, image processing, filter banks and so on.

Professor Aditya Abhyankar obtained his degree of Doctor of philosophy from Clarkson University New York, USA in 2005. He has for quite some time been on the faculty of the University of Pune. In fact, currently he is a professor and head of department in the University of Pune, in the department of technology. In particular as I said, he has worked to a great extent on the themes of signal processing pattern recognition, and I know for sure that he has made some very important contributions to the area of biometrics, specifically finger prints. You will recall that some stage we had said little bit about, the importance that wavelets have had in finger print analysis, finger print pattern recognition and so on.

Professor Aditya Abhyankar has important contributions in this field. In addition, he has been leading several initiatives in the University of Pune. And as I said earlier on, a very important role that he has played in this series, is to review and give us very valuable inputs on these lectures, and therefore without taking too much time in this session, I would like straight away to put before you professor Aditya Abhyankar, will now talk to you on some theme related to wavelets. Thank you.

Hello, and welcome to this session on the topic of wavelets, and in the broader perspective, a topic that deals with joint time frequency analysis. It is my proud privilege that I was called upon to review the beautiful video lectures given by Doctor Gadre; who is a professor at double E department IIT Bombay. And there is a reason as to why am saying this. We have quite a few numbers of books written on wavelets. In fact, today wavelet has become a buzz word. However, almost all the material that has been written on wavelets, it is written by mathematicians and predominantly it is written for mathematicians. For engineers, to understand the concept behind these mathematical formulas, it was required, by someone to simplify those mathematical formulas, and bring out the physical significance, associated with these mathematical formulas. It was pleasant experience, going through the beautiful video lectures recorded by Doctor Gadre, and that is because, he has simplified so many beautiful concepts associated with wavelet transform.

It is also my privilege, that he has given me an opportunity, to record few of my thoughts on this beautiful subject of wavelets, and this very first lecture, we have title this lecture; as zoom in and zoom out using wavelet transform. Let us begin, now a days, wavelet transform has become a buzz word, I honestly believe that if last hundred years were the hundred years of Fourier transform. The coming hundred years are going to be hundred years of wavelet transform, and that is because wavelet transform has so many beautiful facets, so many beautiful characteristics associated with it. And in this session, we try to bring out one such beautiful facet, which is the zoom in and zoom out feature of wavelet transform. If we compare wavelet transform, with the conventional methods of representing the signals or functions, then relatively it is a new field, and then we might pose few questions, and those questions will be answered by a beautiful property, associated with wavelet transform which is known to us as multi resolution analysis or very popularly it is known as MRA. Let’s pose more fundamental question first.

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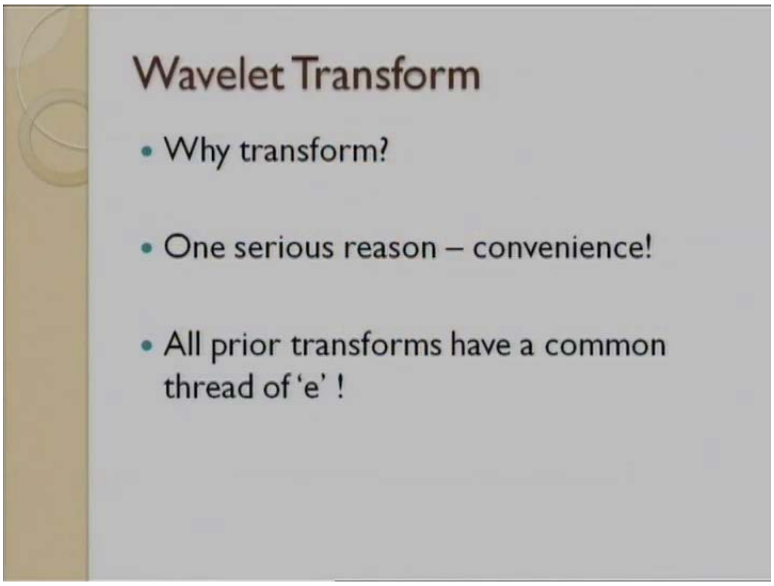

Why transform; not all the operations are called as transforms. For example, when it comes to image processing, there are certain operations like histogram equalization. We do not name that operation as histogram equalization transform. there are only very few certain class of operations which are termed as transforms, and y at all take this pain of transforming information from one domain to another domain. There are multiple reasons; however, the strongest reason is pure convenience, for the analyzer it becomes extremely convenient to understand the representation of the information in one domain rather than another domain, and we have already seen one example; the example of music. We do not understand music as just few time domain signals with varying voltage; not really. We understand music as a sequence of frequencies, and so it makes sense to actually transform this signal into frequency domain, and then the analyzer will be more comfortable dealing with those frequencies.

Ultimately we want to design those beautiful filters, and it is more convenient to design filters in frequency domain, compare to time domain or partial domain. So, the main theme behind doing most of these transformations, is purely the convenience of the analyzer. Wavelet transform in a way is strikingly different than most of the conventional transforms. If we go through most of the conventional transforms that we have studied, with the likes of Fourier transform, Laplace transform, z transform, and probably the first transform to which we get formally introduce to, is the logarithmic transform. For all this transforms, the basis function comes out of a beautiful constant, a constant e. And in a way all this transforms they have some common thread in all of them. All the basis functions: the kernel functions, they are of the nature e to the power some variable. It could be frequency or it could be any variable. However, wavelet transform differs from all these transforms, and we have to go little beyond the purview of all these transforms, to really make sense of the wavelet framework.

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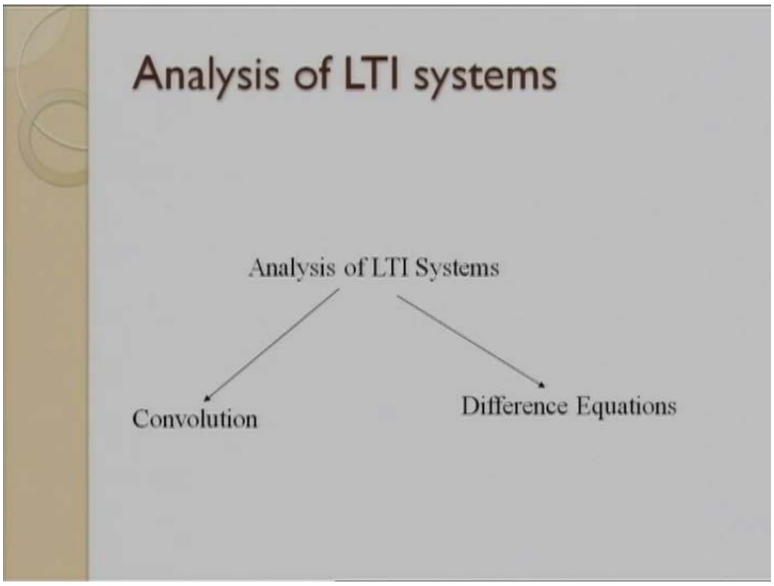

So, why at all do this, we already know how to analyze linear time in variant systems, and if we see this particular slide; traditionally we have two methods, based on convolution or based on difference equations.

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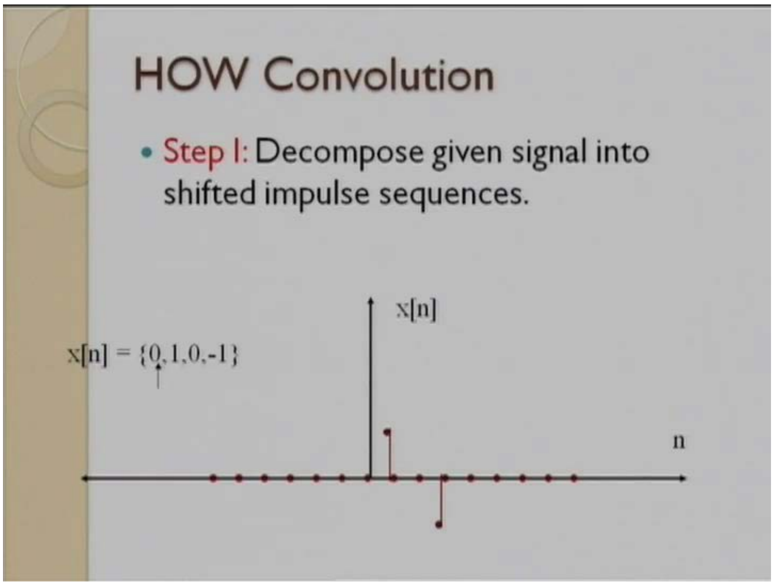

And typically convolution comes out of the fact, that given any signal x of n, I can decompose that signal n 2 sequence of shifted impulses.

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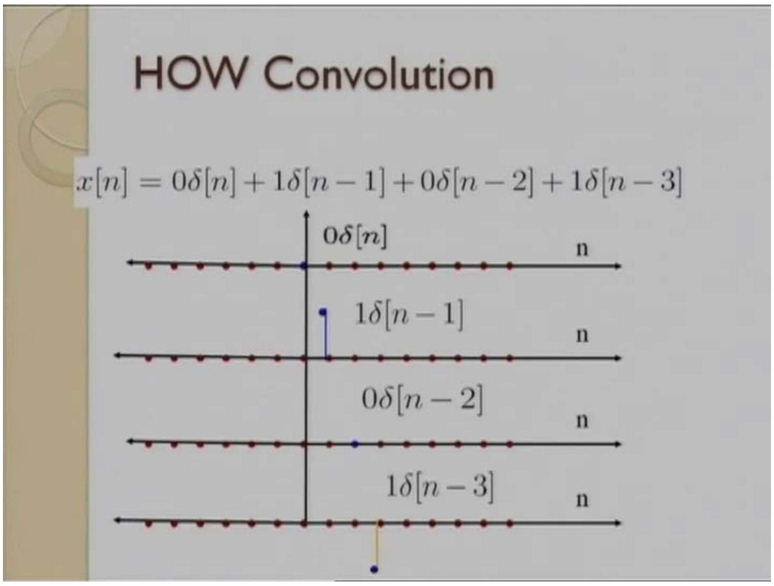

And once I do that, I can very well right x of n as the scale summation of shifted version of impulses. Remember we can do this because we are talking about linear and shift in variant systems.

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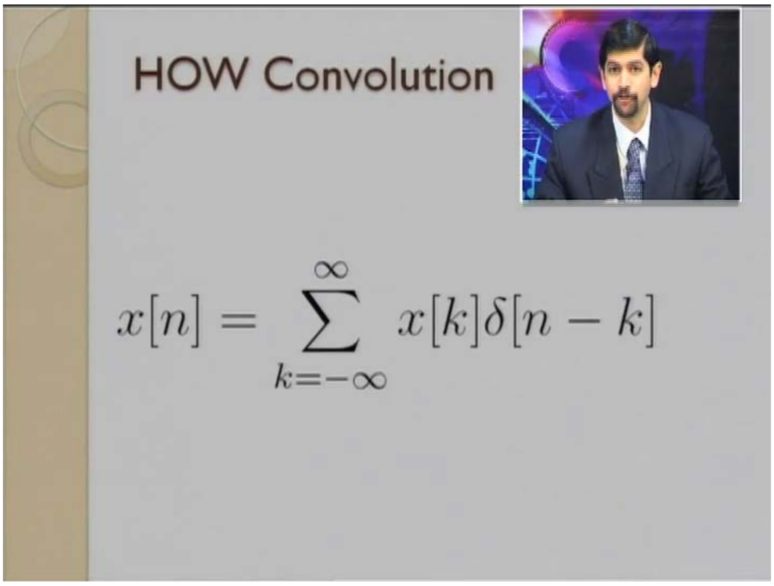

Because my system is linear, I can go on accumulating the information. I can add up everything, because the system would follow supper position theorem; the additive and homoginative properties. And because my system is time in variant or shift in variant, I can go on shifting my impulses.

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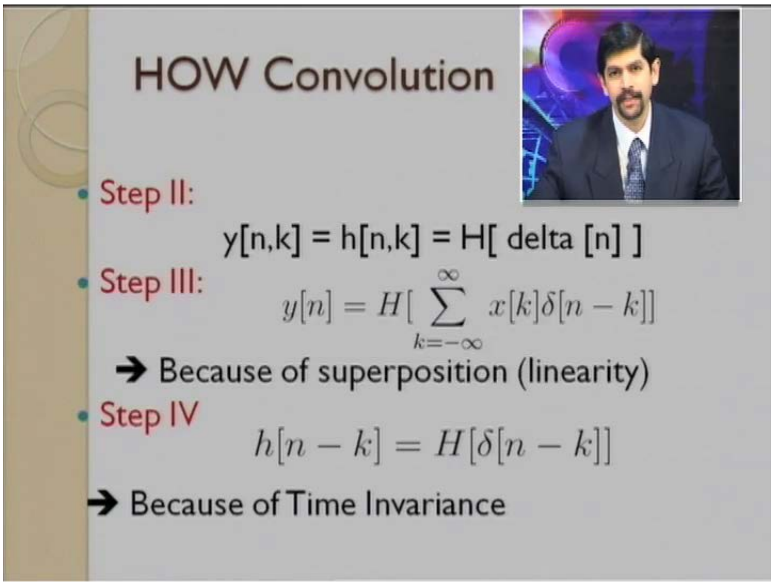

And once I do this, what we say is, if I understand how my system reacts to these shifted versions of impulses, I can characterize my system completely. This is what is known to us as an impulse response.

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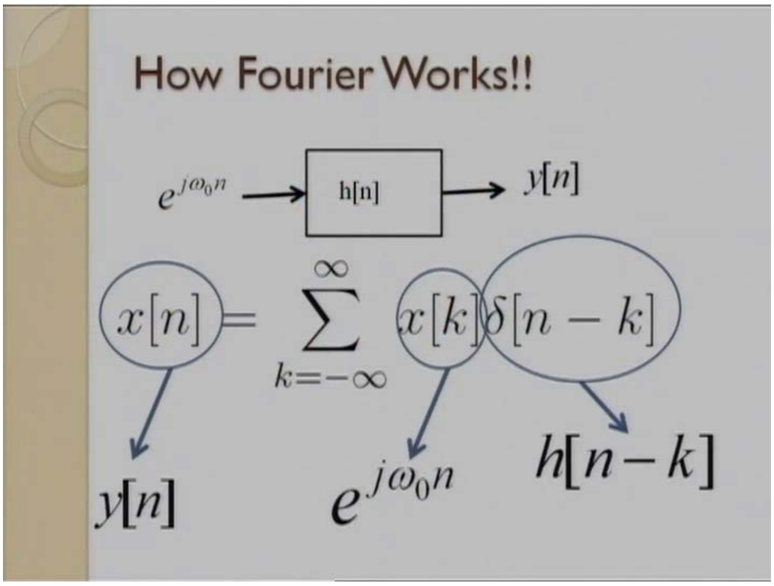

And if I know the impulse response of my system, and let say we call this impulse response as H of n. And if I excide my linear shift in variant system with an exponential. Then what kind of output the system would produce, that was the question that was posed by Doctor Fourier. And if we call this output as why of n, then we realize that this is a very interesting situation that we are in. We have linear shift invariant system. We have characterized that system, by virtue of impulse response. And we are stimulating that system with an exponential, with known frequency, e to the power j omega 0 n and omega 0 is the known frequency. This formulates one very interesting pair of Eigen value and Eigen system for linear and shift in variant systems.

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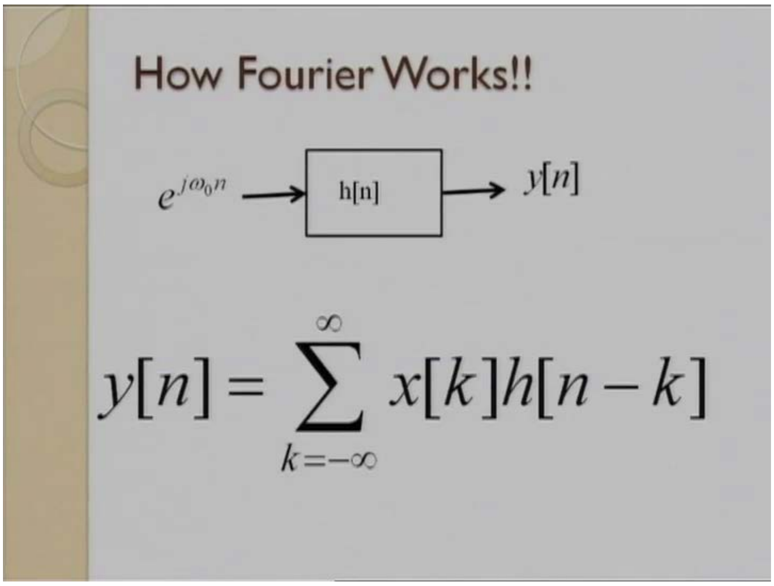

And that is purely, because if I split the term e to the power j omega 0 n minus k. Then definitely the summation happens for variable k, and I can take out e to the power j omega 0 n outside the summation.

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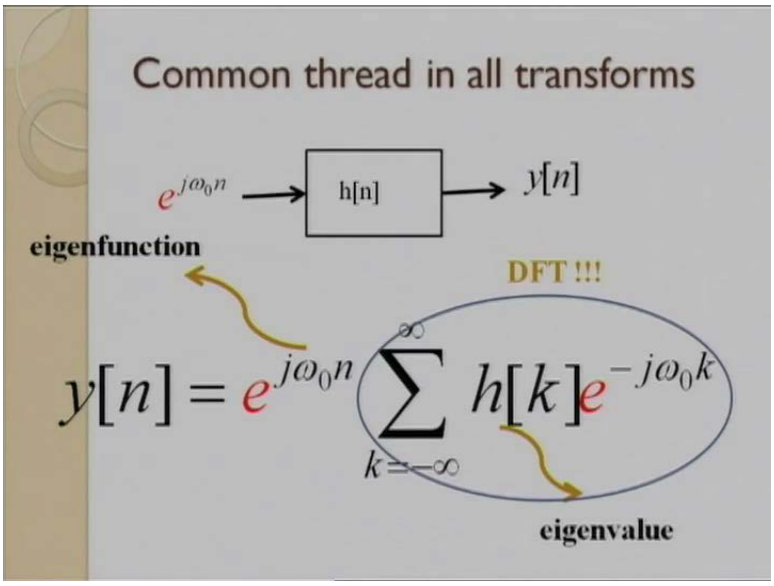

And this is very interesting, the system was excited with e to the power j omega 0 n and what comes out of the system, is again the same exponential, with the same frequency plus something. This exponential e to the power j omega 0 n is known to us as in Eigen function. And mod it produces along with it, is known to us as an Eigen value, and this

Eigen value in broader sense is known to us as furrier transform. What is really interesting, is to observe the common thread in all these transforms, and like we said before for all the different transforms; like Fourier transform, Laplace transform and z transform, the kernel function remains almost same. In case of z transform, the basis function is z to the power minus n, where z is again equal to e to the power j omega. So, we are talking about essentially similar looking kernel function or basis function, same holds to for Laplace transform. And where exactly this constant e comes from, that is an interesting thing to notice.

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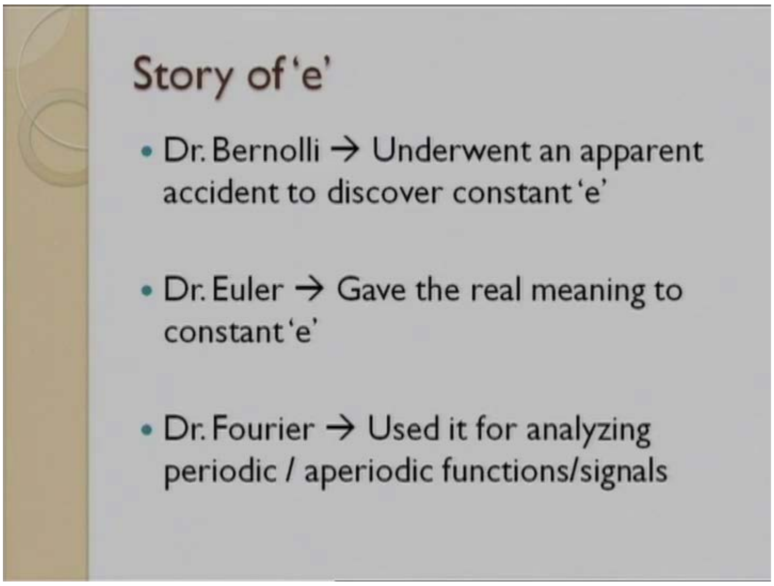

It is a combination of the efforts put in by three great genius scientists. There is a wake story about how Doctor Bernoulli was able to invent this constant e. For that matter, most of the inventions and discoveries are pure accidence, or we can say they are apparent accidents, they appear as accidents. However, there are tremendous dedicated efforts by the respective scientist or researches, that actually goes in and only then the elevated minds will be able to grasp and capture that particular idea. Something similar happened in this case also. This is the wake story and there is no authentic source to the story, however it is very interesting. Doctor Bernoulli was trying to help out his banker friend, and his business was not actually picking up, and he gave him a beautiful solution, which was based on the formula of compound interest, and typically we know, how the formula actually looks. The formula goes like this.

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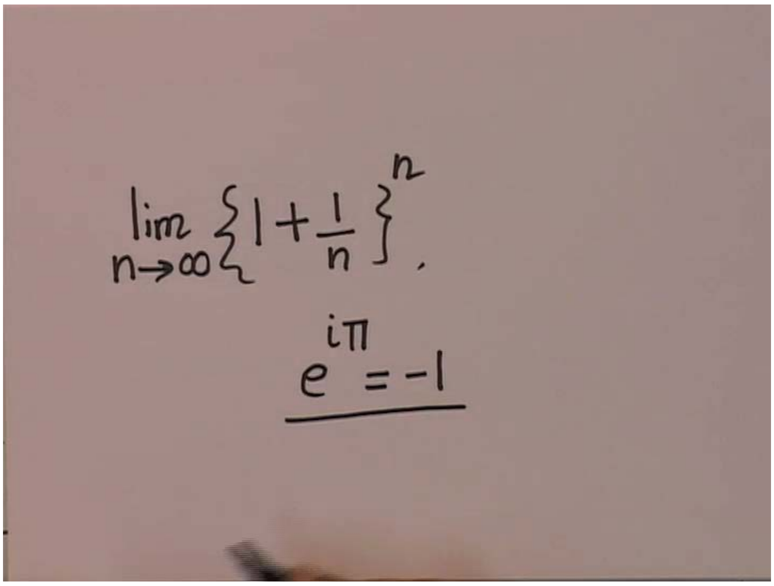

If you are investing one rupee as the principle amount, then the formula found compound interest is, one plus one upon n bracket raise to n, and Doctor Bernoulli simply brought n the series expansion, by putting limit as to when n tends to infinity. We can very well solve this limit. Let's very quickly do a small excise in matlab. Let us define for n is equal to one to some large number, we cannot go all the way till infinity, and then let us implement this formula; one plus one upon n bracket raise to n, and lets end this for loop. And then we will see, that after few iterations it will saturate to value of two point seven one eight three. This is the value which is known to us as the constant e. So, it is simple compound interest formula, and Doctor Bernoulli was able to help out his banker friend, but in the mean while he was also able to discover this beautiful constant e, that was not enough, and then comes the second genius in this story, as can be seen from the slides is name is Doctor Euler. And Doctor Euler, he gave a different meaning all together to this constant e, and he gave us this beautiful identity which is known to us as Eulers identity, and the identity goes like this.

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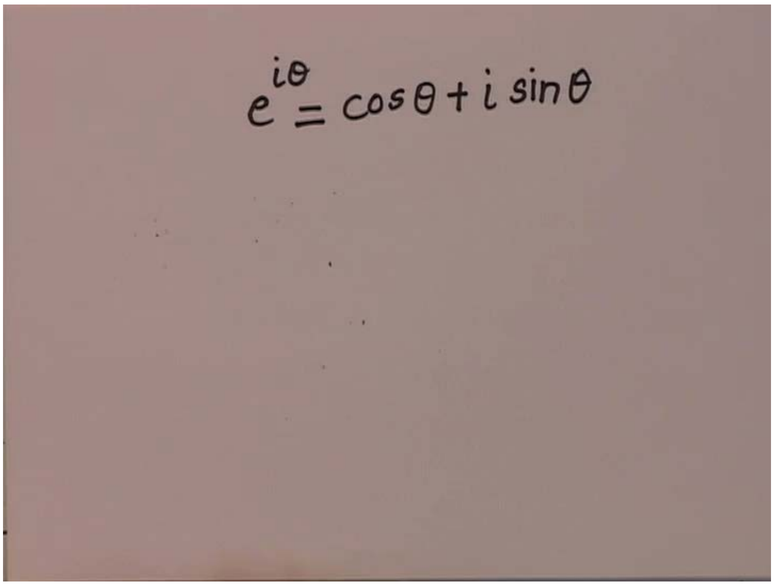

He told us, e to the power i pi is equal to minus 1, this is notice known to us as Eulers identity. For almost last three hundred odd years or so, mathematicians they have been trying there level best, to come up with a formula mathematical proof of this identity, but they are not yet successful. So, once again and elevated mind, and he was able to capture the e sense of that particular idea. What is so special about this identity, is an extended version of Eulers identity. He told us that e to the power i theta, can be disintegrated into cos theta plus i sin theta, and we know cos theta and sin theta together they formed and orthogonal system. So, the beauty associated with this exponential curve is, you can take any point on this curve and draw a tangent. the why intercept and the slope of this tangent will match, and we know an equation of straight line is y is equals to m x plus c, that essentially indicates that on this curve, at any point on this curve I can resolve that entity, along to axis, which are orthogonal in the nature; cos and sign, and it is this beautiful Euler said identity that made all the transforms orthogonal in their nature; that was also not enough, and then came the third genius scientist in the story, his name is, of course Doctor Fourier, as can be seen from the slide, Doctor Fourier told us how to analyze periodic or a periodic functions or signals. The legacy of transforms that we have with us, is a contribution of these three great genius scientists.

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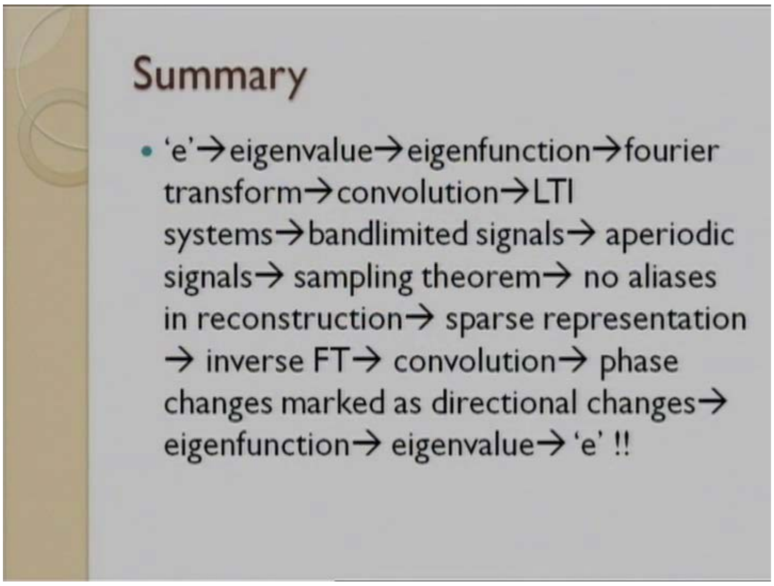

We can summarize and that can be seen from the slide that we have this constant e, which creates and Eigen value, Eigen function system, and the Eigen value is nothing else, but the Fourier transform, which can be implemented using convolution; which is at the heart of any d s p processor. This convolution is possible only if we have linear time invariant system, and then we are talking about band limited, a periodic signals, a specific class of signals. They should obey sampling theorem and then they will be no aliasing in reconstruction, which guaranty is sparse representation, which in a way would guaranty inverse Fourier transform, which can then once again be implemented using convolution. The phase changes are marked as the directional changes, which is once again a property of being in Eigen function of the system; that reduces an Eigen value that takes us back to this constant e. So, at the heart of all these transforms we have this beautiful constant e, and we have this beautiful legacy of transforms. For periodic signals, we have series representation, for a periodic signals or functions, we have transforms. So, we if we already have series representations and transforms, then comes one important question, and that important question is why at all wavelet transform.

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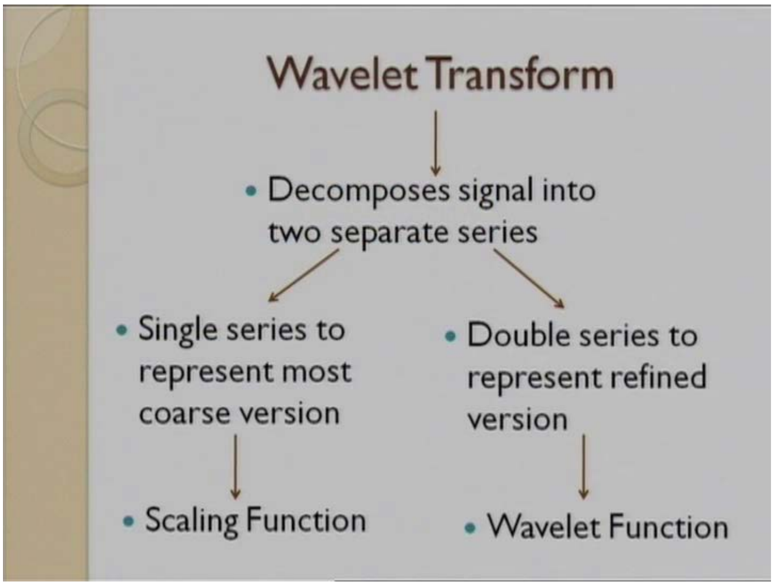

Let us go back to the slides and we will realize, that one beautiful feature associated with wavelet transforms is, it decomposes signal into two separate series, a single series to represent most course version, which leads us to scaling function or which is also very popularly called as the father function. And the double series, to represent the details or the refined version, that leads us to the wavelet function or wavelet function is also popularly known as, the mother wavelet function. And the father and the mother would then in a give the whole wavelet family.

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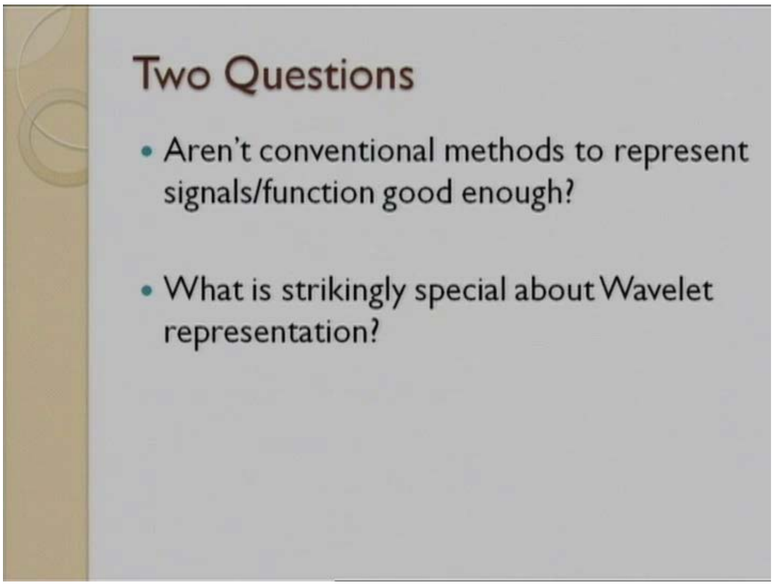

However, there are two fundamental questions, which as still require to be answered; the first is aren't the conventional methods to represent signals or function good enough, and what is strikingly special about wavelet representation. Let us take these questions one at a time, and let us very quickly revisit the conventional methods that we have with us.

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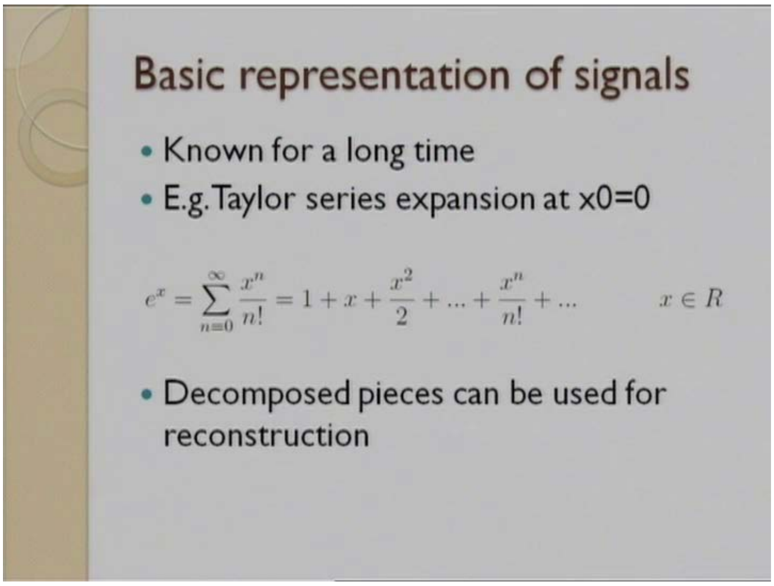

Probably, the most basic way of representing the signals or the functions, is by virtue of using the Taylor series expansion, and we probably learn this at the beginning of our calculus course. We know that signal representation is known for a long time, and one particular example of tailor series expansion at x of 0 is equal to 0, is shown for a function e to the power x, and that gives us in finite coefficients for this particular series; 1 plus x plus x square by 2 and it goes on. Every single coefficient can be looked upon as decomposed piece, and we can make use of these decomposed pieces for doing the reconstruction back of the original signal or function. If we make use of only some finite number of coefficients, let say first three coefficients in the series; 1 x and x square by 2.

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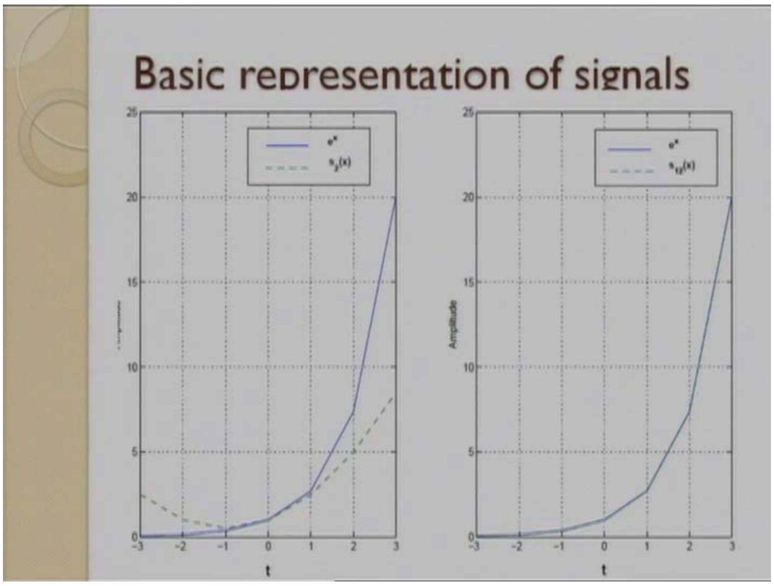

And using only these three coefficients, if we try and reconstruct the original signal, then this is how it is going to look, and it is shown in the diagram on the left hand side. This dotted curve essentially represents the reconstruction using only first three coefficients, and this line in blue essentially is the original function which is e to the power x. Please ignore these discontinuities or here, because the resolution used is lesser, but we can always use a high resolution and we can take care of this part. Now, as against this instead of using just first three coefficients, if I use first well coefficients, then you can clearly see, the representation goes very close to the actual representation.

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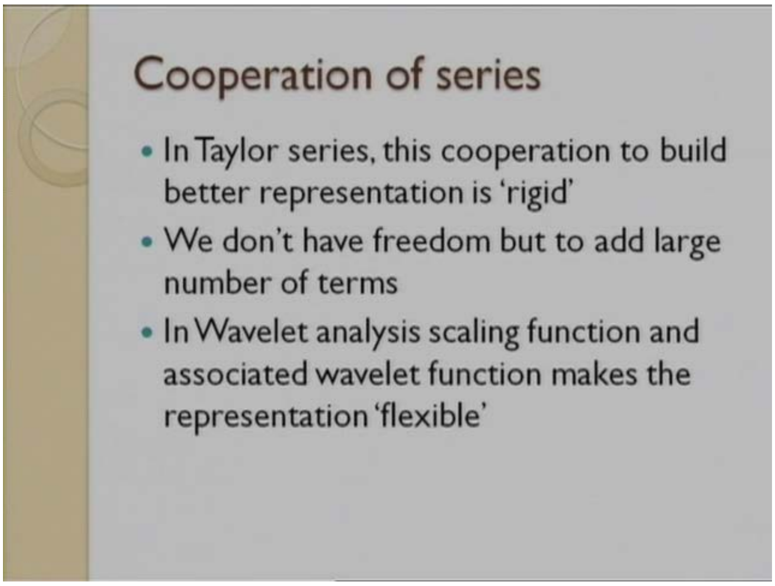

And there comes the question of the cooperation of the series expansion, and what we say is in Taylor series this cooperation to build better representation, is rigid. Why this is rigid; number one, that is because we do not have freedom, but to add large number of terms. I cannot play around with the individual term. I am restricted with the scale and the translation parameters of every single term, and I do not have freedom to change this. In contrast to this, in wavelet analysis, a beautiful combination of the scaling function and its associated wavelet function, makes the entire representation very flexible. I have flexibility in terms of selecting the scale, selecting the translation parameter, and then I can also bring in the dilation, by virtue of which I can then create the nested subsets.

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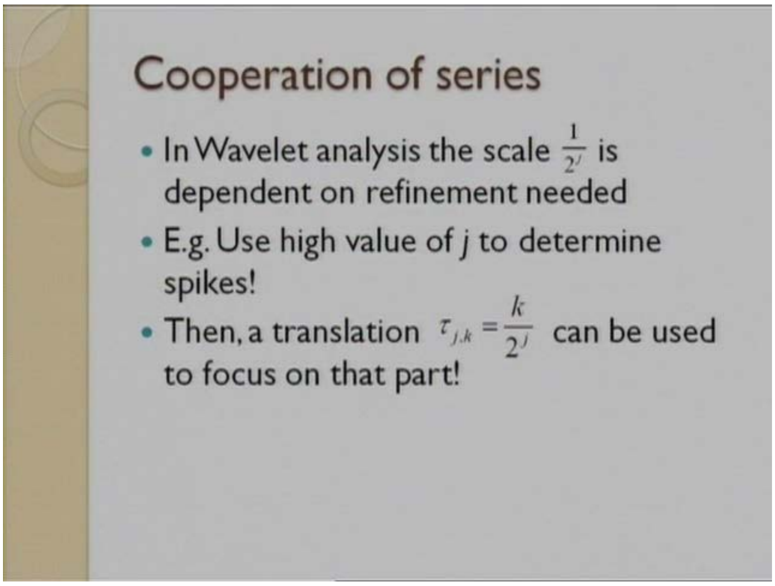

And this is going to lead us to MRA - Multi Resolution Analysis. In wavelet analysis, the scale one upon two to the power j is dependent on the analyzer, to what degree we require the refinement to be added to the actual representation. And one example could be, if we want to determine the spike in the signal, we can think of using a very high value of j. And then we can bring in the translation parameter, tau j comma k which is equal to k upon 2 to the power j, and this can be used to focus on that specific part in the signal. This combination together scale and translation parameter, it is so beautiful that I can go and look at any particular part of the signal, I can change and alter the scale and I can zoom in or zoom out; and by virtue of making use of a beautiful combination of scaling and translation parameter, what we get is the zoom in and the zoom out facility of wavelet transform.

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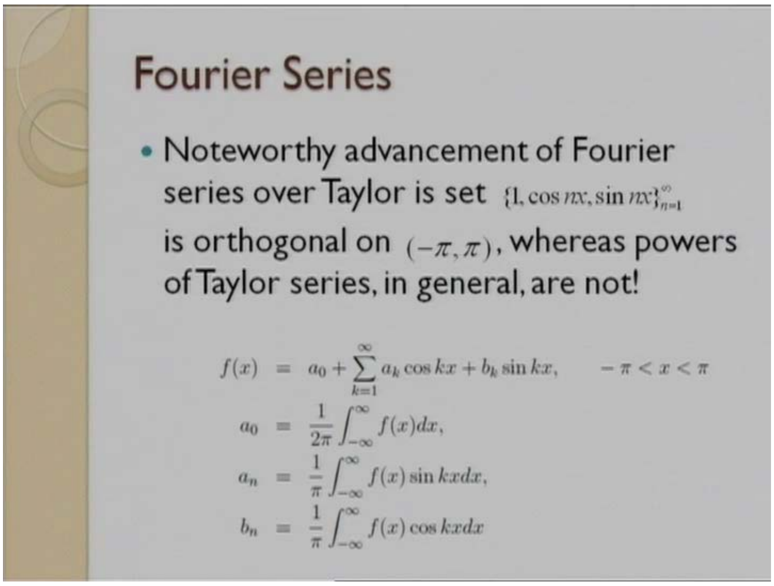

As can be seen from the slides, someone might argue that Fourier series has a noteworthy advancement over Taylor series, and that is all the elements of Taylor series, they do not necessarily and always form an orthogonal set; however, when it comes to Fourier series, the set 1 cos of n x and sin of n x, n ranging from one to infinity is always orthogonal on range minus pi to plus pi, this is absolutely true. However, the fundamental query remains the same. I cannot change the scale and the translation parameters, associated with the basics function or the kernel function when we are trying to represent the information. However, we can derive sub useful information from the Fourier series, and that is why the always say wavelet transform in a way stands on the shoulders of Fourier series and Fourier transform. From the slides, we can clearly see, that a very special relationship actually exists, between the sign and cosine parts of Fourier series. A similar special relationship also exists between the scaling function and the wavelet function, when it comes to wavelet series. This relationship is quite trivial, but very interesting, and the interested viewers can dig further into this.

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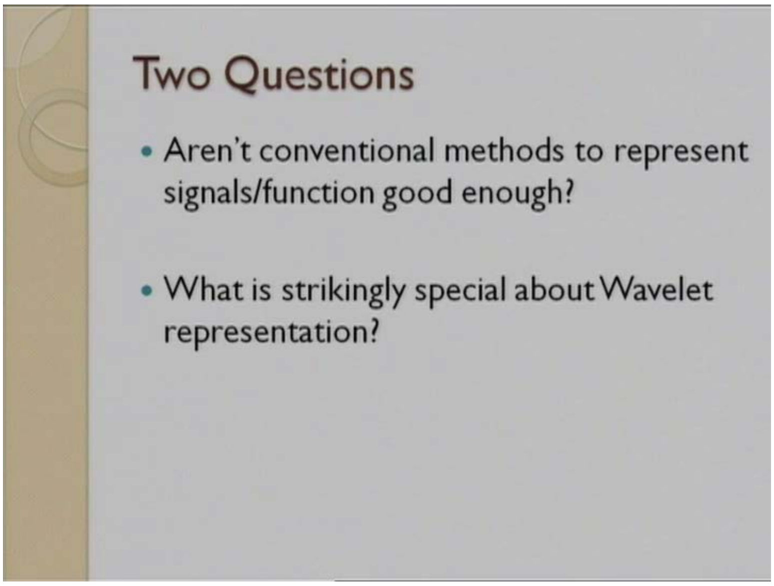

So, in a way we have answered the first question, in a way we have also answered the second question, but will take the second question further, in order to really bring out what is strikingly special about wavelet representation.

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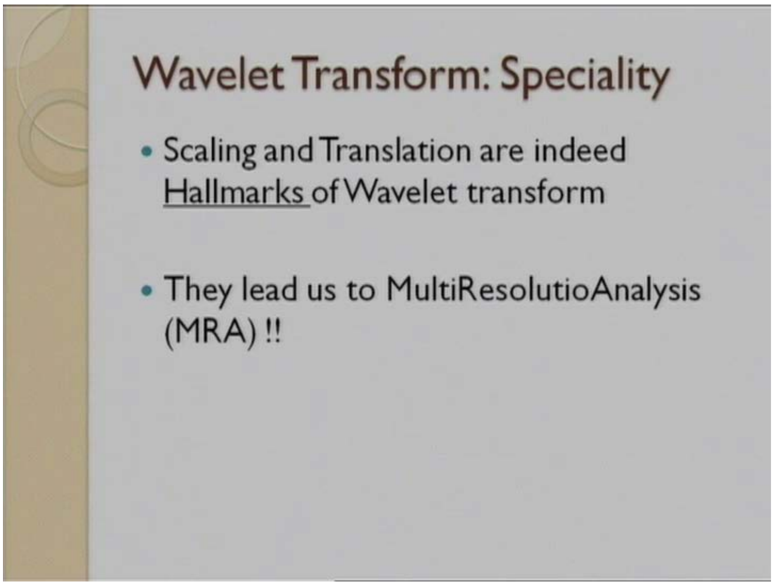

The scaling and the translation parameters are indeed the hallmarks of wavelet transform, and when we add up the dilation parameter, in totality they would lead us to the multiresolution analysis framework, which is popularly also known as MRA. The central theme of MRA is at shown in the slide.

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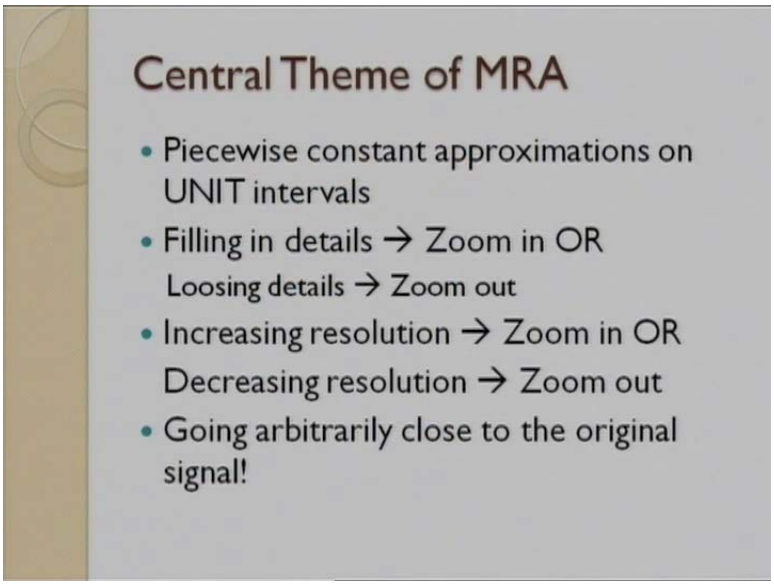

We are talking about the piecewise constant approximations on unit intervals. However, wavelet transform is not just about finding out the piecewise constant approximations, otherwise there would not have been any difference in wavelet transform and a simple process of quantization. Once we carry out the successive approximation, then comes the very interesting part and that is filling in the details. So, the piecewise constant approximation will be given by the scaling equation phi of t, and then the details will be added with the help, coming from the wavelet equation which is psi of t. The filling in of the details can then called as the zoom in feature. Losing out in details can be called as the zoom out feature. Increasing the resolution, would lead us to zoom in and decreasing the resolution, would lead us to zoom out. The very concept of zooming in and zooming out has become a well-known phenomenon these days, and that is because we live an era, where we make use of digital cameras.

In fact, most of the scanners and most of the sensors they have gone digital. And by virtue of using digital camera we often capture digital images and then zoom on to a particular portion in that image to really understand what kind of activity is going on there. Correspondingly, we can also zoom out, by losing out on details, if we want to get a generic field about that particular picture. So, we understand what is zoom in and zoom out. However, when it comes to signal analysis, many of the times it is a requirement, that we should have a frame work by virtue of which, we should be able to zoom on to a specific part in the signal, so as to understand what is really going on there. Consider a case of ECG signal, maybe we have large recording of one hour, and in that large recording of one hour, if there are only few samples which shows some abnormality, then as a signal analyzer we should be able to focus only on that part from the large recording that we have with us. So, this beautiful property of zooming in and zooming out, is of great importance great significance.

From the slide we will understand, that the whole point in doing this exercise, is and ability of the analyzer of going a bit really close to the original signal, and infect n a book return by Stephen Mallette on this beautiful subject of wavelets, he has used one beautiful word. He says one should be able to go tantalizingly close to the original signal, beautiful, how to achieve this; that sport we are going to uncovered in the remainder of this particular lecture. So, in a way the introductory part is over, and now will begin our journey towards understanding, how we can achieve the zoom in and zoom out features. We have understood the requirement. We have also understood why at all it is essential, and now it's actually time to dirty our hands, to actually see the frame work, and then to be able to understand, how we can carry out this task, how we can actually do this zoom in zoom out using the frame work, that we are going to lay down.

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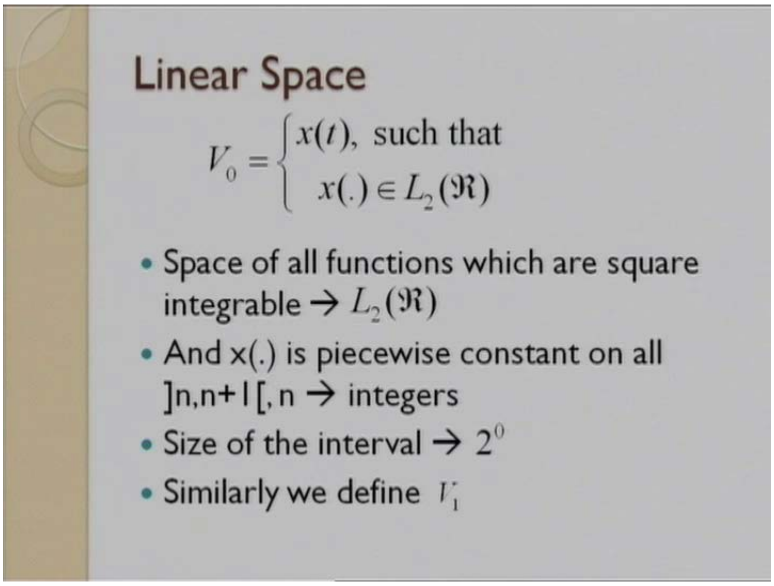

From the slide let us define linear space, and let say its linear space that we have named as V of 0, and then this would contain the functions which are square integrable; that is the functions which in a way obey the L 2 R norm, so to say. And the piecewise constant approximation will be done on an open interval from n 2 n plus 1, where n is n integer. What is really interesting is the size of this interval. If we are in a linear space we of 0, the size of this interval and we can call this as the analysis window. This analysis window will be two to the power 0. From V of 0 we can move on to V of 1,

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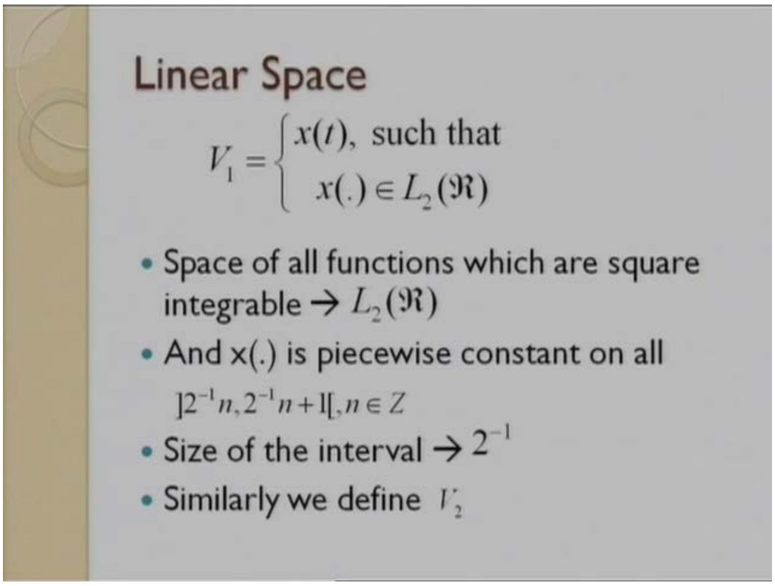

And if we are in linear subspace V of 1, then correspondingly the size of the interval will be two to the power minus 1; that is half, and we can continue doing this activity, and we can generalize this notion.

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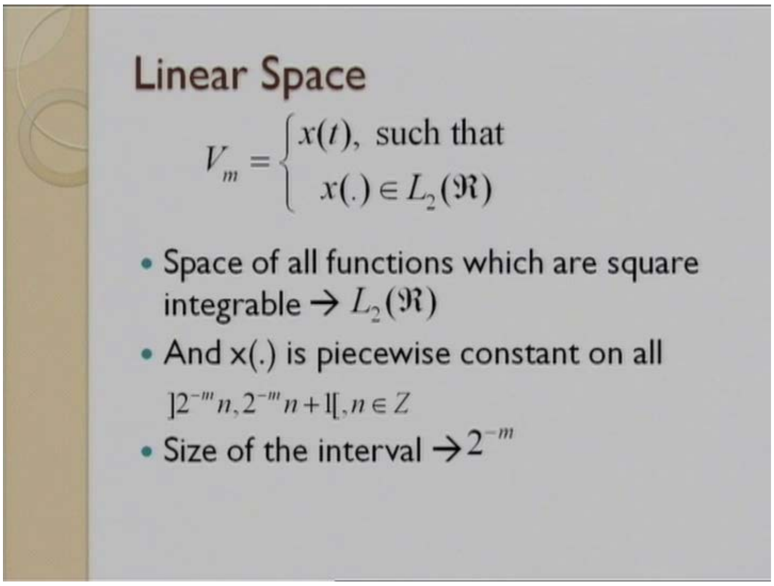

And we can say we are talking about linear space V of m, where the size of interval is 2 to the power minus m.

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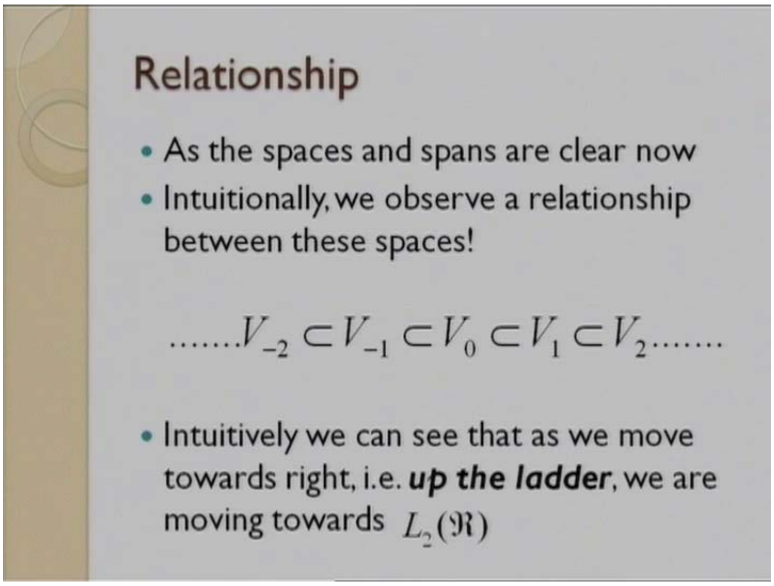

And that leads us to very special relationship, and this relationship in general is called as the nested subsets. And we have already seen in the lectures by Doctor Gadre; that if we are looking at this ladder of subspaces, which are the nested subspaces, then we can think of either moving up the ladder or moving down the ladder. If we move up the ladder, the analysis window will becomes smaller and smaller and smaller, and will go on adding of the details, and eventually we should be able to achieve the L 2 R norm.

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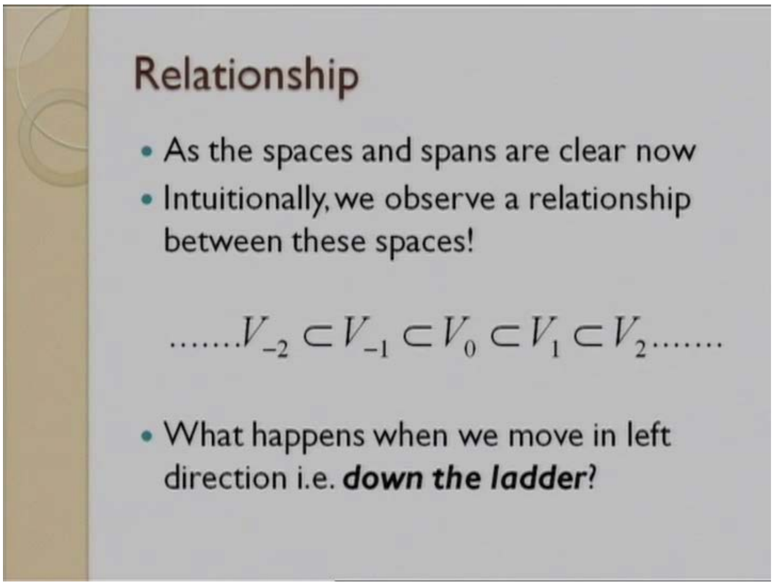

As against that, if we move down the ladder then; obviously, we are talking about the resolution getting coarser and coarser, and eventually that is going to lead us to trivial subspace, and we will probably lose out all the details.

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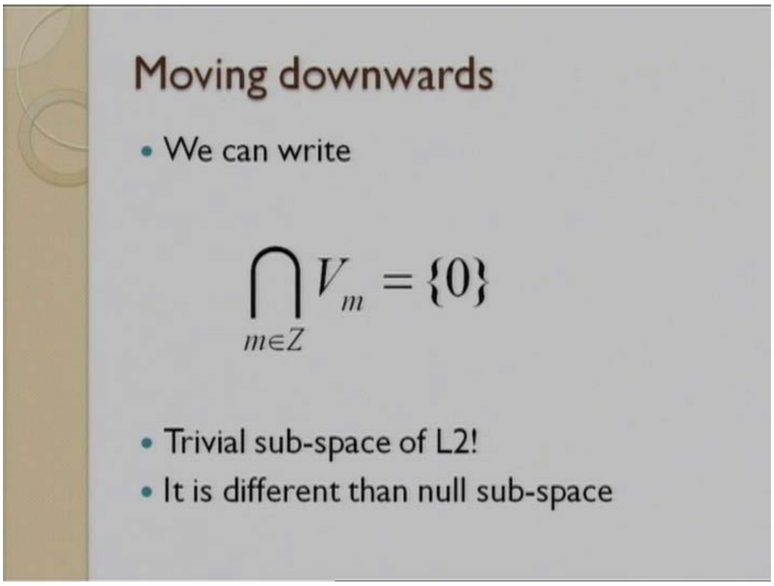

We can convey this mathematically, using this formula and the phenomenon of moving up the ladder with closure, as was given by Doctor Ingrid Daubechies, is captured in this mathematical formula.

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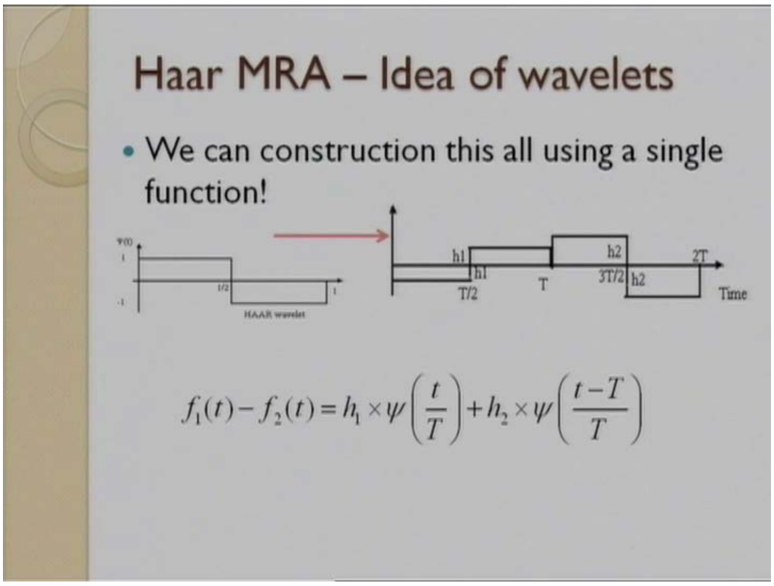

And what is really striking about the wavelets, is you could be talking about a particular function and its corresponding projection in any space, any subspace; that can be very neatly and nicely constructed using just one single function, and that is psi of t, and how to do that, we have already seen this. We can do this, using hallmarks of wavelet transform; that is scaling, translation and also dilation. We will talk about it more, once space starts putting down the framework. well this is all true when it comes to psi of t; that is the wavelet functions, and we can very well span the w subspaces, but who will span v subspaces, where has to be a function who would do that task for us, and that function is phi of t, which is also known to us as the scaling function. Now, who gives us scaling function, how we can go about finding out the coefficients of the scaling function; that is one interesting question, and probably down the line in this series, we are going to also try and uncover this part.

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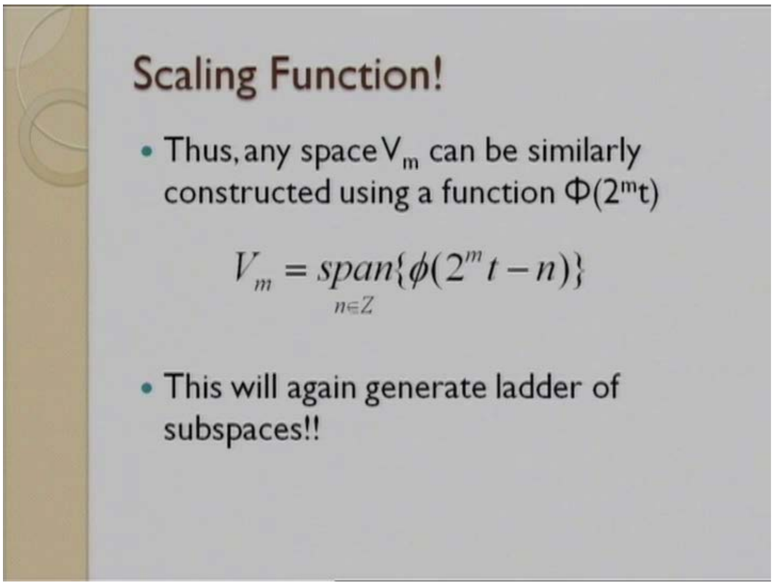

The whole point of having the scaling function, as can be seen from the slide is to be able to span that particular subspace V of m. And this in a way guaranties the generation of ladder of sub spaces, and we are saying this term again and again, because this ladder of subspace is eventually going to lead us to M R A; that is multiresolution analysis.

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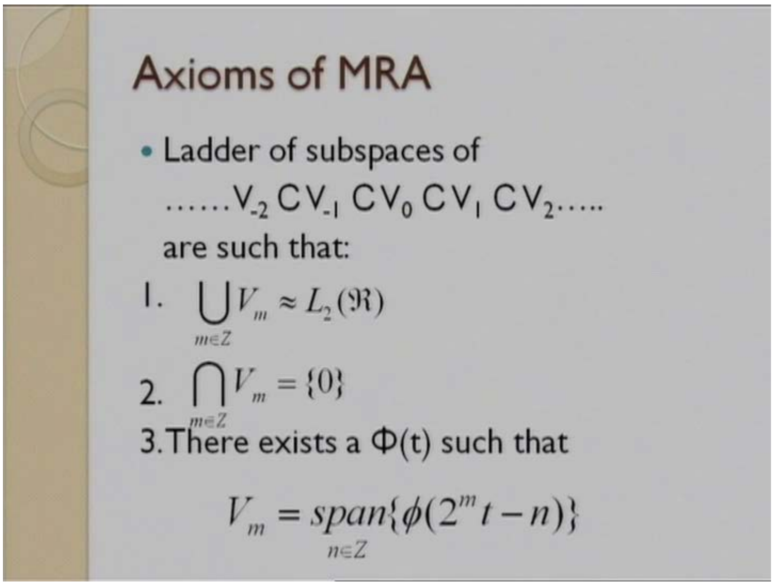

Let us quickly run through the axioms of M R A, because the framework that we are going to see in this particular lecture, in a way depends on these axioms. So, once we understand, how the ladder of subspaces are formed, then the first axiom, is obliviously moving up the ladder. The second axiom is; obviously, moving down the ladder. The third axiom, guaranties the existence of us scaling function; that will help us span all though V subspaces.

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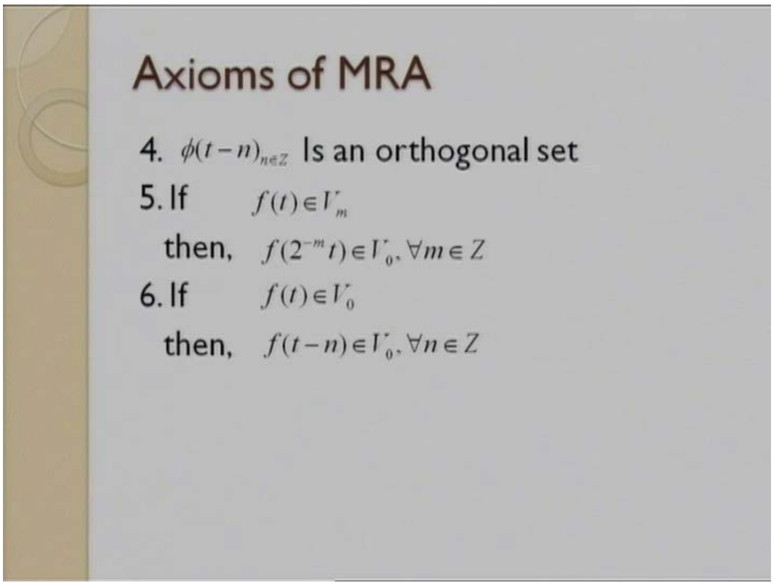

Correspondingly, we have to ensure, that phi of t innovate generates the orthogonal set of series, and then that will lead us to axiom number 5 and 6. The axiom number 5 and 6, we are going to see the direct implication of these two axioms, in the frame work that we are going to go through.

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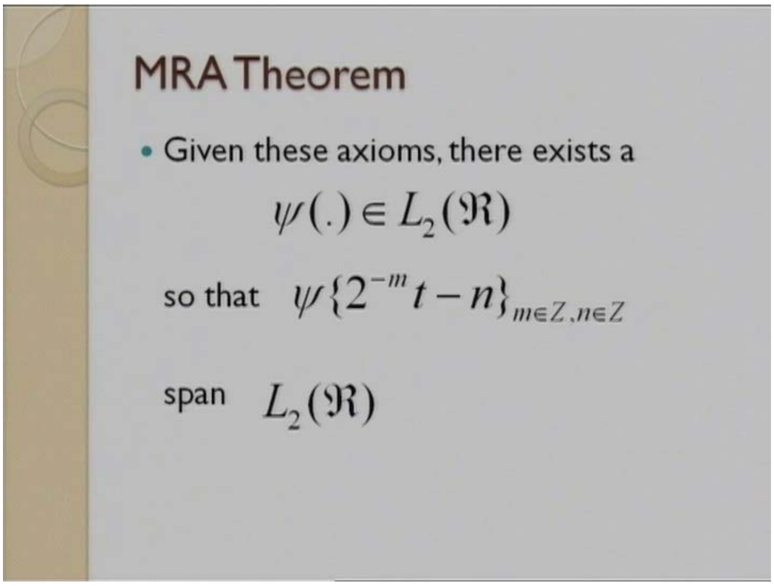

And based on this axioms we have the MRA theorem. They tells us given these axioms there exists a wavelet function, psi, which is once again a square integrable function, and using this function, I can span those w subspaces, and bring out the details in the underlying signal or function.

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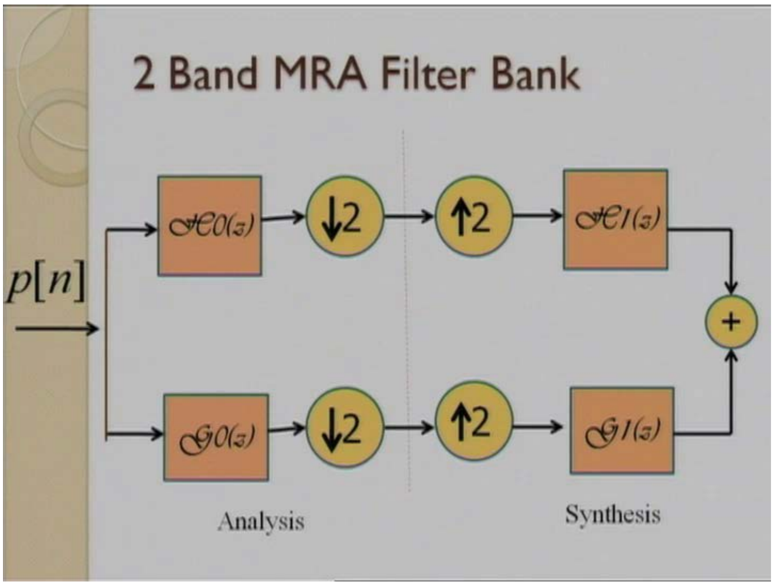

One typical way of implementing this MRA philosophy, is using a two band filter bank structure. And if we have some input; let say p of n. Then I can very well go about doing the analysis, by first of all passing this through the analysis low pass filter, followed by down sampler. Analysis high pass filter which is shown by G 0 of z, followed by down sampler. And if I want to reconstruct back the original signal, then I can run the synthesis phase, where I can have and up sampler, followed by the synthesis low pass filter, which is shown as H 1 of z, and the synthesis high pass filter which is G 1 of z, and by virtue of doing the orthogonal summation, we can very well reconstruct back the original signal, as the two filters are complements or duals of each other. With this introduction, now let us try to understand, how we can go about building the framework.

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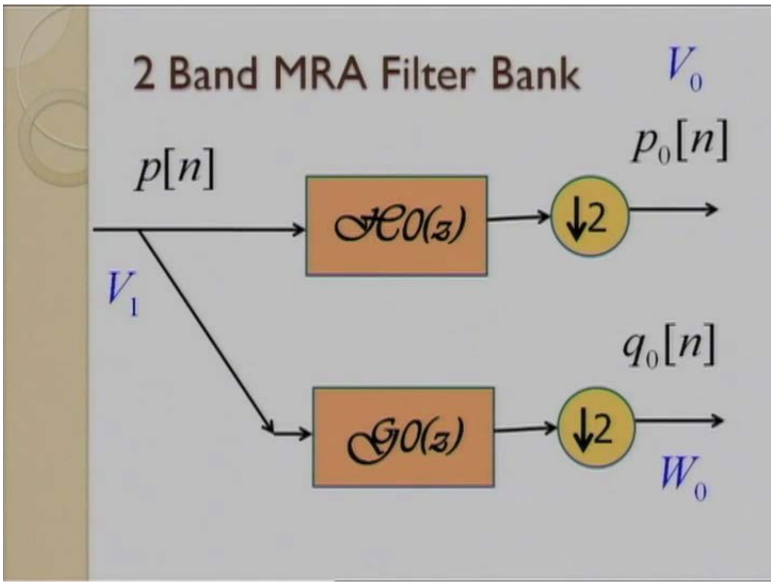

One series drawback of this two band filter bank is as follows; as can be seeing from the slide, if we focus only on the analysis part, and let say input to our system is some signal p of n. It is a digital signal of finite duration, and let say this signal belongs to some sub space V 1. The moment you run the signal through analysis low pass filter, followed by a down sampler by factor of two, and analysis high pass filter and a down sampler by factor of two, you in a way end up with signal p 0 of n; that would belong to V 0, and q 0 of n that would belong to a subspace W 0. So, in this process, let us really try and understand what exactly is happening.

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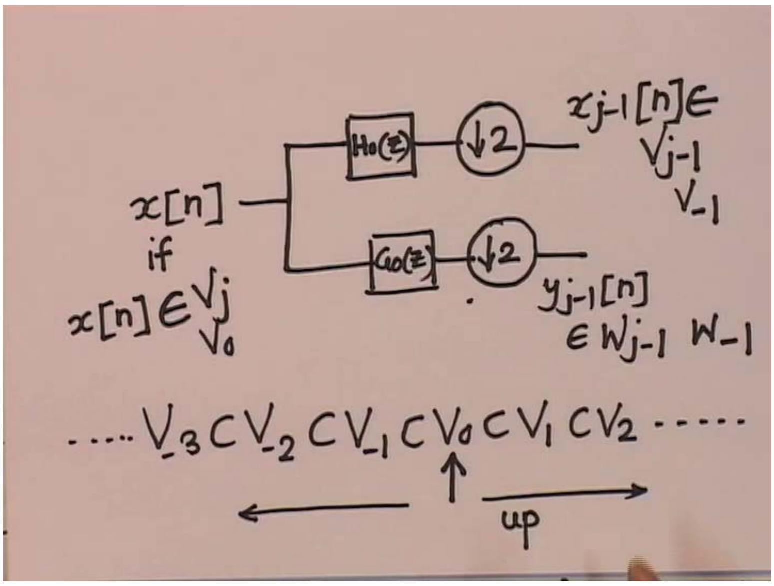

Let us say, you have some signal x of n, and you are passing this signal through the two band filter structure, and this will be followed up with down sampler by factor of two. What we are saying is, if this signal x of n belongs to some subspace say V of j, then by virtue of doing this kind of an arrangement, we end up with x j minus 1 n, which would obviously, then belong to V of j minus 1. And let us say y of j minus 1 n, which belongs to w of j minus 1, and if we try and understand what is happening over here, and if we see the nested sub sets; that we have already seen and so on. And let us say this V of j is V of 0; that means, we are starting over here. Then by virtue of doing these operations, we start moving in the leftward direction, and that is because if I am starting in V of 0 I will generate projections in V of minus 1 and w of minus 1. So, in a way we start moving towards the left direction, we start moving down the ladder, not always it is desirable to move in the downward direction.

In fact, for many of the applications, it is required to actually move up the ladder, and only by virtue of moving up the ladder, we can go on adding up the details; from V of 0, we can move on to V of 1, from V of 1 we can think of moving to V of 2, and we can go on adding up the details, and then we can think of going tantalizingly close to the actual signal and its corresponding representation. This is one interesting journey, and we will have to build the whole framework, in order to be able to achieve this.

What we have seen in this particular lecture, is how we can interpret the well-known two band filter bank structure, and understand it from the point of view of nested subsets, and how typically we end up only moving down the ladder. And for many of the applications it is required, it is desired to move actually up the ladder. And if we want to move up the ladder, we will have to design the framework, and that we are going to cover in the next class. However, we will write down one important mathematical formula, which is going to be at the heart of what, the kind of framework that we are going to build in the next class; and that formula is, by looking at the two band Haar structure.

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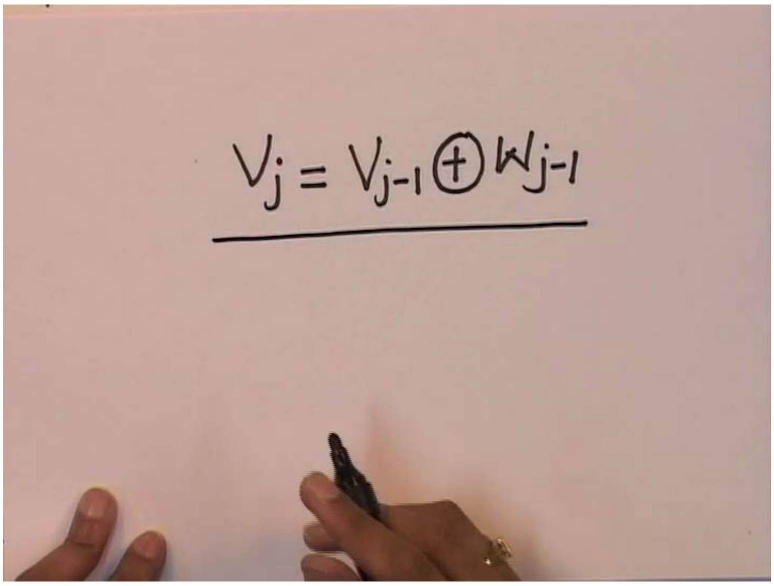

We can very well write down that V of j is equal to V of j minus 1, orthogonally added with W of j minus 1. This is the formula, which is of great importance, which is of great significance, and the whole multiresolution analysis structure is in a way based on this formula. We will stop here, and we will continue building the framework by virtue of which, we can possibly think of moving up the ladder, add the details and really think of going tantalizingly close to the signal or function under analysis. Thank you