

**Advanced Digital Signal Processing - Wavelets and Multirate**  
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**Lecture No. #37**  
**Modulation Analysis and The 3-Band Filter Bank Applications**

So, warm welcome to the thirty-seventh lecture on the subject of wavelets and multirate digital signal processing. Recall, that in the previous lecture, we had discussed one way to analyze the general system with analyses and synthesis filters, namely the approach of polyphase decomposition. Essentially, polyphase decomposition is a time domain approach, where we recognize, that all sequences in question, whether the input sequence, the output sequence, the filter impulse responses could be thought of as comprising of as many subsequences as the number by which the sequence is decimated and interpolated, that is, the down sampling and the up sampling factor.

So, for example, if we have down sampling and up sampling by a factor of 2, we think of the odd and the even numbered points on all of the sequences in question: the input sequence, the output sequence and the filter impulse response sequences.

Based on this decomposition, we identify the relation between the output polyphase components, as we call them, and the input polyphase components through the filter polyphase components. So, essentially, the polyphase approach, which we built up in great detail in the previous lecture, was recognition of the relationship between the input polyphase components and the output polyphase components.

Naturally, this is difficult to do purely in time and therefore, one needs to go to a transform domain. What we have done was to identify this relationship, essentially in the Z domain. Further, we had noted, that the condition for perfect reconstruction amounts to a condition on the product of the polyphase matrices corresponding to the analysis side and the synthesis side. So, when we look at the product matrix and if you wish, that the output and the input polyphase components, essentially, replicate one another, we need to put a condition on the entries and I shall, before I proceed to discuss the other approach today, complete that discussion by putting the condition and the condition, the requirement down explicitly.

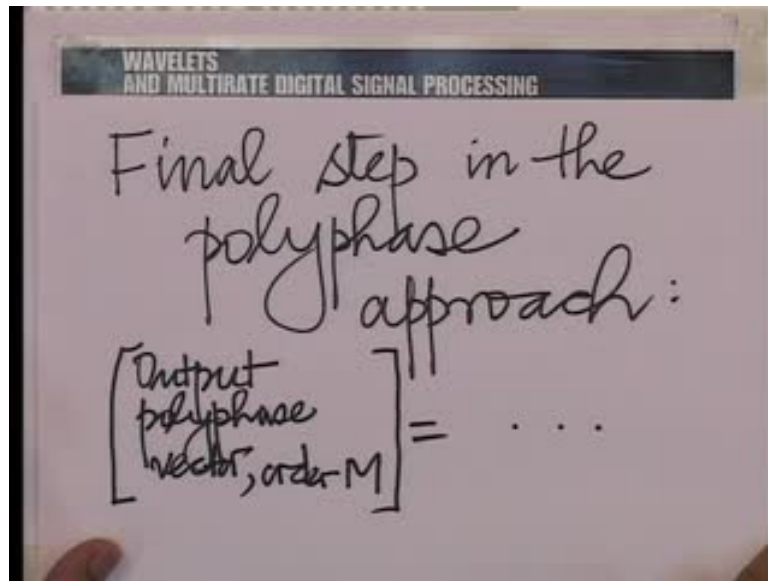
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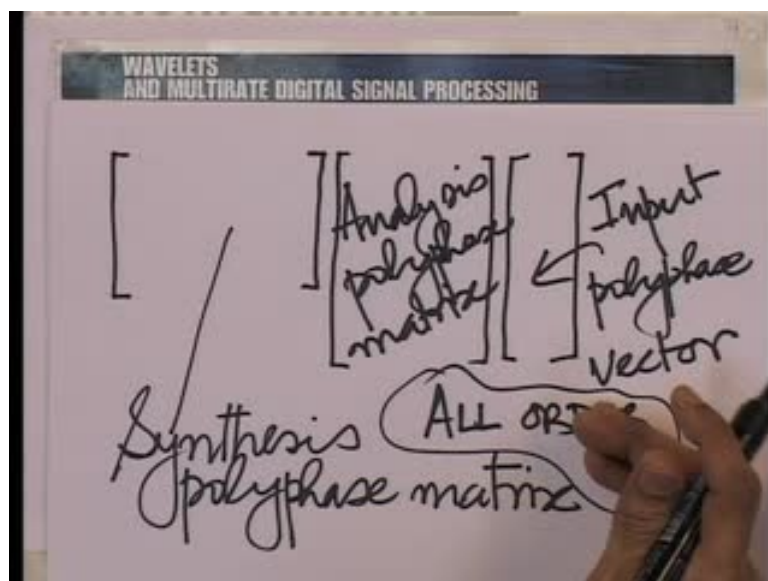
So, in the lecture today, we intend to locate the other approach, namely the modulation analysis, which we shall illustrate using the 3 band filter bank, but before that, as I said, we shall complete the discussion on the polyphase approach by writing down explicitly the requirement on the product matrix.

Further, we shall, in the later part of this lecture, begin a discussion on applications of time frequency analysis of wavelets of filter banks, and so on. And involved in these applications are actually several students who have taken the course for credits in the current semester. I shall put before the audience today, some of the students who have investigated some applications in depth and done a very good job. We shall only look at a very brief introduction to their applications today and discuss them in depth in a subsequent lecture.

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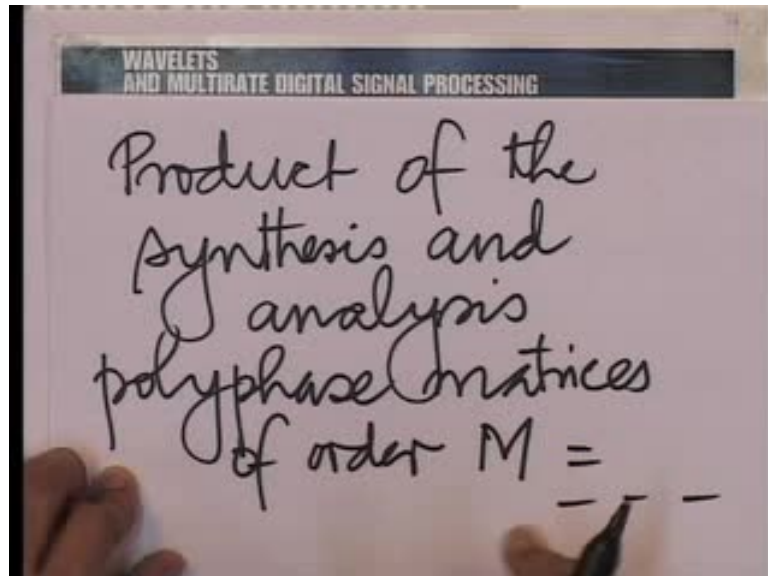


So, with that perspective on the lecture today, let us begin to complete the first part of the discussion, namely the final step in the polyphase approach.

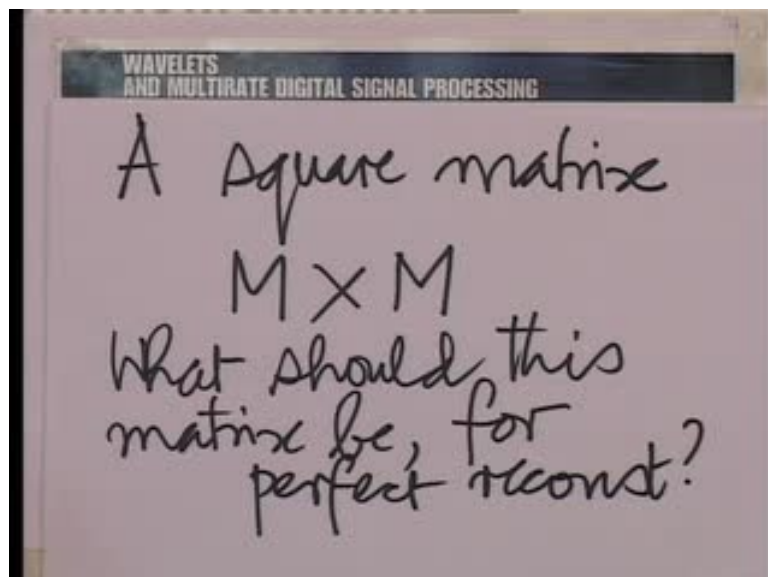
Now, recall, that we had the output polyphase vector of order  $M$ , which was a product of 2 matrices, the synthesis polyphase matrix and the analysis polyphase matrix, and here we had the input polyphase vector, all these were order  $M$ .  $M$ , here is the factor of down sampling and up sampling. Now, we could look at the product, these 2 matrices here, so when we take the product of the synthesis polyphase matrix and the analysis polyphase

matrix, we get a composite matrix square in nature and of size equal to the input of that matter, the output polyphase vector of order  $M$ .

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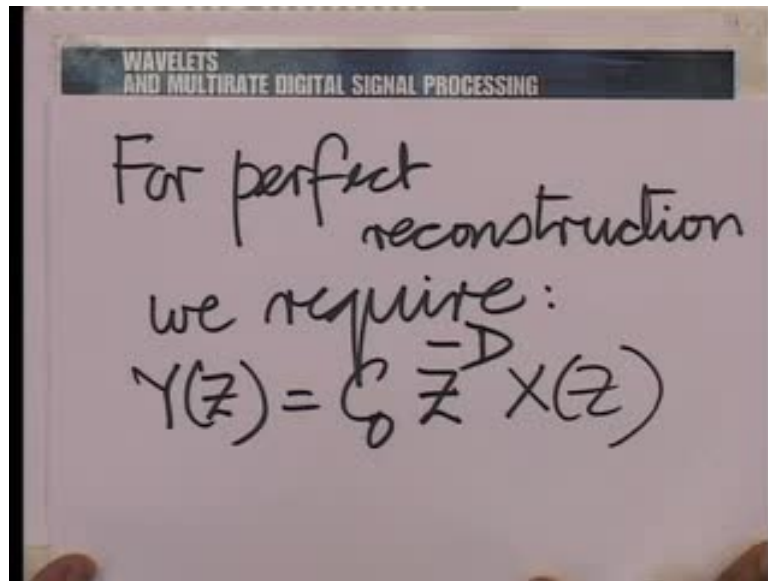


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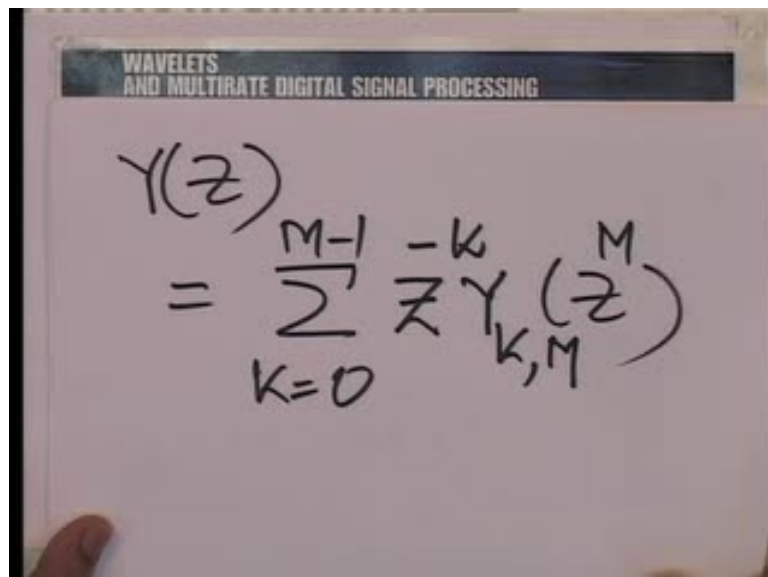
So, it is very easy to see, that the product of the synthesis and analysis polyphase matrices of order  $M$  is a square matrix size,  $M$  cross  $M$ , and the question, that we are trying to answer is, what should this square matrix be for perfect reconstruction? Now, the answer is very easy, after all what does perfect reconstruction mean? It means that the output must be a delayed, possibly delayed and a scaled version of the input.

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In other words, for perfect reconstruction, we require  $Y(z)$  in the  $z$  domain is some constant times  $z$  raised minus  $t$  times  $X(z)$  and all that we need to do is to look at the relationship between the polyphase components on either side.

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So, if we decompose  $Y(z)$  and  $X(z)$ ,  $Y(z)$  in the form summation  $k$  going from 0 to  $M$  minus 1  $z$  raised the power minus  $k$   $Y_{k,M}(z)$  and similarly for  $X$ .

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WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$X(z) = \sum_{k=0}^{M-1} z^{-k} X_{k,M}(z^M)$$

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WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$\sum_{k=0}^{M-1} Y_{k,M}(z^M) \cdot z^{-k} = C_0 z^{-D} \sum_{k=0}^{M-1} z^{-k} X_{k,M}(z^M)$$

We can now establish relation term by term. So, we have summation  $K$  going from 0 to  $M$  minus 1  $Y_{KM} Z$  raise to the  $M$  times raise the minus  $K$  is  $C_0 Z$  raise to the minus  $D$  times summation  $K$  equal to 0 to  $M$  minus 1  $Z$  raise to the power minus  $K$   $X_{KM} Z$  raise to the power  $M$ . Now, all that we need to do is to take this  $Z$  raise to the minus  $D$  inside and combine it with  $Z$  raise to the power minus  $K$ .

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WAVELETS  
AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$= C_0 \sum_{k=0}^{M-1} z^{-(D+k)} X_{kM}(z^M).$$

$D+k$ :  
↑ fixed  $k$

So, we have, that becomes  $C_0$  summation  $k$  going from  $M$  minus  $1$   $z$  raise to the power minus  $D$  plus  $k$   $X_{kM}$   $z$  raise to the  $M$ . And now, all that we need to do is to separate  $D$  plus  $k$ , so you see,  $D$  is fixed for all  $k$ . So, if you look at this  $D$  plus  $k$ , essentially carries out a rearrangement of the polyphase components.

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WAVELETS  
AND MULTIRATE DIGITAL SIGNAL PROCESSING

Example:  
 $M = 3$   
 $D = 5$

So, what it does, you know, let me take an example. Suppose,  $M$  is equal to  $2$ , let us write down to fix our ideas. Suppose,  $M$  for variety instead of  $2$ , let us take it to be equal to  $3$  and let us take  $D$  to be equal to  $5$ .

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The image shows a handwritten table on a slide titled "WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING". The table illustrates the mapping of  $K$  to  $K+D$  modulo 3. The values of  $K$  are 0, 1, and 2. The corresponding values of  $K+D$  are 5, 6, and 7. These values are then reduced modulo 3 to 2, 0, and 1 respectively. The modulus 3 is underlined.

$K$	$K+D$	$\equiv$	
0	5	$\equiv$	2
1	6	$\equiv$	0
2	7	$\equiv$	1

mod 3.

So, let us make a table.  $K$  can, of course, take the value 0, 1 and 2. So,  $K$  plus  $D$  will take the values 5, 6 and 7 and 5 is clearly equivalent to 2, 6 is equivalent to 0 and 7 is equivalent to 1 modulo 3.

Therefore, in this specific example, where  $M$  is equal to 3 and  $D$  is equal to 5, the effect is to take the 0th component of the 0th polyphase component of order 3 of the input and map it to the number 2 polyphase component of the output; the 1 number polyphase component of the input gets mapped to the 0 number polyphase component of the output and number 2 gets mapped to number 1.

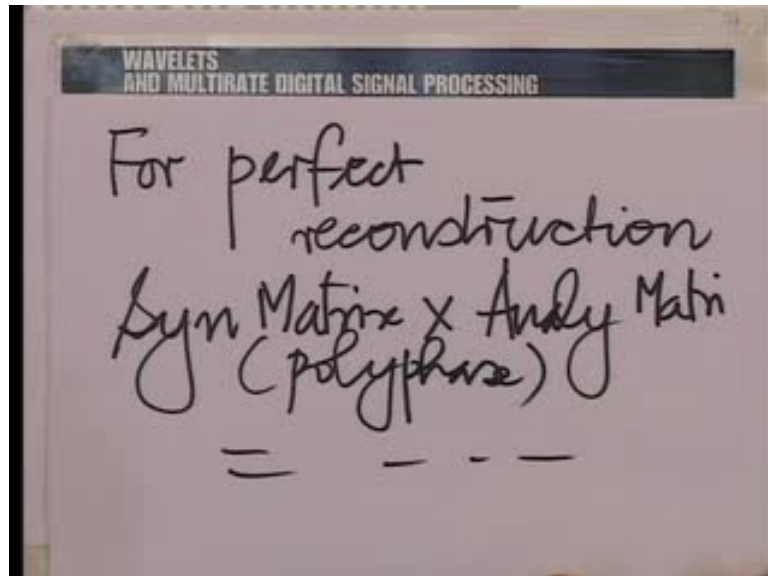
Essentially, what we have is the cyclic rearrangement of the polyphase components and therefore, if we look at the product matrix, what we require is that every row and every column must have only 1 non-zero entry, that non-zero entry must be identical for each of the rows of that matrix, each of the columns. In other words, if you take every row and if you take every column, there is exactly one non-zero entry; that entry is identical. Same entry repeated the constant factor in each of these entries is  $C^0$  and of course, the additional delay or the additional shift, that we have, depends on  $D$ .

For example, in this case, where  $D$  was equal to 5, the additional shift, you know, there will be a factor of  $Z$  raise to the power minus 3 coming in. So, each of the polyphase components, in addition to being rearranged in this way, is multiplied by  $Z$  raise to the power minus 3 on the up sampled scale and if we replace  $Z$  raise to the power of  $M$  by  $Z$ ,

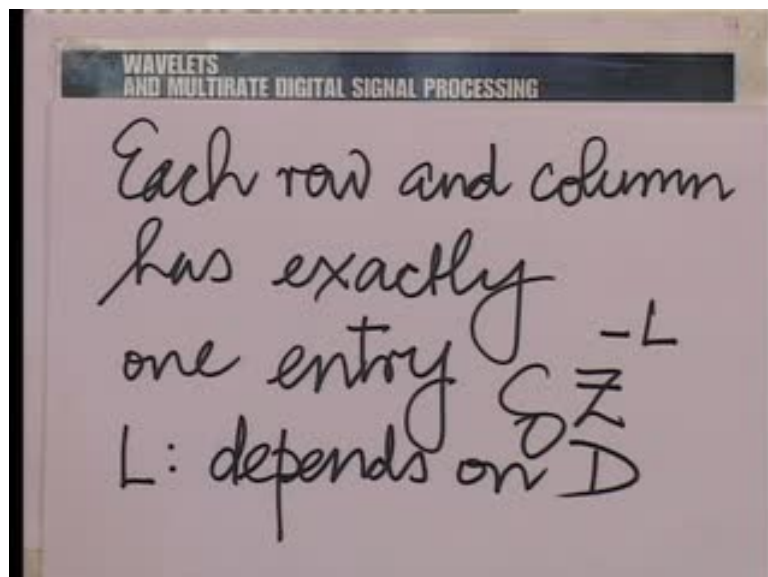


it is equivalent to multiplication by  $Z$  inverse. So, this is the summary of the relationship between the input and the output polyphase component, let us write it down explicitly.

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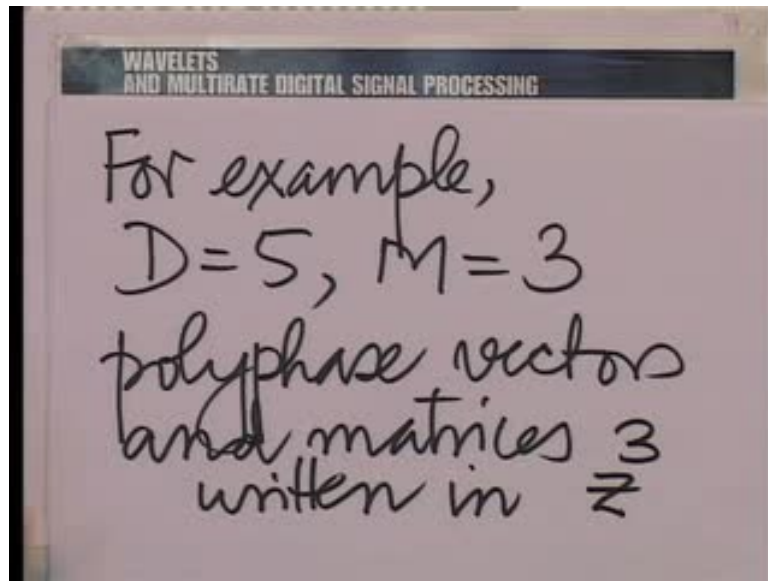


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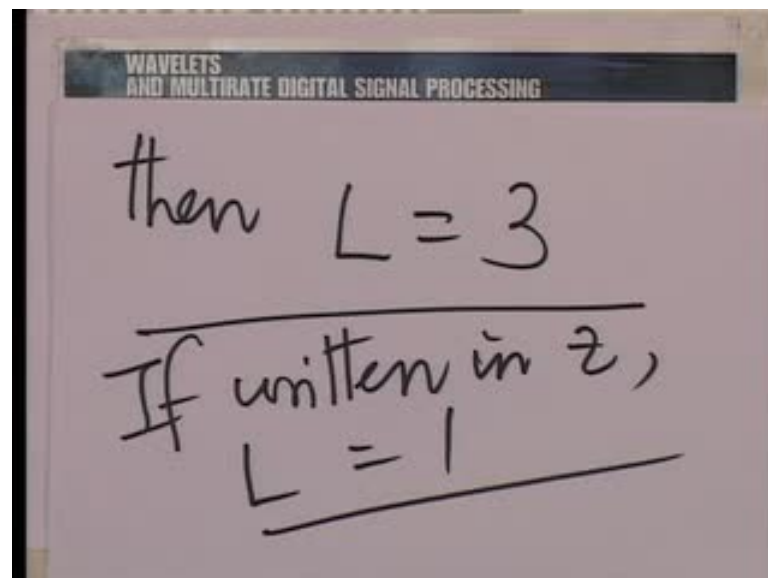


For perfect reconstruction, the synthesis matrix times the analysis matrix polyphase must be of the following form. Each row and column has exactly 1 entry of the form  $C 0 Z$  raise to the power minus  $L$ ;  $L$  essentially comes from  $D$ , it depends on  $D$ .

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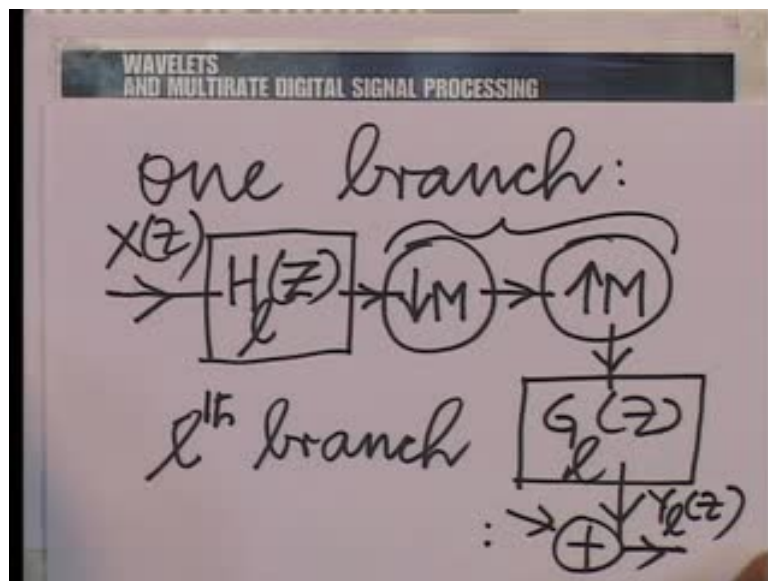
For example, with  $D$  equal to 5 and  $M$  equal to 3 and polyphase vectors and matrices written in  $\mathbb{Z}$  cubes, we have the flexibility of writing the polyphase matrices in  $\mathbb{Z}$  cubed or just in  $\mathbb{Z}$ . If you write them in  $\mathbb{Z}$  cubed, then  $L$  is equal to 3; if written in  $\mathbb{Z}$ , then  $L$  is equal to 1.

Now, of course, this was meant to be illustrative. So, I leave it as an exercise for the student to work out a few more such examples and in fact, at this point, I shall raise a question, but not answer it immediately. And the question is, if we have to obtain perfect

reconstruction, but if the analysis synthesis system together becomes the linear shift invariance system, that means, one can equivalently treat the output as being acted upon, rather the output, as the result of the input being acted upon by linear shift invariance system with a certain transfer function. What can we say about the entries of this product matrix when this linear shift invariance is present in the overall analysis synthesis structure? Can we attribute a certain structure to this product matrix? As I said, I only raise the question at this point, but do not, do not answer it right away, we leave it for reflection and possible answering at a later stage?

Anyway, with that discussion on the polyphase approach, let us now take the 2nd approach, that we had mentioned the last time, namely the modulation approach. Now, as I said, the polyphase approach is essentially a time domain approach; the modulation approach is, as we have seen essentially, a frequency domain approach. If we like to call it that way and the modulation approach essentially proceeds as follows.

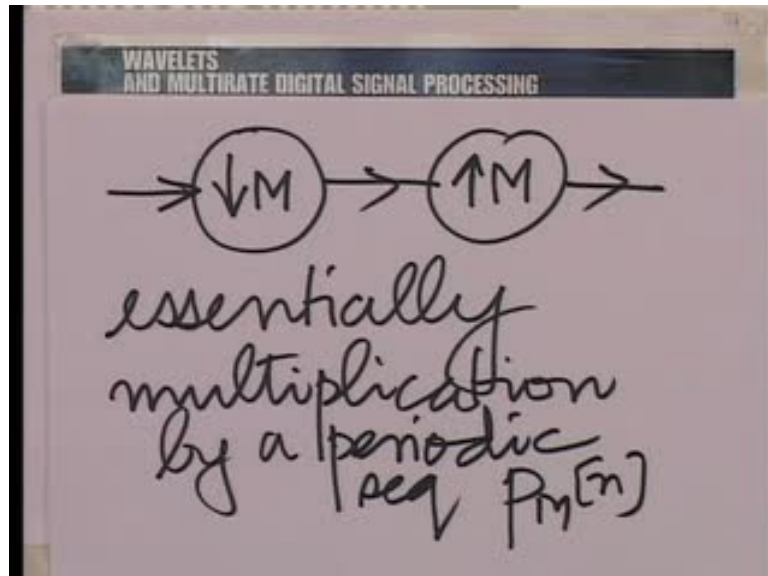
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We look at 1 branch, so let us see the lth branch, small l. We have the analysis filter  $H_l(z)$ , a down sampler by effecter of M and up sampler by effecter of M, followed by a synthesis filter  $G_l(z)$ , and we wish to establish the relationship across this. So, in other words, all these branches with difference synthesis filter  $G_l(z)$  are going to come together in summation to form  $Y(z)$ . So, we could call this  $Y_l(z)$  and we shall write an

expression for  $Y(z)$  in terms of  $X(z)$  here, using a modulation paradigm. Now, the idea behind the modulation paradigm is to look at the combination of these 2 operations here.

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Down sampling by  $M$  and up sampling by  $M$ , as we noted even before, is essentially multiplication by a periodic sequence, let us call it  $P_M[n]$ .

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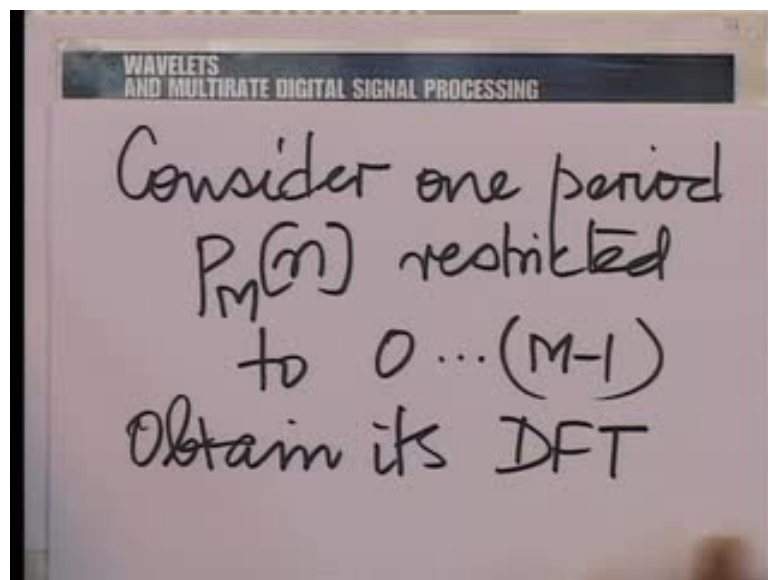
WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$P_M[n] = \begin{cases} 1, & n \text{ a multiple of } M \\ 0 & \text{else} \end{cases}$$

$P_M[n]$  is 1 whenever  $n$  is a multiple of  $M$ , and 0 else.

Now, in the modulation approach, we think of the process of down sampling followed by up sampling as modulation by a sequence and that sequence is again, broken into its component sequence, each of which is an exponential. The reason why we choose the exponential is because it is easy to analyze what happens in the Z domain when we multiply by an exponential and how can we express such a sequence in terms of exponentials. Well, we take recourse to the idea of the discrete Fourier transform. So, we fix our attention on one period of this periodic sequence, we take the discrete Fourier transform of this period, that tells us the components of one period along, so to speak, different modulators; so, let me do that.

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What I am saying is consider one period  $P_M(n)$  restricted to 0 to  $M-1$  and take its DFT; that is easy to calculate, the DFT is capital P.

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WAVELETS  
AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$\tilde{P}_M[k] = \sum_{n=0}^{M-1} P_M[n] W_M^{-nk}$$

$W_M = e^{j\frac{2\pi}{M}}$

Let us write  $\tilde{P}_M$  for convenience,  $\tilde{P}_M$  as a function of  $K$ . If you like, essentially, a dot product of one period of the sequence with the exponentials  $W_M$  here being defined as  $e^{j 2 \pi n / M}$ .

Now, you know how to reconstruct  $P_M[n]$  from its discrete Fourier transform, but before that, let us evaluate this expression. This expression is very easy to evaluate, it has only one non-zero term in the summation and that is  $P_M[0]$ .

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WAVELETS  
AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$\tilde{P}_M[k] = 1$$

$k = 0, \dots, (M-1)$

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WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

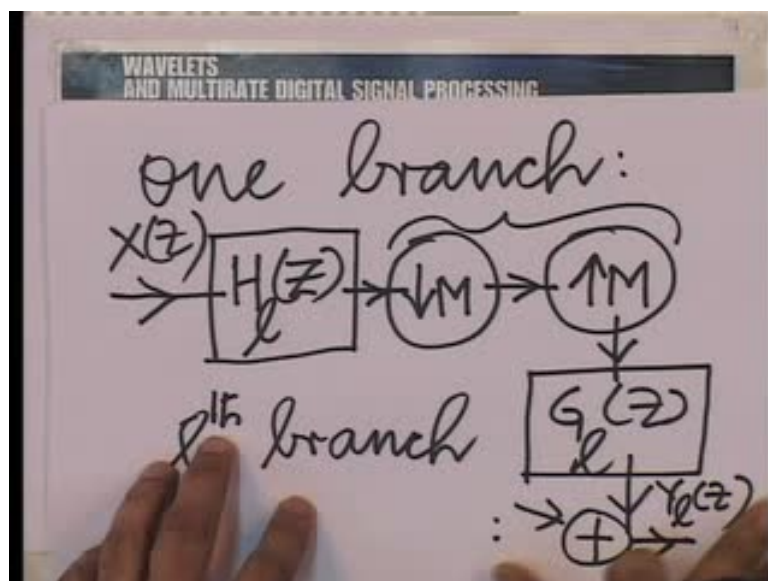
$$P_M[n] = \text{Inverse DFT}$$

$$\frac{1}{M} \sum_{k=0}^{M-1} 1 \cdot W_M^{nk}$$

for all n.

So, in fact,  $P_M[n]$  simply becomes 1 for  $k$  going from 0 to  $M-1$  and therefore, reconstruction is very easy.  $P_M[n]$ , in fact, not just in the region between 0 and  $M-1$ , but everywhere is 1 by  $M$  summation  $k$  going from 0 to  $M-1$ , 1 times  $W_M$  raised to the power  $nk$  for all  $n$ ; essentially, we have done an inverse DFT here. Having set this, now it is very easy to see what we are doing to the input sequence.

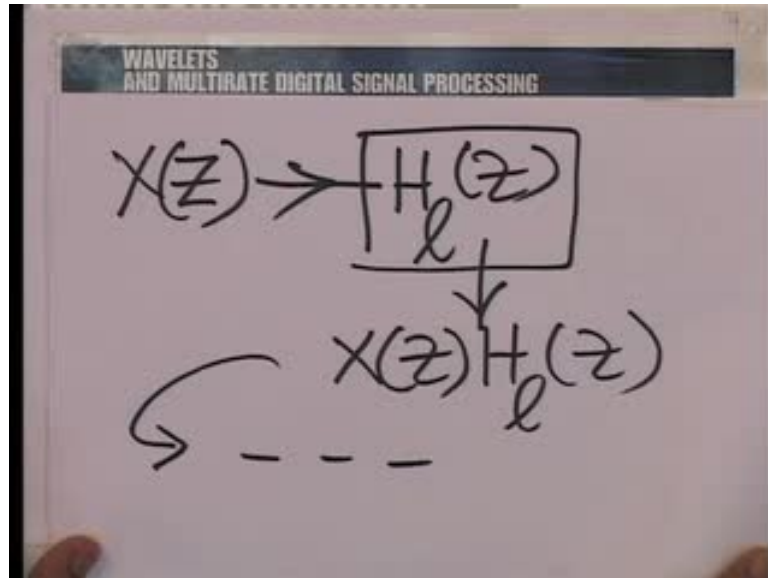
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You see, if we look at this branch once again, if you take this branch and look it at carefully once again, we know what has happened here. Essentially, there is a

modulation of the sequence here by a periodic sequence. We can write down the Z transform of the sequence here with great ease and also write down the Z transform of the sequence here, by using the principle of modulation.

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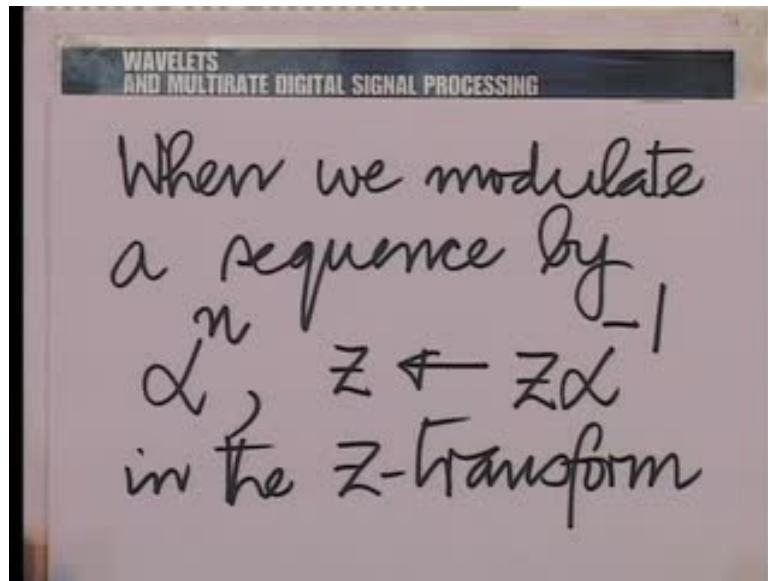


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The slide shows a handwritten equation. At the top, there are two horizontal dashes "--". Below them, the text "modulated by" is written. Underneath, the equation is written as  $\frac{1}{M} \sum_{k=0}^{M-1} W_M^{kn}$ . The background of the slide is titled "WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING".

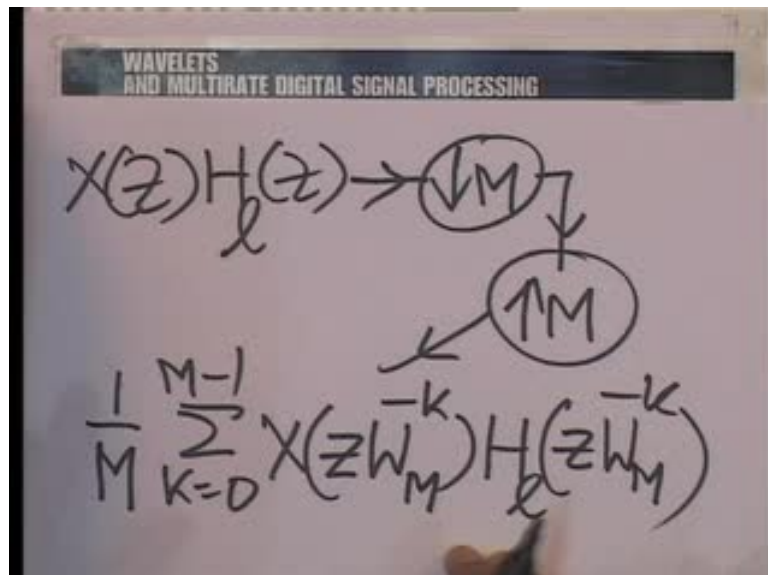


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So, let us write down at each point. We have  $X(z)H(z)$  there, after being processed by  $H(z)$  we have  $X(z)H(z)$ . Now, this is modulated by  $\alpha^n$  by  $M$  summation  $k$  going from  $0$  to  $M-1$ .  $M$  raise the power  $k$ . And recall that when we modulate a sequence by  $\alpha^n$  raise the power of  $n$ ,  $Z$  is replaced by  $Z\alpha^{-1}$  in the Z transform.

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If we use this property repeatedly and note, that Z transform is a linear operator, then we have  $X(z)H(z)$  there. After down sampling and up sampling, what we obtain is  $\frac{1}{M}$

summation K going from 0 to M minus 1  $X(zW_M)^{-k}$  times  $H_l(zW_M)^{-k}$  raise the power minus K.

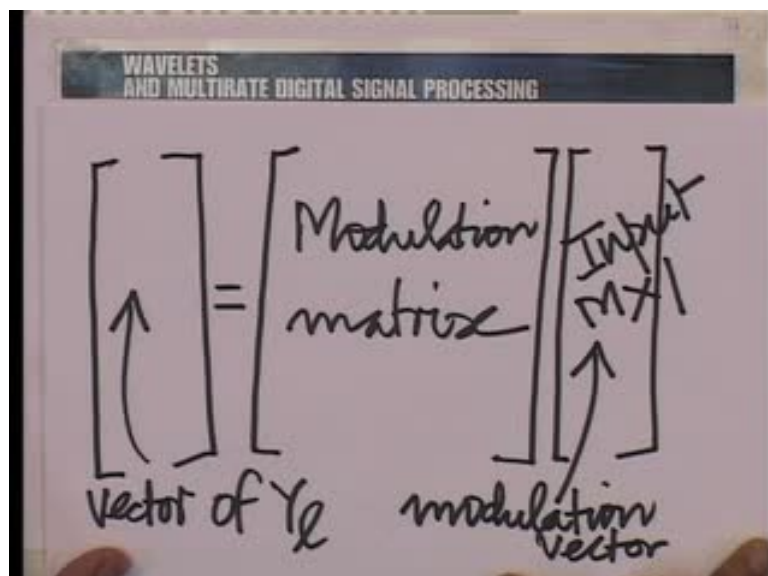
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WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

$$Y_l(z) = G_l(z) \cdot \sum_{k=0}^{M-1} X(zW_M^{-k}) \cdot H_l(zW_M^{-k})$$

So, therefore, we have M modulates of the input being acted upon by corresponding M modulates of the analysis filter. And now, we know, what happens when this is subjected to D Z; that is very easy. So, what we have here essentially is  $Y_l(z)$ , the lth branch carries on it  $G_l(z)$  times summation K going from 0 to M minus 1  $X(zW_M)^{-k}$  times  $H_l(zW_M)^{-k}$  raise the power minus K.

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Now, we could write this in matrix vector form. So, we could write a vector of  $Y_l$  here and we could write down, what is called a modulation matrix and a modulation vector.

The modulation here of course, it is the input, which creates a modulation vector and the modulation vector is going to be of size  $M \times 1$ . The size of this vector here, of  $Y_l$  is going to be as many as the number of branches here and of course, what is then going to be done is to add up all of these to obtain the output.

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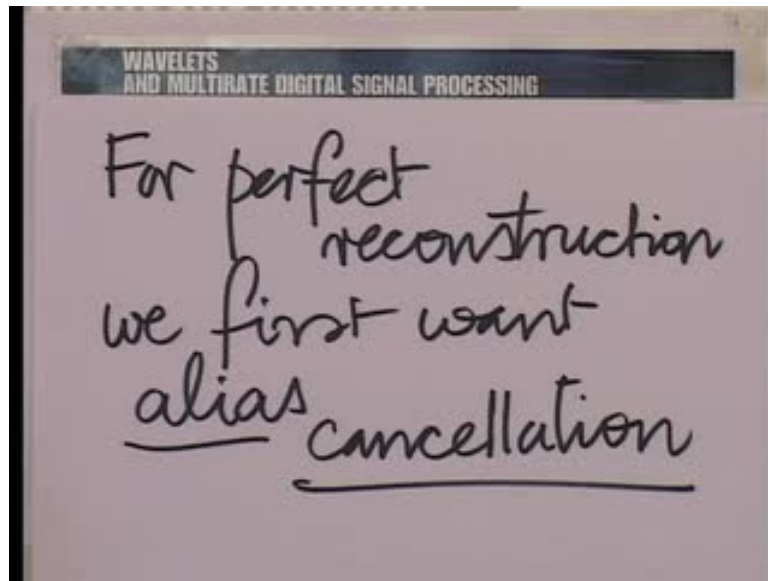
WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

$l^{\text{th}}$  row of modulation matrix

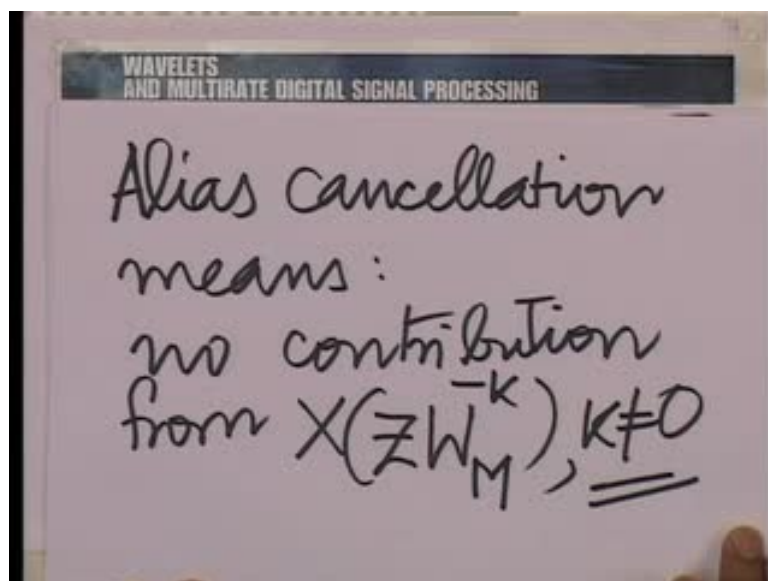
$$= G_l(z) \begin{bmatrix} H_l(zW^0) & H_l(zW^1) & \dots \\ \dots & \dots & \dots \\ \dots & \dots & H_l(zW^{M-1}) \end{bmatrix}$$

Let us write down a typical row of the modulation matrix. The  $l$ th row of the modulation matrix is essentially,  $G_l(z)$  times,  $G_l(z)$  times  $H_l(z)$  to the power  $z$  into  $\omega W M$  to the power of 0  $H_l(zW^M)$  to the power minus 1 and so on, up to  $H_l(zW^M)$  to the power minus  $M$  minus 1; so, essentially, the modulates of the analysis filter multiplied by the synthesis filter. Now, it is very easy to put down a condition for perfect reconstruction or at least shift invariant reconstruction.

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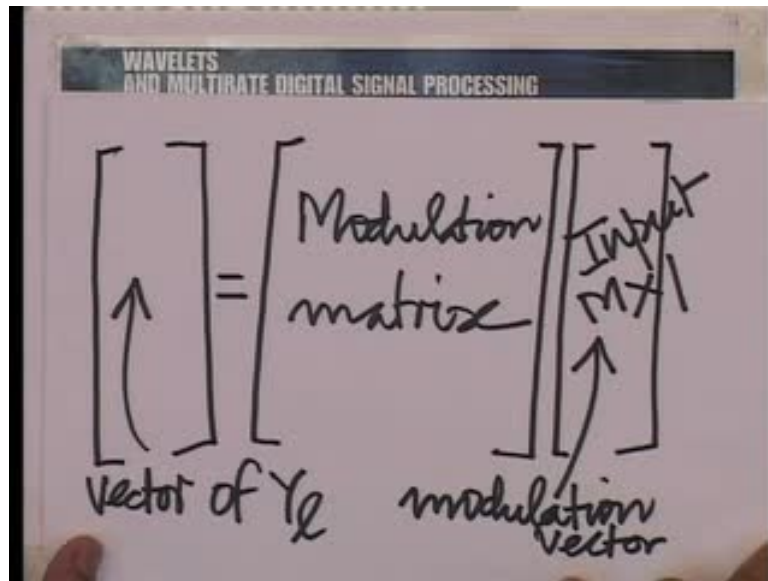


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So, for perfect reconstruction, we first want alias cancellation. An alias cancellation, essentially, means, no contribution from  $X(zW_M^{-k})$ ,  $k \neq 0$ .

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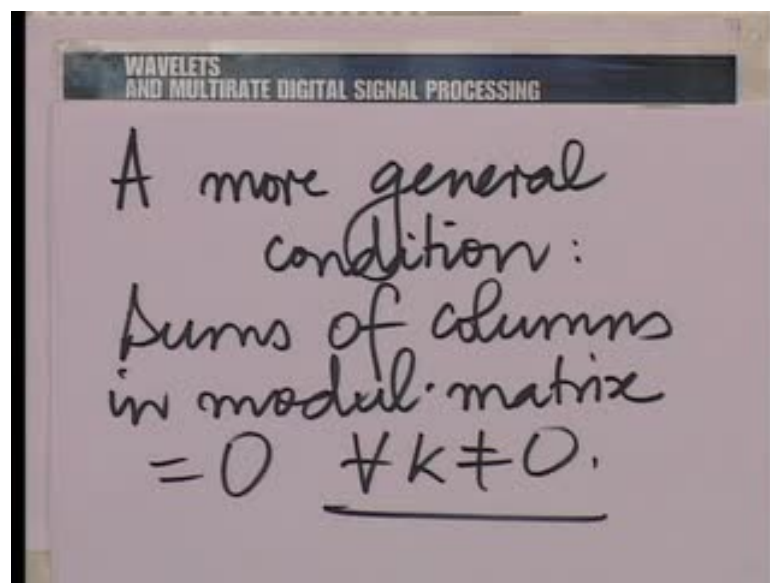
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The text is handwritten on a slide titled "WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING". It reads: "Essentially we ask for: first column of modulation matrix is the only nonzero column".

The only contribution we desire is for the case where  $K$  is equal to 0 and if we go back to the structure of modulation here. So, if we look at the structure once again, what we are asking for is that the 1st element of this vector must encounter a corresponding **none** possibly non-zero column, but all the other elements of this vector must encounter 0 columns here. And therefore, what we are asking for essentially is that the 1st, 1st column of the modulation matrix is the only non-zero column.

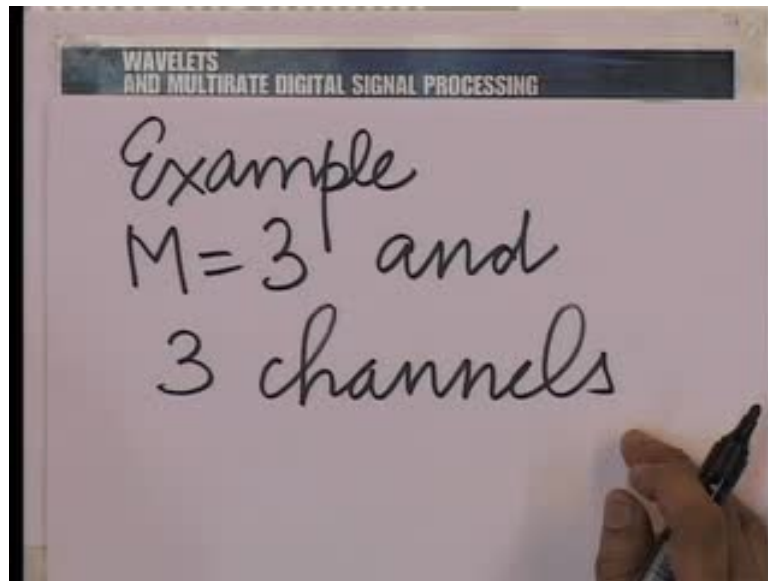
Well, the other way of looking at it is, well this is a very strong generic requirement, so this is sufficient, but of course, as we can see, it is not necessary. The slightly more relaxed condition would be in summing up all the  $Y_l Z$ ; we would like the other terms to be cancelled. So, what we are saying really is, if we take the sums of all the, after all, what we are going to do is take the sums of the columns here in the modulation matrix. When reconstructing the output from the input in these sums, we are going to involve, essentially, terms  $XZ$ ,  $XZ$ ,  $WM$  to the power 1, 2, 3 and up to  $M$  minus 1, the sums corresponding to the terms other than  $K$  equal to 0, must be 0.

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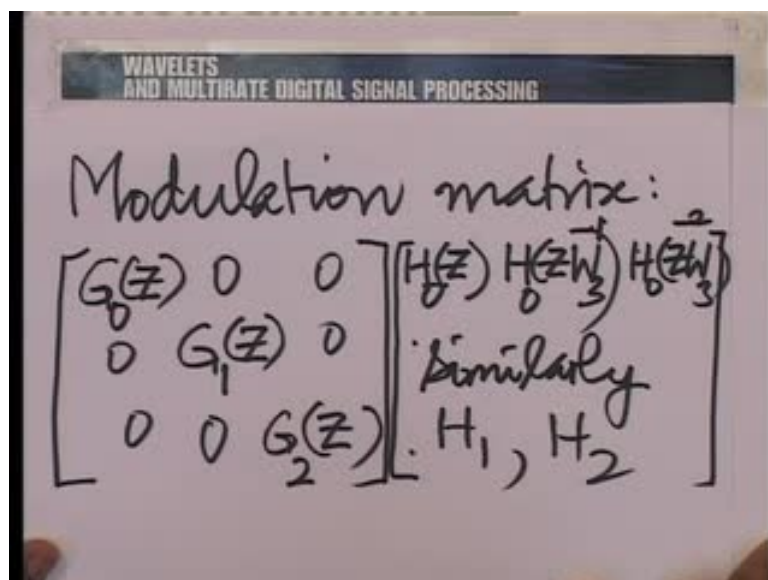
So, although 0 columns is a good thing to have, that would be possible only in the ideal case and therefore, we would, in general, ask not for 0 columns, but columns with sum to 0. So, a more general condition is sums of columns in the modulation matrix equal to 0, for all  $K$  not equal to 0.

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Now, before we can go to the construction on this approach of modulation to fix our ideas, let us consider the concrete case, where we have a 3 channel filter bank with  $M$  equal to 3. Let us write down the modulation matrix explicitly.

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So, modulation matrix is going to **have the form...**, well, you know, it is easy to write it down as a diagonal matrix times another matrix.

$H_0(z)$ ,  $H_0(zW_3)$ ,  $H_0(zW_3^2)$ , if you like,  $W_3^{-k}$  and  $H_0(zW_3^k)$  to the power minus 2. Similarly,  $H_1$ ,  $H_2$  in the column 2nd and the column 3rd here.

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The slide contains the following handwritten equation:

$$\sum_{l=0}^2 G_l(z) H_l(zW_3^{-k}) = 0$$

for  $k=1, 2$

What we would then desire, of course,  $G_0(z)$  multiplies each of the 1st row  $G_1(z)$ , the 2nd row and  $G_2(z)$ , the 3rd row. So, what we would ask for is that  $G_0(z)$  or rather  $G_1(z)$  times  $H_1(z)$  raise the power  $W_3$  to the power minus  $K$  sum for  $l$  going from 0 to 2 must be 0 for  $K$  equal to 1 and 2.

So, essentially what we are asking for is that these columns would sum to 0 here in the product. Now, of course, we can actually carry out an exercise to write down the ideal filters for the 3 band filter bank and build the idea from there, and I encourage the class to work out this exercise.

Consider the ideal 3 band 3 channel filter bank, where the analysis and synthesis filters are each triple band filters, so the low pass filter there is an ideal filter with pass band from 0 to  $\pi/3$ . The middle filter is a band pass filter with pass band from  $\pi/3$  to  $2\pi/3$  and the last one is an ideal high pass filter with pass band  $2\pi/3$  to  $\pi$ , and work out the modulation terms explicitly to see, how they come together and cancel alias terms.

As I said, we have been doing a lot of concepts all this while and my objective in a couple of the subsequent lectures is to introduce some applications, particularly



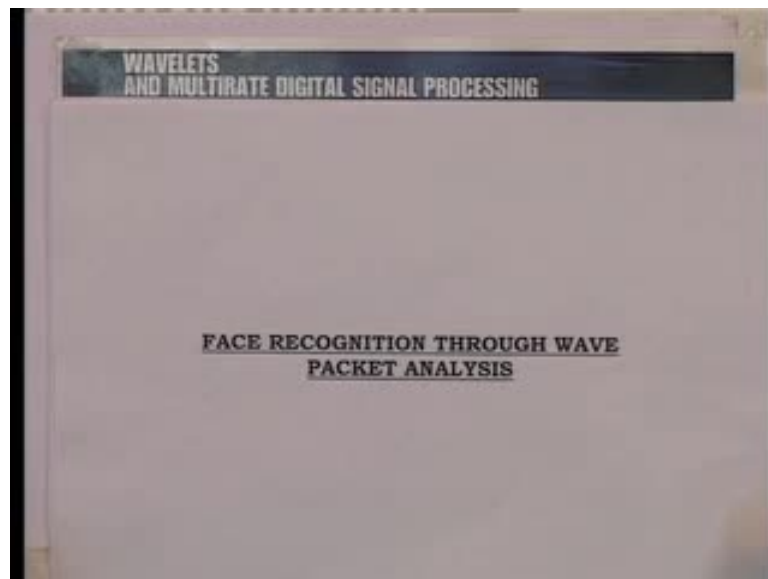
applications that have been investigated by the class that was taught this subject, this semester.

I have invited to this lecture some of the groups of students who have done an excellent job in their application assignments. Specifically, we have 2 groups that will represent themselves today very briefly and in a greater detail in a subsequent lecture. Today's will be a trailer, so to speak, just an introduction to their applications.

The 2 applications, that we are going to look at is first an application on face recognition and other application on databases or data mining. Without taking the thunder away from the people who are going to introduce the application today, let me now handover to the students who are going to briefly introduce the application today and discuss in depth later.

Thank you

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As a part of course work, we the students of this course, I have looked at some of the applications, that include domains like biomedical engineering, image processing, audio signal processing, a few applications in mathematics and a few applications in data mining. In fact, the application today that I am going to present is from image processing. My name is Ronaksha and the topic that we are going to cover is face recognition through wave packet analysis.

So, there are 2 keywords here, one is face recognition and the other is wave packet analysis. Now, why face recognition? We can look this particular keyword as biometric authentication problem, but the problem is face recognition incorporates changing signals. So, faces can be changed abruptly, so this will be difficulty if you go for biometric authentication. Whereas, some other signals, like retina and finger print, they cannot be changed, morphed so easily.

So, face recognition, as that case has limited applications in biometric authentication, so where do we require it? We require face recognition in some part as activity analysis. Let us say we are looking at a surveillance application in which we have to man some particular region in which no persons are allowed. In that case, if we go for face detection and recognition, we can get a very good hand over the particular region, so that we can say, that somebody entered at this particular time and what he was doing. So, we can basically go for activity tracking as well as abnormality detection. So, what that person was doing is also applicable in that case. What we are looking at is face detection as well as recognition, whereas these things are always coupled, face detection and recognition.

Approaches are always different, like face detection, when we go for we are looking for those features common to all faces, whereas in case of face recognition, we are looking at features, which are different across faces. So, this problem can be decoupled at that sense and we are only going to look at face recognition here. So, here it is assumed, that some region of, some region of image, that contains face is given to us and we are only going to do face recognition.

As I said, there are 2 key words, one is face recognition and another is wave packet analysis. Now, why wavelet based approach and not other approach? If we look at typical face image, it contains nose, eyes, mouth, etcetera and these are in focus when we look at different scales in different sub spaces because they contain different frequencies as well as spatial regions. So, we need to go for decorrelation in spatial, as well as, in frequency domain. So, this is the ideal application, this is the signal where we need to look into time as well as frequency localization, so that is what we are looking into wavelet domain.

Now, wavelet gives better representation in terms of time and frequency and that is fine, that will be required for classification. So, why wave packet analysis? Now, as know as, you know, if you pick features, if these features are decorelated as we get in wavelets, but if they are more rich in terms of information, we can classify it better and that is what we do with wave packet analysis.

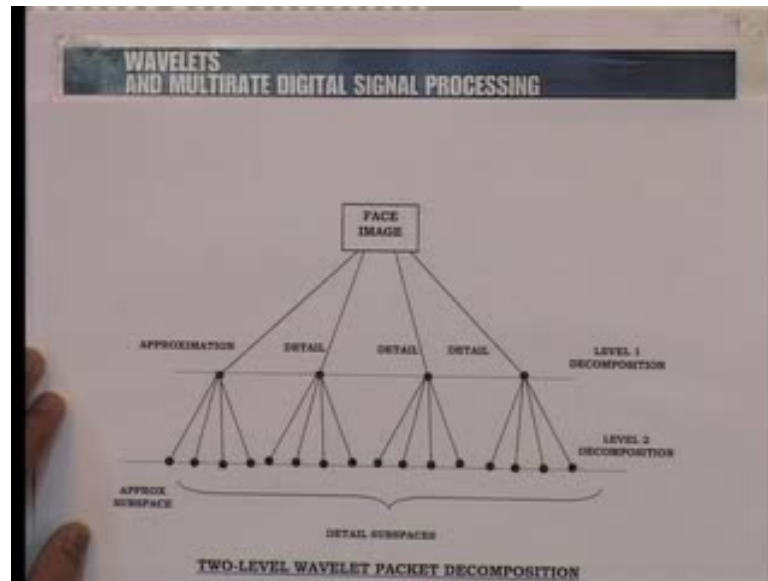
With wave packet analysis we are not only decomposing low frequency bands, but we are also decomposing the bands, which are at high frequencies and this gives richer representation for the face image.

So, there are 2 things here, we are decorelating space, as well as, in frequency domain and we are getting better representation. So, what we need to do is given an image when we would decompose wave packet analysis and then, we can represent it by features and these features are well decorelated, so we can do a very good classification.

The only problem that stands now is with the dimensionality. See, if we decompose this image into, let us say, multiple sub-bands, let us say, some 10 or 15 sub-bands, the data, that we have for classification is immense and as we note, **like curser of dimensionality**, that requires a lot of images for classification and that we cannot do.

So, one thing we can do is we can go for moment based approach and we can take only 1st and 2nd moment mean and variance with features and then, we can classify, that we can do. So, this is just a showcase, so I am going to show a few slides in which there are some results of decomposition and some basic retrieval. We are going to see detailed approach in subsequent lectures.

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So, this is the basic wave packet analysis we can see. So, given a face image, we can decompose into approximation and details of spaces, how this is done and which filters are utilized, that we are going to see in detail in subsequent lectures.

So, this is level 1 decomposition; in level 2 decomposition, we can similarly decompose approximation as well as detailed sub spaces. So, we will get some 16 sub spaces in which 1 will be approximation and some 15 will be detailed sub spaces.

Now, will it do generate features for this? As I said, we are going to use mean and variance only for feature representation. Now, come to think of this, when we utilize (( )) distance, we generally of features, which are not mean and variance together because we need, we are now looking towards PDFs, probability density functions, rather than individual values. So, we need a distance metric, which takes care of mean as well as variance. So, normal (( )) distance will not perform better in that case. So, we need to go for some other distance and that will be utilized later, that I will talk about later in subsequent lectures.

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Now, I am going to show some of the decomposition results, this is just for introduction. So, we can see, given an original image like this, **Lena in the face image**, we can decompose it to different bands. Here, we can see, this is the approximation sub image or rather sub space, so we can see some of the details very clearly and these are the, like all other 15 are detailed sub spaces, so they basically capture some high frequencies.

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Now, the problem with this original images, if we go for approximation, we cannot capture low frequencies, the DC, because we need to take care of lightning condition as

well. So, this approximation, so in that case, we cannot utilize low and high frequencies as we traditionally use in wave packet analysis. So, what we are going to use will be again in subsequent lectures and I am going to show some of the reasons. And I will conclude this showcase for this particular application. We have used a yield database, so given an image like this, which has different expression like, **an image**, a database contains some 15 subjects and 11 different expressions and lighting conditions.

So, if you get better results here, like your algorithm must be robust in that case. So, given a query image like this, the identity to retrieve images of the same class like this, but as we can see, expressions are different all the way and lighting conditions are also different, or for other application we might go for given a single image, where either only interested in, let us say, the class to which it belongs or we are interested in all the images. So, in later case, we can go for activity analysis and 1st case, we are going for recognition problem traditionally. So, this is what I am going to present in subsequent lecture in detail and with this, I will conclude the showcase.

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Good morning, the topic of our presentation for this application assignment is application of wavelet in data mining. I am Archo Choudhry and my other group member is Kunal Shah.

In today's world, we need to deal with the huge amount of data for a lot of practical purposes and data mining is a subject, better to say, it is an ensemble of the methods or tools, which we use to deal with huge amount of data efficiently for our own purposes.

Now, this dealing up with data involves 2 things, one is storing it with the help of an efficient data structure and 2nd is a retrieval of data from the data structure, as for our own purpose.

Now, for this purpose, when we have to store and retrieve the data, the user basically interacts with the data structure by the help of some queries and responses. Now, in practice, when we have a large amount of time series data, the queries, that are generally encountered are not point queries, rather they are the queries, which spends over some larger duration of time, such as I am, if I am having a data structure, which says, that I am having the stock prices, the daily stock prices of a company over a particular period of time, then the queries, which generally we encounter are something, like this, on which months the stock prices had a rising trend or on which weeks the stock prices suddenly had a deep dip.

So, in order to get this data, we need to do some post processing on the raw data and we get the output of this query. This way we interact with the data structure and we get the output.

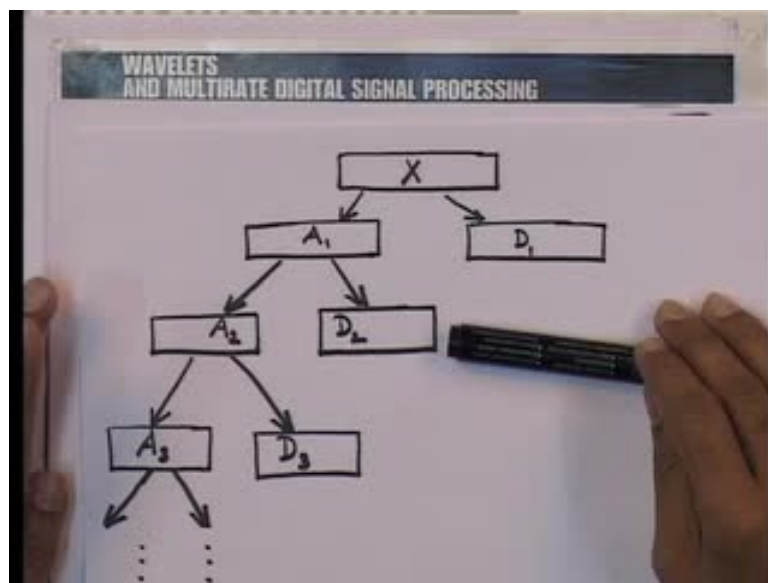
Now, here we can think, that now, in this case of this application assignment, we can think, that why we are correlating wavelet with this type of situations. Now, we all know, that wavelet transform on a signal gives us the information at various levels of trends, better to say, at various levels of abstraction and with the various translates and various scales.

Now, if you practically look at a data structure, rather raw data can be thought of as a sequence or signal, better to say, then we can think, that we are also interested in various translates and scales of the signal, in particular. If we say, our daily stock page data as a sequence, then depending on our queries, we can say, that we are interested in various translates on the data, such as we are interested in a data 2 months back or we are interested in a data 6 months back. So, this 2 months, 6 months, these are different translates.

At the same time, we can also explain in it, whether we are interested in a data at different scales, such as we are interested in a weekly data around 2 months back, so in that case, that 2 months becomes the translate and the week becomes the scale. Similarly, we are interested in a monthly data 2 years back, so in that case, that monthly data becomes the scale and the 2 years back, that 2 years becomes the translation parameter. So, based on this analogy, we found, that wavelet can be used very efficiently in this situation to deal with this type of analysis. And in this case, if we use wavelet, then we need a very little post processing to analyze this type of data because whatever data mining technique we use, our main aim is detected on 2 things - the 1st one is that we need a least memory crunchy data structure and the 2nd one is we need the least amount of post processing, that is, the post processing complexity is very less.

So, if you use wavelet in this case, our post processing complexity reduces drastically because if we have to do the response, if we have to answer to the query based on only the raw data, then we have to do a lot of (( )) post processing.

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Now, let us have a deeper look in what we have exactly done in our application assignment. We have basically done the trend and surprise analysis of a time series data; we have done the trend and surprise analysis of a time series data at various levels of abstraction and for this purpose we have used a particular data structure, which is called a trend and surprise abstraction tree, in acronym it is called TSA tree and it is, it looks



like something like this, at this X node, it is something like a tree and the root node at the top level. This root node, which is the X node, it is the original sequence itself and now all the subsequent nodes, this one, A1, D1, A2, D2, these nodes are obtained by discrete wavelet transform of its parent node, and in this way we construct a tree, which is called a TSA tree.

And as we go down this tree by taking the discrete wavelet transform at this level, again we take the DWT on A and we get this A2 and D2, again we take DWT on A2, we get A3, D3 and so on. We can proceed in this way the number of levels, as what you want.

Now, when we go down each level, it is something like we are increasing our level of abstraction by 1 level each and whenever we need a trend or surprise at any level, say at this level, so 1st we need to extract out this node here. Each node corresponds to a one particular sequence, so first we need to extract out this node and from this node with some further post processing, we have to find the trend information corresponding to this level.

The almost similar rule applies to that surprise information also. If we have to extract out the surprise information at this abstraction level, so we need to first figure out this D2 node and from D2 node we have to do the post processing to get a surprise extraction.

Now, the next part of our job is that we have to implement this thing in a memory efficient manner. Now, for this purpose, it is important to look into that matter, that at the first appearance it may appear, that we need to store whole wavelet into a memory for all operation, but its deeper look will say, that we do not need all the nodes to store in the memory. If we store only these leaf nodes, we can generate any of the intermediate nodes from which we can start our calculation and so only these leaf nodes are enough to get any of the intermediate nodes. So, we just need to store only the leaf nodes and further on, we have compared 2 more algorithms, which are known as node dropping and coefficient dropping, which talks about further compressing the leaf node information, so that to get a more memory efficient implementation of this method. So, this is the whole outline of our work that, which we have done and we will elaborate it in further details in the subsequent presentation. Thank You.

So, we saw 2 very interesting introductions, 2 applications by Ronaksha and Archo Chowdhry. Of course, both of these students and their groups would present the

application, the results, the concepts in much greater depths in a subsequent lecture, what we intend to do today was to raise the curiosity of the student to want to understand these applications better and we shall satisfy that curiosity in a subsequent lecture.

Thank You.