Advanced Digital Signal Processing-Wavelets and Multirate

Prof.V.M.Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Module No.# 01
Lecture No.#28
JPEG 2000 5/3 Filter Bank And Spline MRA

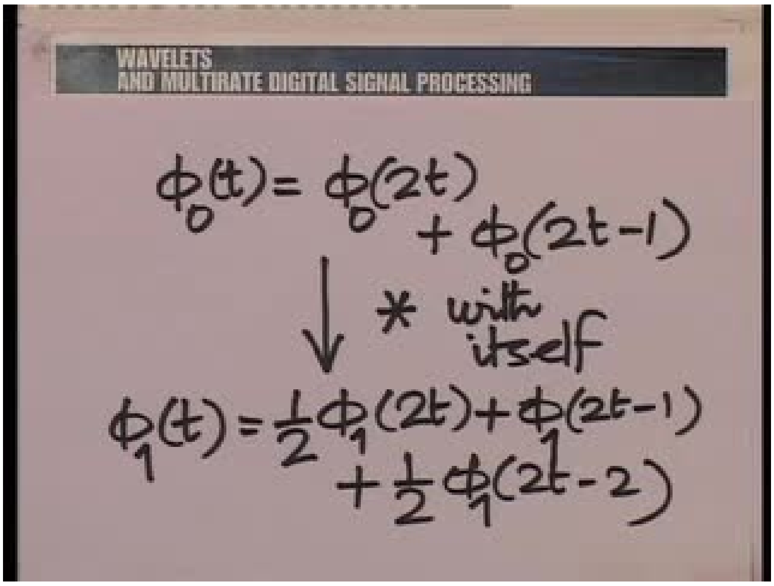
A warm welcome to the 28th lecture on the subject of wavelets and multirate digital signal processing. Let us spend a minute onto in recalling what we did in the previous lecture; we had in the previous lecture, briefly introduced the need for thinking out of the box in the context of multi resolution analysis. Specifically we had introduced some variance of the idea of multi resolution analysis and these variance that we had talked about, where one where we do not insist on the synthesis and the analysis filter banks to be the same of a that matter, we do not insist to an orthogonal filter bank. The second, where we do not insist to an iterating only on the low pass branch, and third, where we do not insist on f i r filters.

Today, we shall explore in greater depths, the possibility of allowing different analysis and synthesis filter banks, and in fact, related to that is the idea of having different links low pass and high pass filters. After we complete our discussion on one such filter bank, which has been accepted in the standards, we shall also make it clear why such filter banks are attractive.

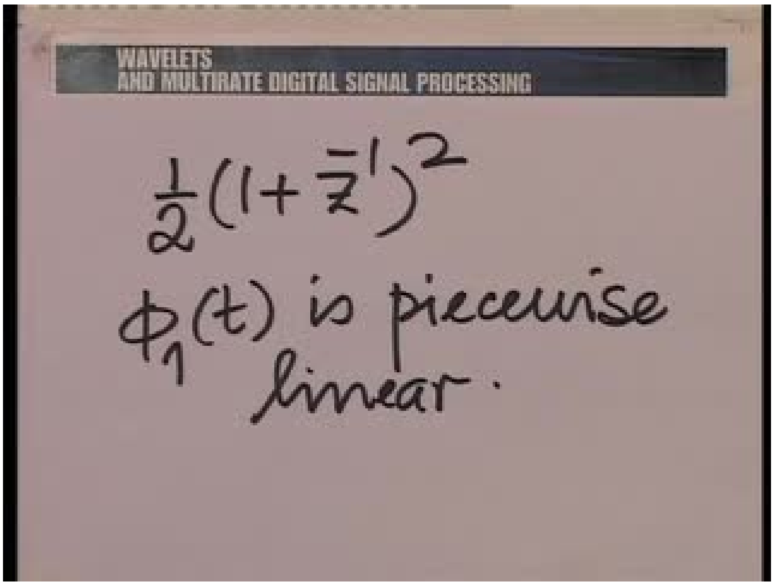
(Refer Slide Time: 02:08)

So, with that thing let us proceed to the main theme of the lecture today. You will recall that in the previous lecture, we had briefly introduced the on the recent standard for data compression called JPEg 2000, joint photographic experts group standards introduced around the calendar year 2000. And we were going to talk about what is called the 5 3 filter bank and associated with it what is called the spline multi resolution analysis.

(Refer Slide Time: 03:01)



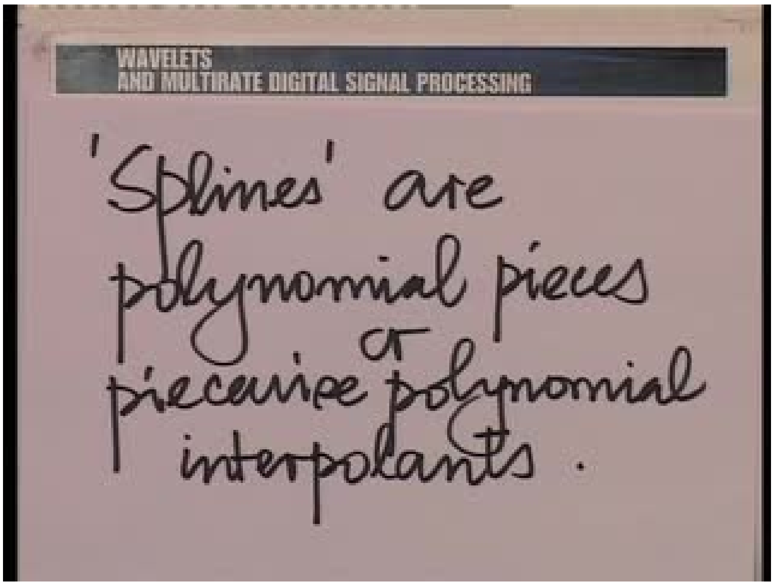
(Refer Slide Time: 04:04)



Well, just to put back a few points that we had talked about towards end of the previous lecture, we had seen that we could extend the dilation equation constituted for the hour m r a, namely phi 0 t is phi 0 2 t plus phi 0 2 t minus 1 by convolution with itself to get phi 1 is half phi 1 2 t plus phi 1 2 t minus 1 plus half phi 1 2 t minus 2. And we had noted that if we look at the coefficients of this dilation equation, dyadic dilation equation, the coefficients essentially represent the filter half 1 plus z inverse the whole squared and in fact, phi 1 t is piecewise linear.

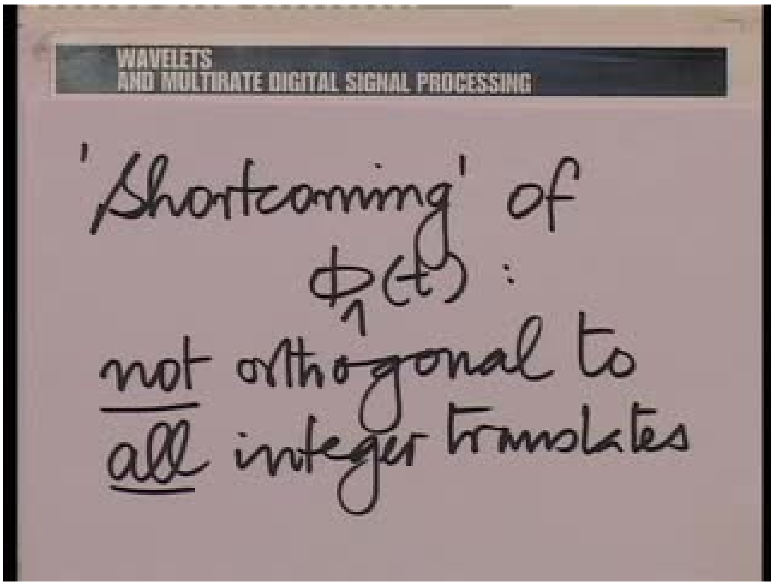
Now, an important observation is that as we convolve phi 0 t with itself repeatedly, we seem to get piecewise polynomials of higher and higher degree and this notion of piecewise polynomial of higher and higher degree is what is captured in the word spline. So, let me put it down a not so formal, but a reasonably clear definition of the word splines.

(Refer Slide Time: 04:56)



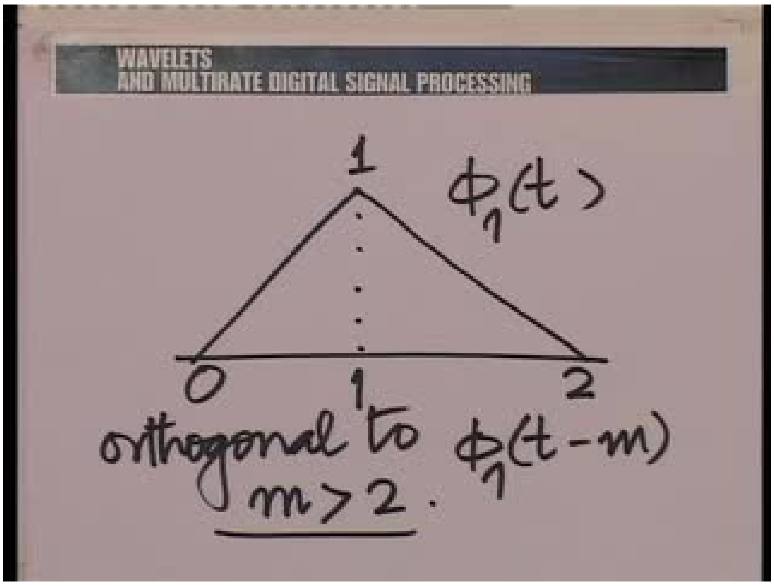
Spline are polynomial pieces of piecewise polynomial functions, in fact to be more precise, they are piecewise polynomial interpolants. Functions that interpolate the joined, that complete in between samples using piecewise polynomials and as you can see there is a generalization here from the idea of piecewise constant approximation with which we began the discussion on the haar multi resolution analysis.

(Refer Slide Time: 06:04)



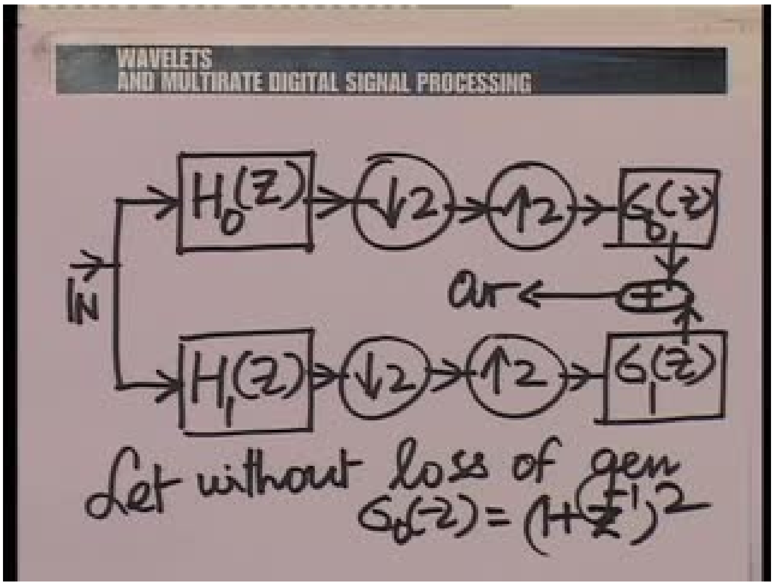
Now, we also noted the shortcoming of phi 1 t, the shortcoming of phi 1 is that it is not orthogonal to all integer translates, I must make one thing clear here, phi 1 t is of course orthogonal to integer translates, where the integers are multiples of 2, let me sketch phi 1 t for you once again.

(Refer Slide Time: 06:50)



Phi 1 t looks like this and it is of course orthogonal to phi 1 t minus m for m greater than 2, but it is not orthogonal to phi 1 t minus 1 and that is the catch point or that is a shortcoming here. When we say orthogonal to all integer translates, we mean all integer translates, we cannot excuse 1, but then you know we will see after a while that although we cannot excuse that 1 integer translate where it is not orthogonal, we can indirectly incorporated to build a different paradigm of orthogonal multi resolution analysis, but that is a more difficult exercise. We agreed towards the end of the previous lecture first to take a slightly out of the box approach, not to insist anymore on orthogonal multi resolution analysis and we shall take that approach first. In other words, let us put down 1 of the low pass filters either on the synthesis or the analysis side to be exactly the filter that constitutes the coefficients of this dilation equation.

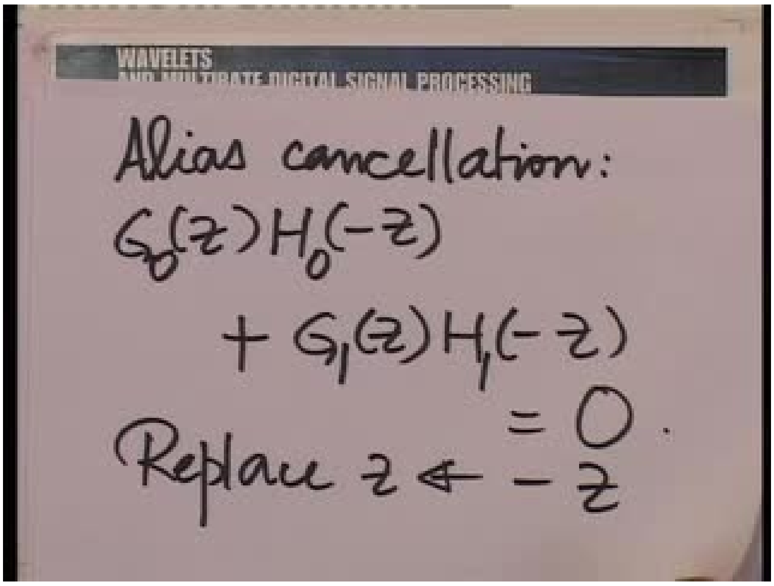
(Refer Slide Time: 08:40)



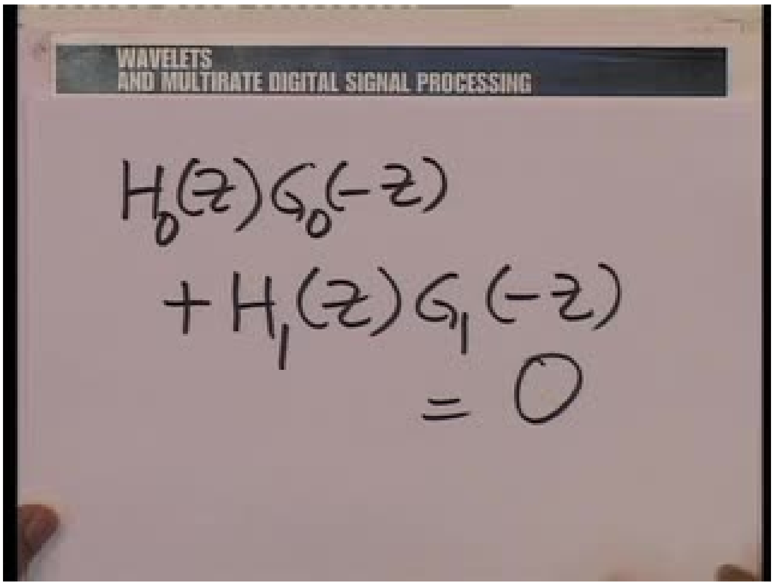
So, let us consider a filter bank, a two band filter bank. Just to recaptulates some of our ideas, what we are saying is we have this two band filter bank, where the low pass filters are h 0 z and g 0 z on the analysis and synthesis sides respectively and the high pass filters are h 1 z and g 1 z, again on the analysis and the synthesis sides respectively, in here, out there.

So, what we are saying is, let without loss of generality g 0 z equal to 1 plus z inverse the whole squared. Now, I must before I proceed, make an important remark about perfect reconstruction filter banks, you know the question is we have chosen g 0 z to be 1 plus z inverse the whole squared, we could have as well chosen h 0 z to be 1 plus z inverse the whole square. An important question that we must answer is if I interchange the analysis and the synthesis side what difference does it make?

(Refer Slide Time: 10:43)

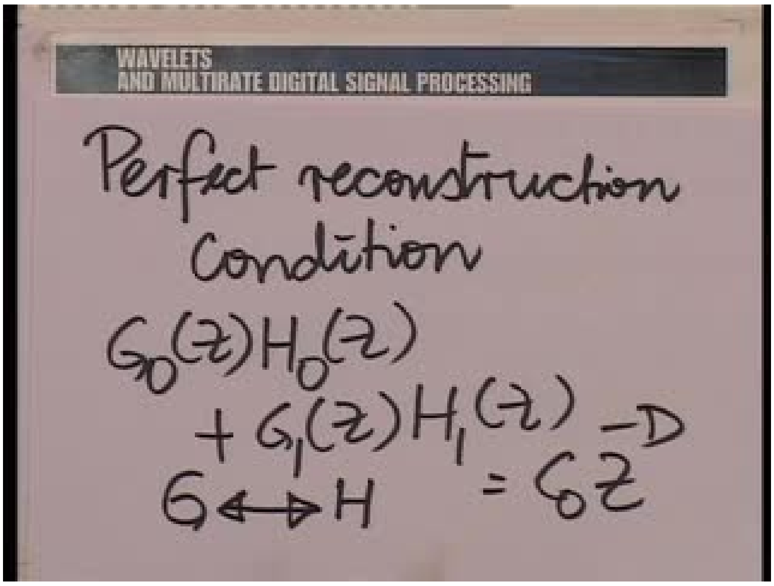


(Refer Slide Time: 11:22)

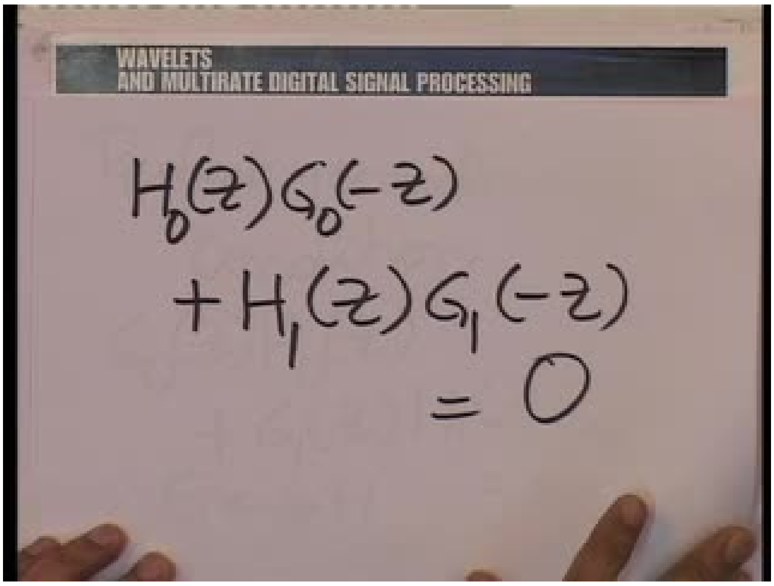


Now, to answer that question, let us look at the requirements of aliasing cancellation and perfect reconstruction. The alias cancellation requirement essentially says g 0 z h 0 minus z plus g 1 z h 1 minus z is 0. Now, if we replace z by minus z we would get h 0 z g 0 minus z plus h 0 z g 1 minus z is equal to 0.

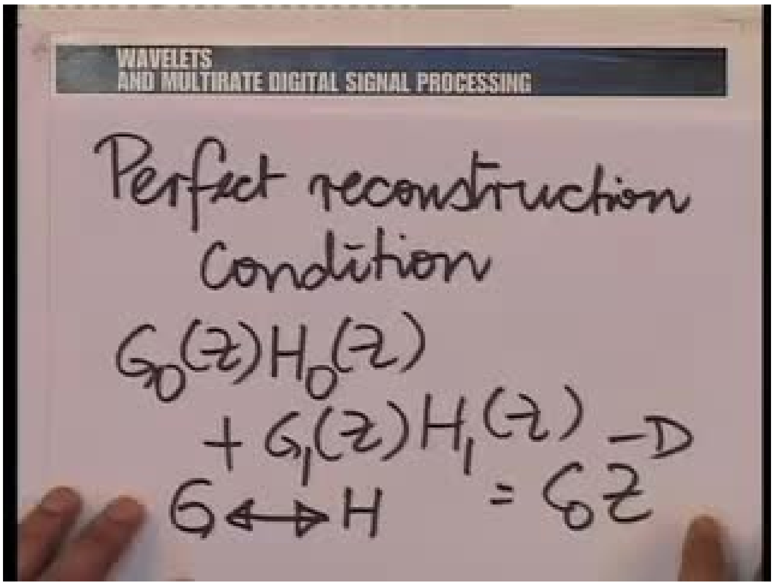
(Refer Slide Time: 11:50)



(Refer Slide Time: 12:41)



(Refer Slide Time: 13:14)

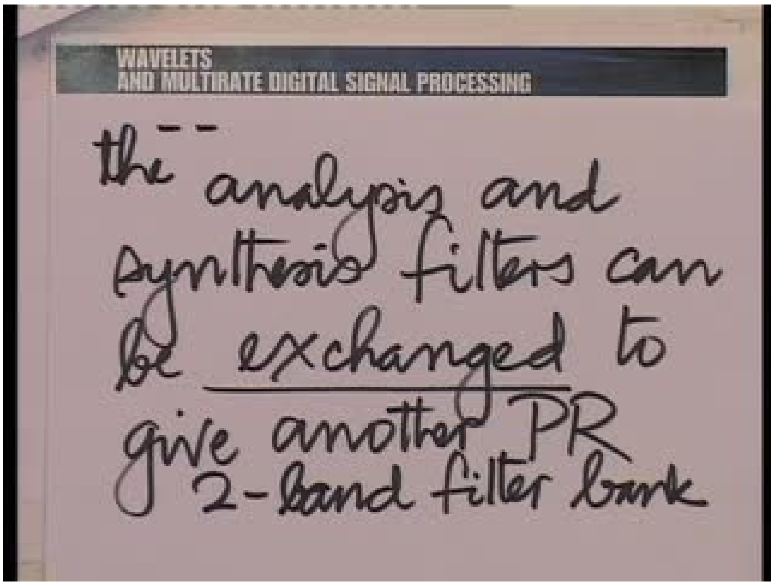


Now, let us keep this as one a far observations and take the second observation now, namely that of the perfect reconstruction condition. The perfect reconstruction condition says g 0 z h 0 z plus g 1 z h 1 z is essentially of the form a constant and then a delay. Now, here, we can interchange g and h as you notice, in other words, we could say h 0 z g 0 z plus h 1 z g 1 z is the same thing, trivially true. And now, if I combine these two observations that I have made, you will notice that this equation is an alias cancellation equation for the case when you interchange the analysis and the synthesis side. So, here the synthesis low pass filter as become the analysis low pass filter and the synthesis high pass filter has become the analysis high pass filter. And this then becomes the alias cancellation condition for this interchanged filter bank and as far as the perfect reconstruction condition goes, we do not need to do anything at all, it is already an interchanged condition.

So, if h 0 where the synthesis low pass and g 0 the analysis low pass, and correspondingly if h 1 where the synthesis high pass, and g 1 the analysis high pass, we would still have this perfect reconstruction for this rearranged or exchanged 2 band filter bank as the case is and therefore, we have a very an important conclusion that we draw here.

(Refer Slide Time: 13:51)

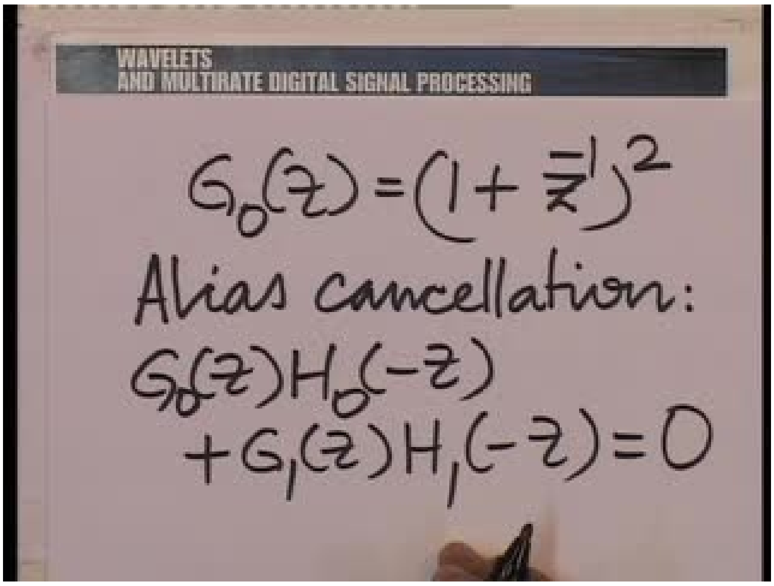
(Refer Slide Time: 14:29)



Once the perfect reconstruction conditions are satisfied in a 2 band filter bank, the analysis and synthesis filters can be exchanged to give another perfect, the p r stands for perfect reconstruction, perfect reconstruction two band filter bank.

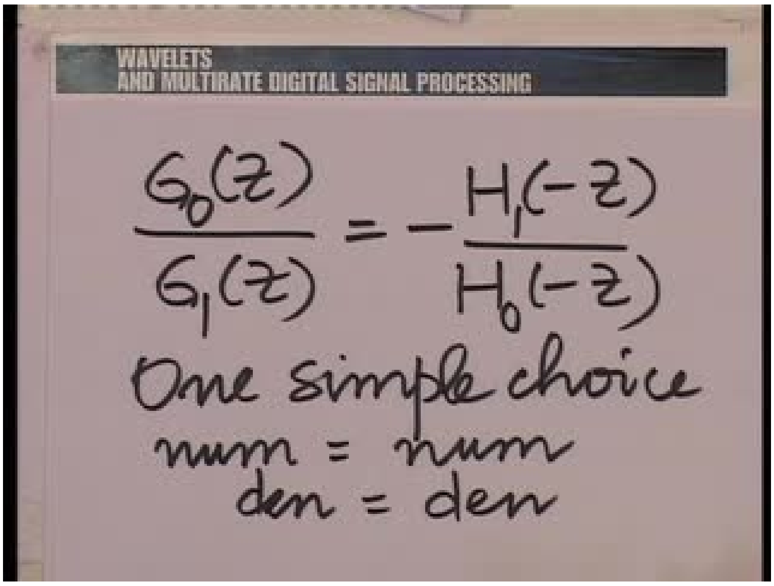
A very important observation, because it tells us that it is not so critical, it is not so important, which pair of filters we put on the analysis side in which pair of filters we put on the synthesis side, as far as perfect reconstruction goes, but there is a settle point here. And that settle point is, if you look at the lope, if the low pass filters happen to be different on the two sides and if you use the low pass filters to construct a dilation equation, a dyadic dilation equation, I mean the coefficients of the low pass filter could be thought of as representative of the coefficients of a dilation equation that operates on the scaling function. So, if you visualize the two different scaling functions that come out of the analysis and the synthesis side, it is quite possible that one of them is core and core smoother than the other and in that case, one might want to decide which scaling function to put on the analysis side and which one to put on the synthesis side.

(Refer Slide Time: 17:00)

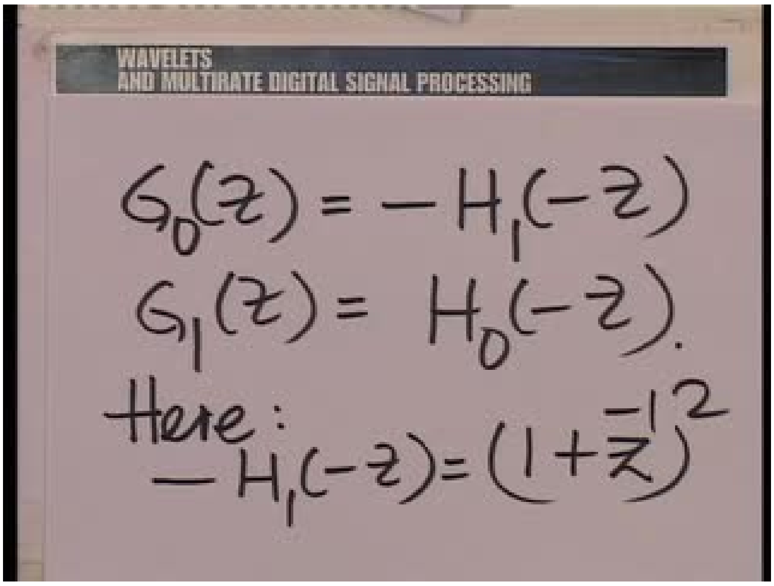


A broad guideline is that it is desirable normally to put the smoother function on the synthesis side, because if one uses smoother function to synthesize one would get more a appealing reconstruction after quantization, that is in a side, but an important observation. But, coming back to this spline filter bank that we are now going to build, let us consider a two band filter bank as we have here, with g 0 z as it where equal to 1 plus z inverse the whole squared.

(Refer Slide Time: 17:57)

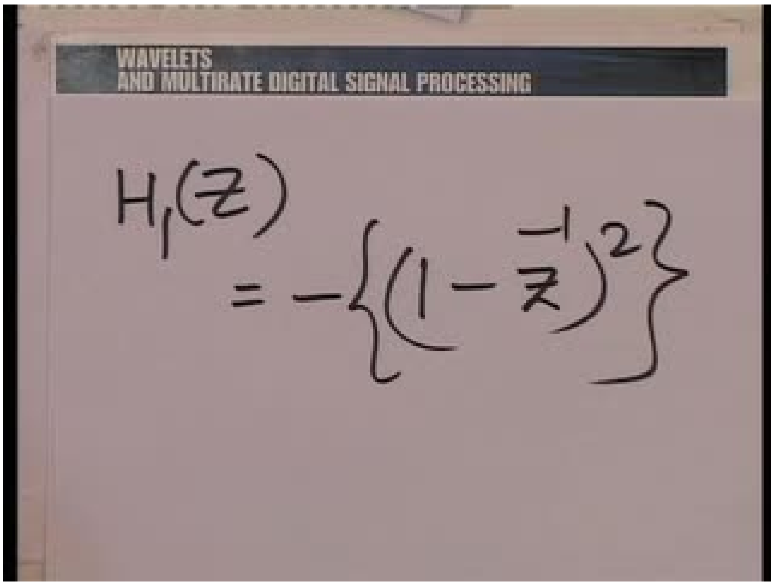


(Refer Slide Time: 18:30)



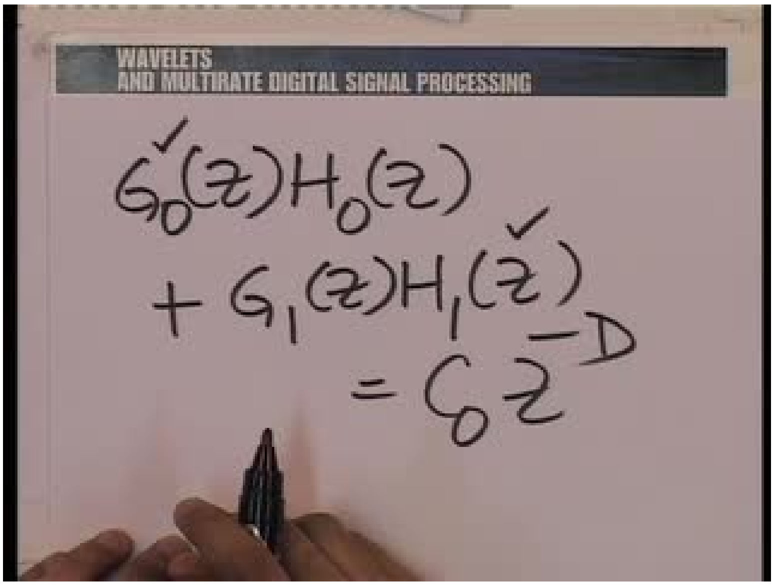
Now, we shall take the alias cancellation equation first and we shall insist initially only on satisfying the alias cancellation equation, which tells us that g 0 z h 0 minus z plus g 1 z h 1 minus z is equal to 0. Now, we can rearrange this as you know to the following equation, g 0 z by g 1 z is minus h 1 minus z by h 0 minus z. And one simple choice to ensure alias cancellation is to choose numerator equal to numerator here and denominator equal to denominator and that is exactly what we shall do. We shall choose g 0 z equal to minus h 1 minus z and g 1 z equal to h 0 minus z and therefore, the equation that we would get, now we want to write all the equations in terms of g 0 and h 1, because once we know g 0, we know h 1.

(Refer Slide Time: 19:22)

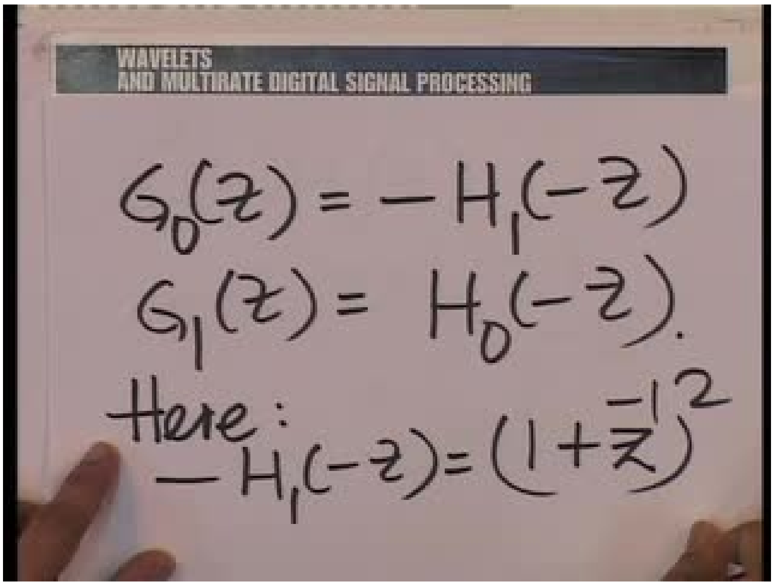


Therefore, in our discussion here we would have minus h 1 minus z is 1 plus z inverse the whole square from here and therefore, we would have h 1 of z is minus, of course take the minus sign on the other side and replace z by minus z, so 1 minus z inverse the whole squared, this is what we have.

(Refer Slide Time: 19:48)

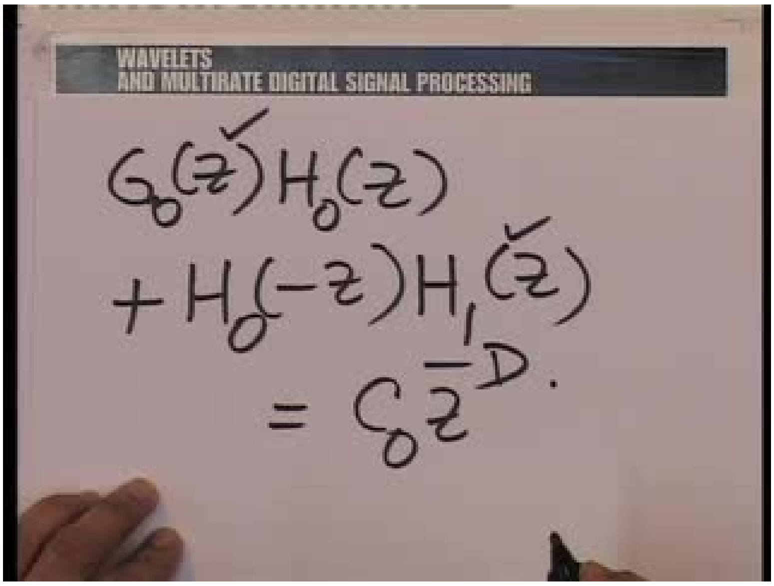


(Refer Slide Time: 20:22)

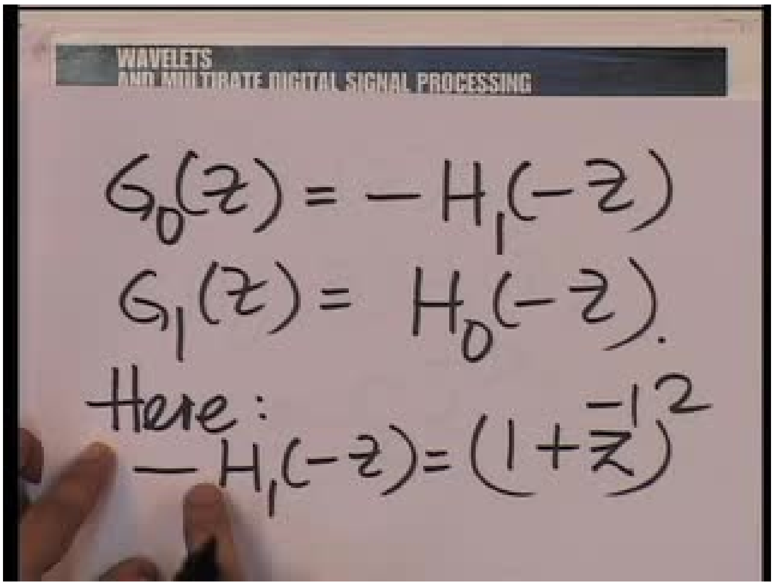


Now, we write down the perfect reconstruction condition as well. The perfect reconstruction condition says g 0 z h 0 z plus g 1 z h 1 z is a constant and a delay. And we know h 1 and we know g 0, we are required to find h 0 and g 1. Here again, we know that g 1 can be expressed in terms of h 0, in fact we have the expression here, so let us make a substitution of g 1 in terms of h 0, we essentially wish to obtain h 0.

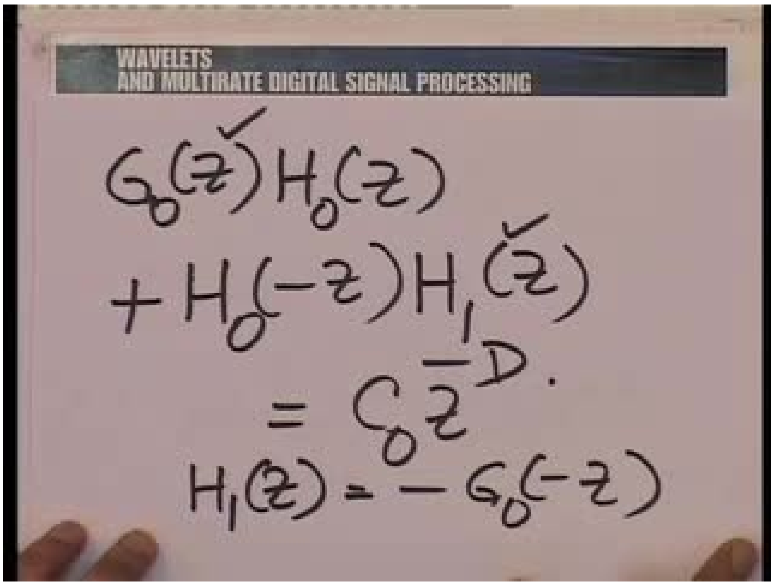
(Refer Slide Time: 20:44)



(Refer Slide Time: 21:28)

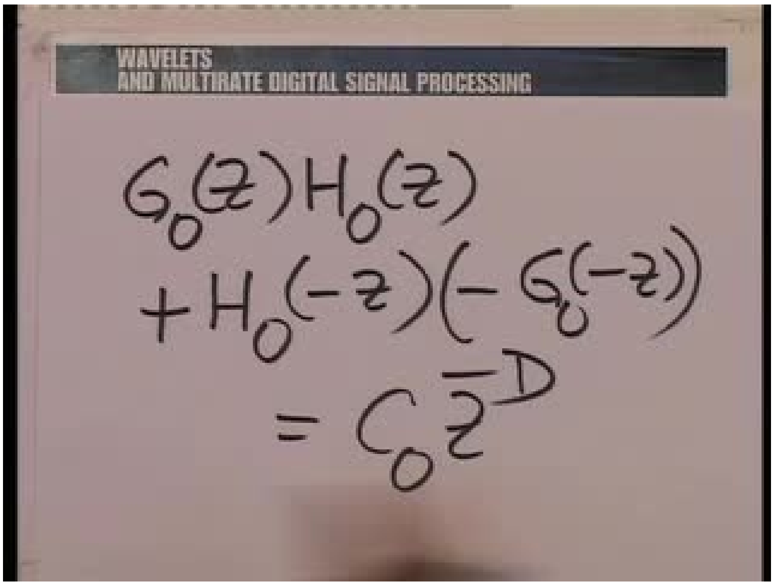


(Refer Slide Time: 21:39)

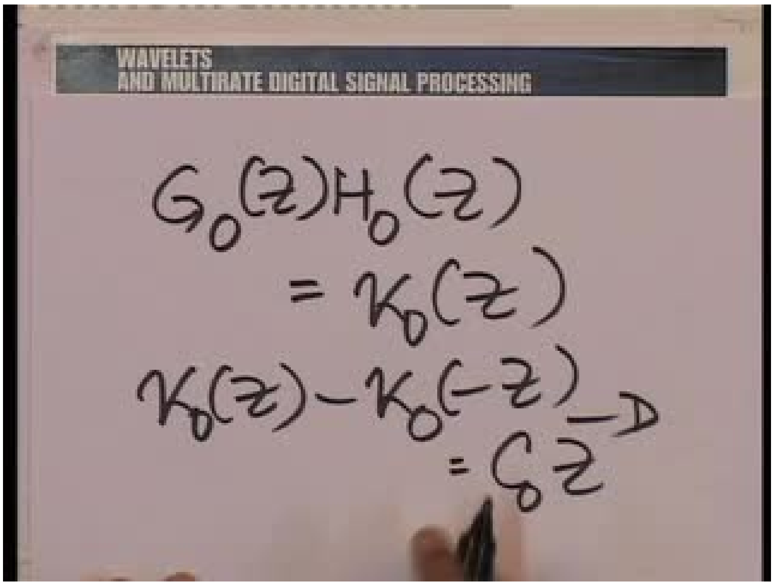


So, making this substitution we would have g 0 z h 0 z plus h 0 minus z, well h 1 z, but h 1 z can again be expressed in terms of g 0 z, so we leave it like that, we know it, is c 0 z raise the power minus d. In fact, let us write down h 1 in terms of g 0 notionally first, so you know if you recall g 0 z is minus h 1 of minus z, now, of course, we have an explicit expression for h 1, but what I have should do is to bring out a property of this product here, g 0 h 0 by making an explicit substitution. So, let me take h 1 z and rewritten in terms of g 0 z, so h 1 z would clearly be minus g 0 minus z and we can make that substitution here.

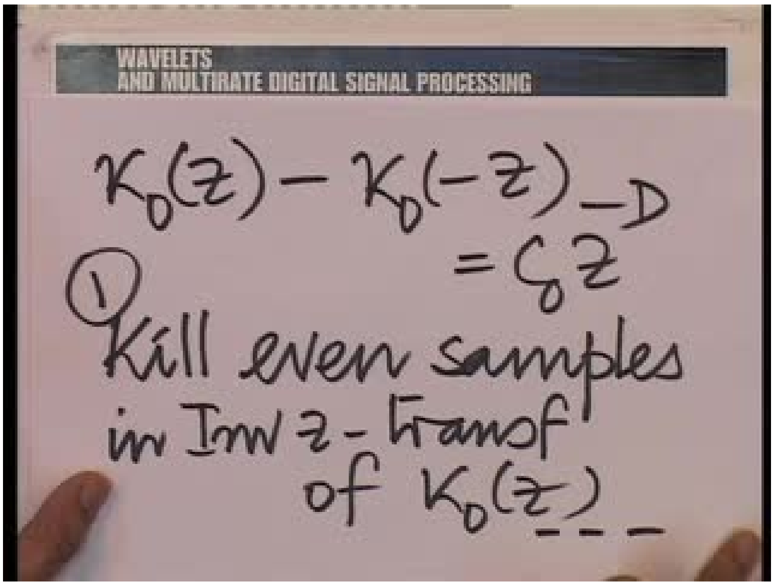
(Refer Slide Time: 22:13)



(Refer Slide Time: 23:12)

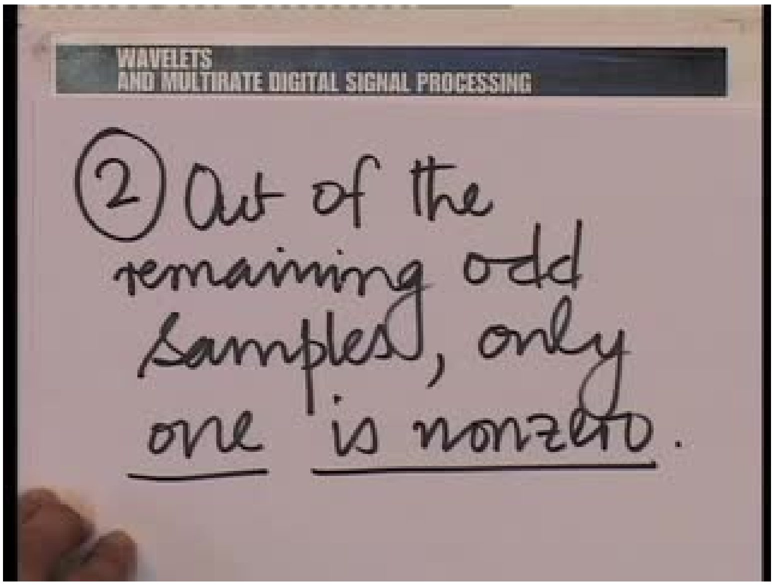


(Refer Slide Time: 23:45)

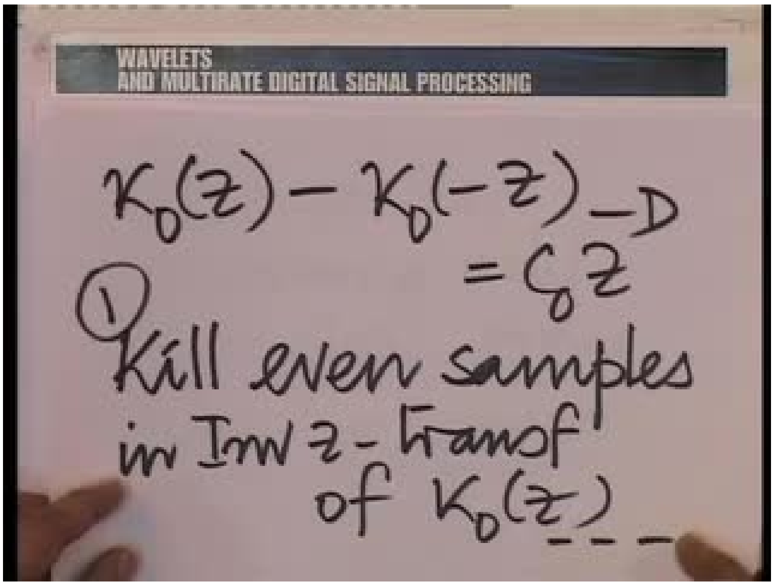


So, we will substitute h 1 by minus g 0 minus z, and all in all we then have something very interesting in terms of the samples of this product. Now, again, we have a situation where we have a product of z transforms and then the same product of z transform should z replaced by minus z, let us see what we have here. So, notice, we have g 0 z h 0 z minus g 0 minus z h 0 minus z is essentially a constant and a delay. And if we choose to denote g 0 z h 0 z by another kappa 0 z what we saying in fact is kappa 0 z minus kappa 0 minus z is essentially a constant and a delay. And recall that this operation, kappa 0 z minus kappa 0 minus z essentially brings up the odd samples and destroys the even samples, so kappa 0 z minus kappa 0 minus z is c 0 z raise the power of minus d, means that if we kill the even samples in the inverse z transform of kappa 0 z.

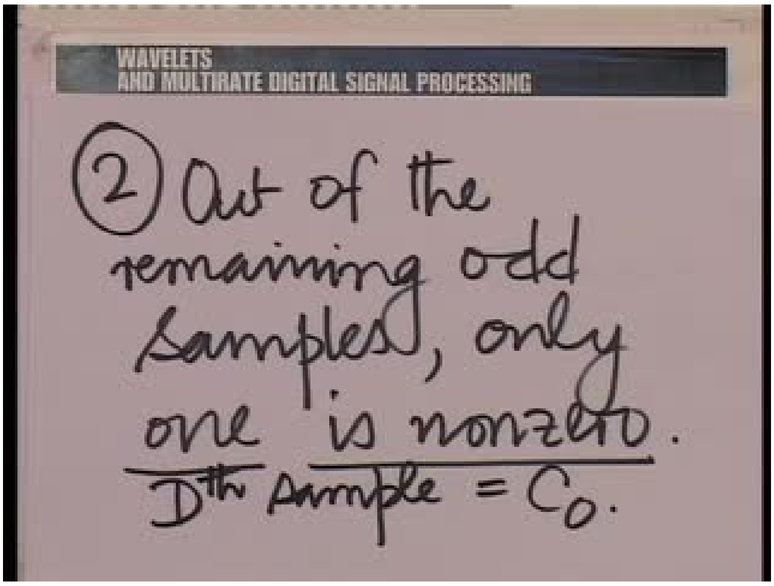
(Refer Slide Time: 24:29)



(Refer Slide Time: 24:54)



(Refer Slide Time: 25:21)

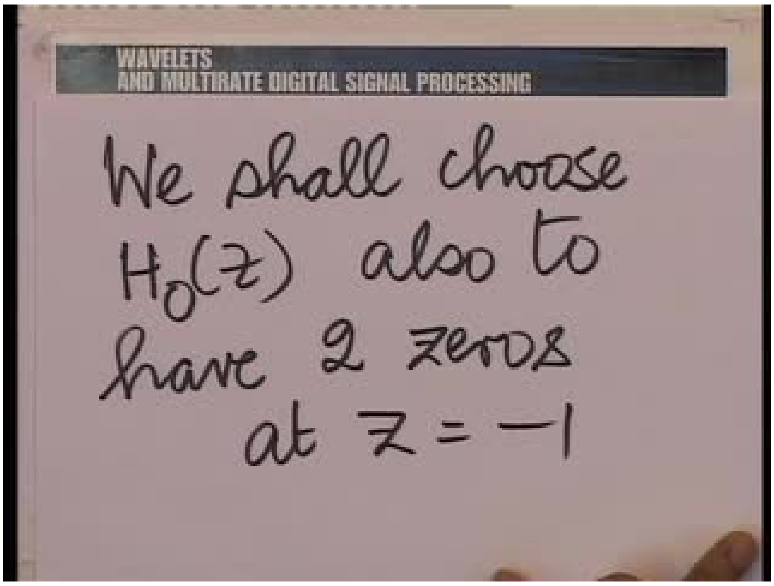


We first kill the even samples, because this operation kappa 0 z minus kappa 0 minus z. Second, out of the remaining odd samples only one is non 0 and that is captured in this c 0 z raise the power of d, this c 0 z raise the power of d is the z transform of essentially the inverse z transform of kappa 0 with all the even samples killed and the odd samples left, that means among the odd samples, the d th sample is equal to c 0. In fact, we can say something specific here, we can say the d th sample is equal to c 0 and now our whole exercise is to impose this condition on the product kappa's h 0 z g 0 z amount into kappa 0 z.

Now, to do that let us of course recognize that we have too much of freedom here. In imposing alias cancellation we have put down only two conditions, we have constraint one of the filters, the other low pass filter is free for us to choose, but we must choose it strategically. Now, you will recall the importance of the factors 1 plus z inverse or their higher powers, a factor of the form 1 plus z inverse ensures a 0 at z equal to minus 1 or e raise the power j omega equal to minus 1 or omega equal to pi.

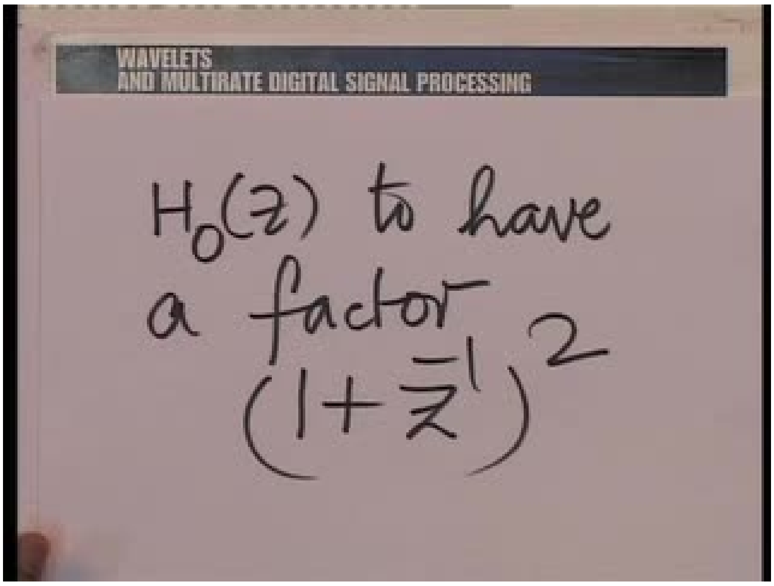
So, on the unit circle, we are putting a 0 and null at pi, pi or a minus pi that is not an issue. Now, this as you recall is one of the sufficient conditions to ensure that when we iterate such a low pass filter to produce a scaling function, that iteration converges to a core and core smooth scaling function, so it is one of the condition that ensures what is called regularity of the iterated scaling function.

(Refer Slide Time: 27:50)



Now, in our choice of g 0 z, we already have two factors, 1 plus z inverse the whole squared. If we wish to bring at least some degree of symmetry on the analysis and synthesis side, although we do not intentionally wish to bring complete symmetry, only a little bit of symmetry, at least we can ensure that they have the same degree of regularity, the same degree of smoothness in iteration.

(Refer Slide Time: 28:16)



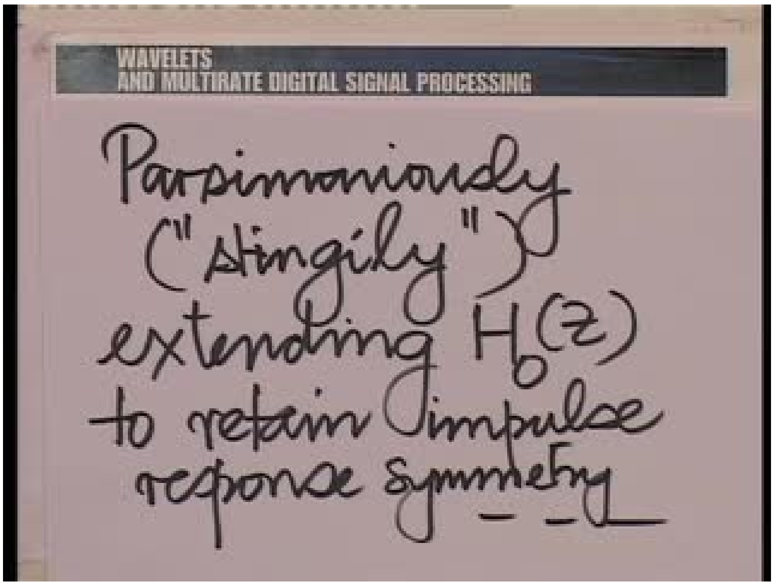
And therefore, we shall choose h 0 z also to have two 0s at z equal to minus 1, in other words, we shall choose h 0 z to have a factor 1 plus z inverse the whole squared. Now, what we wish to do next is to look for the minimal possible solution, which we can get to satisfy perfect reconstruction. So, as is always the case, we start very parsimoniously, very stingily, we want to construct kappa 0 z. Now, we already know that we have a 1 plus z inverse squared corresponding to g 0, we have a factor of 1 plus z inverse the whole squared in h 0 and we wish to introduce just as many additional 0s in h 0 as are required to satisfy the perfect reconstruction condition.

And by the by one of the shortcomings of an orthogonal filter bank, in fact if you look at the (( )) series of orthogonal filter banks, often that matter if you look at the very conditions on perfect reconstruction and alias cancellation, when we look at the filters in an orthogonal filter bank, one can with the little effort. It is a slightly difficult exercise, but with a little effort one can show that one cannot possibly get what is called linear phase, in other words, the impulse response cannot possibly be symmetric. I will give you hint why that is so, for a symmetric impulse response in a finite impulse response filters, the 0s must occur in reciprocal pairs, so for every 0 inside the unit circle there must be a 0 outside the unit circle.

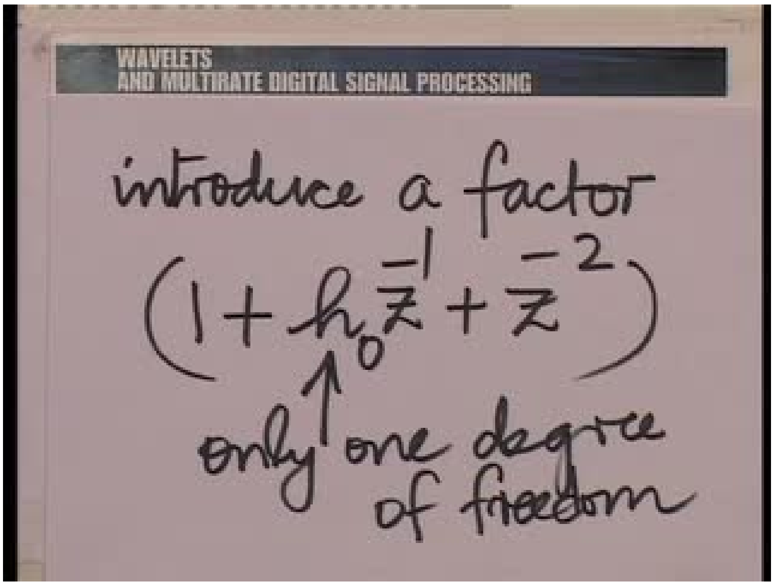
Now, this kind of a choice of reciprocal pairs of 0s is just not possible when we construct the low pass filter in an orthogonal filter bank. I shall not dwell any further on the proof of why there is this is not possible or why this is true, but this is a central reason why one cannot get linear phase in an orthogonal filter bank, beyond the very trivial in simple case of haar, where the 0 lies on the unit circle. So, it is 1 plus z inverse there and 1 minus z inverse for the high pass filter, 0 lies on the unit circle. But, for any higher order multi resolution orthogonal analysis, one just cannot get linear phase and this is one of the reasons why as I said we wish to think out of the box, we do not wish to confine ourselves to orthogonal multi resolution analysis, we would like to explore other options, so that is what we are doing here.

So, if we do wish to bring in that additional degree of freedom, we wish to explore the possibility of bringing in, in such a way that linear phase can be ensured. So, what we shall do now is to introduce just one mod degree of freedom in constructing h 0 in such a way that linear phase is maintained, in other words, that the impulse response continues to be symmetric.

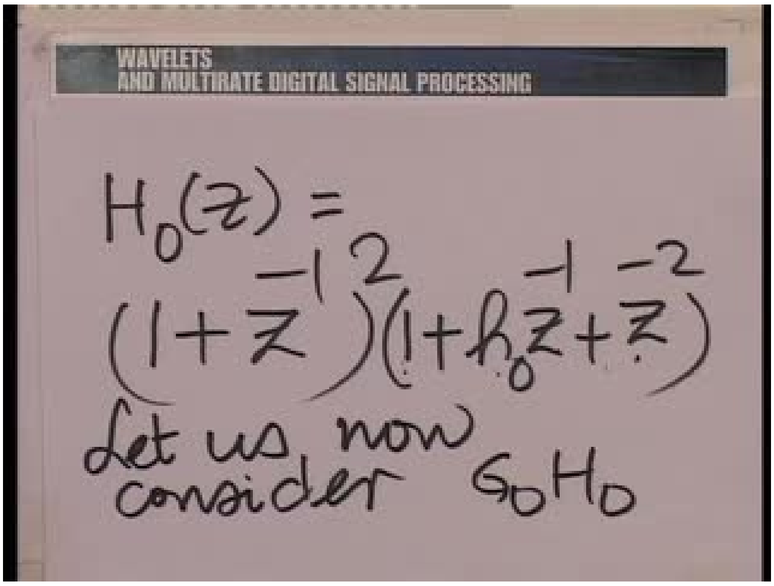
(Refer Slide Time: 31:56)



(Refer Slide Time: 32:44)



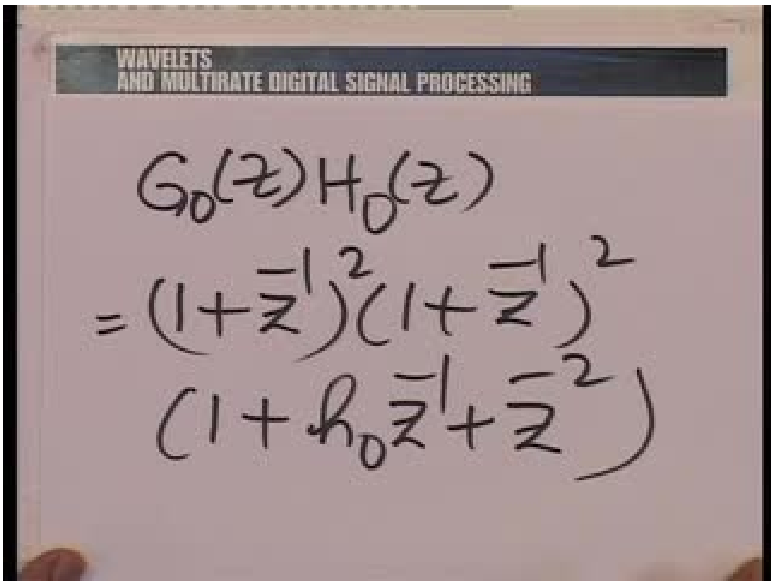
(Refer Slide Time: 33:26)



So, parsimoniously, in informal language stingily, extending h 0 z to retain symmetry, alright. We shall therefore introduce as a just one parameter, so we will introduce a factor 1 plus h 0 small h 0 z inverse plus z raise the power minus 2, so what I am saying is retain symmetry. This is a symmetric factor and the only degree of freedom is this and therefore, we have h 0 z is essentially 1 plus z inverse the whole squared times 1 plus h 0 z inverse plus z raise the power minus 2. Now, couple of remarks you know in retaining symmetry, I could have used the same constant, let us say h 1 here and here and kept a constant h 0 here.

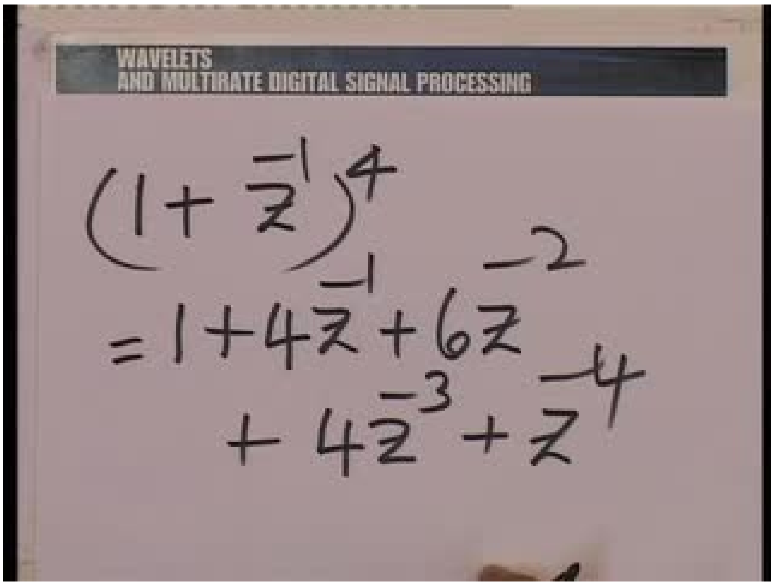
So, apparently I would had two degrees of freedom, but this factor h 1 here essentially scales the whole filter by a constant. Now, I am not really interested in scaling, you know I could scale all the impulse responses by an appropriate constant and thus change the constant c 0 that is not going to materially affect the nature of my filter bank. So, I am introducing only so many degrees of freedom as give me something novel in terms of nature of frequency response, not in terms of the scaling constant c 0, which I can always adjust in the end.

(Refer Slide Time: 34:49)

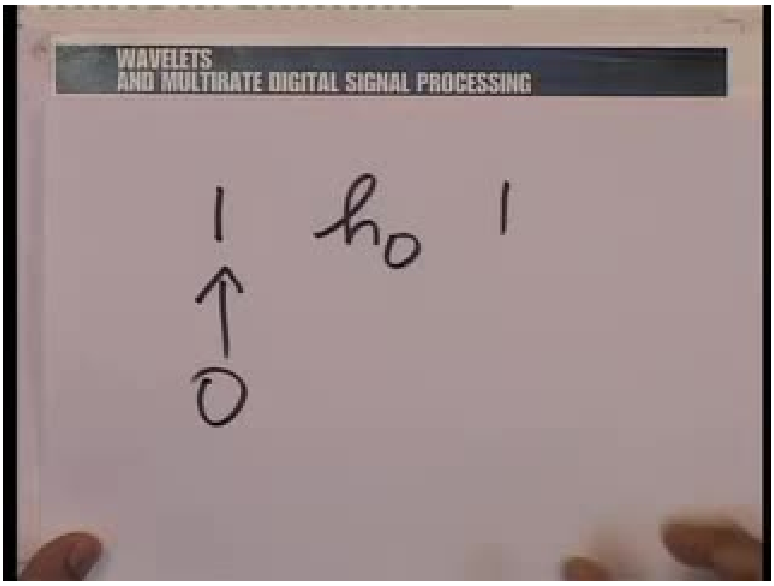


So, therefore, keeping only this degree of freedom here, let us now impose the condition on g 0 h 0. Let us now consider the product g 0 h 0 and that product is going to be g 0 z h 0 z given by 1 plus z inverse the whole squared times 1 plus z inverse the whole squared times 1 plus h 0 z inverse plus z raise the power minus 2. And let me expand this, 1 plus z inverse the whole squared times this is 1 plus z inverse the power 4, so let we first expand 1 plus z inverse the power 4.

(Refer Slide Time: 35:19)

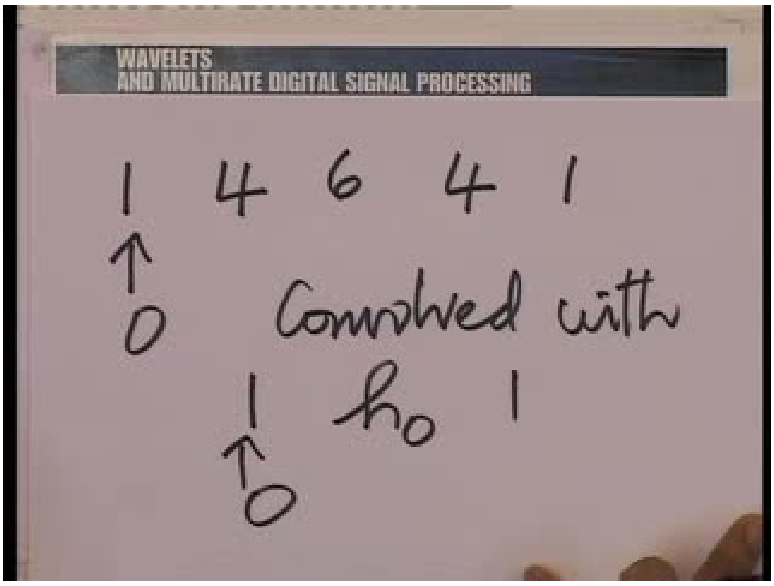


(Refer Slide Time: 35:49)

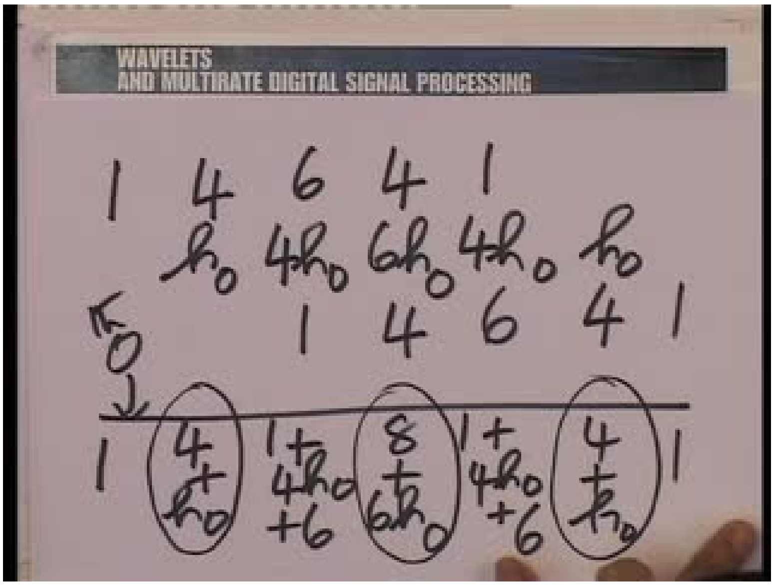


Essentially 1 plus 4 z inverse plus 6 z raise the minus 2 plus 4 z raise the powerminus 3 plus z raise the power minus 4. And now, I shall use the convenient notation of coefficients only, so when I multiply this factor by the factor 1 h 0 1 you know so let us use this representation.

(Refer Slide Time: 36:01)

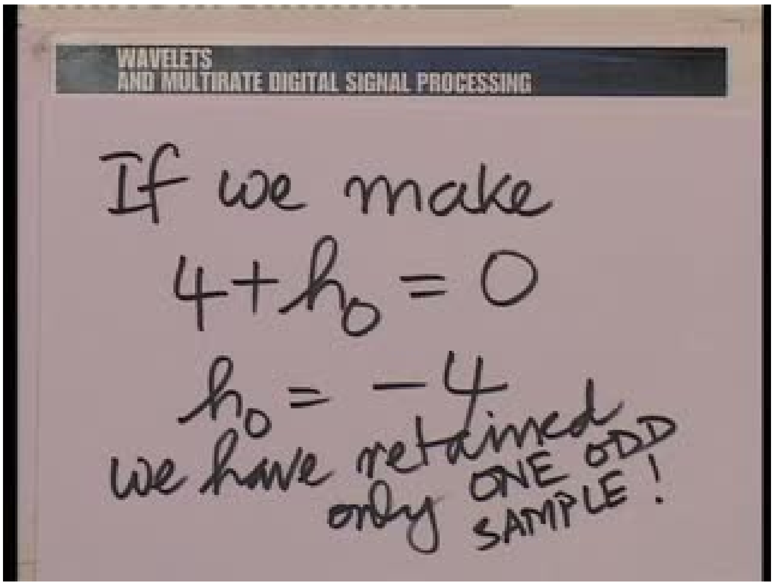


(Refer Slide Time: 36:23)

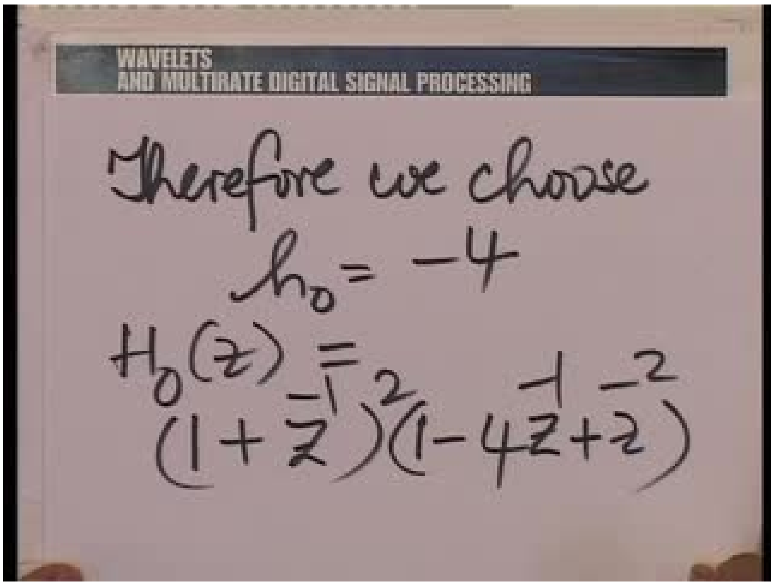


So, I have 1 4 6 4 1 convolved with essentially you multiplying to z transform amounting to convolving the sequences, it would give me essentially 1 4 6 4 1 h 0 4 h 0 6 h 0 4 h 0 and h 0 and back again 0 4 6 4 1. And when I add them, I have 1 4 plus h 0 1 plus 4 h 0 plus 6 and you have 8 plus 6 h 0 1 plus 4 h 0 plus 6 4 plus h 0 and 1, this is the 0 th sample and therefore, you have the even samples here, the second sample here, fourth sample here, the sixth sample here. We are not worried about the even sample, we are going to kill them anyway in kappa 0 z minus kappa 0 minus z, what worries us is the odd sample, let’s mark them. And we are required to retain only 1 odd sample, how can we do that? Notice that these are essentially the same sample, so if ianal this sample, I have achieved by objective.

(Refer Slide Time: 38:05)

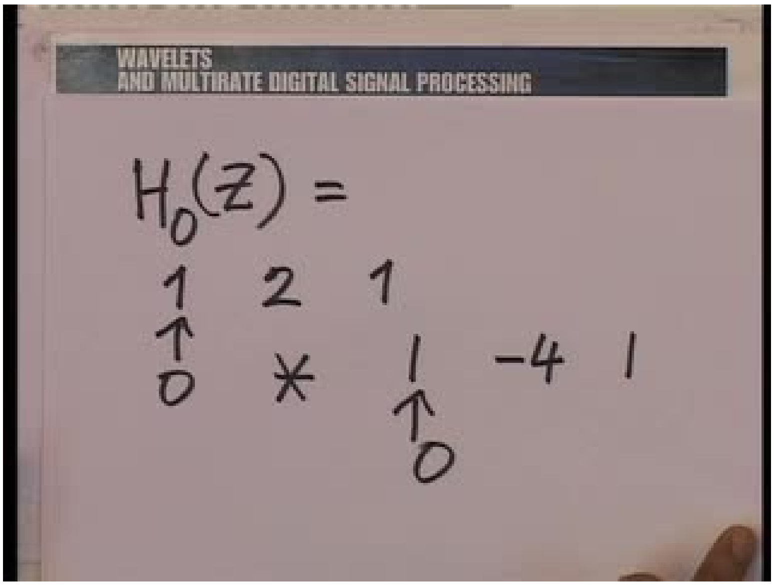


(Refer Slide Time: 38:42)



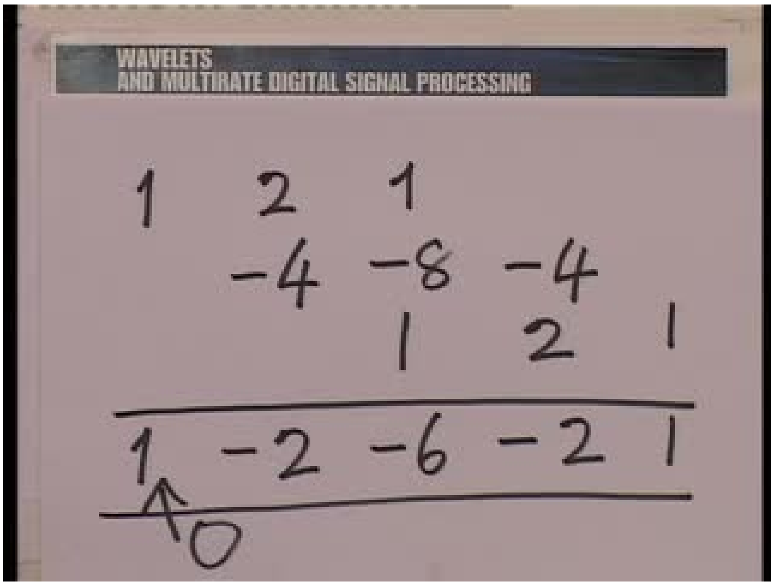
Essentially if we make 4 plus h 0 equal to 0, in other words, h 0 equal to minus 4, we have retained only 1 odd sample and therefore, let us make precisely that choice h 0 equal to minus 4. And therefore, h 0 z is essentially 1 plus z inverse the whole squared times 1 minus 4 z inverse plus z raise the power minus 2.

(Refer Slide Time: 39:23)

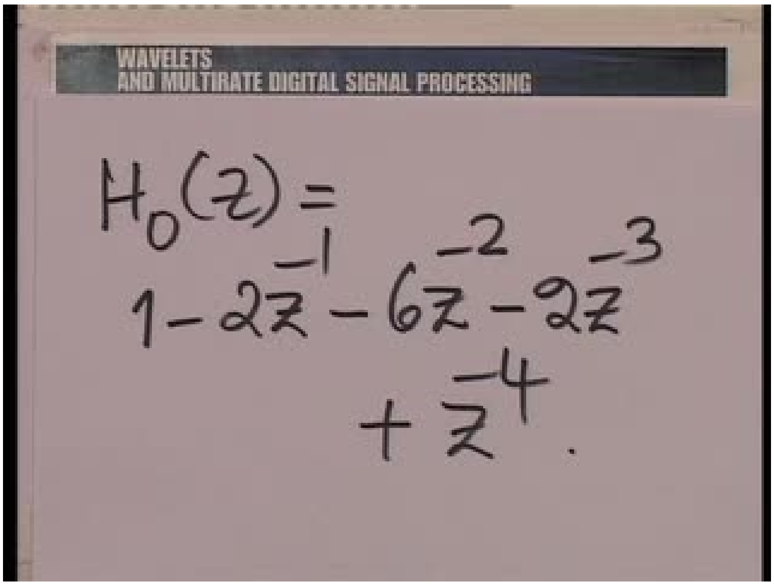


Now, let us look at the degree of h 0 here, h 0 z can be of course expanded. Now, you know we use the same strategy, we convolve 1 2 1 convolved with 1 minus 4 1.

(Refer Slide Time: 39:47)

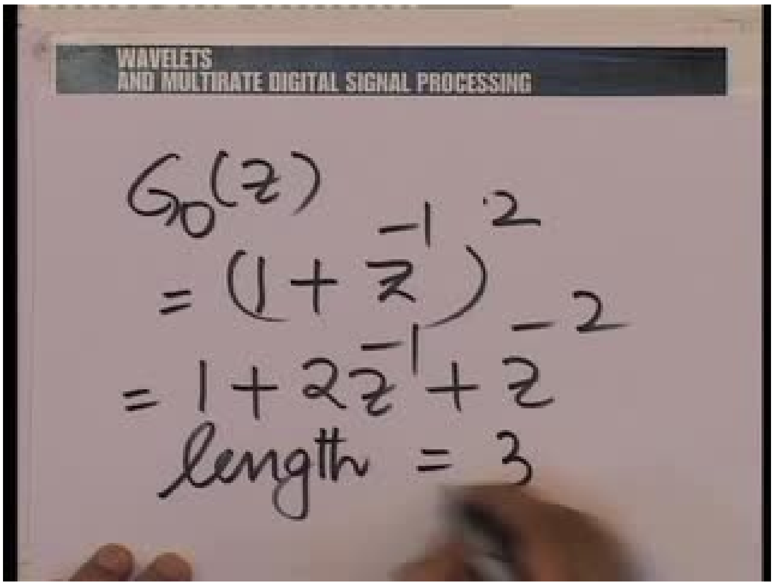


(Refer Slide Time: 40:19)



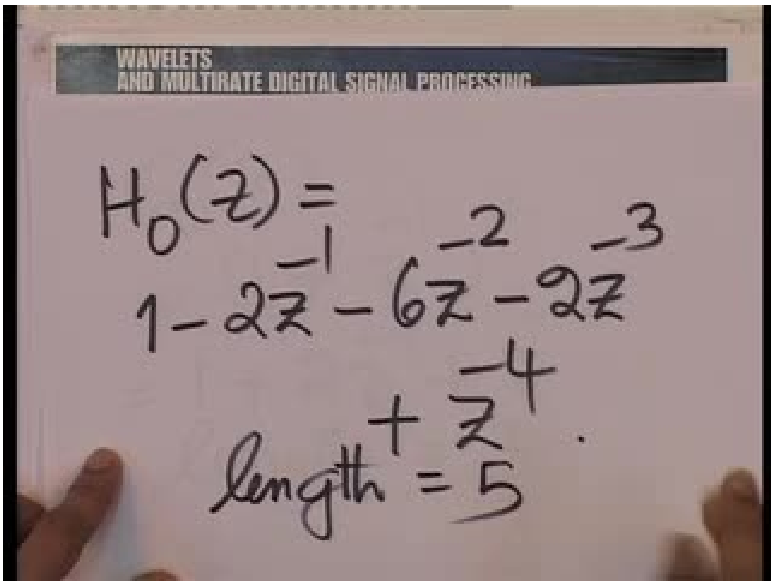
So, we have 1 2 1 minus 4 minus 8 minus 4 and 1 2 1, 1 minus 2 minus 6 minus 2 and 1, this is the impulse response of the other low pass filter, as you notice again symmetric. And therefore, h 0 z would turn out to be 1 minus 2 z inverse minus 6 z raise the minus 2 minus 2 z raise the power minus 3 plus z raise the power minus 4.

(Refer Slide Time: 41:04)

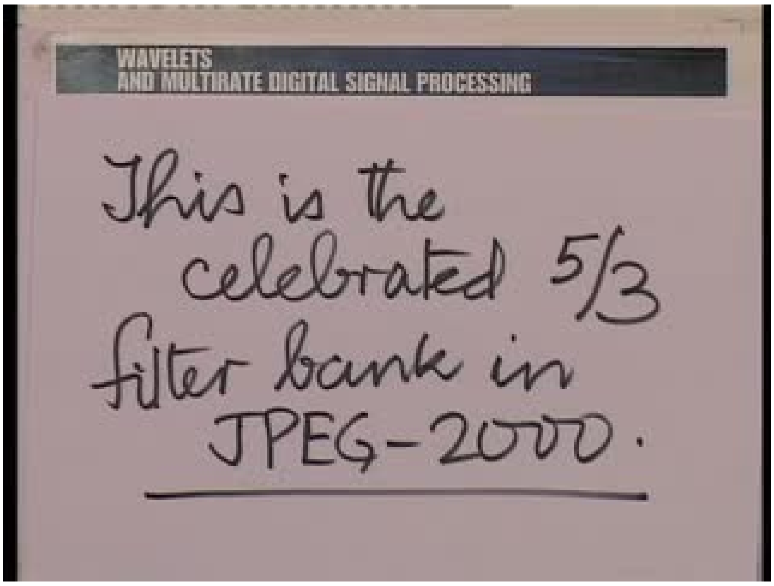


Now, what we have just derived in this discussion is a very important biorthogonal filter bank. Let us look at the length of the impulse response of h 0, obviously the length is five and let us look at the length of the impulse response of g 0, let me put down g 0 for completeness here, obviously the length of the impulse response is 3.

(Refer Slide Time: 41:22)



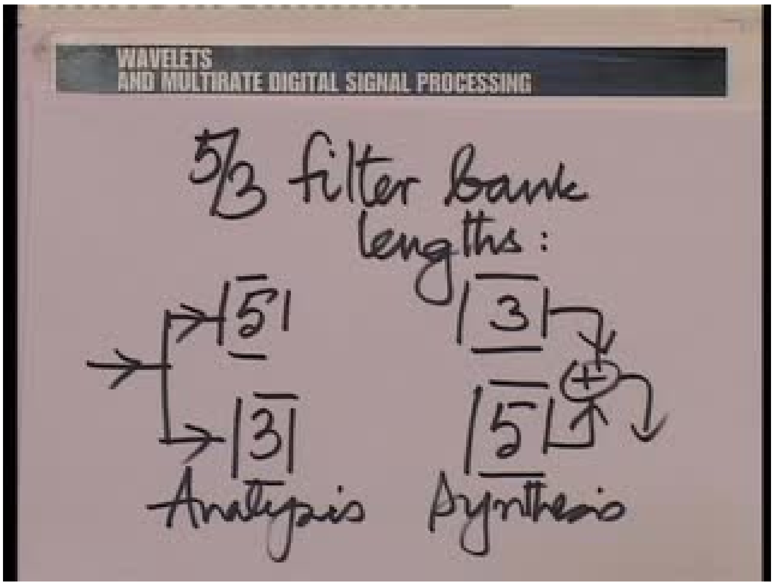
(Refer Slide Time: 41:47)



So, let us write that explicit, length 5 and length 3 and that is precisely the reason why this filter bank that we just derived, this perfect reconstruction filter bank that we just brought out in this discussion is called a 5 3 filter bank, this is the celebrated 5 3 filter bank in JEPG-2000.

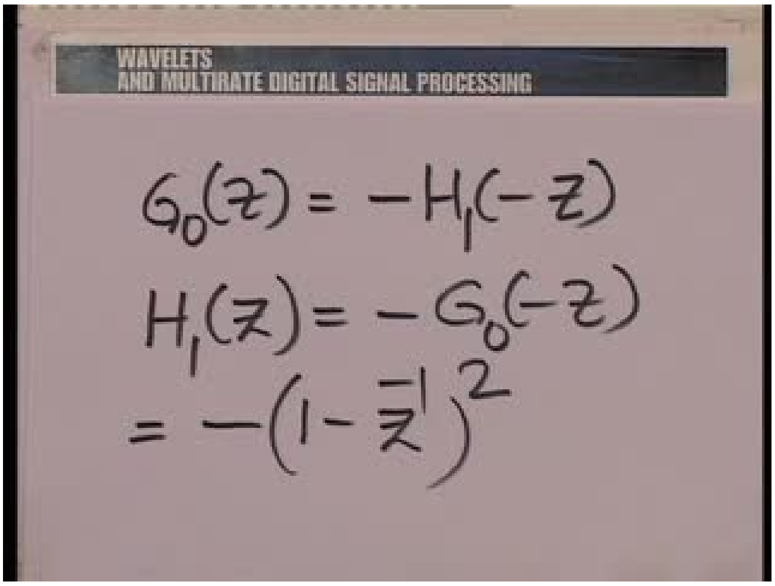
Now, you know, JPEG-2000 admits two kinds of filter banks a 5 3 filter bank and a 9 7 filter bank and now we know what 5 3 and 9 7 mean they essentially refer to the lengths of the impulse responses. 5 3 means the impulse responses or of lengths 3 and 5, now which impulse responses, as you can see, you could think of them, as the impulse responses of the two low pass filters or you could think of them as the impulse responses of the two high pass filters, it does not matter.

(Refer Slide Time: 43:11)

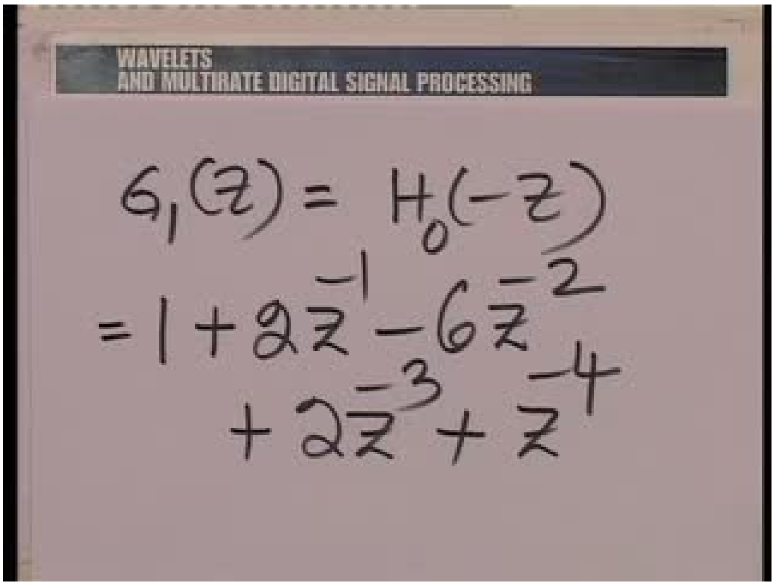


You could also think of them as the lengths of the impulse responses of the analysis filters or the lengths of the impulse responses of the synthesis filters. In fact, let us put it down explicitly, what the lengths are in this 5 3 filter bank. So, in the 5 3 filter bank, as you recall h 0 has a length 5, g 0 has a length 3, now if this has a length 5, then the high pass filter on the synthesis side would also have a length 5. And if this has a length 3, the high pass filter on the analysis side would also have a length 3, analysis, synthesis, so this is the lengths distribution in a JPEG-2000 5 3 filter bank.

(Refer Slide Time: 44:37)



(Refer Slide Time: 45:03)



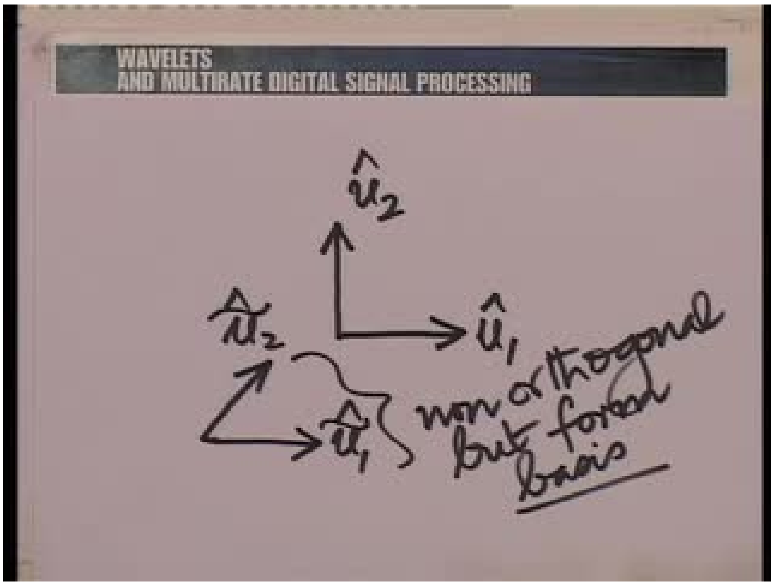
Now, you know let us put down explicitly all the filters in the JPEG-2000 5 3 filter bank, and for that lets just recall the relationships, we had g 0 z is minus h 1 of minus z, in other words, h 1 z is minus g 0 minus z and therefore, h 1 z is going to turn out to be minus 1 minus z inverse the whole squared. Further g 1 z is h 0 of minus z and that would turn out to be 1 you know, so wherever you have odd powers of z inverse, the signs are going to get reversed.

So, 1 plus 2 z inverse minus 6 z raise the power minus 2 plus 2 z raise the power minus 3 plus z raise the power minus 4, this is the high pass filter. You know, in fact if you care to look at the impulse response and if you add up the impulse response coefficients, 1 2 minus 6 2 and 1, you can see their sum is 0 and that is to be expected. After all the impulse response of a high pass filter which has a null at 0 frequency as many good high pass filters have must sum up to 0. It is very often the case that a high pass filter has its impulse response summing to 0 indicating that it has a null at 0 frequency and that is true in this cases well. In fact, here, the high pass filter has a double null at 0 frequency, because it has a factor of 1 minus z inverse the whole squared, anyway that a part. We have now put down very clearly what the impulse responses or on the entire 5 3 JPEG-2000 filter bank. Now, the aim of this exercise was to show you how one builds what are called a biorthogonal filter bank and I wish to spend some time on discussing this notion of biorthogonality.

You know in the axioms of a multi resolution analysis, we let us put one of the axioms to be the scaling function, must be orthogonal to its integer translates. Now, if you construct the scaling function from g 0 here, namely 5 1 t, that piecewise linear triangular function, that we started with in this lecture, it is obviously not true for that scaling function.

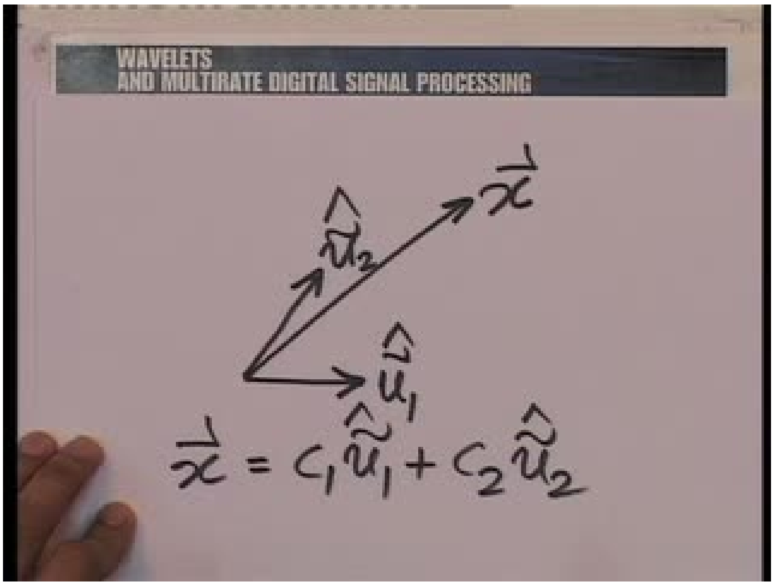
So, this g 0 by itself is not going to give us a multi orthogonal resolution analysis. What is the slight generalization that we need to make here? Well, the generalization is in terms of dealing with a pair of scaling functions.

(Refer Slide Time: 48:35)



So, you have one scaling function emerging from the synthesis side and one scaling function emerging from the analysis side, from g 0 and from h 0. The orthogonality is actually when you take them together and so let us to explain the idea little better, let me start from a two dimensional space. You know in a two dimensional space, if you have two orthogonal unit vectors, let us say u 1 cap and u 2 cap then this forms an orthogonal system by itself, but suppose you do not have two orthogonal vectors, you have a u 1 cap and a u 2 cap like this, a unit vectors, but not orthogonal.

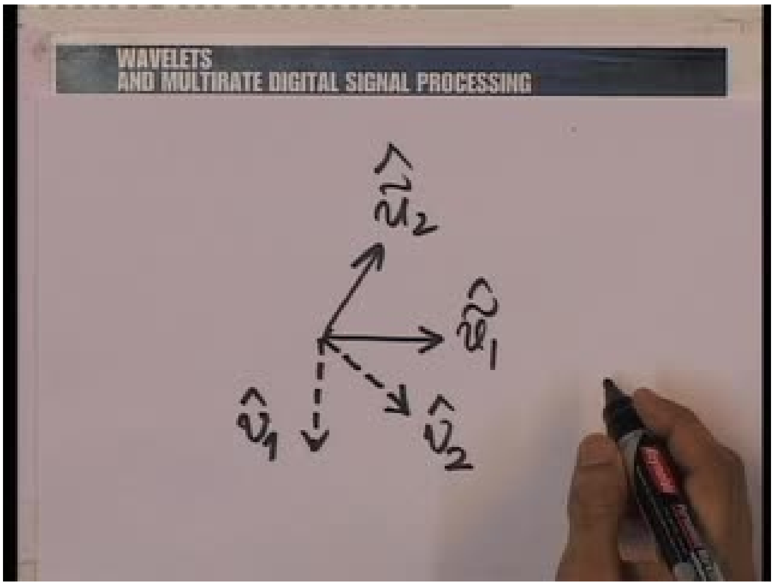
(Refer Slide Time: 49:38)



However, of course a linearly independent, obviously these two unit vectors u 1 cap, let us call them u 1 tilde cap and u 2 tilde cap, u one tilde cap, u 2 tilde cap do form a basis in two dimensional space, because they are linearly independent, but if

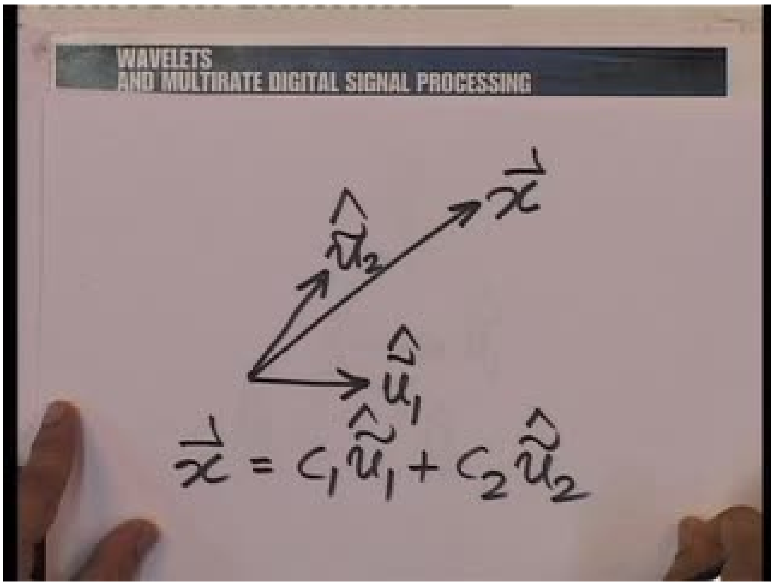
I wish to obtain the coordinates, so suppose I wish to take any vector, let us say x in two dimensional space and I have this pair of unit vectors here, u 1 tilde cap and u 2 tilde cap. And I wish to express x as c 1 times u 1 tilde cap plus c 2 times u 2 tilde cap, how would I obtain the constants c 1 and c 2, I cannot do them by taking the dot product with respect to u 1 tilde u 2 tilde cap, because they are not orthogonal that is the convenience of an orthogonal basis. What I can do however is to construct what is called a biorthogonal basis.

(Refer Slide Time: 50:26)

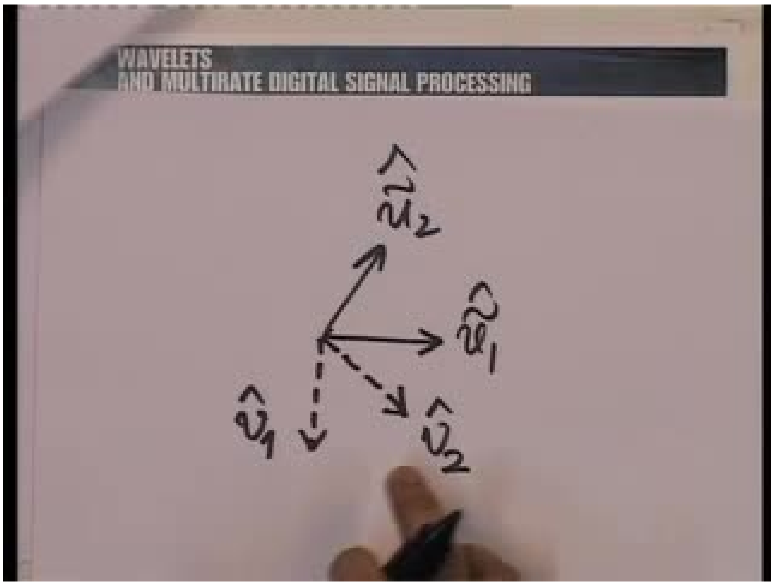


So, I could instead start from this basis, u 1 tilde cap and u 2 tilde cap, now I could take a vector which is perpendicular to u 2 but not to u 1and let me show that one, let me call this v 2 cap, so v 2 cap is perpendicular to u 2, but not to u 1, similarly I could have a vector perpendicular to u 1, but not to u 2 may be this vector let we call it v 1 cap.

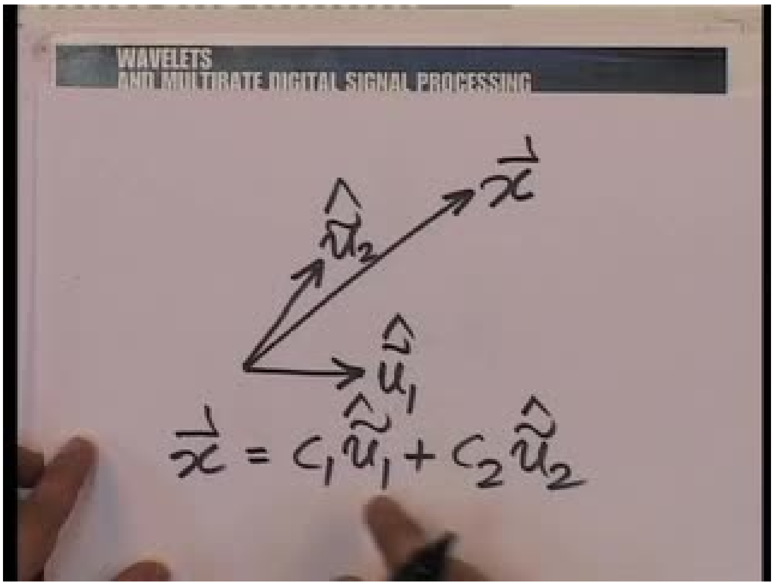
(Refer Slide Time: 51:12)



(Refer Slide Time: 51:18)



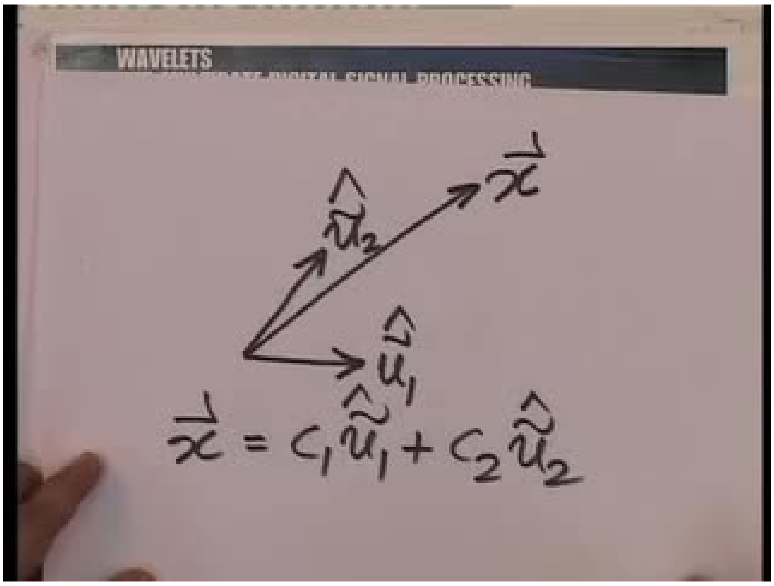
(Refer Slide Time: 51:21)



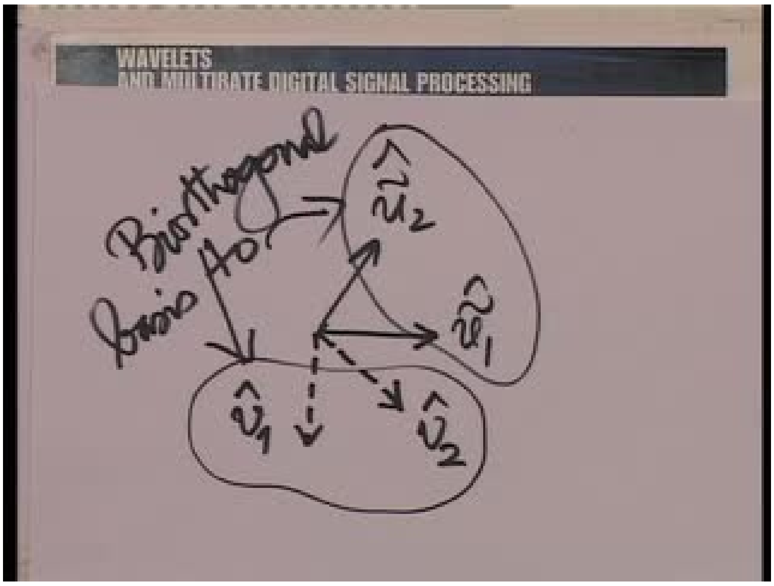
(Refer Slide Time: 51:33)

Now, if I were to take the dot product here, if I come back to this, if I take the dot product of x with v 1 cap here, there would be u 1 would go away, because the dot product, because of v 1 being perpendicular to u 1 cap, but this c 2 would remain and similarly, even I take the dot product with respect to v 2 cap here.

(Refer Slide Time: 51:57)



(Refer Slide Time: 52:04)



So, if I take the dot product of x with respect to v 2 cap, it could destroy the term of u 2 tilde, but retain the term of u 1 tilde, namely I would be able to obtain c 1 by taking the dot product of both sides of this equation with v 2 cap. And I would be able to obtain c 2 by taking the dot product to both sides of this equation with v 1 cap and therefore, we say v 1 cap v 2 cap is a biorthogonal basis a biorthogonal basis to this basis. In fact, they are mutually biorthogonal, each of them is biorthogonal together and in fact this is the idea that is generalized when one (( )) a biorthogonal filter bank.

This is one generalization one variant that we have introduced in two band filter banks. In the next lecture, we shall introduce another variant; thank you.