**Advanced Digital Signal Processing - Wavelets and Multirate**
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**Module No. # 01**
**Lecture No. # 27**
# Introducing Variants of the Multiresolution Analysis Concept

A warm welcome to the 27th lecture on the subject of Wavelets and Multirate Digital Signal Processing. In the previous lecture, we had completed or almost completed the proof of the theorem of multiresolution analysis. So, in a certain sense, we are at a cross point, a juncture, where we need to explore more avenues. We have understood a whole approach to building multiresolution analysis; we have talked about our orthogonal filter banks, in great depth.

We have also established the connection between the continuous time functions phi t and psi t, and the filter banks. And we have brought out a proof, that given the axioms of the multiresolution analysis, we are guaranteed the existence of a wavelet psi t, whose dyadic dilates and translates, can span the whole of L 2 R, and we did that by essentially considering 1 peal. We looked at one incremental subspace and we showed that incremental subspace namely w 0, could be spanned by the translates of a single function psi t, and in fact, the function psi t was a linear combination of the basis elements in V 1 as expected and we could also characterize the coefficients of the linear combination. So, in that sense, the proof that we had in the previous lecture was constructive.

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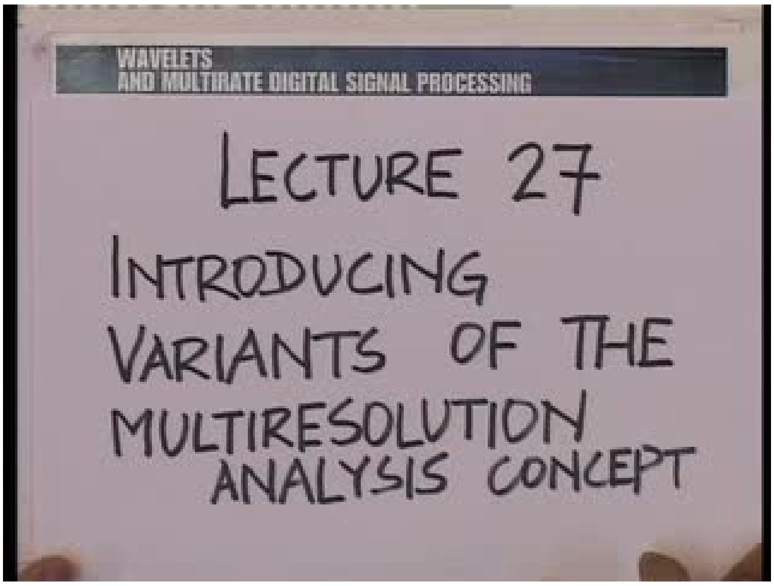


Now, the aim of the lecture today, is first to complete a few details in the proof that we attempted the last time, and to take further, by introducing variants. So, what we intend to do today is first to begin with a few details of completion of the proof, and that would in fact, lead us to the possibilities of varying the motion of multiresolution analysis in ways that will make our time frequency possibilities richer.

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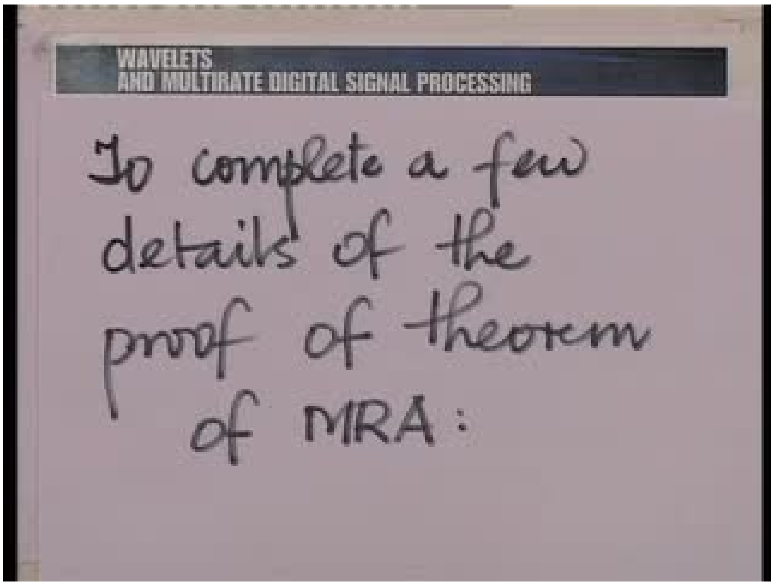

So, we take the first task; that is, to complete a few details of the proof of the theorem of multiresolution analysis. The first thing that we need to complete is to prove that psi t is orthogonal to phi t and its translates.

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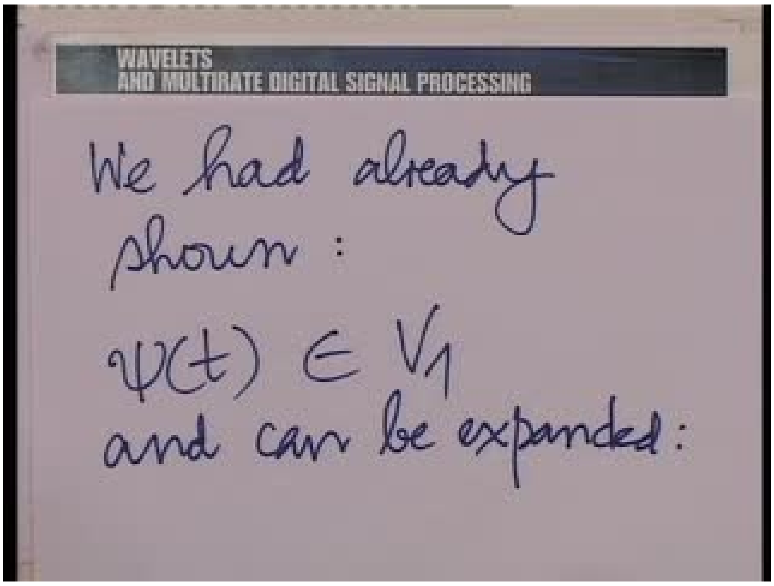


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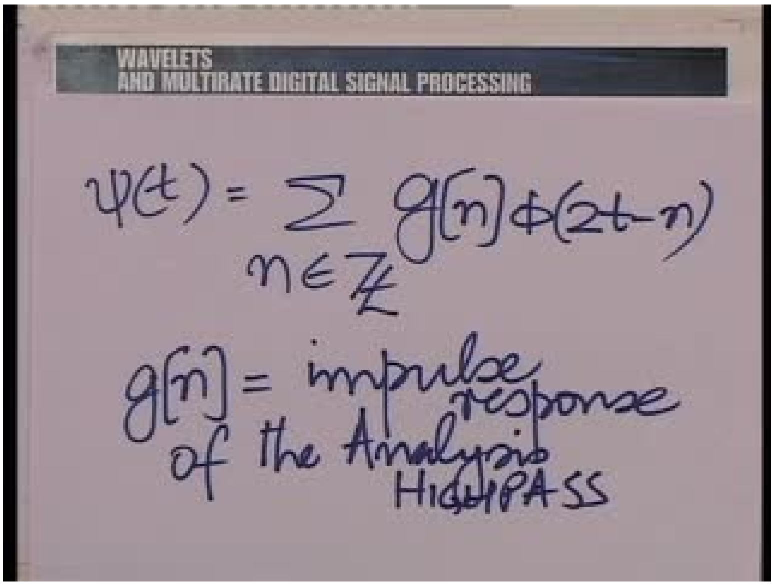


We had already shown psi t belongs to V 1 of course, and can be expanded as follows: psi t is summation on all integer n g n phi 2t minus n, where g n is the impulse response of the analysis high pass filter; that is interesting. Expressing phi t in terms of the basis of

V 1 involve the coefficients of the low pass analysis filter, and expanding the wavelet in terms of the basis of V 1, involve the impulse response of the analysis high pass filter.

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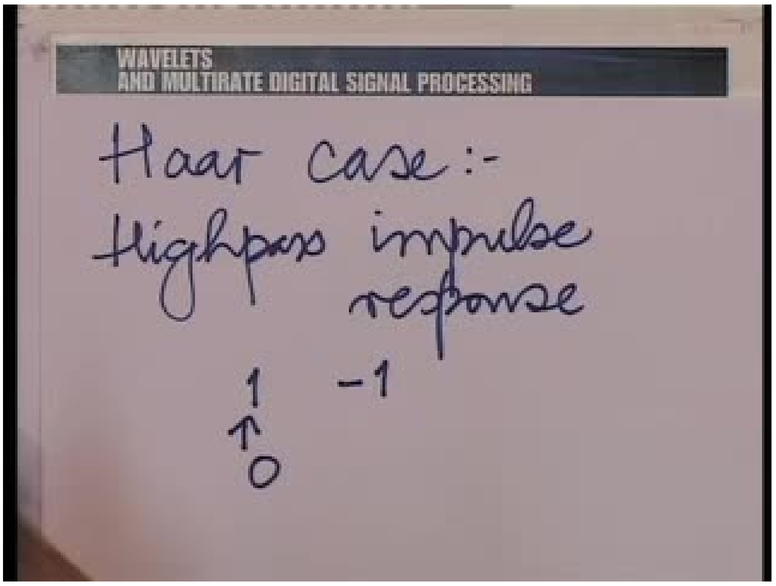


In fact, just to complete an example, let us look at the Haar case. In the Haar case, the high pass impulse response is essentially 1 and minus 1.

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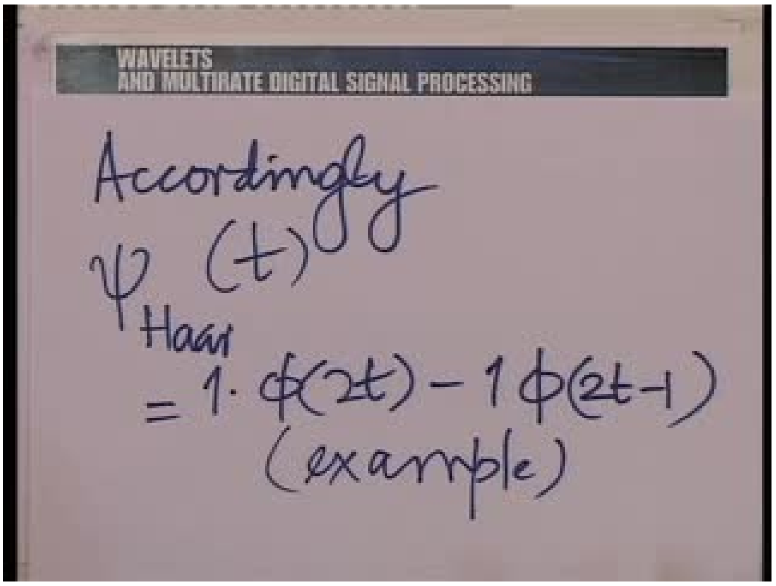


And as expected, the Haar wavelet is indeed 1 times phi 2t minus 1 times phi 2t minus 1, as an example.

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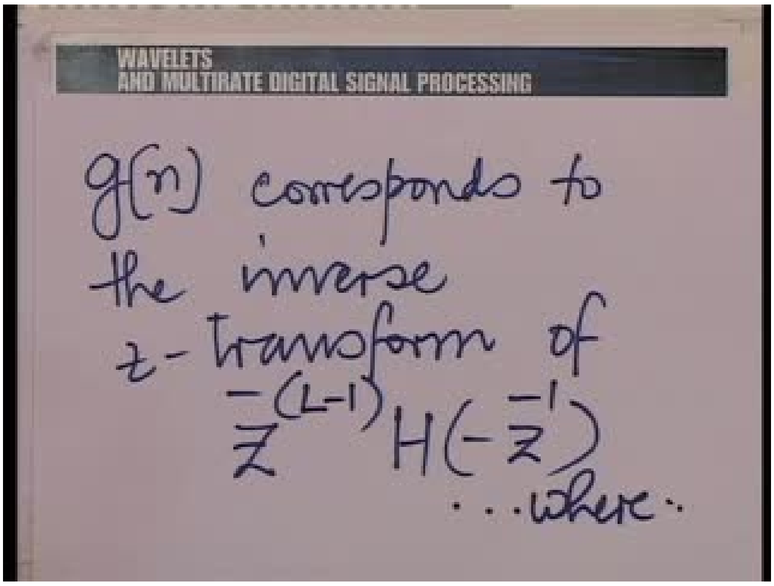


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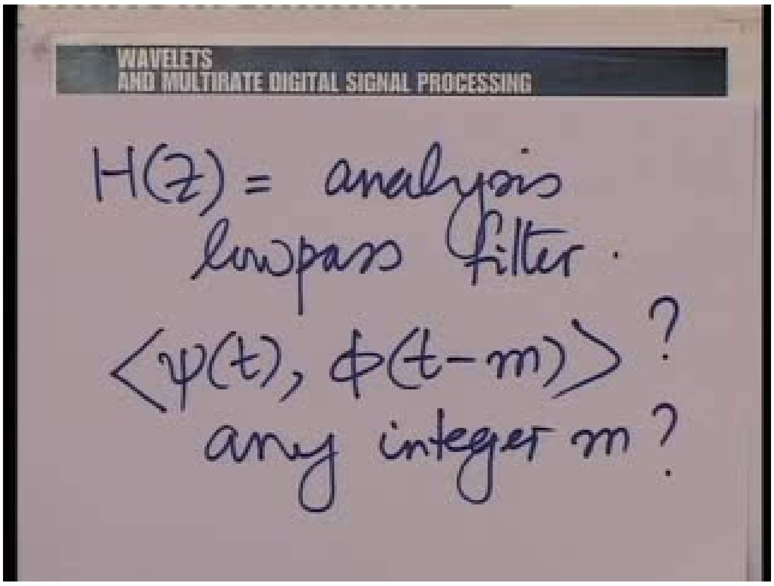


Now, let us make an observation in general, here. So, what we say in effect is - if we take g n essentially to correspond to be inverse Z-transform of Z raised to the power minus l minus 1 H of minus Z inverse, where H of Z is the analysis low pass filter. Then, what we wish to do is essentially to analyze the following dot product; the dot product of psi t with phi t minus m for any integer m, and this is easy to do.

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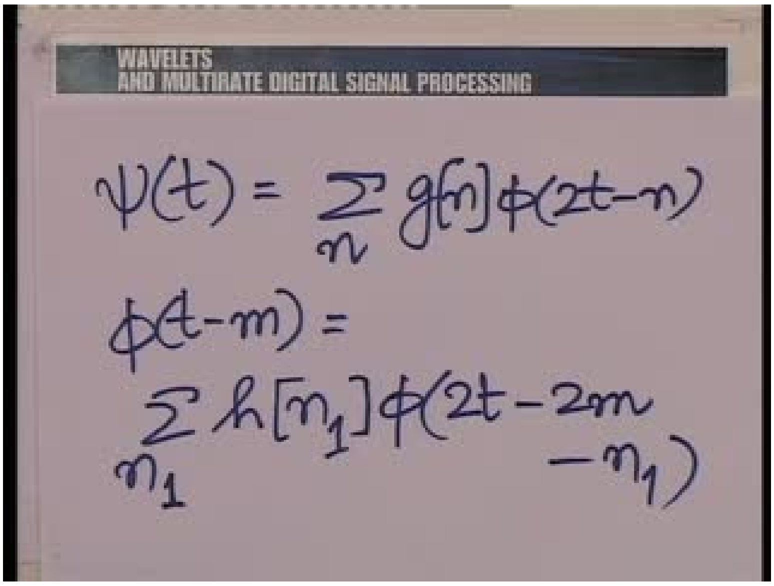


Indeed, psi t by itself is summation on n g n phi 2t minus n. Phi t minus m is simply summation on n, and here again, I imply summation on all integer n. So as So, not to confuse these two indices, I shall write n 1. So, n 1 h n 1 phi 2t minus 2m minus n 1 taking a cue from expanding phi t minus m in the basis of V 1, as we can do.

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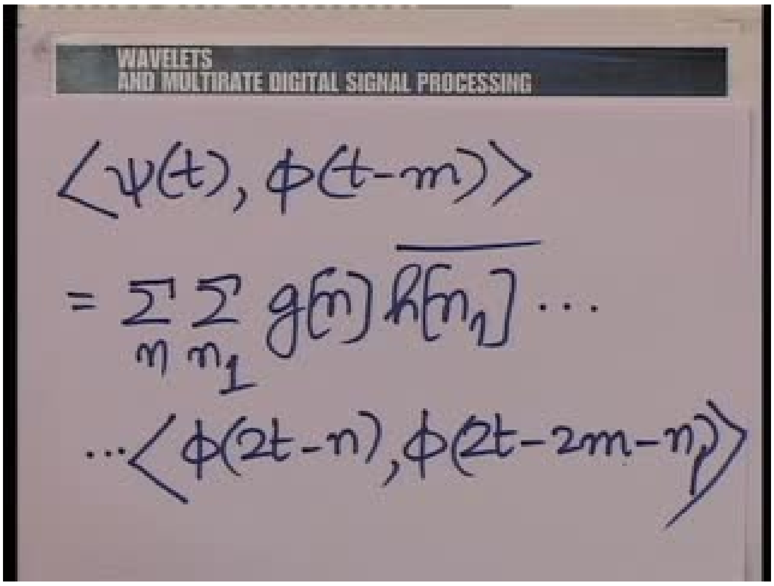


Where upon, this dot product is essentially a summation on n, a summation on n 1 g n h n 1 complex conjugate times the following dot product: phi 2t minus n phi 2t minus 2m minus n 1. Now, once again, without going through the whole derivation again, I might observe that this dot product is 0, say for the case, when n is equal to 2m plus n 1, and in fact, that means that we can drop one of these two summations.

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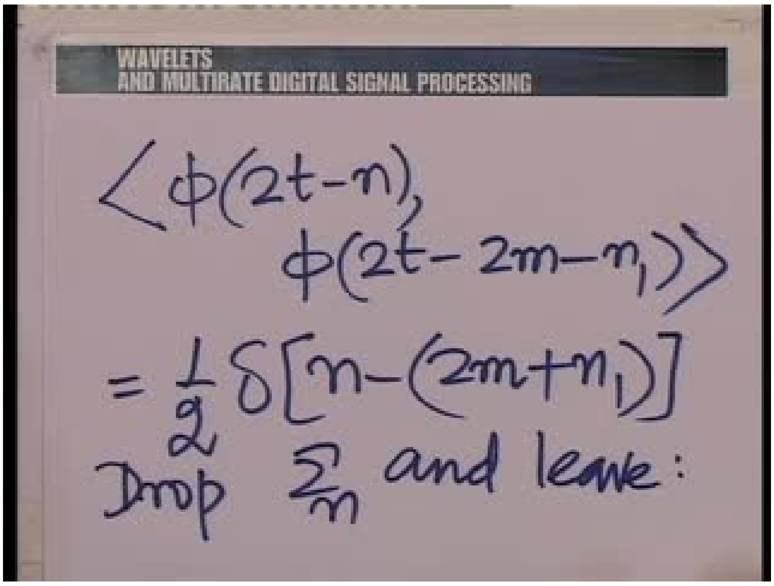


We could, if we like, drop the summation on n of your saying, essentially is that, phi 2t minus n dot product with phi 2t minus 2m minus n 1 is half delta n minus 2m plus n 1. And therefore, we can drop the summation on n and leave summation on n 1 g. Now, you know, this is a discrete impulse which has a non-zero value only when n is equal to 2m plus n 1. So, in place of n, we substitute 2m plus n 1.

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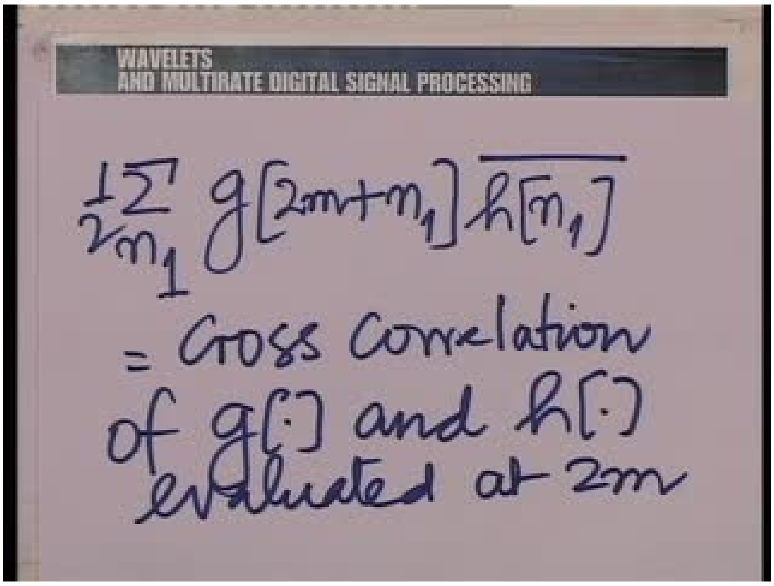

Of course, h n 1 complex conjugate and a factor of half which is not a terribly serious matter at all; essentially, this is the cross correlation of the sequences g and h evaluated at 2m, for all integers m; looks familiar, does not it; we have been doing this frequently.

Now, one can again evaluate the cross correlation by going into the Z domain.

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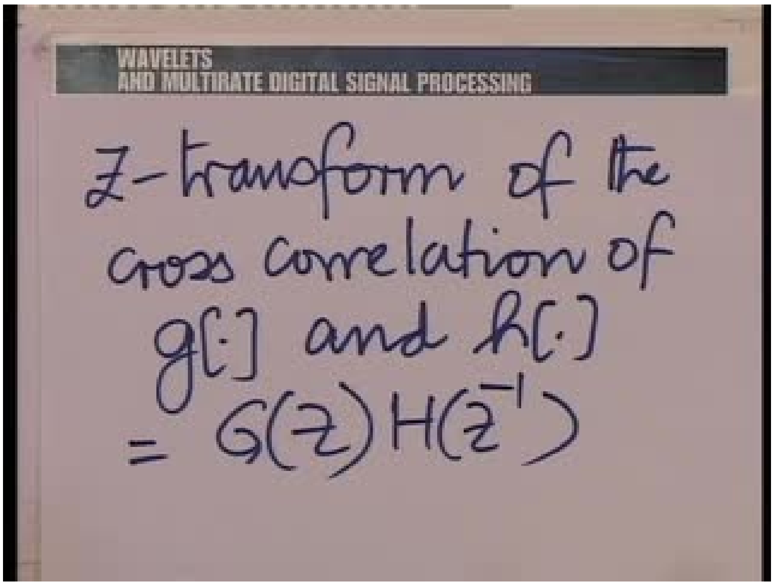


So, obviously, the Z transform of the cross correlation can be seem to be essentially G of Z times H of Z inverse, and we know what G of Z is.

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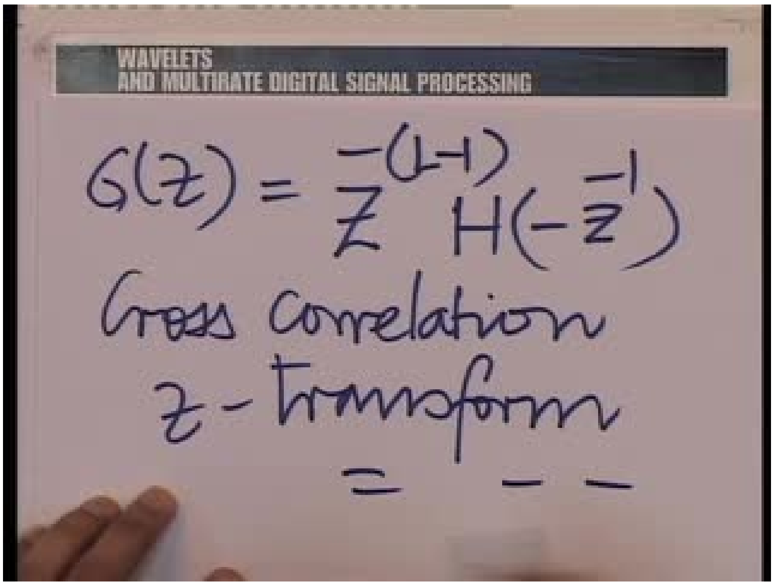

G of Z is essentially the system function of the analysis high pass filter with the Z raise to the power minus L minus 1 H minus Z inverse. And therefore, we have the cross correlation Z transform.

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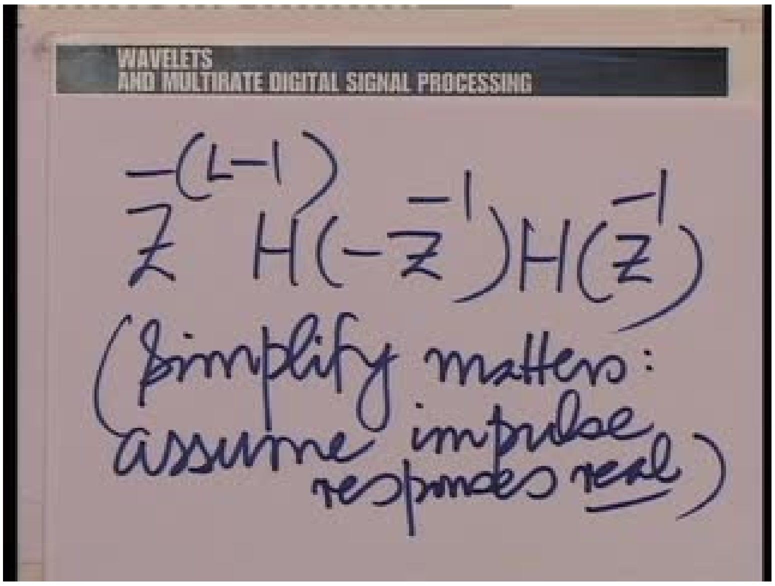


It essentially becomes Z raised to the power minus L minus 1 H minus Z inverse times H Z inverse. Now, for the moment, let us simplify matters; assume the impulse response is real.

We can, of course, generalize all these results when the impulse response is complex, but that is not the critical issue here. Let us not get confused by the complex conjugate every time. Anyway, now, all that we need to do is to look at the values of the cross correlation at even locations, and we know how to do that.

If a Z transform of the cross correlation is some function of Z, we take the same function of Z, with Z replaced by minus Z, add the 2 together, and see what happens. That tells us about the values of the sequence at even locations. Let us do it for this cross correlation sequence.

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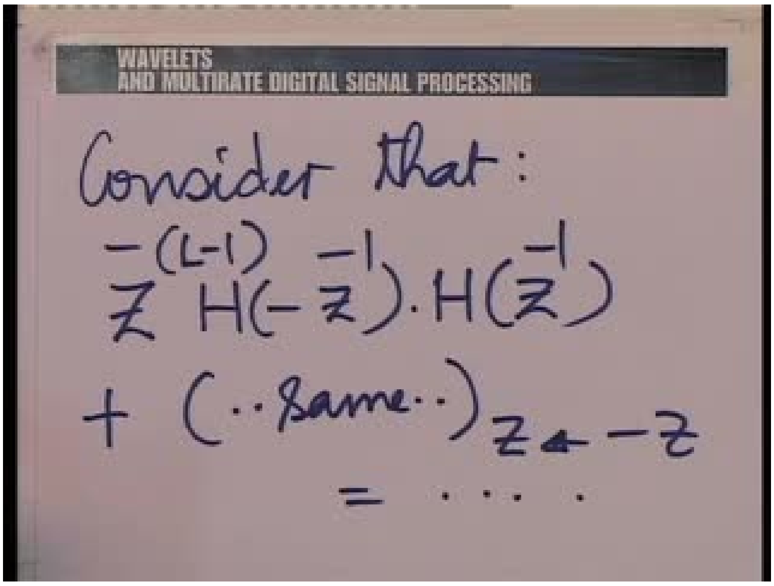


Consider that Z raise to the power minus L minus 1 H of minus Z inverse times H of Z inverse plus the same thing with Z replaced by minus Z is as follows:

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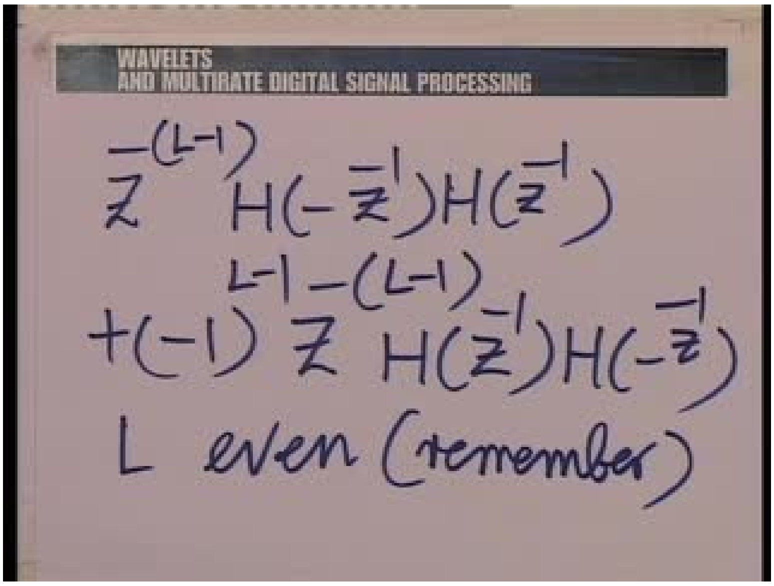


It is essentially Z raised to the power minus L minus 1 H minus Z inverse H Z inverse plus minus 1 raised to the power L minus 1 Z raised to the power minus L minus 1 H Z inverse times H of minus Z inverse; this is very interesting; remember, L is even. We have agreed that are orthogonal filter bank is going to have an even number of samples in the impulse responses. If it does not, then we are going to have a problem in the requirement of orthogonality to even shifts. So, take for example, the daub ash series; obviously, their impulse responses are of even lengths.

And in fact, it is not too difficult to show that, even if you assume an odd length, you will find the last sample must be 0, for the requirement that the impulse response be orthogonal to its even shifts.

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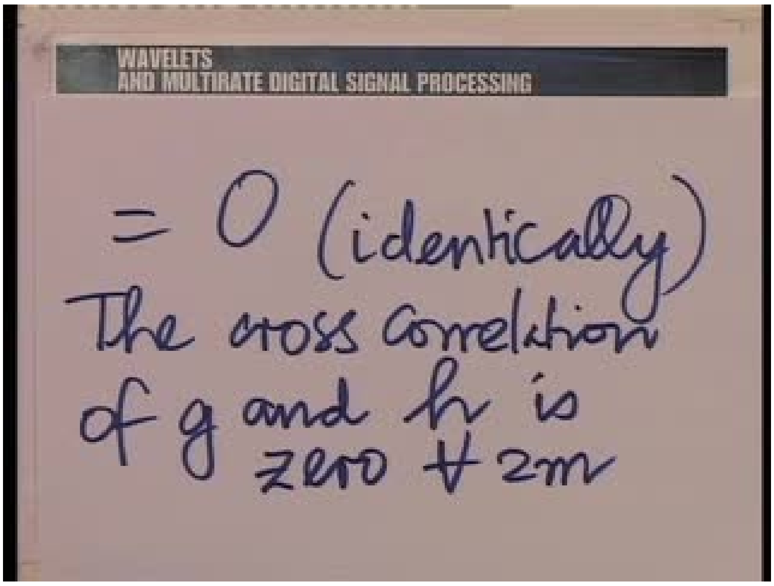


Anyway, so, coming back to this, since L is even, L minus 1 is odd, and therefore, minus 1 raised to the power L minus 1 is minus 1. And also, you can see, this term and this term (Refer Slide Time: 15:53) are exactly identical if you just interchange the order of the products, and therefore, these two cancel; so, this is identically 0, and therefore, the cross correlation of g and h is 0 for all 2m.

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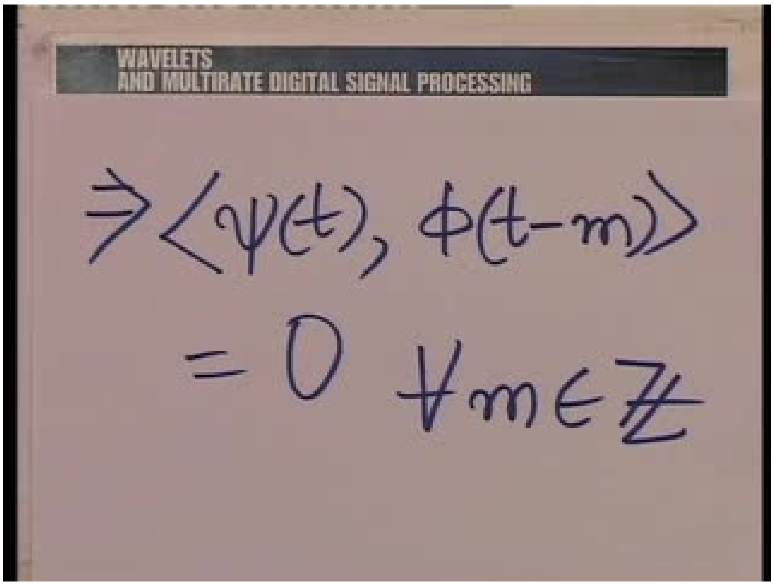


That amounts to saying that the dot product of psi t and phi t minus m is equal to 0 for all integer m, which is exactly what we are expected to prove. So, if completed that little detail and in completing the detail, you have also brought out an important issue namely the intimate connection between the Z domain representation of the filters and orthogonality of the functions.

We had seen this implicitly and proof had points. In fact, now, to take this discussion little further and to give you a feel of the connection in straightly greater depths, let me again take the issue of orthogonality of phi t with zone integer translates. How does that manifest onto the behavior of the autocorrelation of the low pass filter impulse response?

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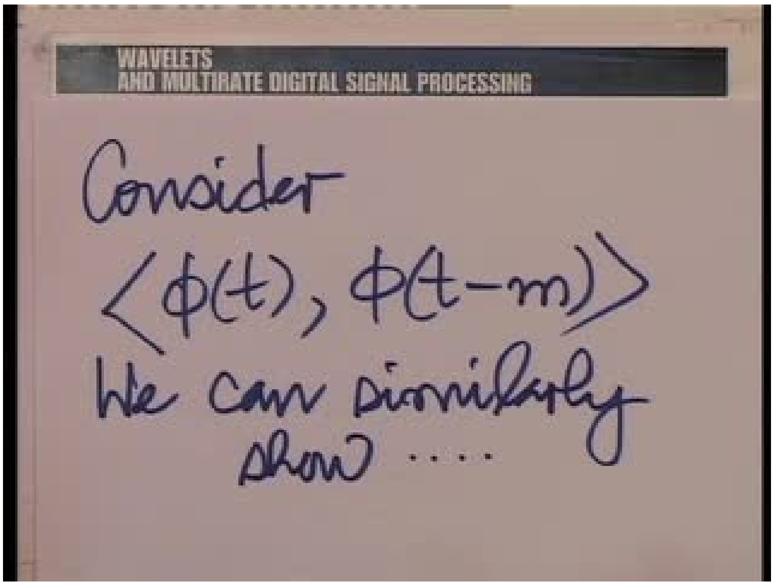


So, in fact, we look at the function phi t and its integer translates. Consider, the dot product of phi t and phi t minus m. We can use a similar set of steps as we have just done.

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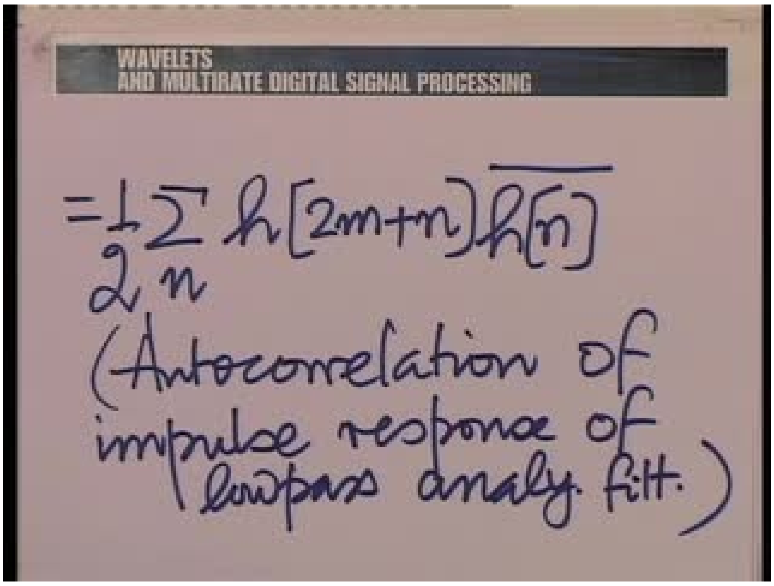


So, I would not repeat all the steps, but this is equal to summation on n h 2m plus n h bar n, and then, of course, you have a factor of half there. So, it is essentially the cross or the autocorrelation of the impulse response of the low pass filter. Now, once again, we are evaluating the impulse response at only even locations.

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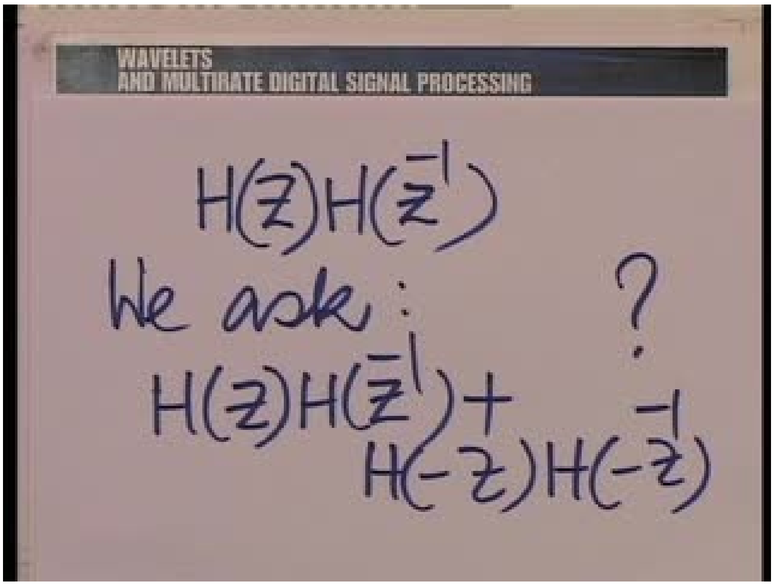


So, let me first write down the Z transform of the impulse response. You will remember, the Z transform of the autocorrelation of the impulse response would essentially be H of Z times H of Z inverse, and what we are asking is what is H Z H Z inverse plus H minus Z H minus Z inverse? This is essentially what would tell us about the values of the autocorrelation at even locations. Now, we know the answer to this. By the very design of an orthogonal filter bank, this is essentially a constant.

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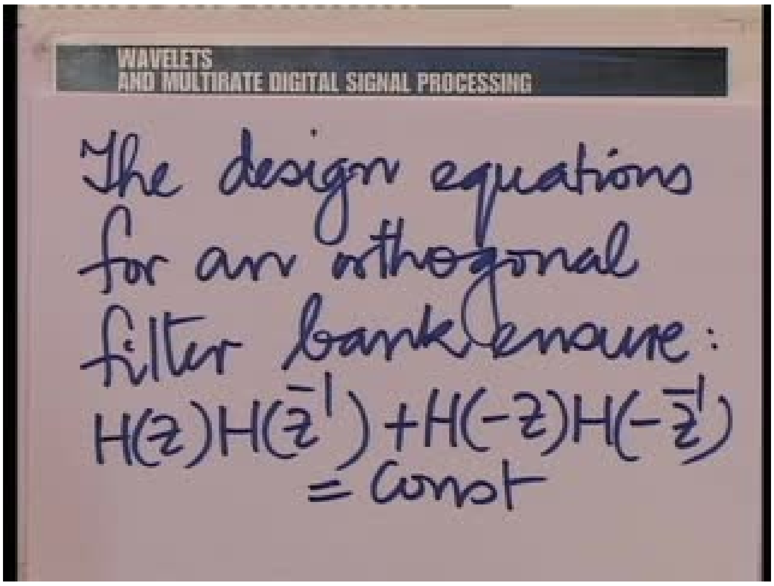

So, the design equations for an orthogonal filter bank ensure that, this quantity H Z H Z inverse plus H minus Z H minus Z inverse is a constant.

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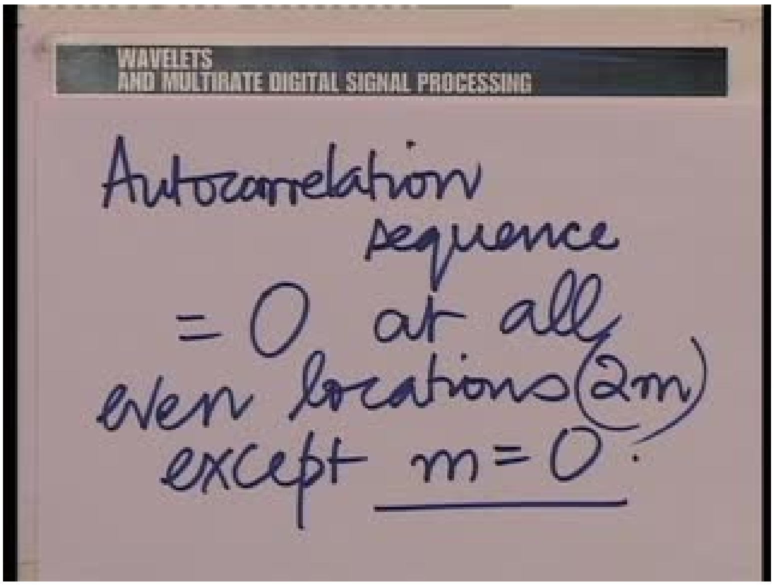


Therefore, the autocorrelation is 0 at all even locations. Let us call them 2m, except for m equal to 0; that is why, you get that constant; non-zero constant.

And so, there is a very beautiful observation we have made, and so called power complementary property, that is also consequence of this. This essentially manifest as the orthogonality of the impulse response to its own even translates. And that also becomes the part of establishing the orthogonality of phi t to its odd integer translates. So, they are all related. Now, this again brings out the intermittent connection between the autocorrelation behavior of the filters in the filter bank, and the orthogonality requirement that we insist upon in the underlying continuous functions that are generated on the iterations of the filters in the filter bank, as we have done before. You know how to iterate the impulse responses convolution of the impulse responses, to go towards phi t and psi t.

Well, now, what are the variants that we can introduce at this point? You see, when we look carefully at the equation; that means we put them all down together like this; I mean, we look at them from these perspectives. We see, there are many things which we have not done to date which we can probably explore, and let us put them down one by one.

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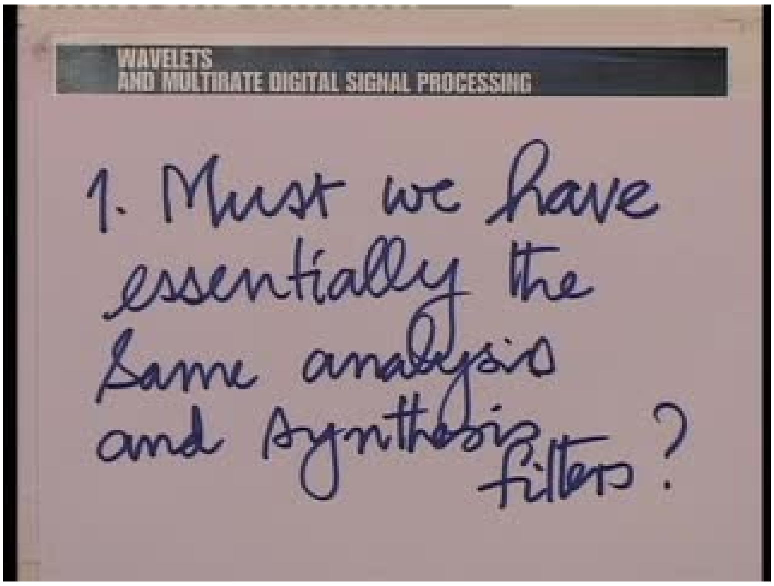


The first thing we need to explore is - Must we have essentially the same analysis and synthesis filters? Of course, this leads to an orthogonal filter bank, but then, remember, when we talk, we moved from the continuous wavelet transform towards the discrete wavelet transform, going through one step where we discretized the scale parameter logarithmically, we had made a remark.

We had come to a point where we said there are two options: either have different wavelets psi and psi tilde on the analysis and synthesis side, and build different analysis and synthesis filter banks, and get perfect reconstruction; or, use the same wavelet on the analysis and synthesis side, but that so called quote and quote, same wavelet would be different from the wavelet from which we started.

In case the discretization of the scale parameter was such that the sum of the highlighted spectra did not come to a constant for all frequencies, but lay between two positive bounds, we could bring in a new wavelet which gave us an orthogonal analysis and synthesis filter bank. The wavelet would be the same psi double tilde, but different from the original wavelet psi, from where we started; there were two options.

Now, we must explore the same two options in the context of discrete filter banks. Establishing the connection between the continuous case and the discrete filter bank is an involved exercise, but taking inspiration from the continuous case, from the continuous wavelet transform, and realizing that there are these possibilities. To be more general, namely that you could have different analysis and synthesis filters, and come up with a variant of the concept of multiresolution analysis is certainly something that we should explore in depth. And in fact, that needs to be explored in depths for another reason; one of the most recent data compression standards actually employs a filter bank with that perspective.

Let me say a little more about this.

You see, wavelets have been rather successful in the area of data compression. In fact, the central idea in wavelet based representation is that you can represent both global and local information efficiently with a with a few coefficients.

So, you could take transient information or short lived information, and represent it with few coefficients in the details. And long lived information are information spread all over the interval, over which in the last and also represented with few coefficients in the approximation part, and therefore, wavelets are an attractive proposition for a data compression.

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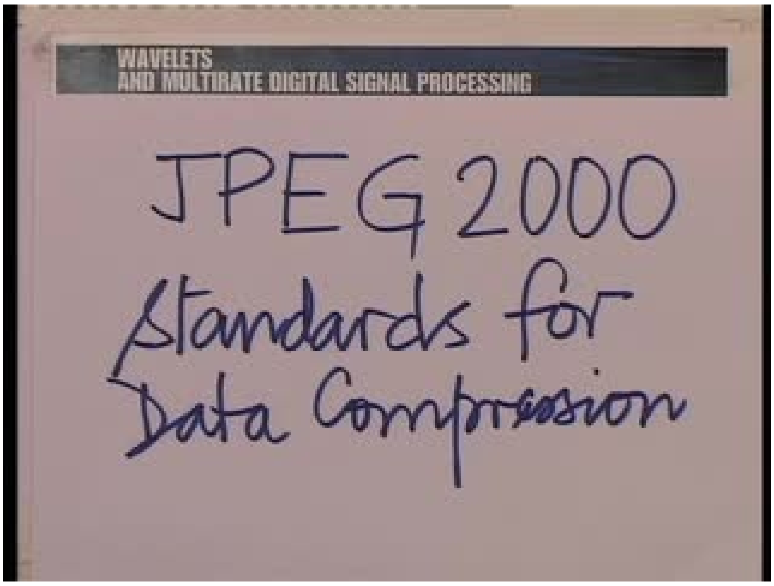


Now, using this fact, the joint photographic experts group or JPEG as it is called for short, came up with what are called the JPEG 2000 standards; 2000 refers roughly to the year around which this standard was finalized.

So, JPEG 2000 standards for data compression actually employs what are called biorthogonal filter banks, and in the lecture today, we shall look at this biorthogonal filter banks in some depths. So, biorthogonal filter banks are filter banks where the analysis and the synthesis filters are not quite the same. You note the words; you do not just have one analysis low pass filter from which everything else is derived by small variations of replacing Z by minus Z or Z by Z inverse, and so on. There are essentially, in fact, in the JPEG 2000 standards. There are essentially two filters and the other two are derived out of them. So, we come to that in slightly greater depth in a few minutes, but before that, let us look at some other variants set up possible.

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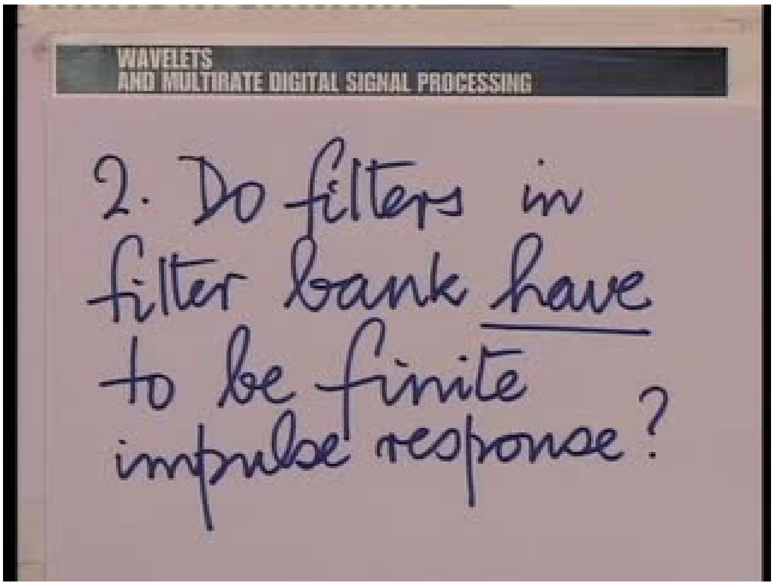


Now, so far, we have been looking at finite impulse response filters. Do the filters in the filter bank have to be finite impulse response? Obviously, the answer is no.

But does not have to have a finite impulse response which one could certainly conceive of an H Z of that matter a G Z I mean H 0 H 1 G 0 G 1 where, the sequence underlying the Z transform is infinite in lengths as an infinite number of non-zero samples nothing stops us from considering such H Z and G Z. So, we have to admit that possibility. And actually, when we look at the phi 3 filter bank, as it is called in the JPEG 2000 standards, we will see that, if I insist on getting an orthogonal filter bank solution for this kind of a paradigm, we need to go to an infinite length impulse response filter bank.

We shall build that up in a subsequent lecture. So, this is the second variant that we wish to explore; must restrict ourselves to finite impulse response filter banks. The third variant which we need to explore is as follows.

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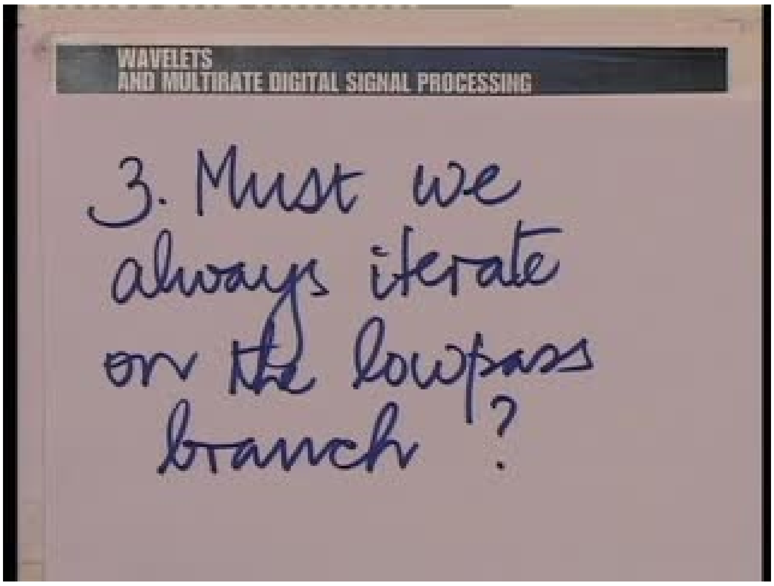


Must we always iterate on the low pass branch? And obviously, again, the answer is no. In principle, nothing stops us from putting the whole filter bank back on the high pass branch.

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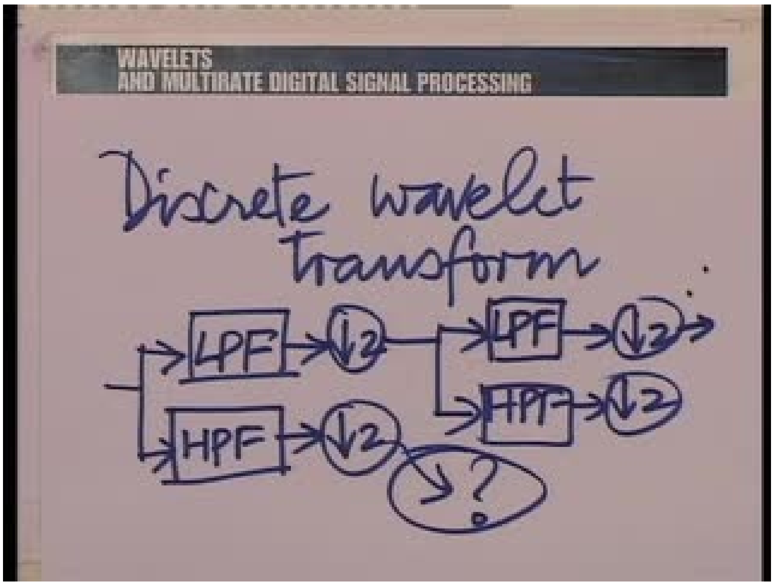

Let me put before you, what I am saying, graphically. What I am saying is this. The discrete wavelets transform looks like this

Essentially, we have a low pass filter and a high pass filter; whether finite or infinite length, that is not the issue followed by a down sampler, and we keep iterating here. We put the low pass filter here and the high pass filter there, and down sample once again, and keep doing this.

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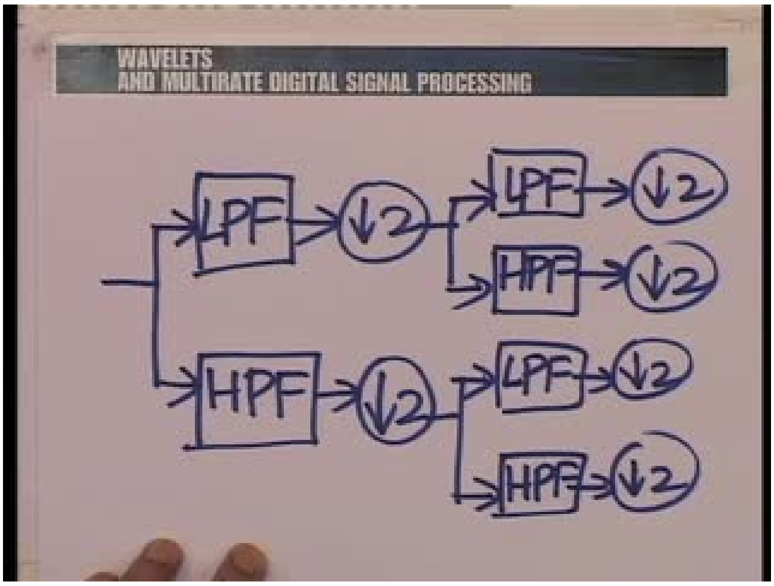


Now, what about this? Can we put this filter bank here? Nothing stops us in principle. In fact, it is obvious; that is, we have to take one stage of this, namely, if I were to do the following: I were to take the input; I were to subjected to the action of a low pass and a high pass filter, followed by down sampling, as usual followed by iteration; here LPF HPF down by 2, and down by 2 there, but also to the same here (Refer Slide Time: 32:29 to 32:59). I could get back here by using the corresponding synthesis filter bank for this and I could get back here by using the corresponding synthesis filter bank for this.

And once I get back here and here, now please remember, of course, I would get back with the constant multiplier and a delay, but that is not such a serious issue. I mean I can always take both the constant and the delay across the down sampler, and I could actually observe it with these filters here (Refer Slide Time: 33:19). And finally, I could, in principle, reconstruct this once I have got here.

So, whatever argument allowed me to decompose on the low pass branch every time, can also allow me to decompose on the high pass branch. So, there is a new possibility that is opened down.

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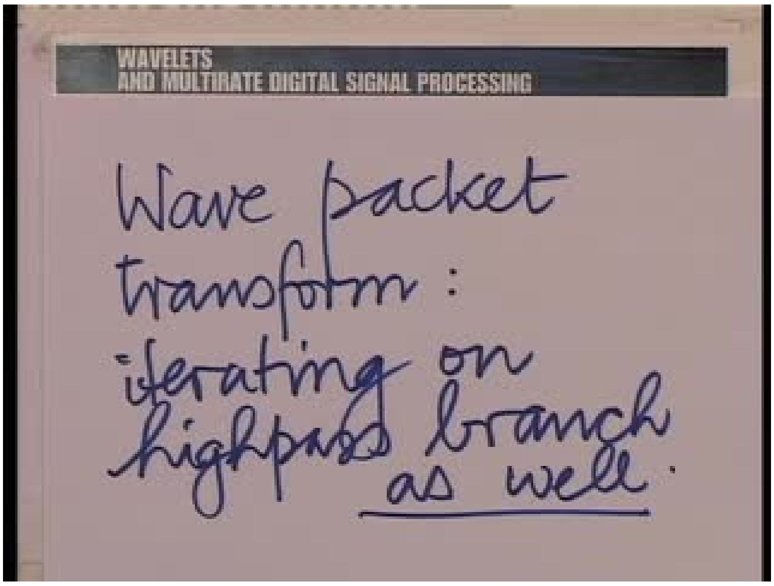


Suppose, I were to conceive of decomposing the high pass output, what does it imply for the underlying functions? And that would lead us to the idea of what is called the wave packet transform. The central idea in the wave packet transform comes from iterating on the high pass branch as well.

Not only the high pass branch, but a loving for iteration on the high pass branch also. That would lead us to the idea of the wave packet transform. So, this a brief introduction to the variants of the idea of multiresolution analysis; that, we wish to explore one by one.

Now, let us begin with the first of the variants; namely, the filter bank, used in JPEG 2000; let us not be too general; let us consider one particular filter bank which is used in JPEG 2000, where the filters are of different lengths; the analysis filters are of different lengths. And that would lead us to the idea of what is called a biorthogonal filter bank, or a filter bank, where we do not quite have orthogonal or scaling functions or scaling functions which are orthogonal to its to their translates.

Now, you know, the inspiration for the phi 3 filter bank in the context of JPEG 2000 could be seen to be the following. There are various ways to explain the inspiration, but one of the inspirations can be seen to be the following.

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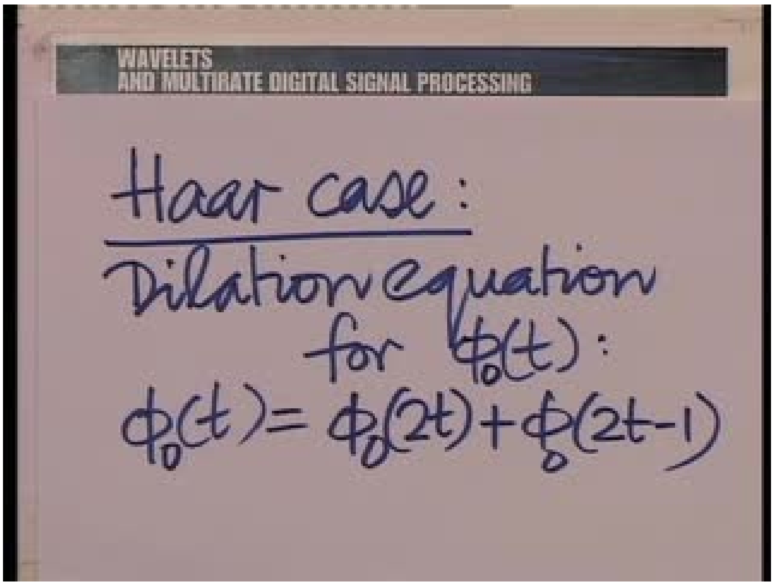


Let us begin with the Haar case, where we have the dilation equation for phi t given as follows: phi of t; now, just to ease or notation, we shall call the corresponding scaling function for the Haar MRA as phi 0 t. We would understand why the 0 in a short while; phi 0 t is phi 0 2t plus phi 0 2t minus 1.

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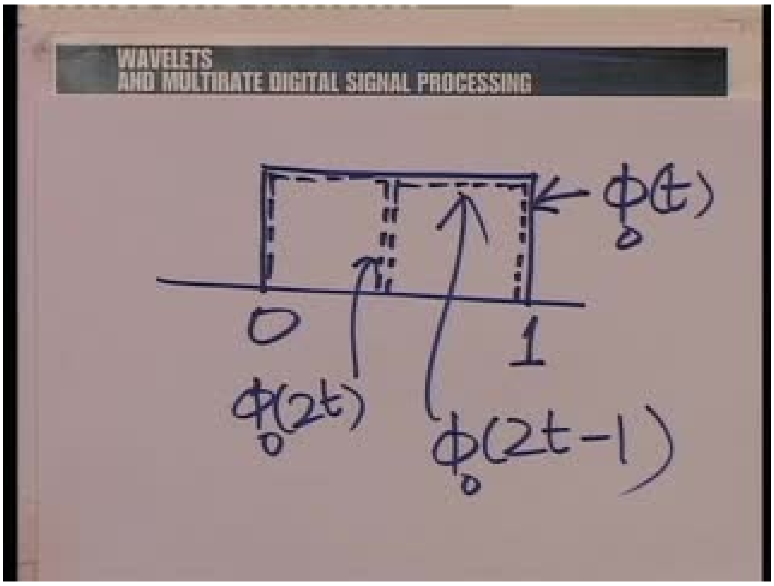

And we will recall that this translates graphically to the following: This is phi 0 t, this is phi 0 2t, and this is phi 0 2t minus 1. So, this explains what we are saying, graphically.

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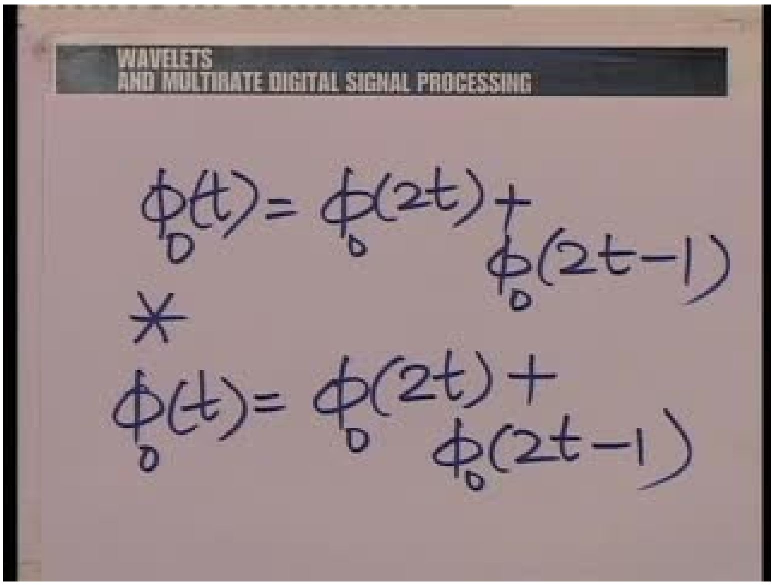


Now, suppose we convolve this equation with itself. So, we write the equation down twice and convolve with itself; what would happen? Now, you know, the central ideas what happens when you convolve say a term like phi 0 2t with a term phi 0 2t minus 1.

In other words, we are asking the following general question.

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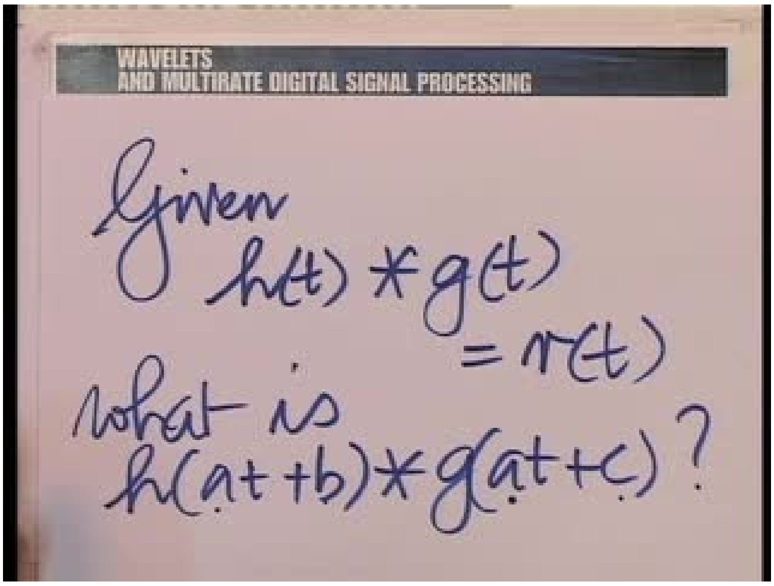

Given h t convolve with g t to be r t, what is h at plus b convolve with g at plus c.

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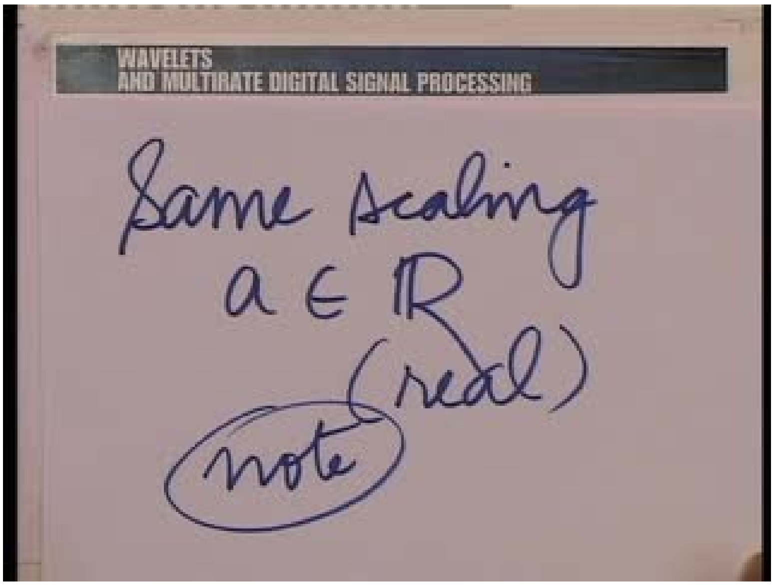


Please note, a - the same a, is used here; b and c can be different, but the same scaling factor is used; a, of course, belongs to the set of real numbers.

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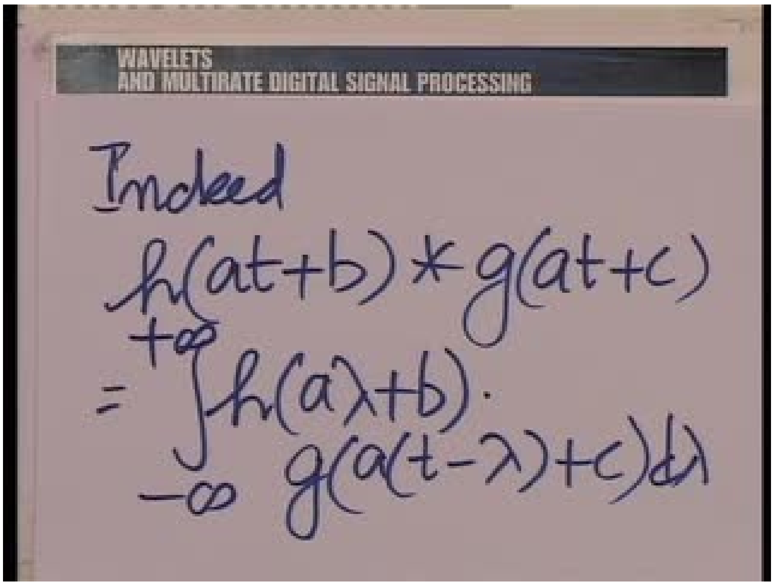


Same scaling that should be emphasized, and a is real. In fact, let us evaluate; indeed, h at plus b convolve with g at plus c as we desire here is essentially integral h a lambda plus b g a t minus lambda plus c d lambda, integrated over all lambda.

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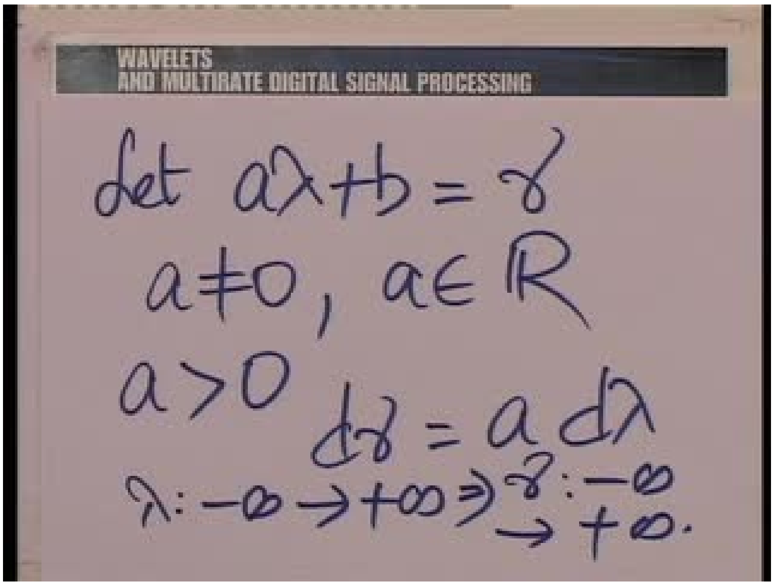


And we use the standard principle of substitution of variables. So, let a lambda plus b be another variable; let us call it gamma. Another two possibilities; a, is of course, not equal to 0, and a, is real. So, when a, is greater than 0, d gamma is of course, always a d lambda. And lambda going from minus to plus infinity translates to gamma also going from minus to plus infinity.

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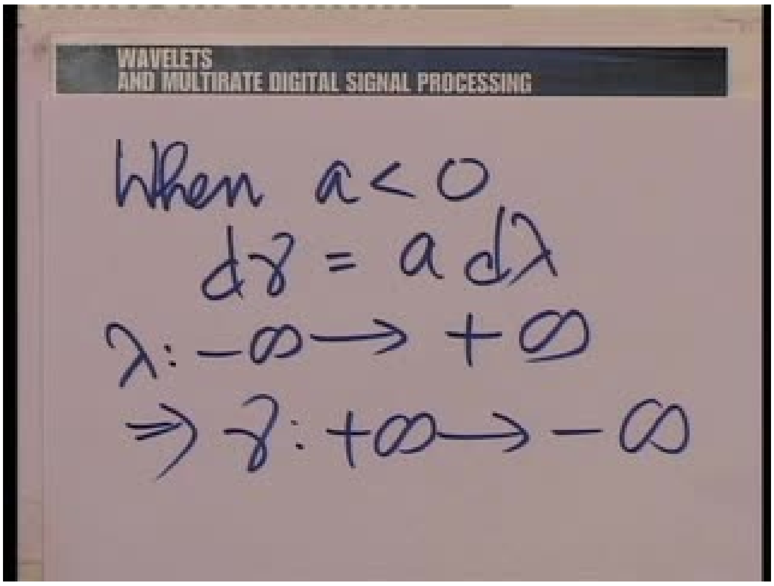

On the other hand, when a is negative, of course, d gamma is as usual a d lambda, but lambda going from minus to plus infinity translates to gamma going from plus infinity to minus infinity.

However, if you look back at the limits of the integral, here (Refer Slide Time: 40:21). So, you know, when a is positive, d lambda is going to be, you are going to have d gamma as e d lambda. So, you are going to have a factor of a coming here in general, or 1 by a.

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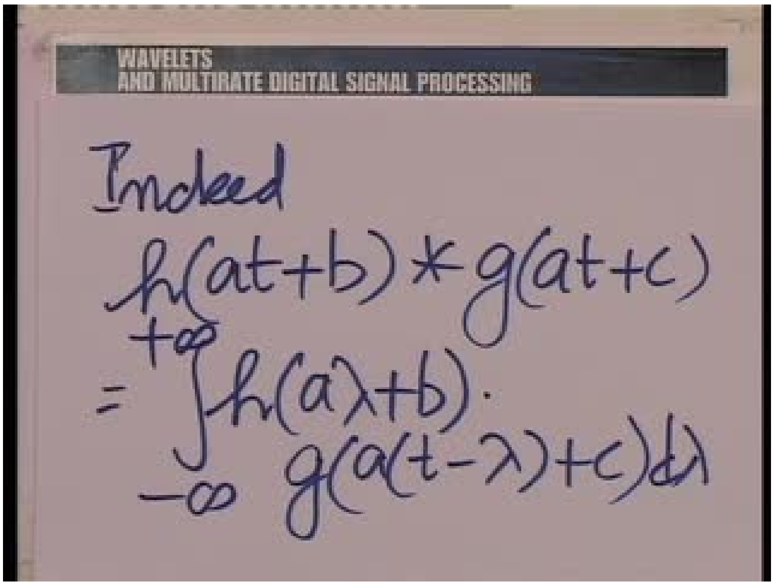


So, here, d lambda is going to be d gamma by a (Refer Slide Time: 40:36). So, 1 by a is going to be positive, if a is positive; 1 by a is going to be negative if a is negative. When a is negative, these limits are reversed; when a is positive, the limits are where they are.

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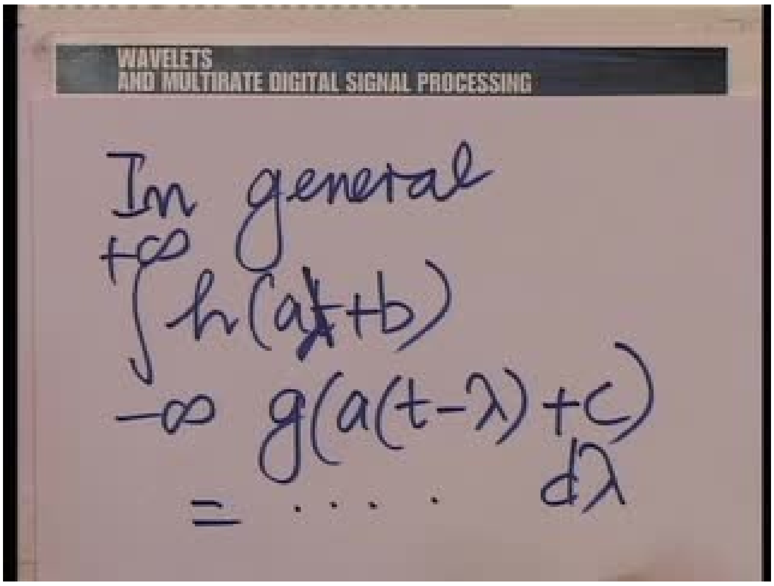


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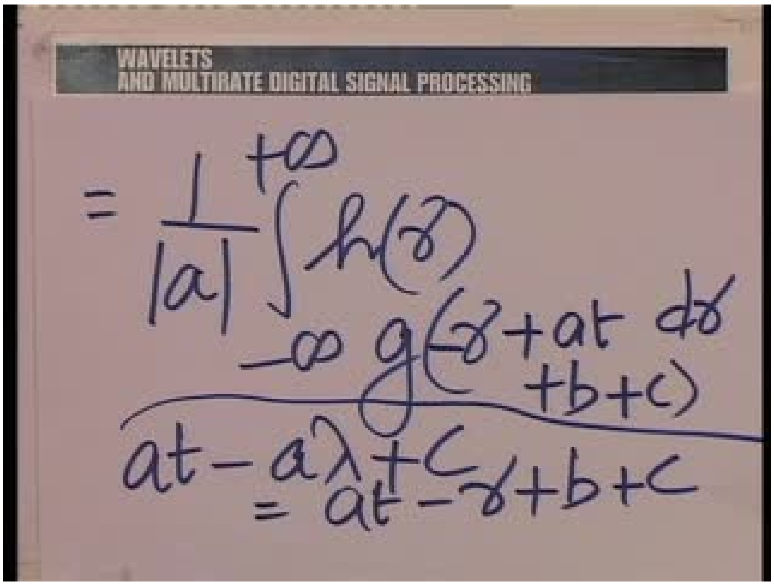


So, all in all, we can write down the following: We can say, in general, or rather a lambda, it is make it a lambda is equal to 1 by mod a integral minus to plus infinity h gamma g gamma, when actually, let me let me work this out; so, at minus a lambda plus c is essentially at minus gamma, and minus gamma means minus a lambda minus b plus b plus c. So, I have a minus gamma here plus at plus b plus c and a d gamma there (Refer Slide Time: 42:13). Let me write this down neatly.

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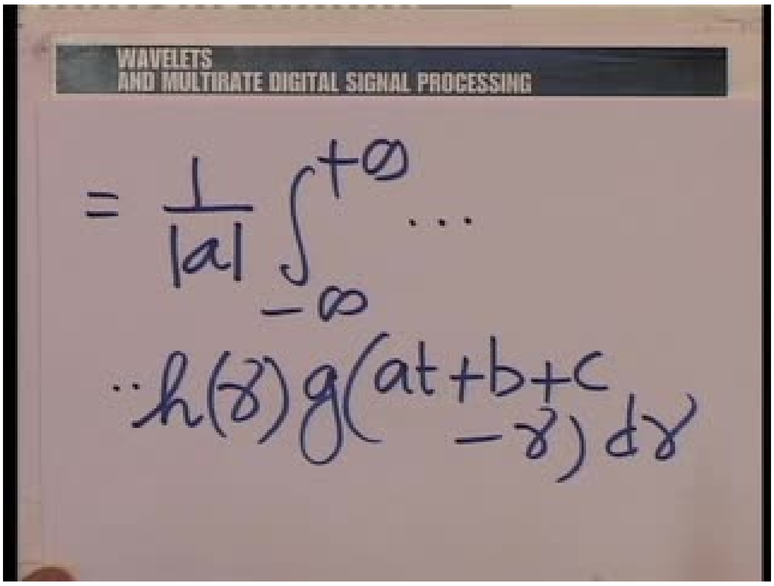


So, I am saying, essentially, it is 1 by mod a integral from minus to plus infinity h gamma g at plus b plus c minus gamma d gamma and this is obviously the convolution of h and g evaluated at at plus b plus c.

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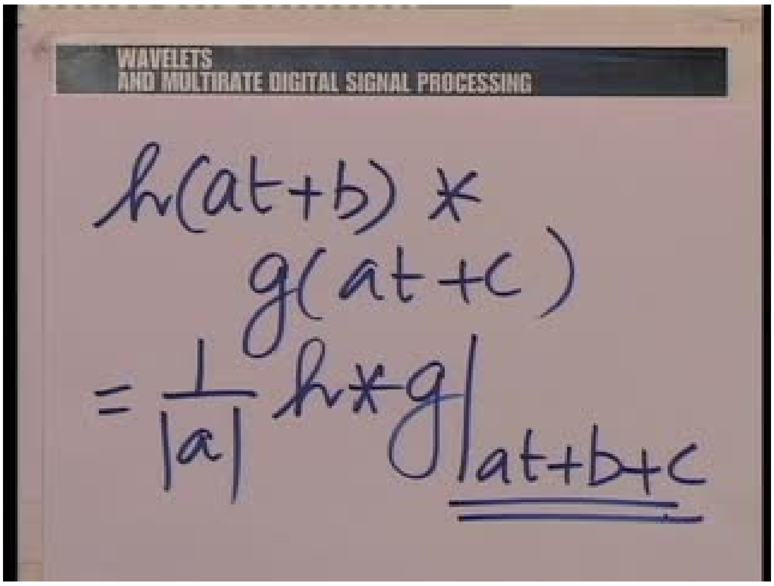


So, we have the answer now. h at plus b convolve with g at plus c is essentially 1 by mod a h convolve with g evaluated at at plus b plus c.

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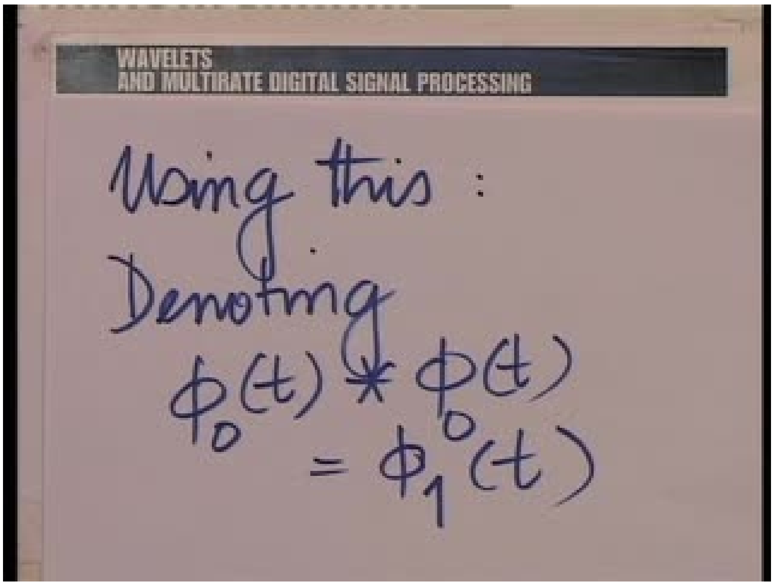


And we can use this in the dilation equation that we had a few minutes ago. Therefore, using this and denoting phi 0 t convolve with phi 0 t as phi 1 t, and now, I will explain why I am calling it phi 1 t.

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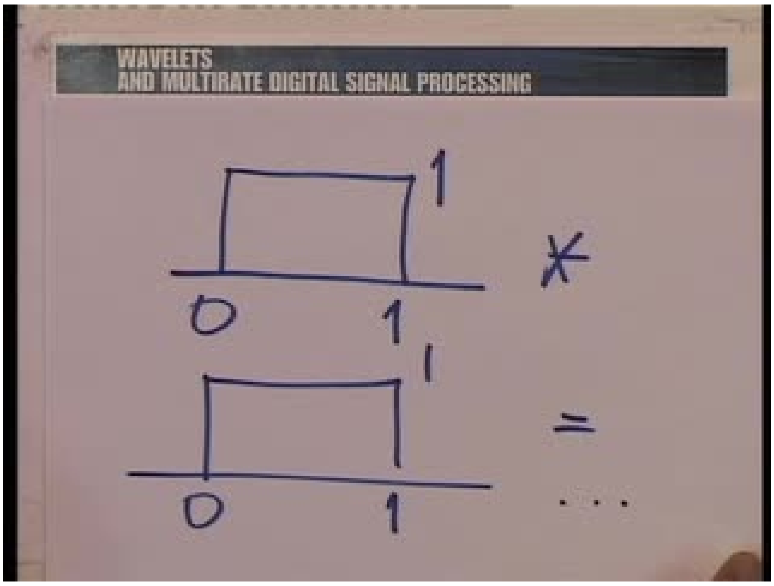

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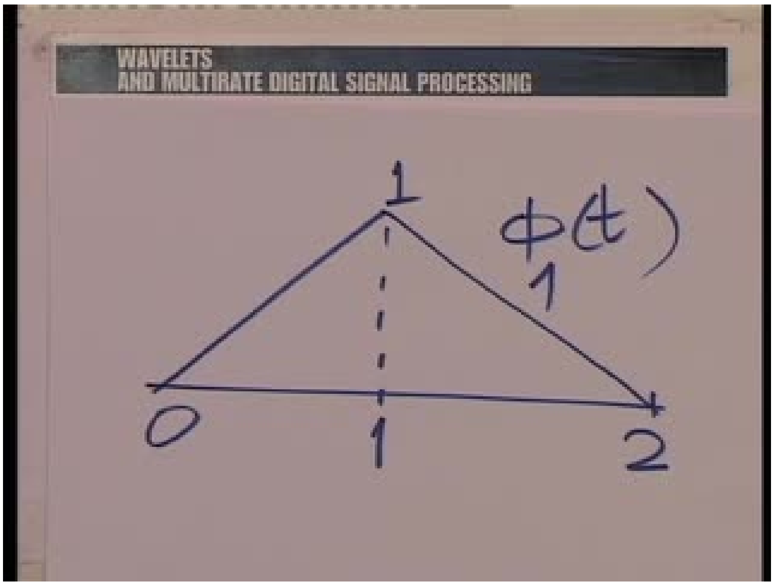


What is phi 0 t convolve with phi 0 t? This is phi 0 t. When you convolve it with itself, it gives you a function that looks like this. This is phi 1 t. What is 1 about this and 0 about this? (Refer Slide Time: 44:33) The degree of the polynomial. If you look at it, phi 0 t is, essentially piece wise polynomial with the polynomial of degree 0; constant phi 1 t is piece wise polynomial with polynomials of degree 1 linear. So, you could similarly conceive, piece wise polynomials of higher and higher degree. So, you could have a phi 2t; in fact, if you take phi 1 t and convolve it with phi 0 t once again, you would get a so called phi 2t. A piece wise polynomial with polynomials of degree 2, and if you keep repeating this, the degree of the polynomial increases by 1 every time. So, that is the reason why 0, 1, 2, and so on.

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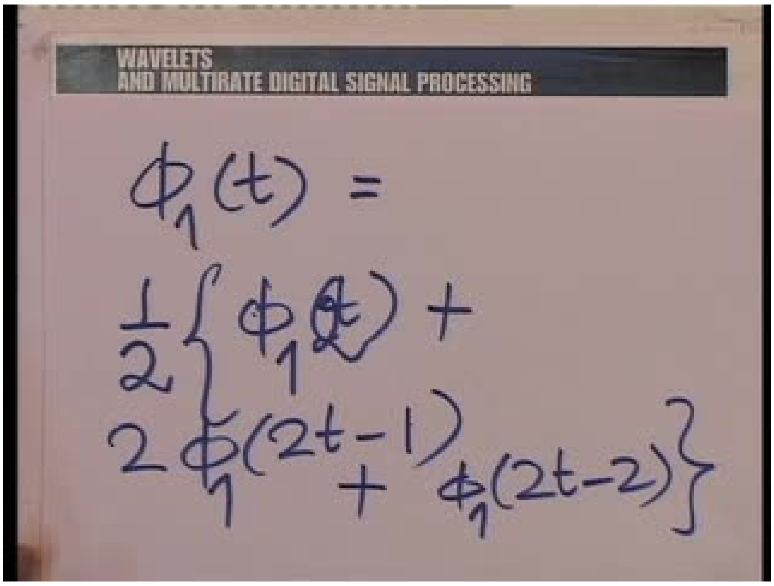


Anyway, coming back to the dilation equation, phi 1 t can easily seem to be half phi 1 t plus 2 phi 1 2t; wait, phi 1 2t; I am sorry, plus 2t times phi 1 2t minus 1 plus phi 1 2t minus 2.

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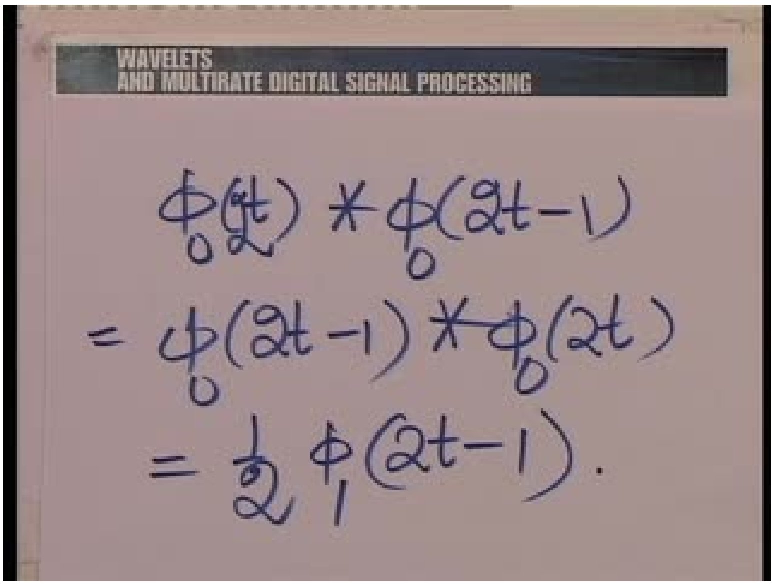


In fact, what we are saying essentially is that phi 0 2t convolved with phi 0 2t minus 1 is the same as phi 0 2t minus 1 convolve with phi 0 2t, and both of them are equal to half phi 1 2t minus 1. This is the simple application of the result that we derived, a few minutes before.

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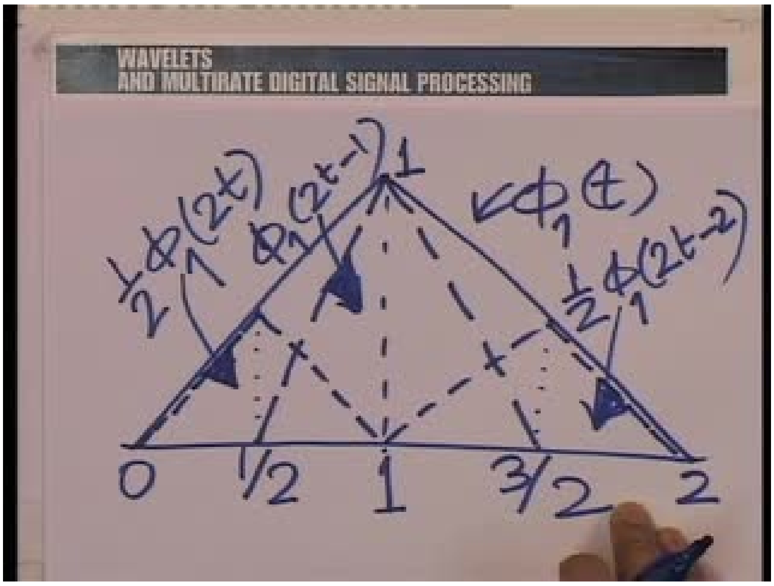


And in fact, this can be seen graphically. You know, if you draw phi 1 t, let us draw it large and big. It is not very difficult to see that if you were to mark the half points here, and draw these triangles. This is essentially half phi 1 2t; this is essentially half phi 1 2t minus 2 and this is essentially phi 1 2t minus 1. I mean, this means this here, this means this here, and this refers to this here (Refer Slide Time: 48:01 to 48:28). So, this piece wise linear function plus this piece wise linear function plus this piece wise linear function would obviously give you this solid piece wise linear function. It can easily be seen graphically.

In fact, linear functions are easy to add; you need to add them only at end points; for example, here, the sum is this; at this point the sum is essentially this; at this point the sum is essentially this (Refer Slide Time: 48:44).

So, you can see that these three dilates and translates of phi 1 t give you back phi 1 t, and therefore, phi 1 t obeys the dilation equation in its own. Right and What are the coefficients of the dilation equation?

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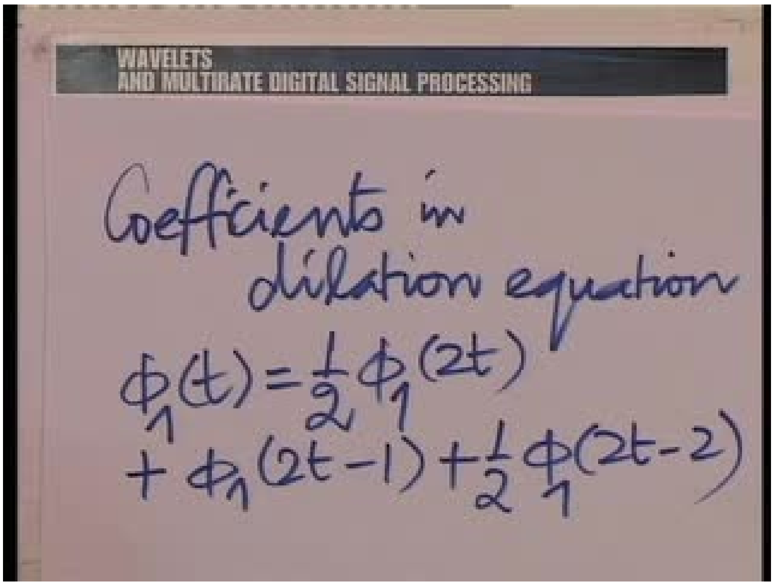


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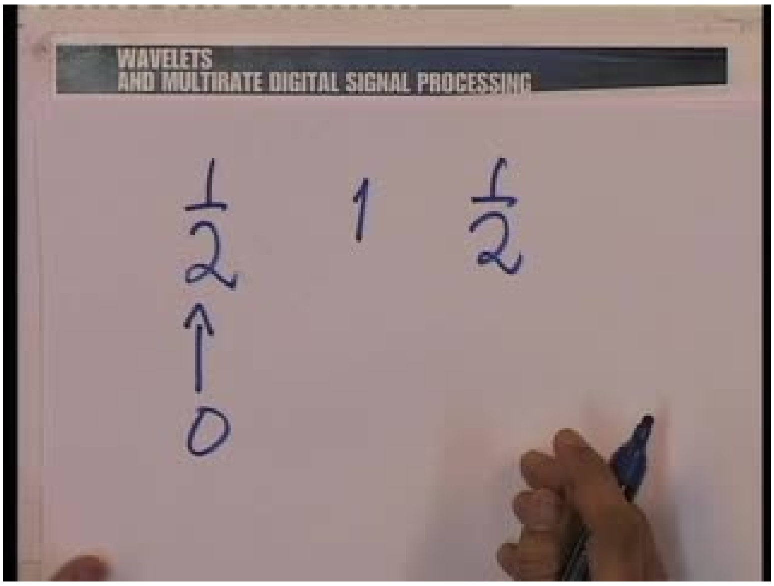


The coefficients in the dilation equation phi 1 t is half phi 1 2t plus phi 1 2t minus 1 plus half phi 1 2t minus 2 are essentially half, 1, and half. And if you put a filter to represent these coefficients, that filter would essentially be the following.

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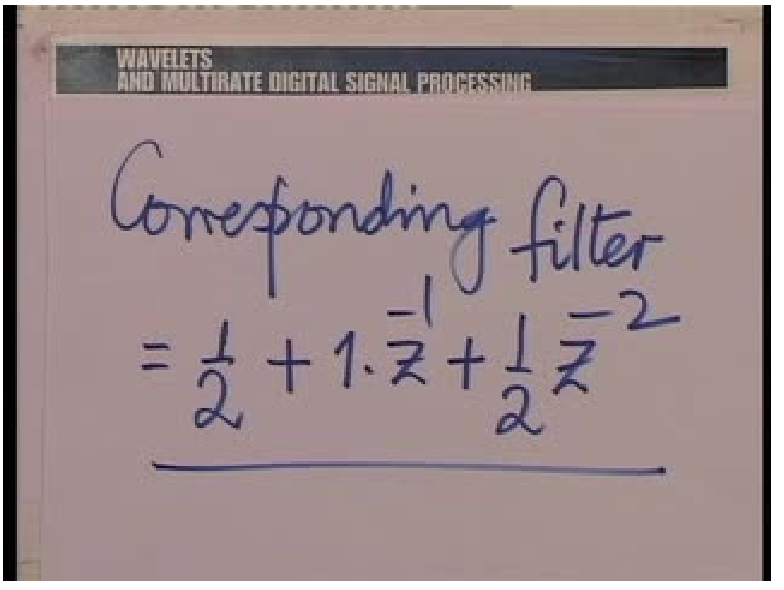


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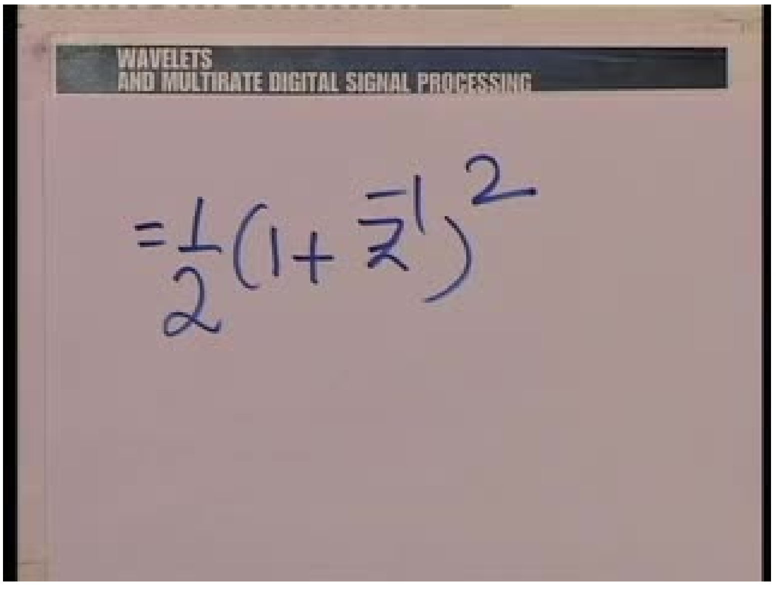


And this filter is easily seem to be 1 plus Z inverse the whole squared, but there is just one short coming here.

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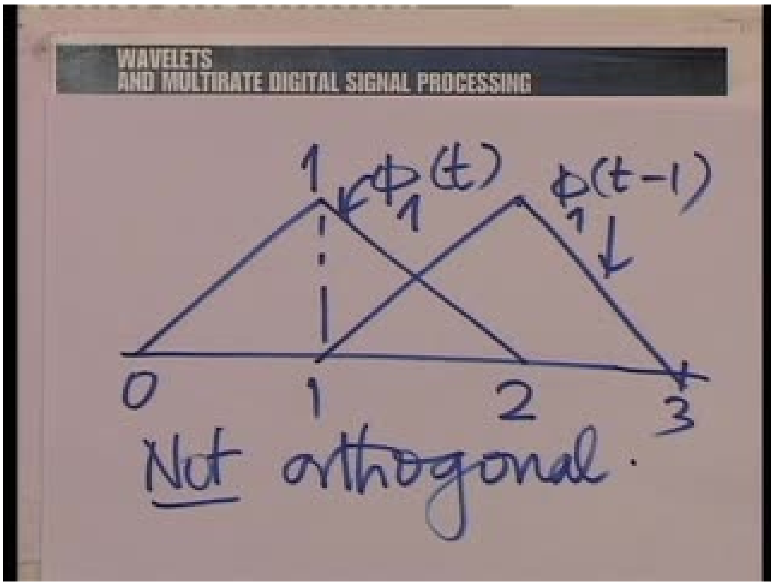


Unfortunately, phi 1 t is not orthogonal to it is integer translates in general. Let me show this to you graphically if you consider phi 1 t and phi 1 t minus 1. This is phi 1 t and this is phi 1 t minus 1; they are not orthogonal. So, we have a problem. Although phi 1 t obeys a dyadic dilation equation, and in fact, in principle, one could also start investigating the basic axioms of a multiresolution analysis, the axiom of orthogonality does not obey it.

Now, we have two options. We can ask whether we can do something similar to what we did in the discretization of scale; if not quite a constant. When I am talking about the some of dilated spectra, we said, if that some of the dilated spectra is not a quite a constant, could be allowed to be between two positive constants; something similar could possibly be investigated here; that is one option. The other option is - think out of the box, and in fact, we will do that first and come to the JPEG phi 3 filter bank.

Why stick to the same analysis and synthesis filters? Can we allow different lengths for the low pass and high pass filter, analysis filter, and then, therefore, derive the synthesis filter in a slightly different more relaxed way from the analysis filters, and establish a perfect reconstruction filter bank? And that is what we shall do in the next lecture. Investigate that variant; think out of the box; do not insist on orthogonality or do not insist on using the same axioms of an MRA; build a perfect reconstruction filter bank where you have different length analysis and synthesis filters. I mean different length analysis, high pass and low pass filters, and therefore, different analysis and synthesis filters, and build different paradigm for multiresolution analysis out of it. So, we shall proceed the next time, to derive the JPEG 2000 phi 3 filter bank.

Thank you.