Advanced Digital Signal Processing –Wavelets And Multirate.

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Lecture No. # 24
Logarithmic Scale Discretization Dyadic Discretization

A warm welcome to the twenty fourth lecture on the subject of wavelets and multirate digital signal processing. In the previous lecture, we had look that the admissibility condition for the continuous wavelet transform in depth. The admissibility condition was essentially a condition for reconstruct ability from the continuous wavelet transform, as we saw.

The continuous wavelet transform - we note it was extremely redundant to use a continuous scale and continuous translation, meaning essentially a two dimensional representation. When what we are dealing with is a one-dimensional entity is extremely redundant, very wasteful, and therefore, we began to ask the question of discretization, can we discretize the scale and can we discretize the translation parameter?

We first addressed the question of discretizing the scale, and in that context, we had noted what happens when we change the scale. We shall proceed in this lecture today to build on this discretization of scale in great a depth and to consider one particular kind of discretization of scale, with which, we have been dealing in the first half of the course, namely: dyadic discretization.

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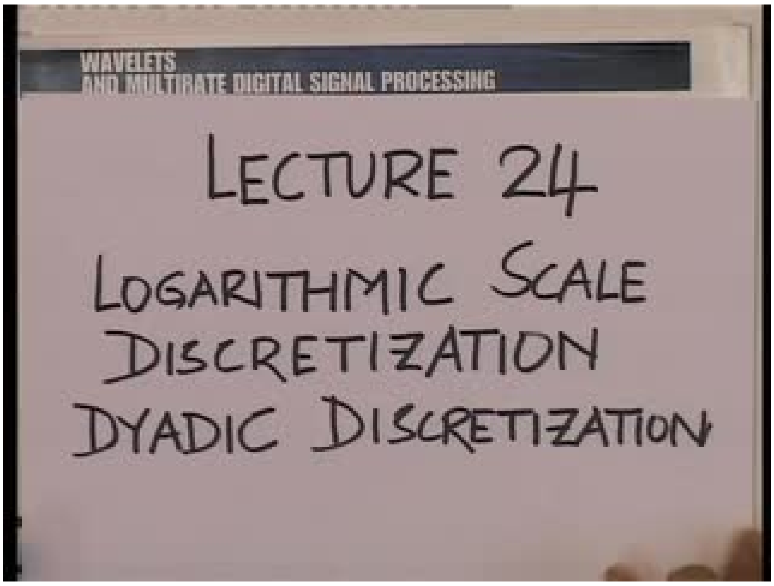

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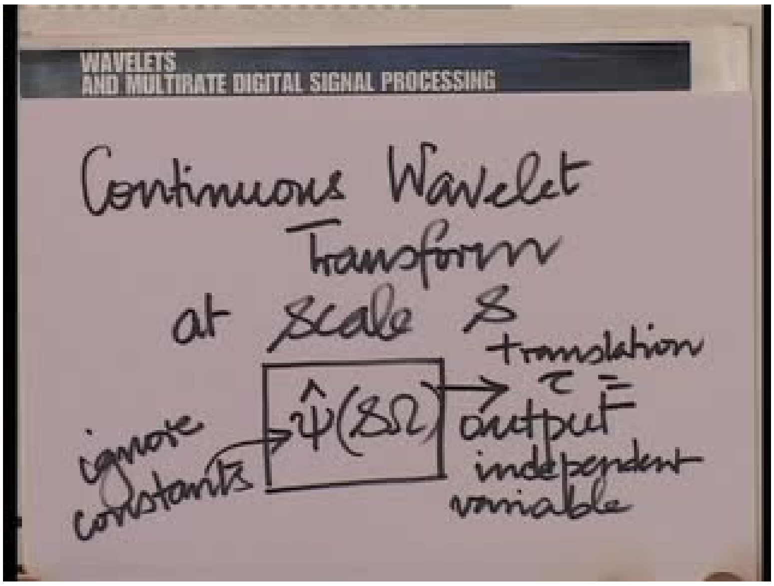

Therefore, I intend in the lecture today to talk about logarithmic scale discretization in general and dyadic discretization in particular. With that little background, let us look once again to what the continuous wavelet transform does. We had noted that a continuous wavelet transform essentially operates like a filter, both in synthesis and in analysis. So, we had noted that the continuous wavelet transform, at scale s is essentially a filtering operation.

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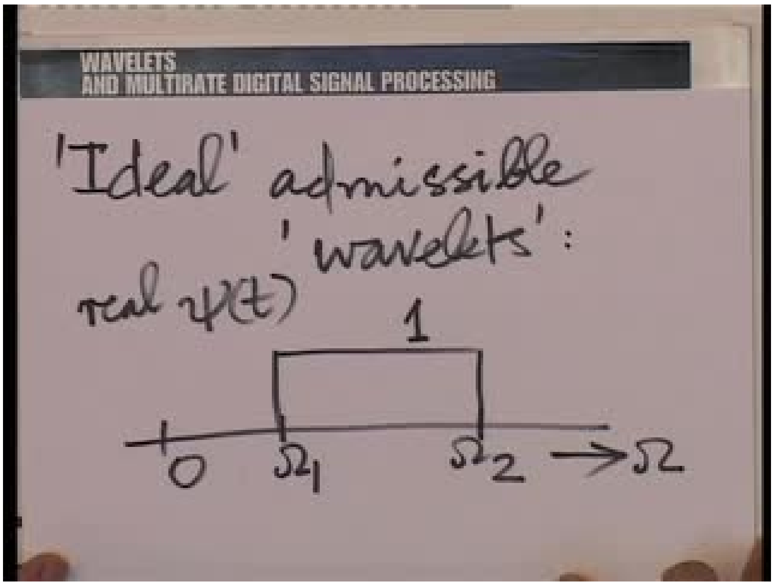

The filtering operation with the frequency response given by psi cap s omega with some constant of course, at the moment, let us simply ignore the constants. So, you see, the output independent variable here can be interpreted as the translation. The output independent variable is tau here, alright? Well, then, if we take an ideal filter or an ideal wavelets, how to speak? We are taken examples of ideal frequency response that were admissible and those was strictly band limited.

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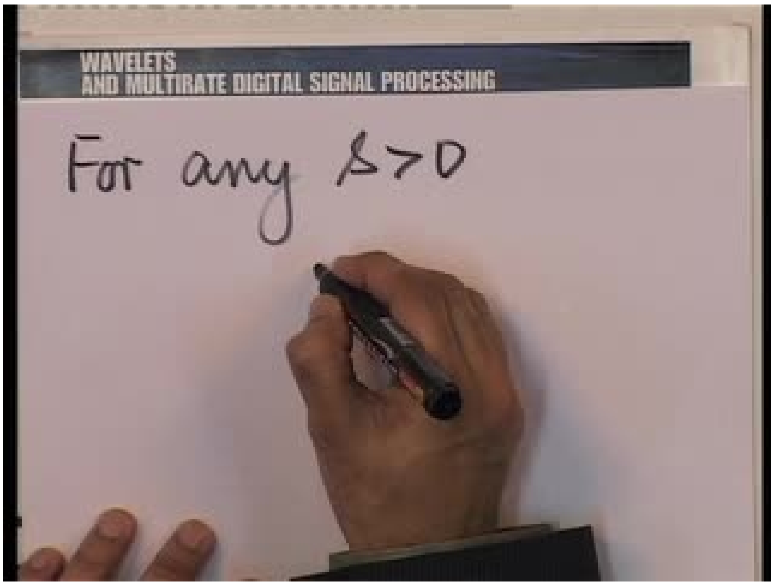

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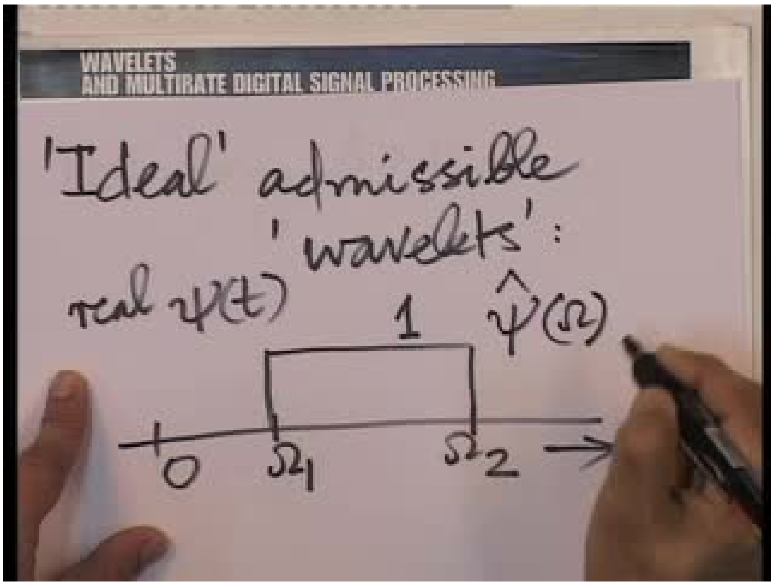

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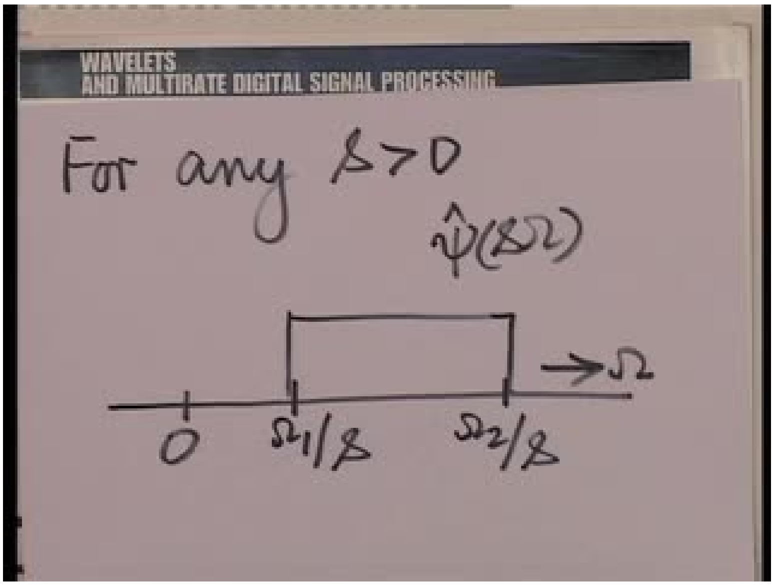

They had a psi cap, which look something like this. Of course, I am assuming that the wavelets are real, and, and looking only at the positive side of the frequency axis. So, we had note it, that, that the admissibility condition is essentially a statement of band pass character, and therefore, if we consider an ideal band pass function like this and take a dilation of the same, we again get a band pass function. So, for any s greater than zero as we should have it, psi cap s omega. This was of course the Fourier Transform - psi cap omega. Psi cap s omega would look like this, and we once again note that there is a contraction or expansion dilation in general, both of the band and the center frequency, and that is the reason why we are argued that logarithmic discretization make sense.

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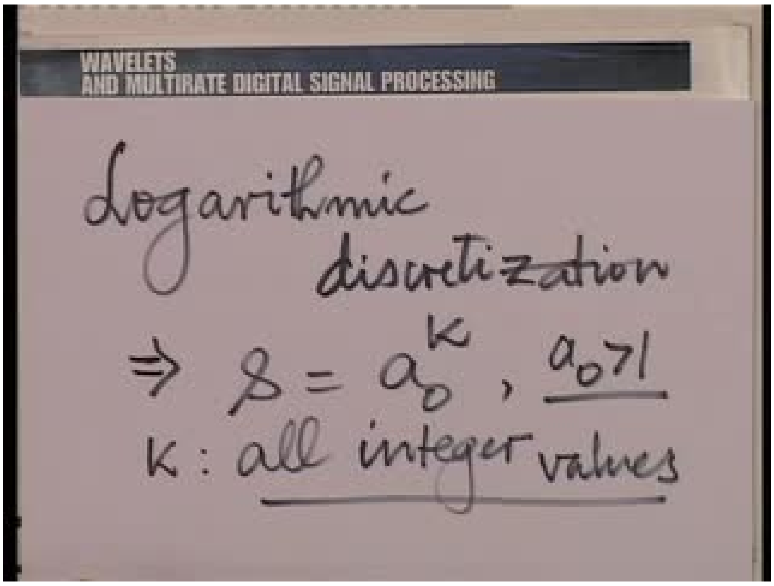

Logarithmic discretization means s should be discretized; s should be discretized according to a naught to the power of k, where k runs over all the integers, and we said: as long as a naught is positive and greater than one and k runs over all the integers. We do not need to take a naught less than one also as the possibility, because of after all, when k runs over all the integers, the less than one possibility is taken care of. I had given you the example of two, for example, two raise to the power of k over all integer k is the same thing as say half raise to the power of k over all integer k. With that little recapitulation of our discussion, let us go again in depth to analyze, what will happen when we discretize the scale? So, in other words, when we discretize the scale, what we are trying to ask is - what you do to each of these filters and how do you recombine what you have done to each of these filters to get back the original x of t.

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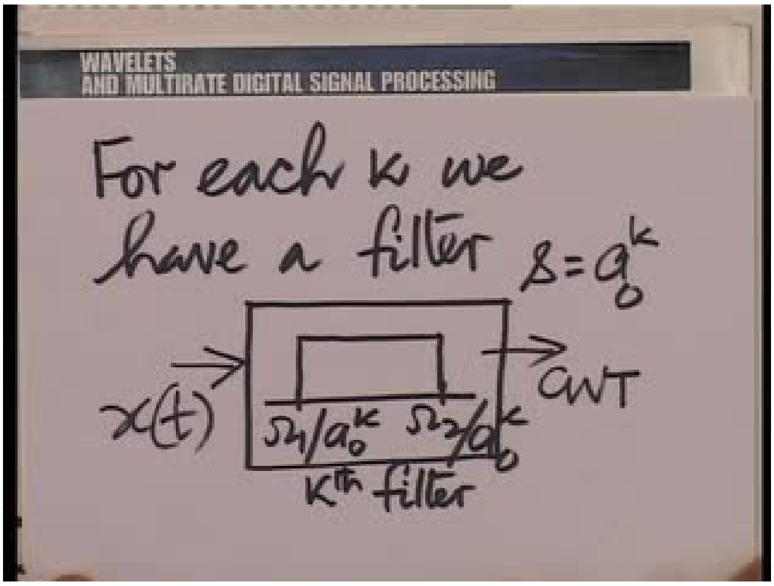

In other words, for each such k, we had a filter, and you see, how the frequency response of the filter looks; I mean let start with the ideal situation, and then, lets the great to the practical situation eventually. In the ideal situation, the filter, the kth filter so to speak would have a frequency response like this. So, it would have a band going from omega one divide by a naught to the power k to omega two divide by a naught to the power of k, where s is, of course, a naught to the power k.

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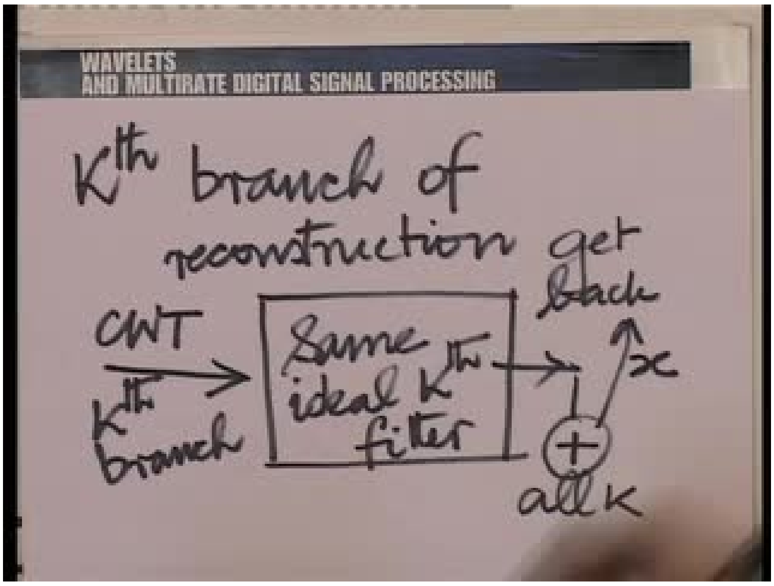

Now, it is obvious how we can reconstruct this part after all If things are ideal, and in fact, if you do pass x t, the input through an ideal filter like this to get the continuous wavelet transform here. The reconstruction should also proceed by doing the same thing. So, in fact, the kth branch of reconstruction should essentially have the continuous wavelet transform coming in here, the kth branch; I mean the continuous wavelet transform being fed to the same ideal filter, and if we add this over all k, we should get back x t.

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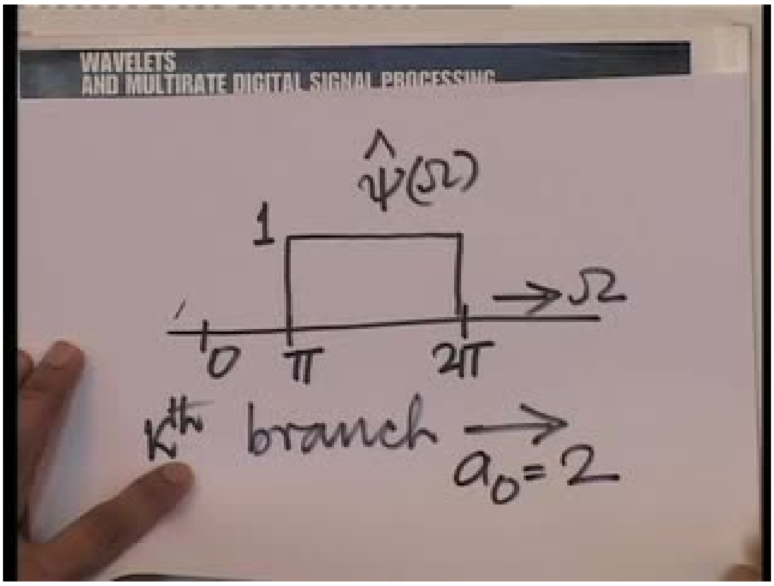

So, in the ideal situation, the answer is very simple. What each branch, so to speak each kth branch in some sense of this continuous wavelet transform does is to extract a particular band on the frequency axis, and now, we can also see very easily, interpret very easily what each branch should do. Each branch should have a separate non overlapping band. Just to take an example, suppose you had a band pass filter of the following kind, so, if you had psi cap omega looking something like this, suppose this was psi cap omega one between pi and two pi and zero everywhere else, and naturally, mirrored on the negative side of the omega.

Then the kth branch were essentially do the following: the kth branch would cover the band between pi divided by two to the power k if a naught where equal to k. So, I must also specify what a naught is - kth branch with a naught equal to two.

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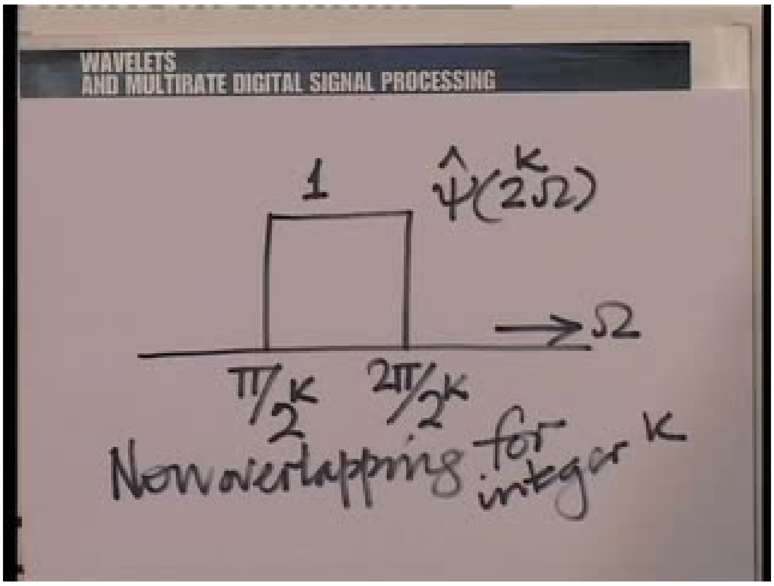

So, essentially, the kth branch would take the frequencies from pi by two raise the power of k to two pi by two raise the power k, and we can see that in this particular case for different integers k, these bands would be non overlapping. So, it is easy to fix our ideas. In fact, what I have done is to put before you the very idealization of the dyadic multi resolution analysis which we been looking at all this while.

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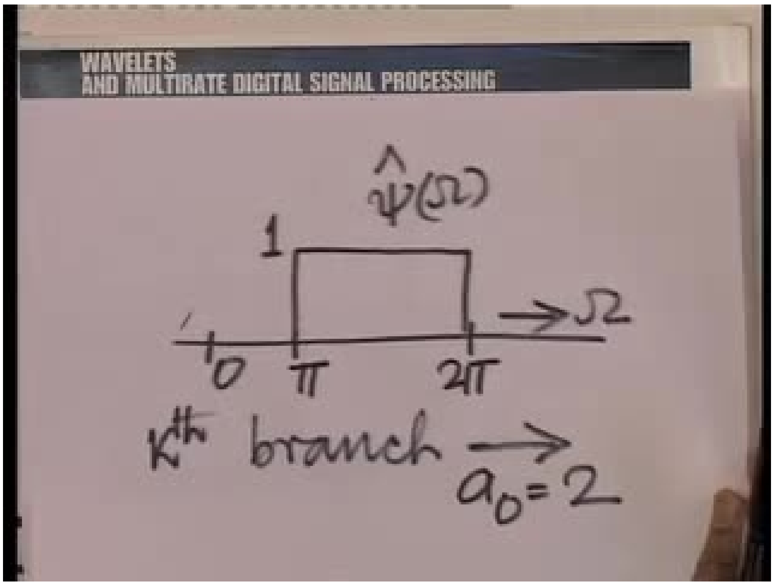

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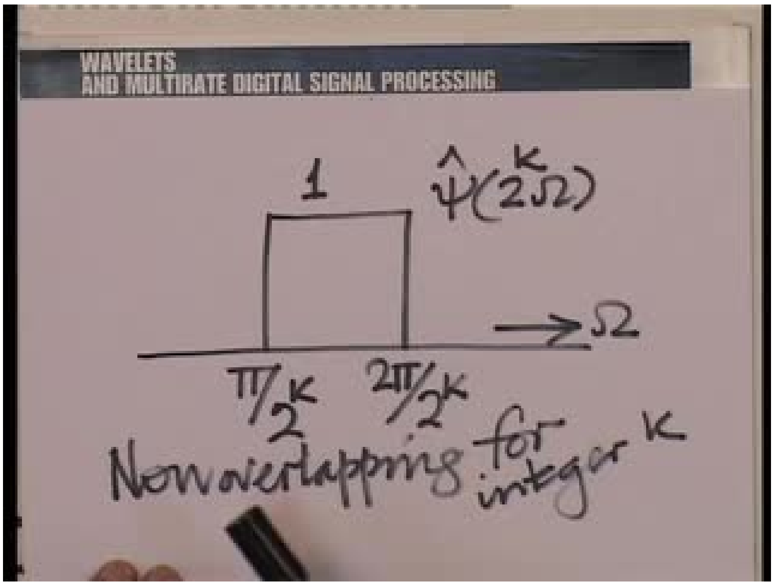

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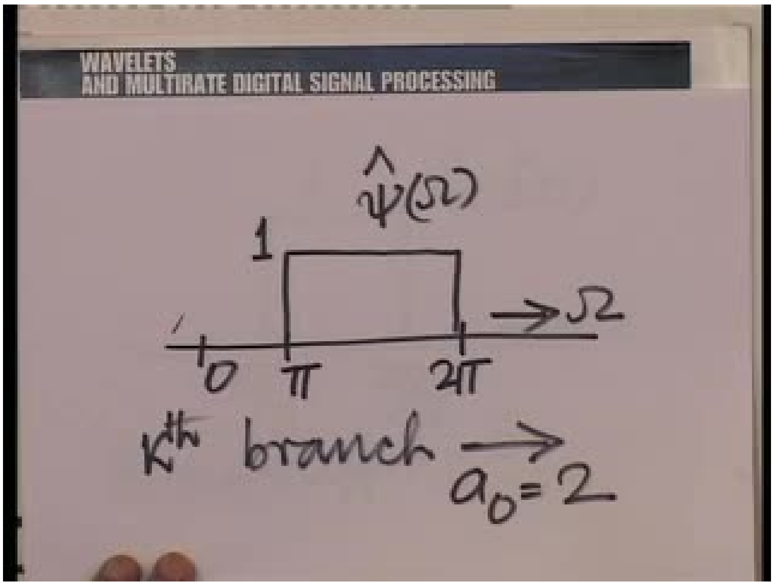

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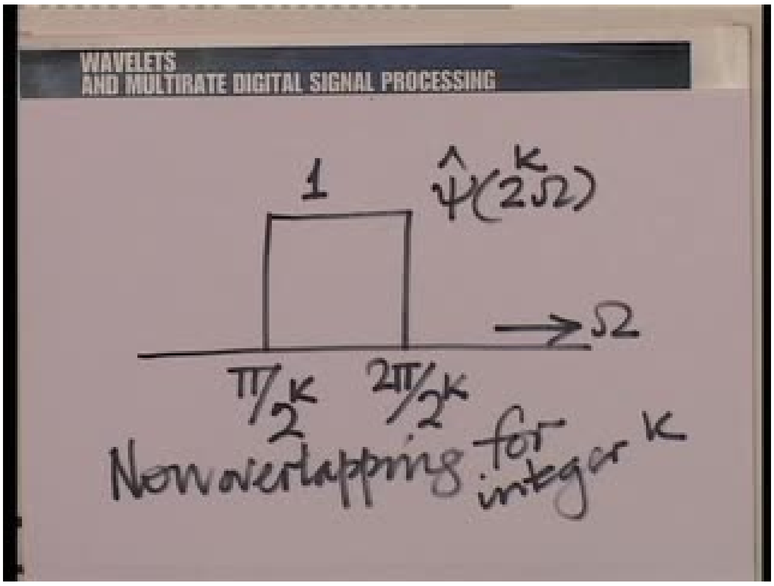

In fact, let us look back at this frequency domain here. You see, take for example, k equal to one k equal to one would take to over the band pi by two to pi, and if you look at the original band, its pi to two pi k equal to minus one would take you from two pi to four pi. Again non overlapping with pi to two pi, and you can continue in a similar way k equal to two k equal to minus two and you can proceed; k equal to zero needless to say, keeps you in the original band - pi to two pi.

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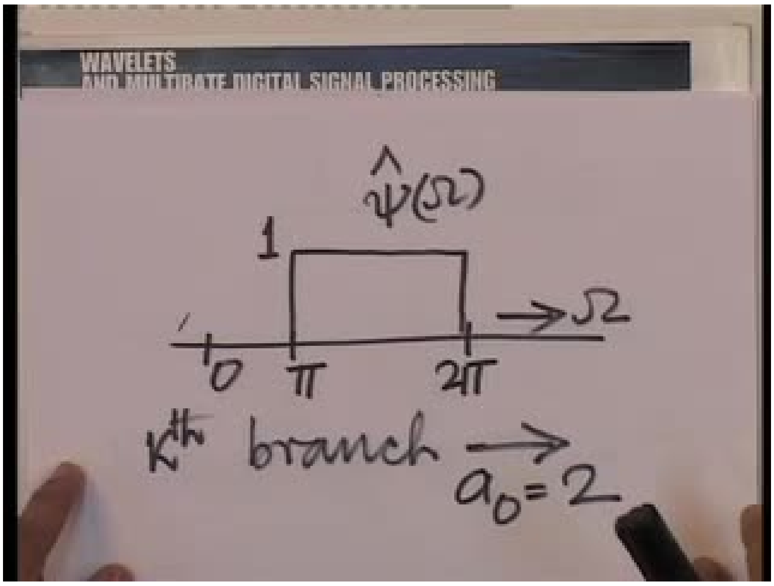

So, now, we have beginning to understand what the continuous wavelet transform does in an idealization sets. You see, in that case, how we really just talking about doing this only in the frequency domain? Well, we must not forget what we are trying to do and which we can never do because of the uncertainty principle is to do what I have just shown you on the frequency axis ideally, but do it with time limited functions, and alas if you look at the inverse Fourier Transform of this psi cap omega that we which just drew here, it is far from time limited. In fact, it decays as slowly as the reciprocal of time terribly, anyway.

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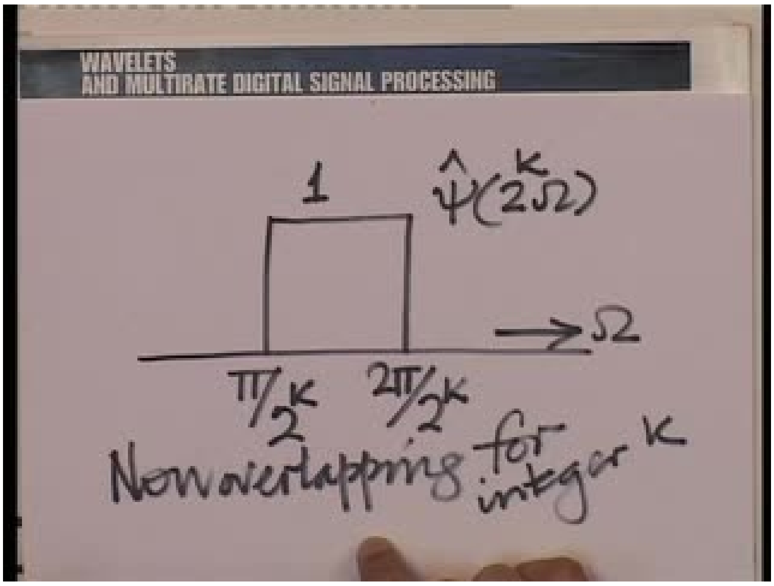

So, this is easy now to interpret and understand, and now, it is also easy to see that if you take the ideal situation, when you could simply put the same filter on the analysis side and the synthesis side. This also makes it clear how we get the notion of filter banks when we discretize.

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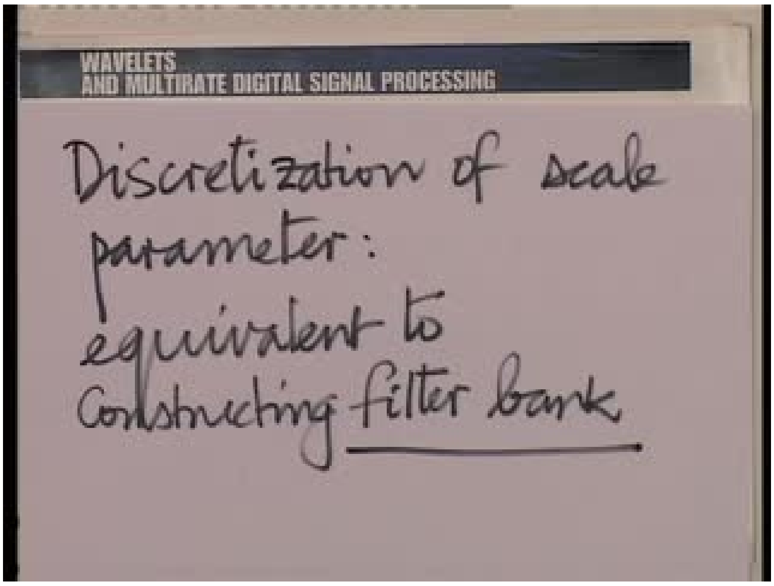

So, you see, the discretization of the scale parameter is equivalent to constructing a filter bank. Recall what a filter bank is. A filter bank is a collection of filters although with the common input or with all the outputs sum together to get a common output, and of course, more subtlety the filters are interrelated.

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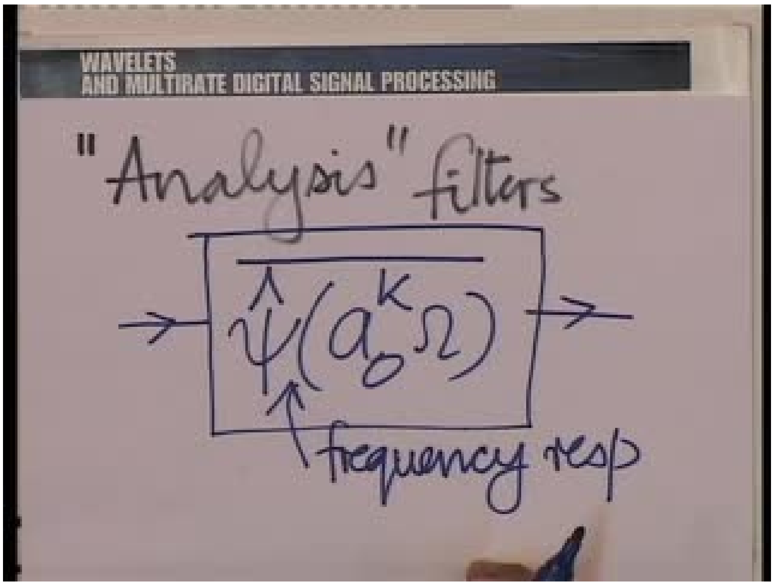

Now, all these qualities are satisfied in the kth branches that we are talking about here. In fact, in the particular kth branches that we speak of here are analysis filter have the following pattern. The analysis filter is the ones that create the continuous wavelets transform. Strictly speaking, we should put a complex conjugate here as was the case with the continuous wavelet transform.

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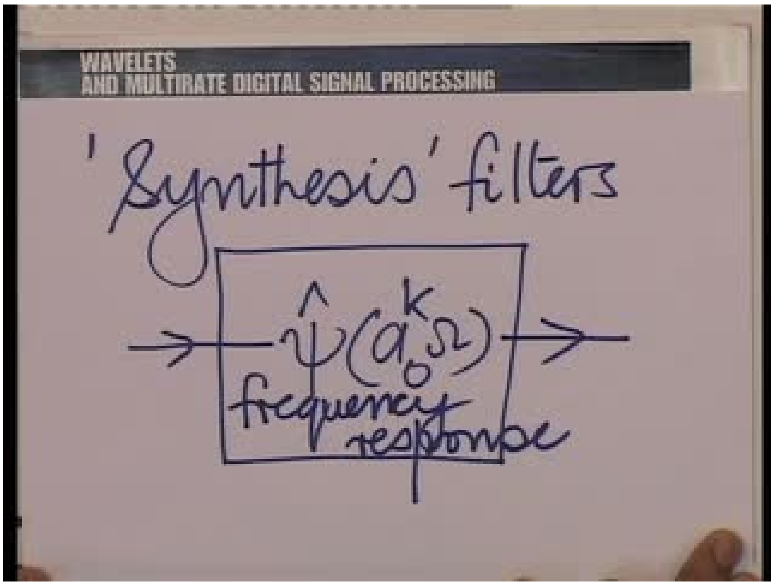

So, these are the analysis filters, and this is the frequency response if you ignore. The constants the syntheses filters, again if you ignore the constants, have a frequency response that looks like this. This is the kth branch, and therefore, if you analyze what you are doing all over the, k, kth branches for k going from minus to plus infinity, you are doing the following.

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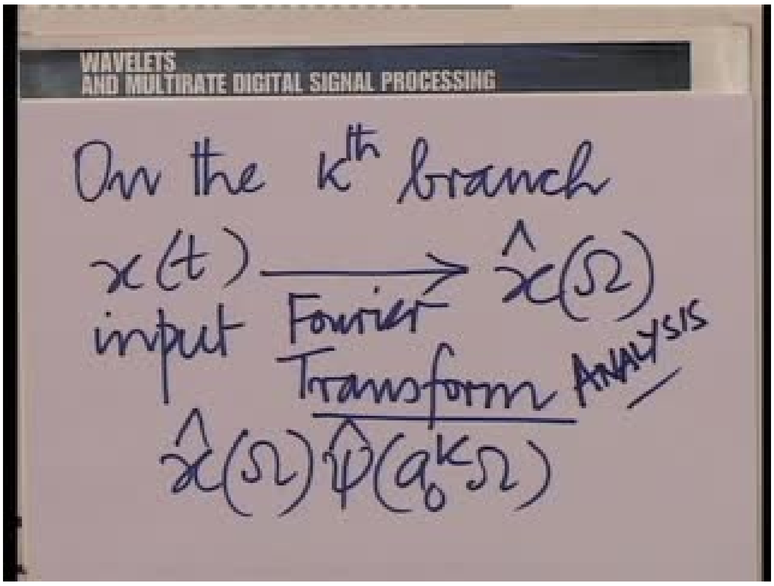

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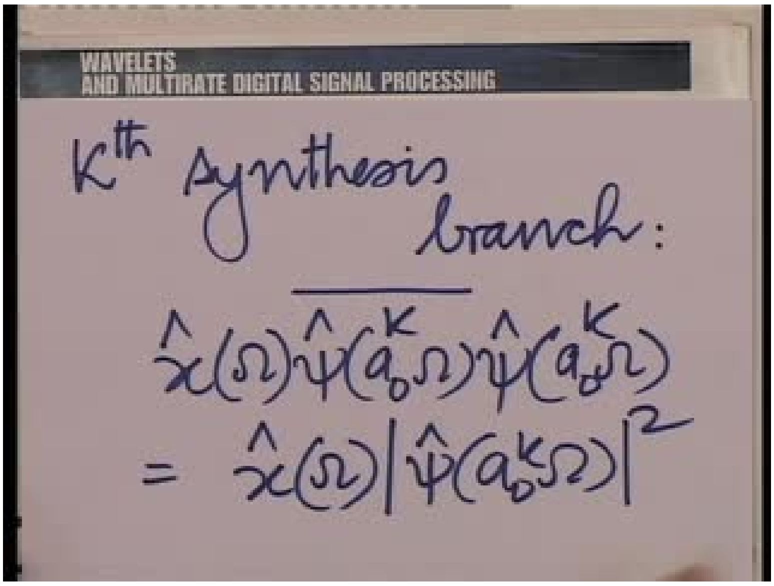

On the kth branch, if the input signal x of t has the Fourier Transform x cap of omega, then we are actually constructing x cap omega times psi cap a naught to the power k omega complex conjugates, and therefore, this is on the analysis side I mean, and on the synthesis side, the kth branch multiplies this again by psi cap a naught raise to the power of k omega, which essentially amongst to multiplying the original Fourier Transform by a modulus squared of psi cap a naught raise to the power of k omega, and on the output side, one is going to sum up all these outputs of the branches.

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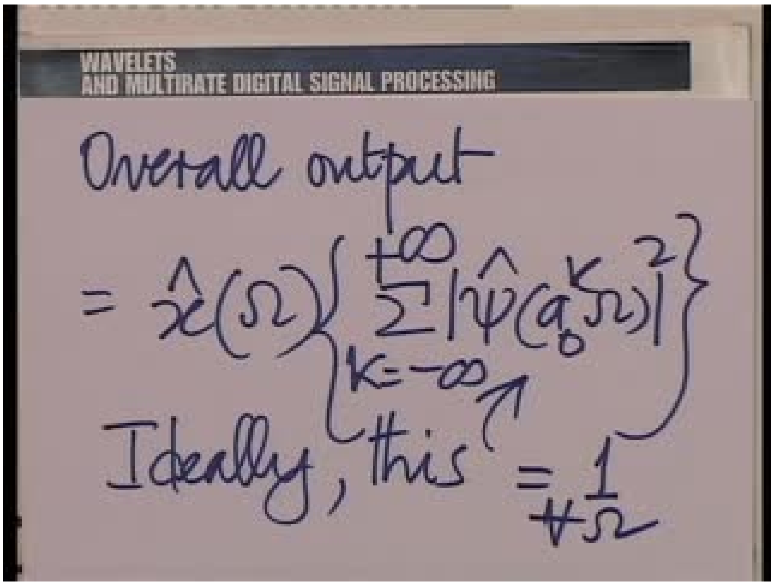

So, the final output, overall output is going to be x cap omega multiplied by a summation on all k psi cap a naught raise the power of k omega modulus the whole squared, and a minute ago, we saw what this was for the ideal situation. Ideally, this should be one; one for all omega. Of course, that is where the challenge is. We wanted to be one for all omega with time limited functions, and that is the whole challenge.

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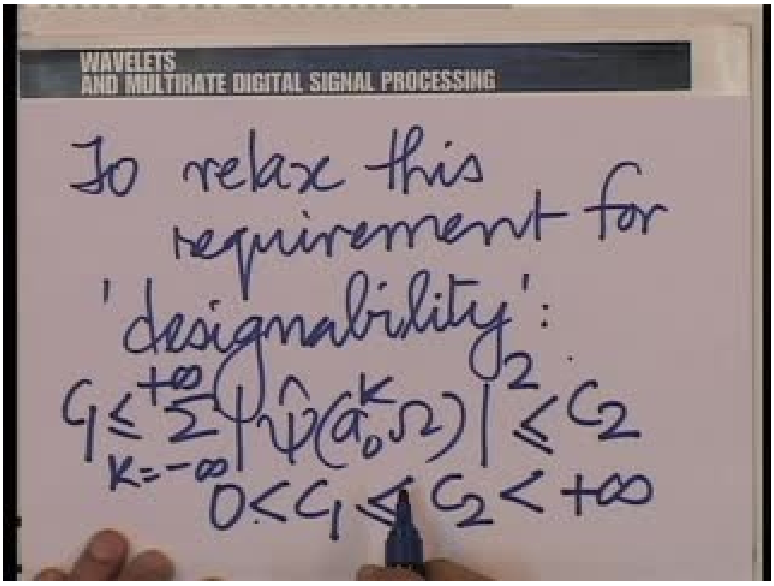

Anyway, how do we relax in the frequency domain if we want to meet this challenge? Well, relaxation means do not quite insist, it must be a constant. Allow it to be between two positive constants. Now, I shall insist on strictly positive constants and we will see why. What I am saying is to relax this requirement, for design, for designability. Let this quantity psi cap a naught to the power k omega mod squared summed over all k all integer k be between two constants. So, the greater than or equal to a constant, let say c one, and less than equal to a constant, let say c two, and we are told that c one is of course, less than equal to c two; c two is less than infinity and c one is greater than zero, not this is greater than, not greater than or equal to, and of course, this is strictly less than, not less than, whereas, this cannot go to infinity.

So, here, there is the possibility of equality; here there is not. These are strict in equality; that means c one is strictly positive, c two is strictly finite, and of course, it is not difficult to see that this is the nonnegative quantity. So, it cannot possibly become negative.

So, all that we are saying is instead of insisting that, this be a strict constant, allow it to be between two constants, and this is the case, then can be make a small change to the synthesis filter and reconstruct? That is the first question that we shall answer That is very easy to say to see.

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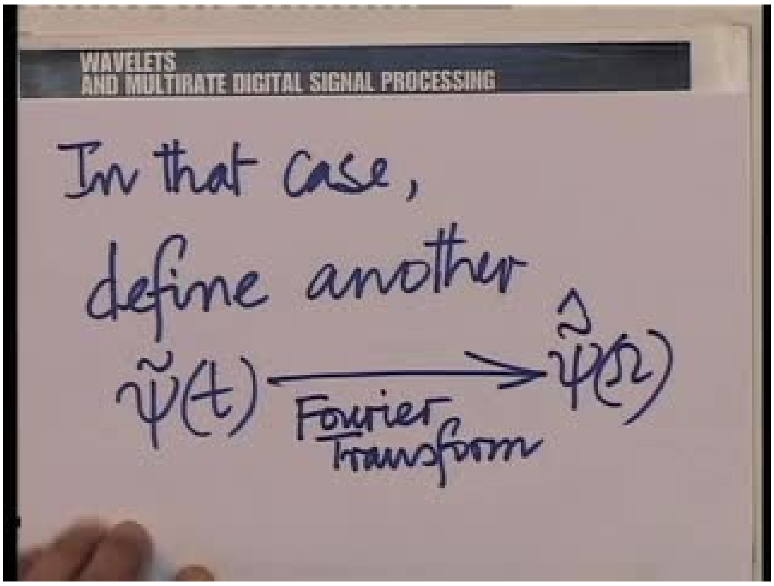

You see, in that case, let us define another function psi tilde from psi. Now, psi tilde shall be defined in the frequency domain. So, let psi tilde t have the Fourier Transform psi tilde cap omega. So, we will show that, you know, even if the wavelet psi t does not quite obey this strict constancy of the sum, we can still build a reconstruction approach. That is what we are trying to show.

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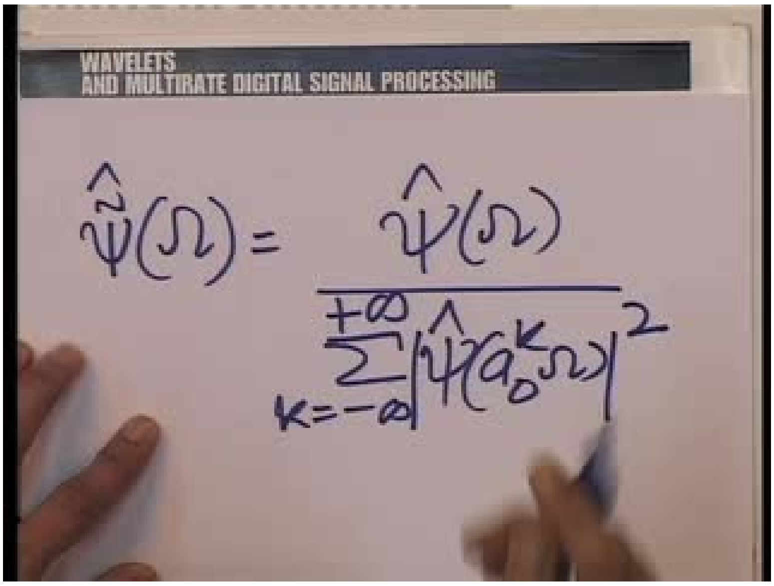

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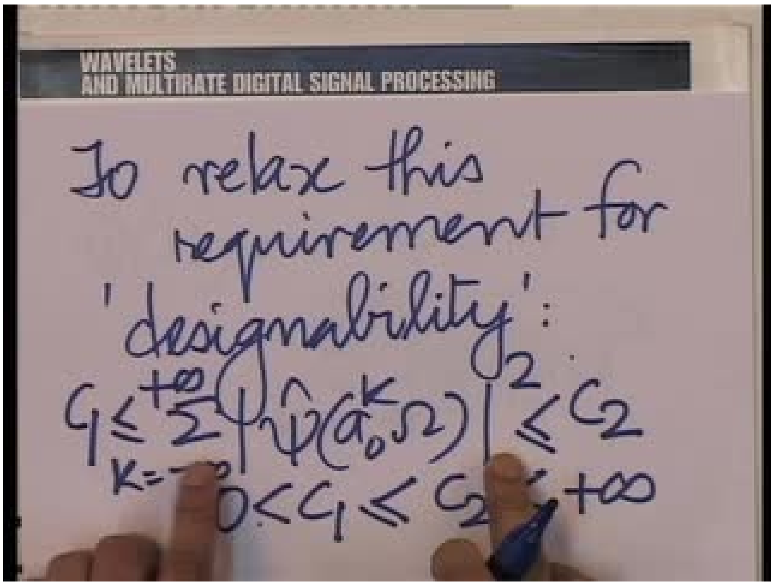

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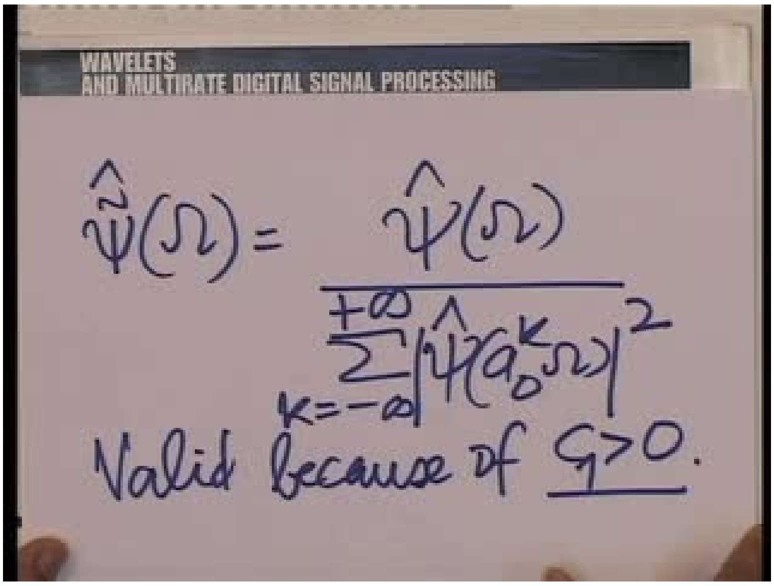

So, you see, suppose this is a definition of its Fourier Transform, define psi tilde cap omega to be psi cap omega divided by the sum here, and please note that since we have this condition here, this sum lies between two positive constants, and what is important here is the fact that it does not go to zero. The c one constant is important here, because the c one constant, this denominator never goes to zero, and therefore, this is valid because of c one being greater than zero. Otherwise, this definition would be meaningless.

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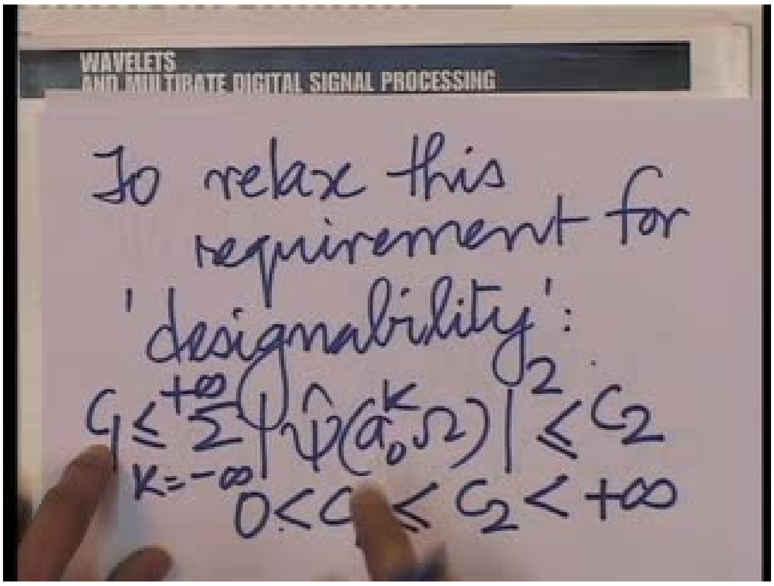

So, the c one is required for this reason to be able to build this psi tilt to cap. Now, we will show something interesting. You see, we will show that we could use psi on the analysis side and psi tilde on the synthesis side, and we will first show that if psi obeys this requirement, then psi is admissible. So, we will show that now we have use c one to define psi tilde. We will use c two to prove admissibility. So, once we ensured this, we also have admissibility. Let us prove that.

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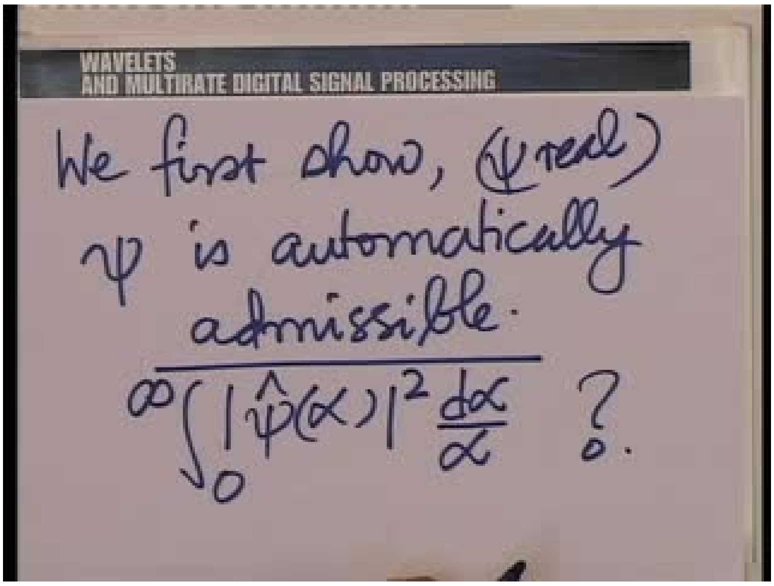

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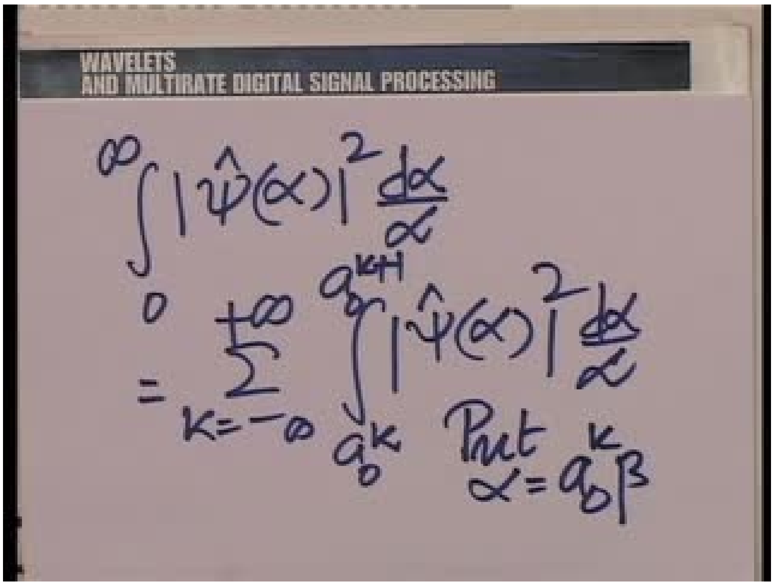

By admissible, I mean we need to consider this integral, and we have of course, agree that psi t is real. So, let us make the remark here psi is real; I mean it is a real wavelet. So, you see, let us brake this integral into partial, but finite integrals of the following form summation k running from minus to plus infinity integral from a naught to the power k to a naught to the power of k plus one.

Now, let me take an example suppose for example, a naught is equal to two, then the integral from zero to infinity is the same as the integral from two raise to the power k to two raise to the power k plus one with k running over all the integers. As k takes negative integer values, you are covering the range below one; as k takes positive integer values, we are covering the range from one and above one. That is the interpretation here.

So, for example, k equal to zero would cover the range, one to a naught in general or one to two of particular a naught is equal to two, and k equal to two will cover k equal to one will cover the range from two to four and so on and so forth, and k equal to minus one will cover the range from half to one and so on so forth in particular with a naught equal to two. Anyway, with that, we keep the integral as it is, but as usual, we make a substitution on the integral. So, put alpha equal to a naught raise to the power of k times beta and make a change of variable in the integration.

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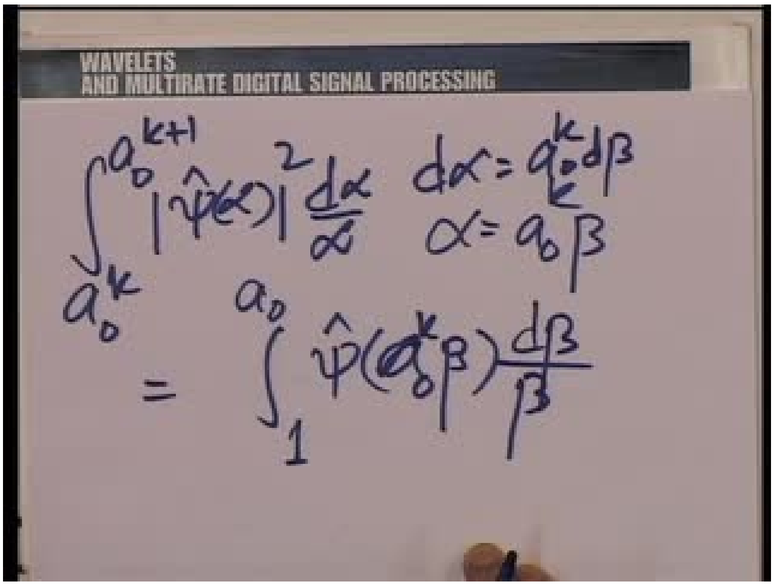

Now, when alpha is equal to a naught raise to the power of k times beta, this limit simply becomes one and this limit becomes a naught. (Refer Slide Time: 29:25) So, we have a naught raise to the power of k beta here, and of course, interestingly, d alpha is also a naught raise to the power of k d beta, and therefore, one can easily see that d alpha by alpha is equal to d beta by beta. So, in place of d alpha by alpha, I can simply write down d beta by beta.

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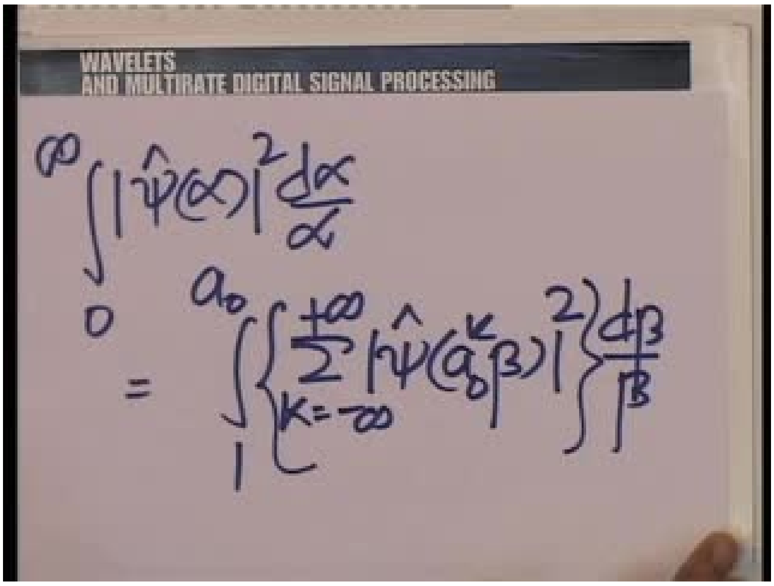

So, now, we have a beautiful summation coming in. What it means really is that the integral from zero to infinity psi cap alpha mod squared d alpha by alpha is equal to the integral from one to a naught summation if I interchange the order of the summation, and the integral which I can do simply, because this is a finite integral here, summation from k running from minus to plus infinity. In fact, I could interchange this order, because of I am guaranteed that I have a finite integral here and I am guaranteed that this is convergent for all beta; it lies between two positive constants. So, it is meaningful to do this, and in fact, now we have a condition on this. This is a nonnegative integrand here, and this is of course, you know, integral over a nonnegative quantity one by beta between one and a naught. If I can pull an upper bound on this, the upper bound on this is known to us - its c two.

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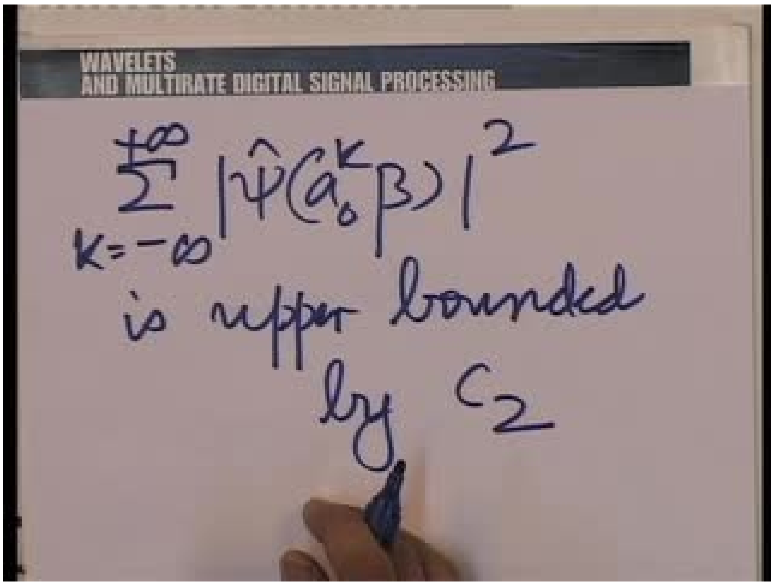

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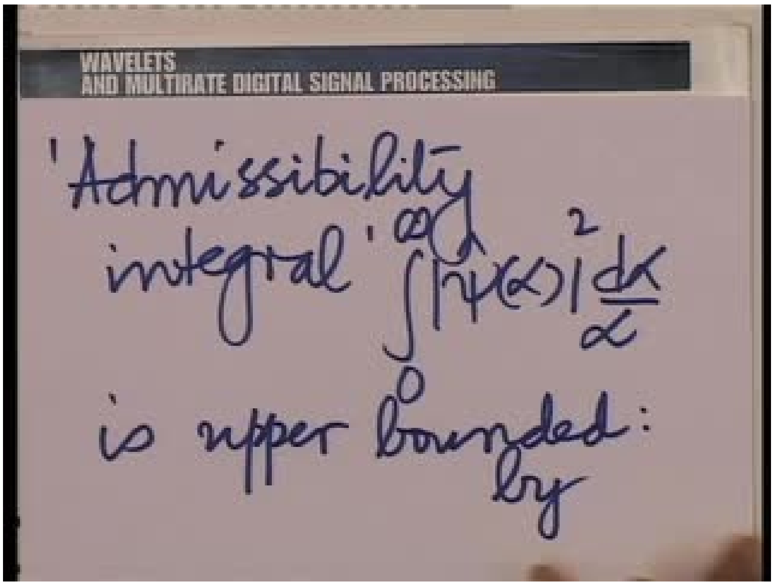

So, the summation k running from minus to plus infinity psi cap a naught to the power of k beta mod squared is upper bounded by c two, and therefore, the so called admissibility integral as we call it. We shall now introduce this term.

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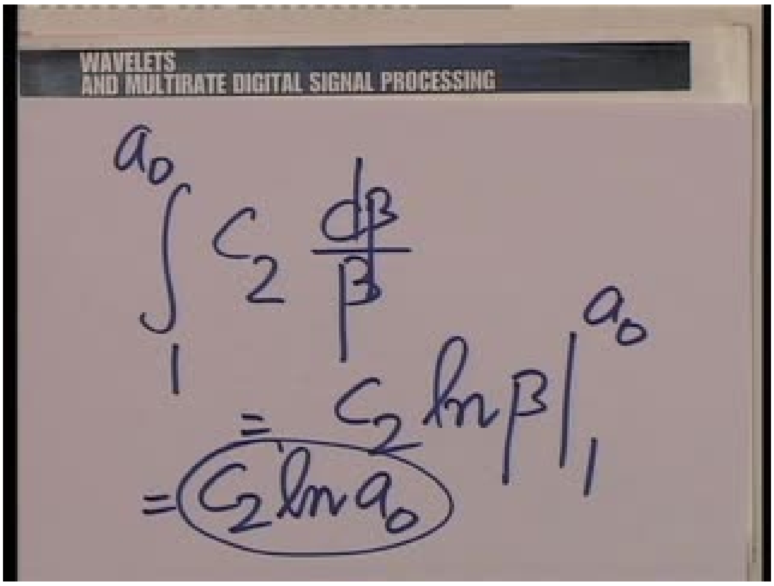

The admissibility integral is upper bounded also by integral from one to a naught c two d beta by b beta, and this is a very easy integral to evaluate. It is essentially c two integrated log natural beta from one to a naught and that is c two log natural of a naught, a neat and elegant result.

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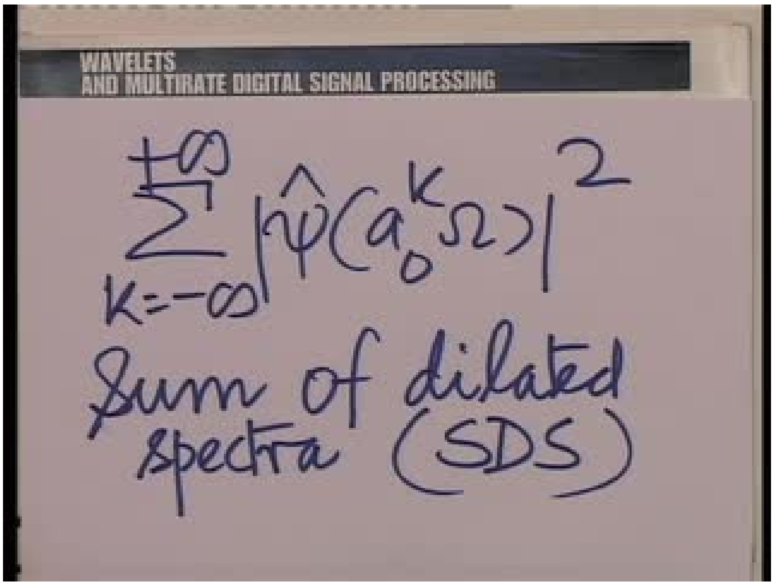

So, once I have the upper bound on the summation, I am guaranteed admissibility. So, we have shown that if psi obeys this requirement, and now, we will introduce some terminology. We shall call this quantity which we are talking about all this while the summation. We shall call this quantity a sum of dilated spectra and we shall abbreviated by s d s, and just as in the case of continuous wavelet transform, we have primary and secondary arguments for the sum of dilated spectra.

The secondary argument is psi, because it is for psi that we are constructing a, dilate, dilated or sum of dilated spectra, and of course, the primary arguments are a naught and omega. Well, actually, a naught is a gray area; you could treated as secondary or primary. You know, in the continuous wavelet transform, for example, we distinguish primary and secondary by calling those arguments primary which did not change in the discussion in a particular context, and secondary for those that did or rather other the other way around primary for those which did change, which were important in a particular context and secondary for those that did not.

So, for example, in the continuous wavelet transform, the primary arguments where the translation and the scale, those where important in a given context, they changed in that context, and we were looking at the variation of those in a context, and the secondary arguments where the essentially the wavelet with which we are constructing the c w t or the continuous wavelet transform and the function on whom the continuous wavelet transform is being constructed.

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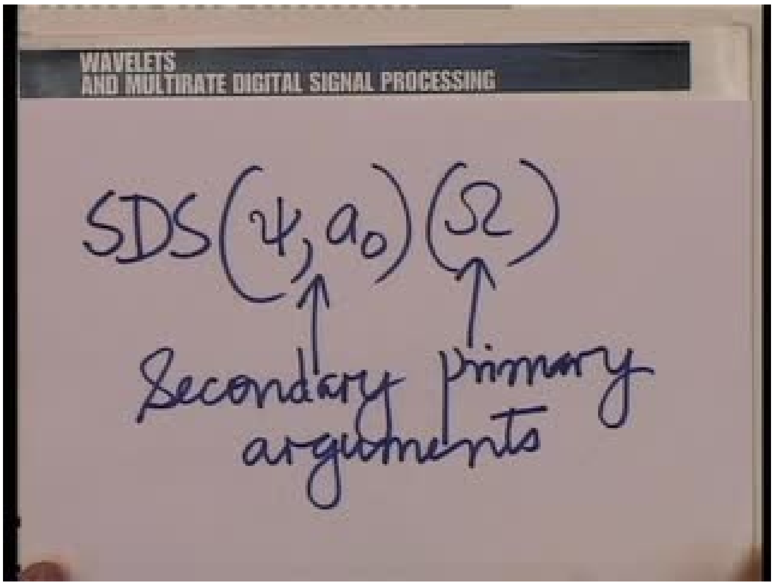

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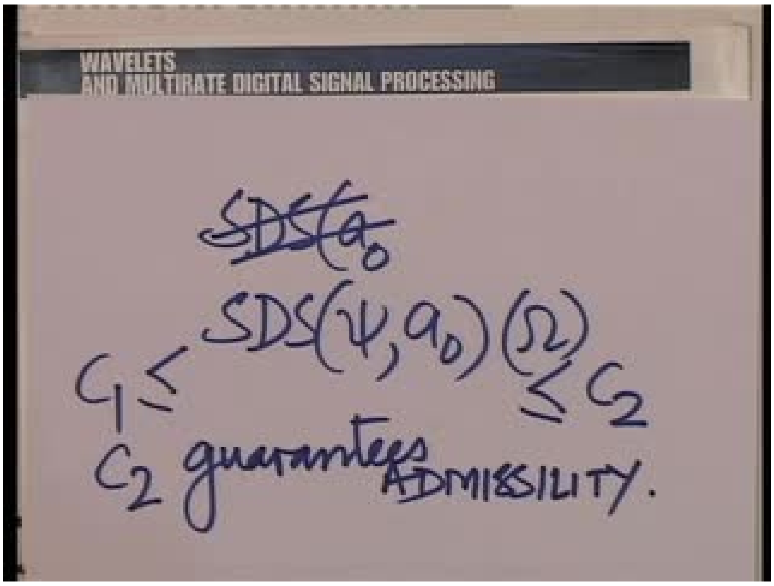

So, here to, we shall talk about s d s with secondary arguments of psi and a naught and primary argument omega, but of course, we could shift this, and what you are essentially saying is, s d s, s d s psi a naught as a function of omega needs to be upper bounded and lower bounded, and the upper bound guarantees admissibility.

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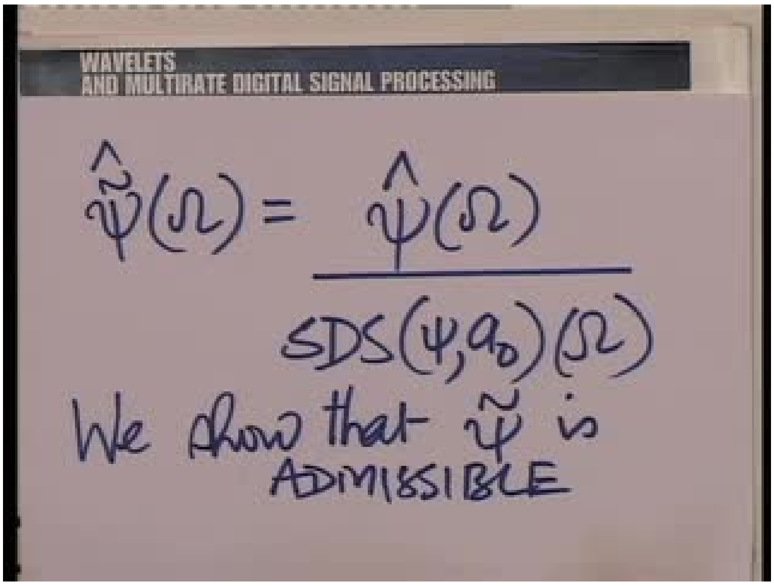

Now, we soon see in a minute that c one guarantees reconstruction, and in fact, we are going in that direction. We have already constructed a psi tilde, where psi tilde cap omega is essentially. Now, with this language psi cap of omega divided by s d s psi a naught omega, and it is very easy to see that if s d s obeys the upper lower bound condition, psi tilde is also going to be admissible.

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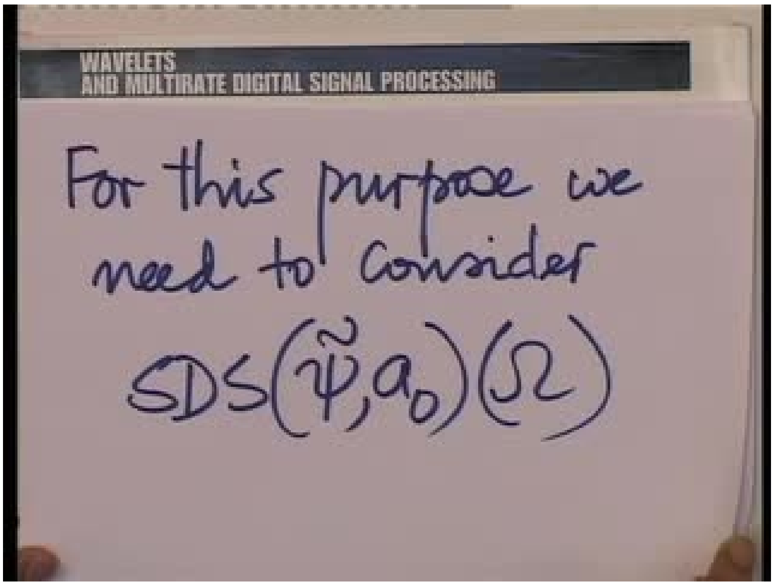

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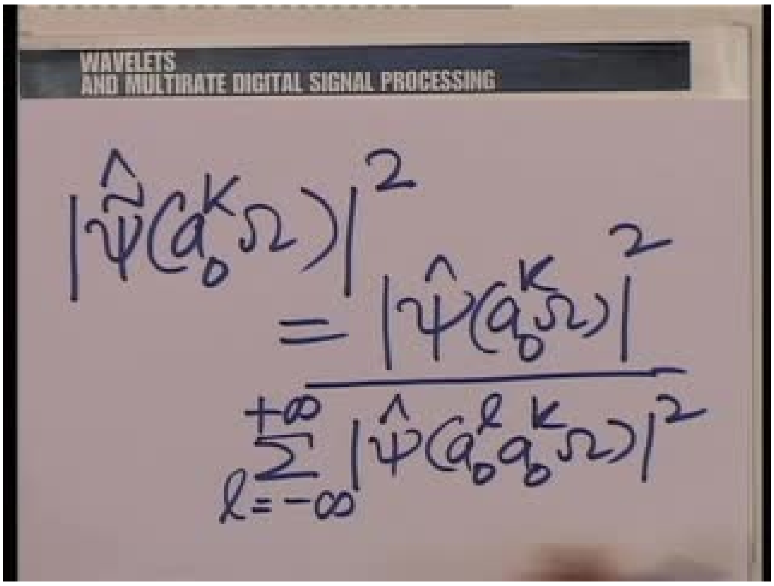

So, we show that psi tilde is admissible, and to prove that, all that we need to do is to try and look for bounds on the sum of dilated spectra of psi tilde. So, we need to consider for this purpose - the sum of dilated spectra psi tilde naught as a function of omega, and this is not at all difficult to compute. Indeed, let us in particular construct psi tilde cap a naught raise to the power k omega, which is the typical term required or rather the modulus of this squared, and the modulus of this squared is very easily seen to be the modulus of psi cap a naught raise the power of k omega the whole squared divided by, that is the interesting thing.

Let me expand the denominator. I first try to summation on l to distinguish it from this k. Summation on l going from minus to plus infinity psi cap a naught raise to the power of l. Now, in place of omega, I need to write a naught raise to the power of k omega, so I will write a naught raise to the power of k omega there, and now, this is easy to evaluate. In fact, you can see that this becomes a naught raise to the power of l plus k, and please remember the summation is on l and k is fixed. So, for a fixed k when l runs over all the integers as it does here, l plus k also runs over all the integers.

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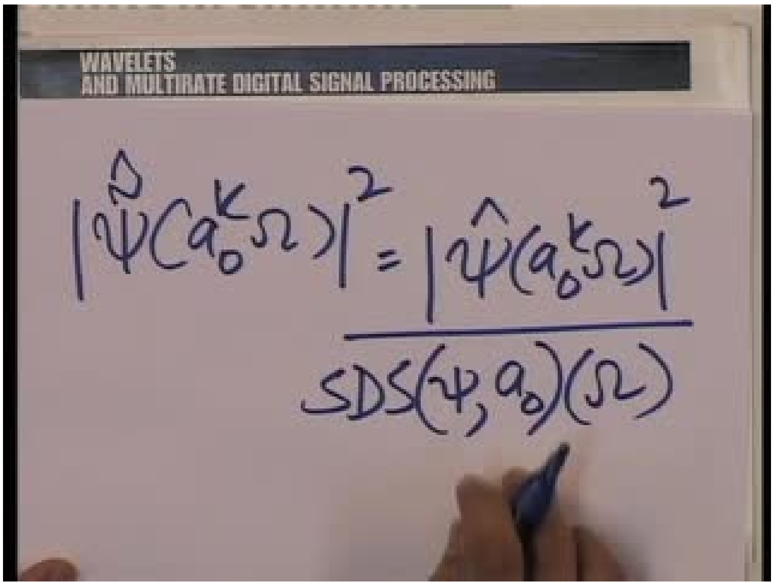

So, what we have in the denominator is once again the sum of dilated spectra of psi a naught evaluated at omega. That makes my life very easy. What I am saying in effect is psi tilde cap a naught raise to the power of k omega mod squared is simply psi cap a naught to the power of k omega mod squared divided once again by s d s psi a naught evaluated omega.

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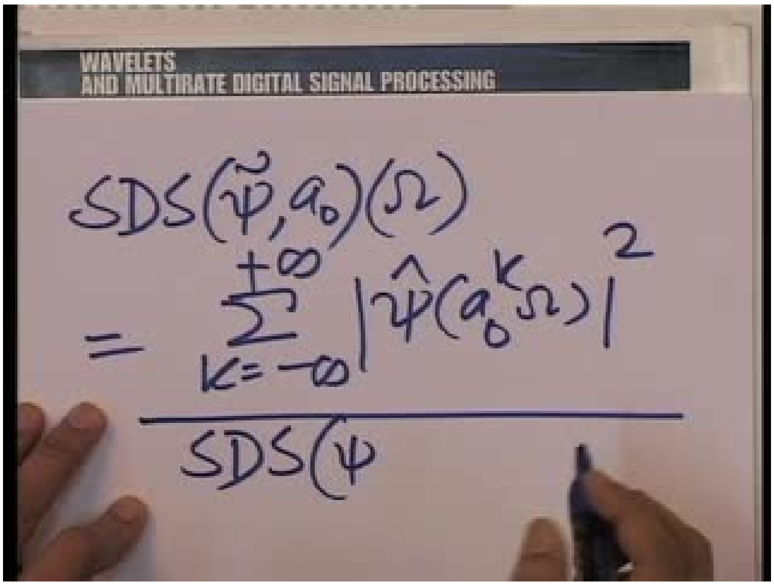

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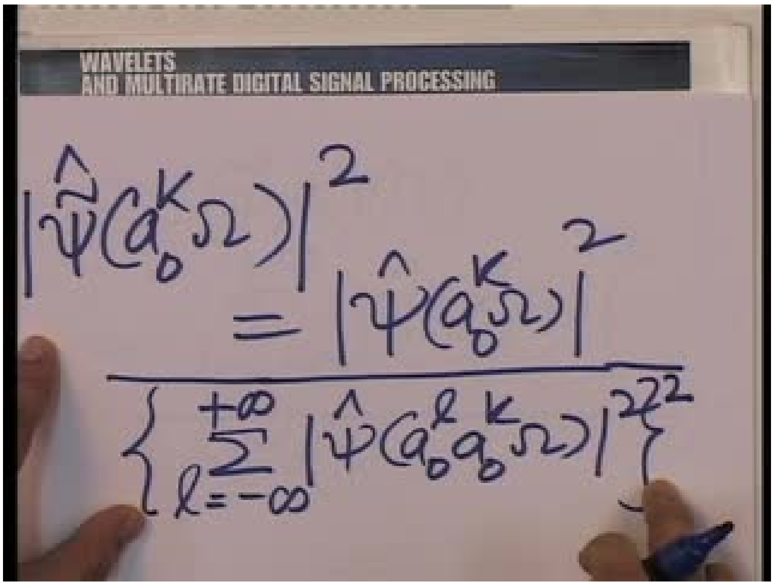

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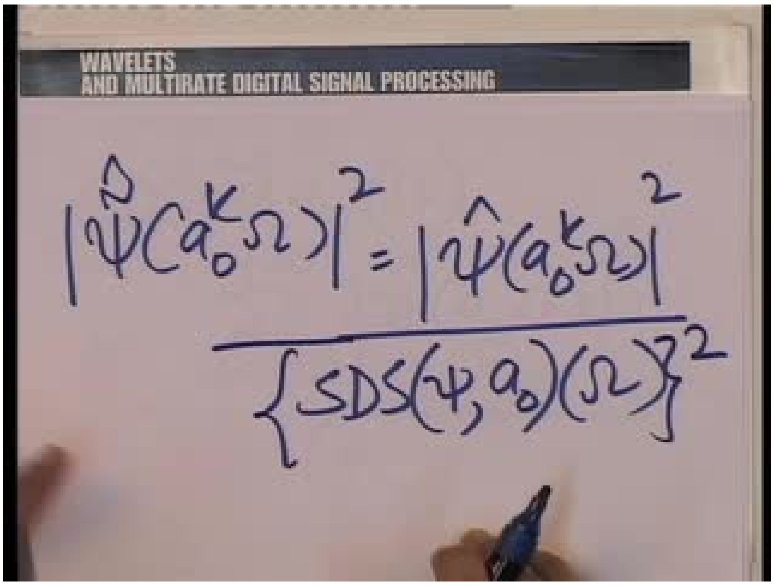

It makes my summation very easy to do. So, what I am saying in effect is that the sum of dilated spectra of psi tilde with parameter a naught evaluated at omega. In another words, summation only on the numerator. You see, the denominator is independent of k. In fact, a little correction which we need make here, you see, here, I need to put a square. So, although I had discuss this in the context when I took the modulus squared, I should also put a square here. I do not need to put a modulus, because this the anywhere nonnegative. So, with that little correction, what I have in the denominator, in fact, here also I need to make the correction, so I have s d s psi a naught omega squared here; a little correction is required.

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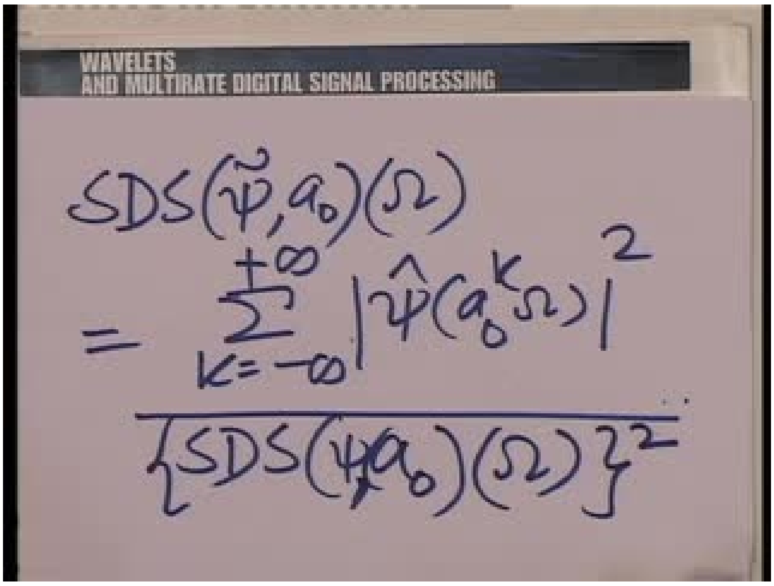

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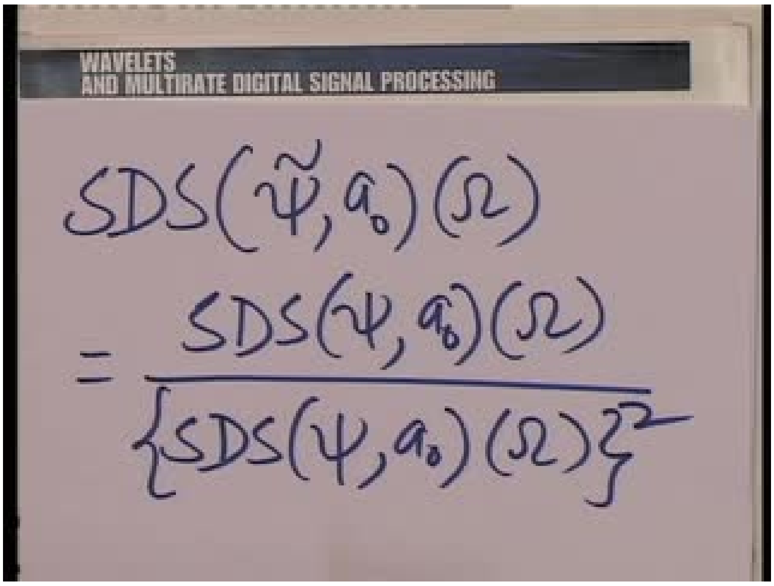

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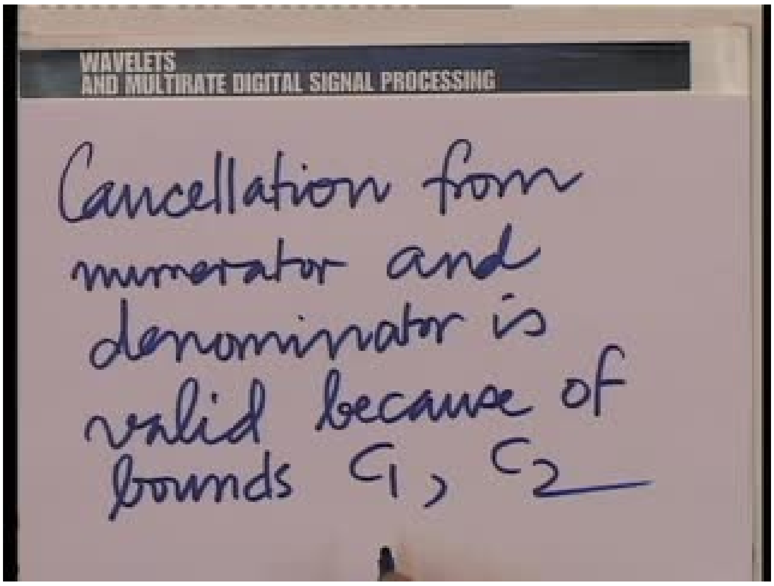

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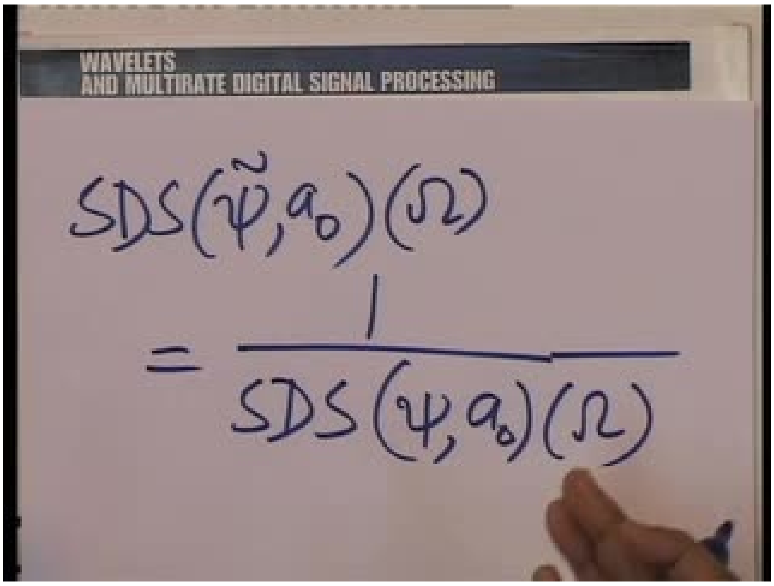

So, here we have psi a naught omega squared, and now, this is familiar; this is essentially s d s psi a naught once again. So, what I have in effect is that s d s psi tilde naught evaluated at omega is just s d s psi a naught evaluated omega divided by s d s squared, and this is where the lower bound becomes important. Since this is lower bounded by c, one never goes to zero, and it is also upper bounded by c two, so never goes to infinity. It is valid to cancel. Cancellation from the numerator and denominator is valid because of the bounds, and on cancellation, we shall get s d s the sum of dilated spectra of psi tilde evaluated omega with parameter a naught is just the reciprocal, simple, and in fact, now, I can go a step further. I can use the bounds on the sum of dilated spectra for psi to come up with bounds on the sum of dilated spectra of a psi tilde. Let us do that.

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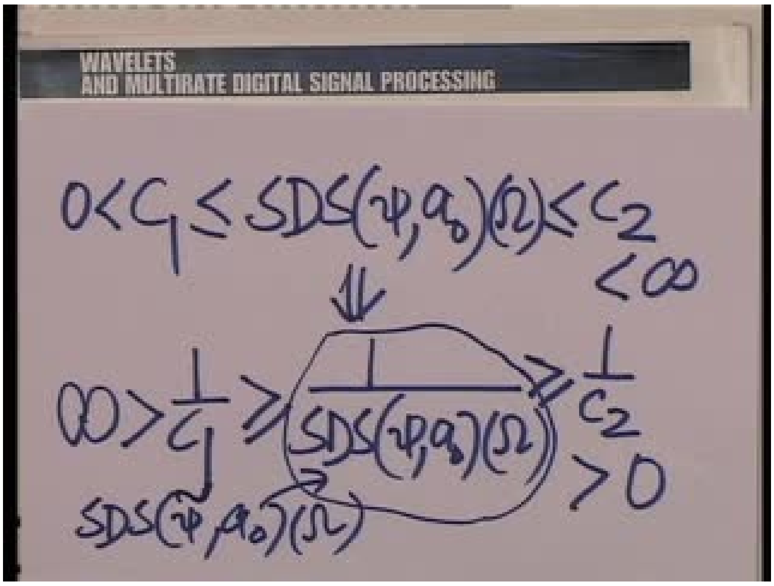

So, let us come back to this expression. You see, all that we need to do is to note that if s d s psi a naught omega is bounded between c two and c one, this one greater than zero and this one less than infinity. Then, I can take the reciprocal on all side, because these are all nonnegative quantities and I can get infinity is greater than one by c one is greater than or equal to one by s d s psi a naught evaluated omega is greater than equal to one by c two which is in term greater than zero, and this of course, is the sum of dilated spectra that we are looking for here. This is nothing but s d s psi tilde naught evaluated omega.

So, we have a bound; the bounds on the sum of dilated spectra of psi give you the bounds on the sum of dilated spectra of psi tilde, and the same arguments that we used to show that psi is admissible can be used to show that psi tilde is also admissible.

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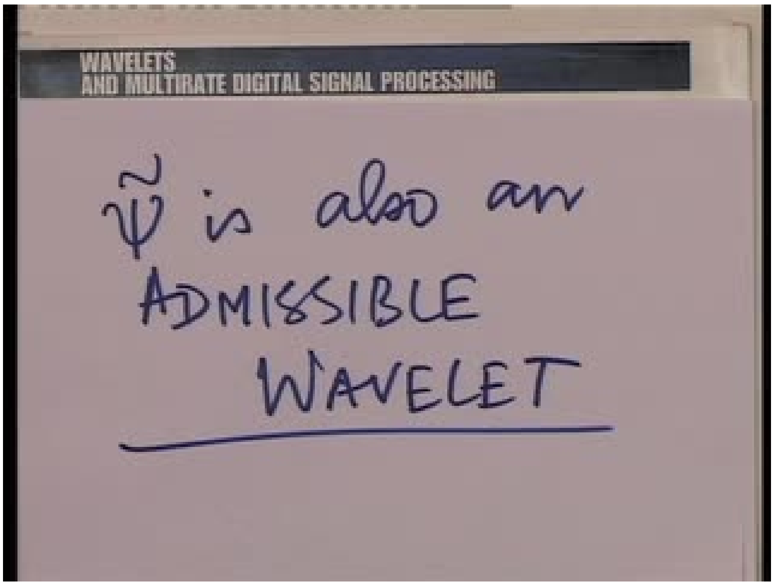

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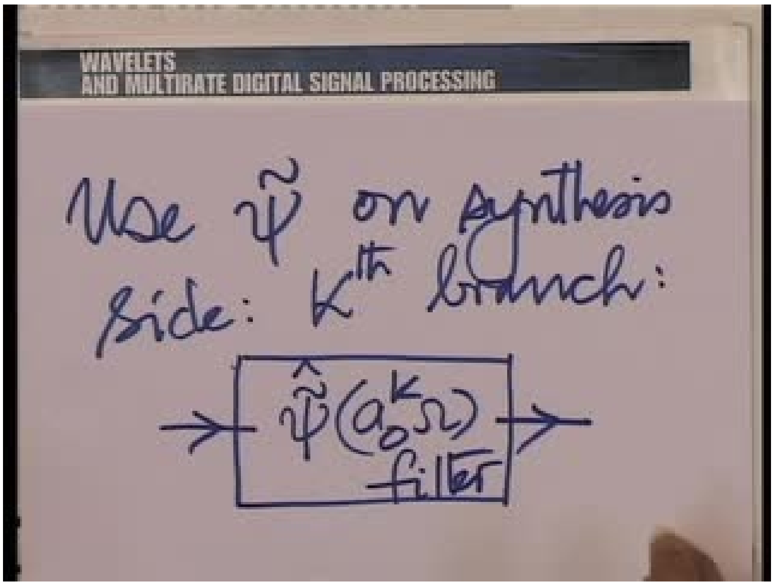

So, psi tilde is also an admissible wavelet, and therefore, I could not construct and asymmetric analysis and synthesis pyridine. On the analysis side, I would use the wavelet psi; on the synthesis side, I would use the wavelet psi tilde.

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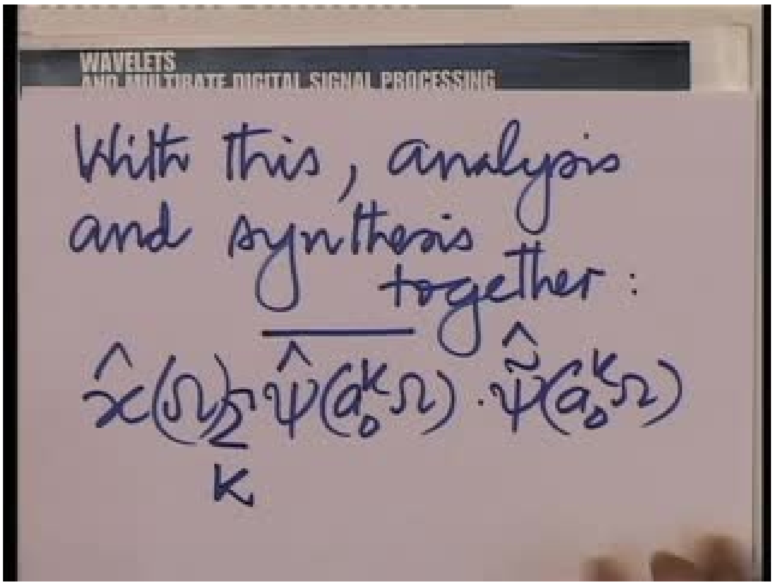

So, all that I am saying is use psi tilde on the synthesis side; that means the kth branch is as follows: it has a filter with frequency response given by psi tilde cap a naught raise to the power of k omega, and now, we can see what happens to x when it goes to the analysis and synthesis side. With this, analysis and synthesis together would produce x cap omega times psi cap a naught raise to the power of k omega complex conjugate multiply by psi tilde cap a naught raise to the power of k omega, and this is easily seen to be, of course, this is summed over all k.

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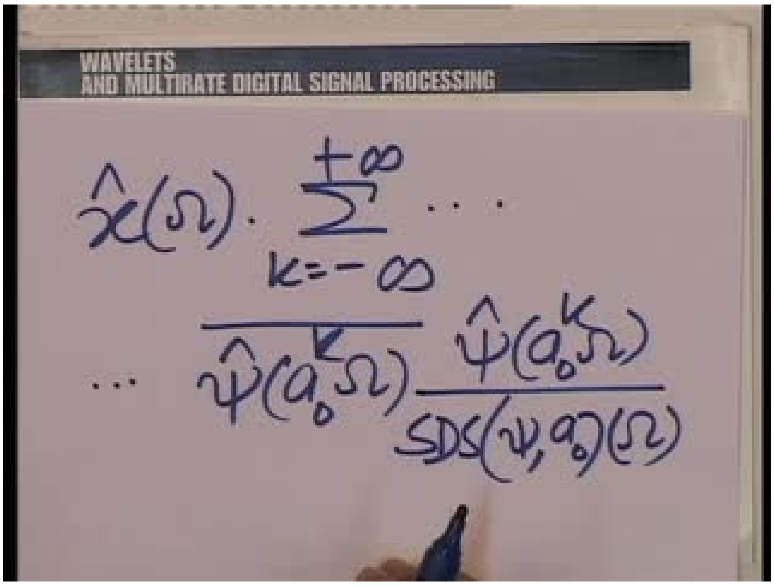

This is easily seem to be x cap omega times sum for k for going from minus to plus infinity psi cap a naught raise to the power of k omega complex conjugate psi cap a naught raise to the power of k omega divided by the sum of dilated spectra psi a naught evaluated at omega, and once again, you can see what all this is leading to. This is independent of k.

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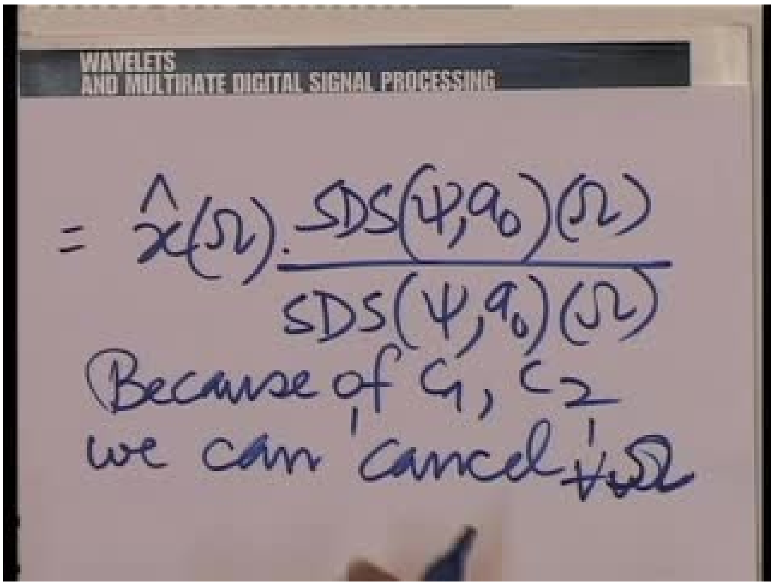

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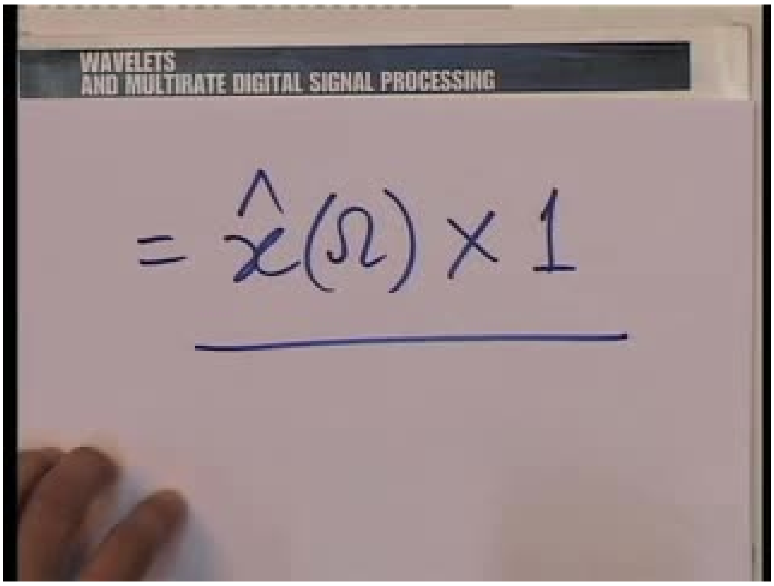

This simply gives you x cap omega into sum of dilated spectra psi a naught omega divided by the sum of dilated spectra psi a naught omega, and once again, because of c one and c two, we can cancel for all omega and that gives us just x cap omega.

So, we reconstruct. Now, we have also brought in a new notion here. If we have made this relaxation, namely that the sum of the dilated spectra can lie between two positive constants, we also need to generalized are notion of analysis and synthesis a little bit.

We need to generalize it by allowing a different wavelet on the analysis side and on the synthesis side. You have psi on the analysis side; psi tilde on the synthesis side. So, we are slowly leading to a different pyridine in the context of filter banks. All this, while the wavelet on the analysis and the synthesis side has been the same.

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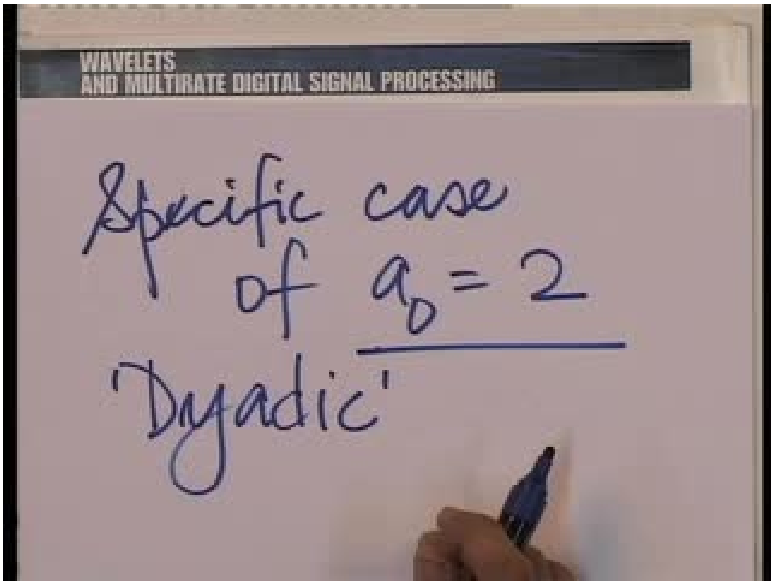

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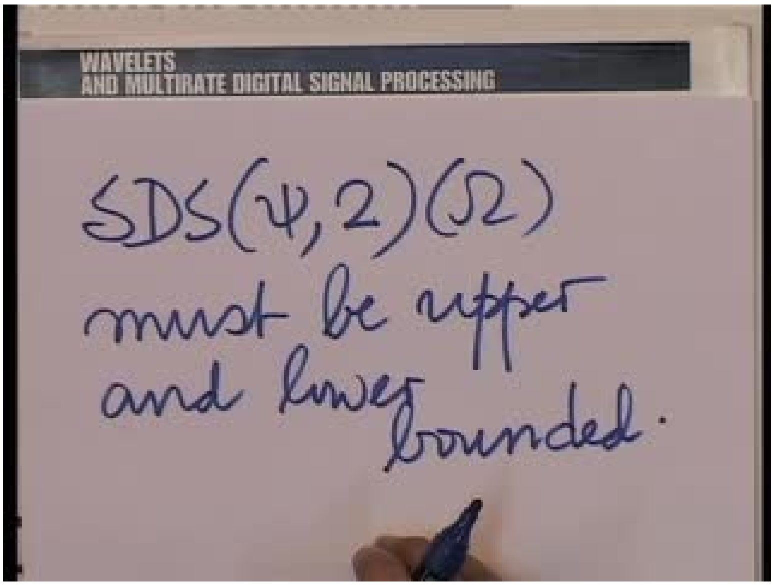

Now, we are allowing them to be different if we want to relax the context of design. Anyway, that remark apart. Let us come back to one specific case. The specific case a naught equal to two. In the specific case of a naught equal to two which is also called the dyadic case; dyadic refers to this - a naught equal to two. What we are asking for is that the sum of dilated spectra of psi two omega must be upper and lower bounded. Now, in particular, if one considers the haar wavelet, one can show that this is true. I do not wish to go through the details, but rather to leave it to you as an exercise.

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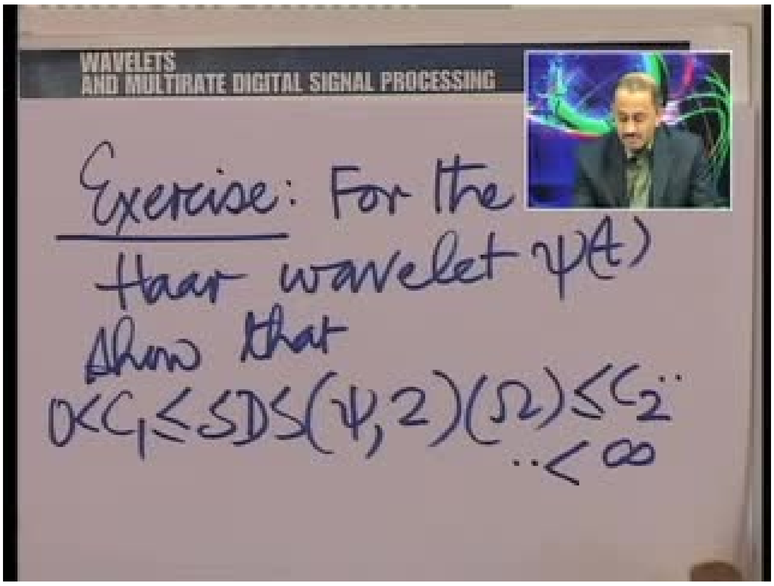

So, exercise for the haar wavelet, for example, show that the sum of dilated spectra as we have constructed lies between two positive bounds. Now, you know, to give your hint, it is clear that the upper bound essentially gives you admissibility, and it is not very difficult to show the haar wavelet is admissible. So, first, if you can show the simply haar wavelet is admissible, then this is kind of done.

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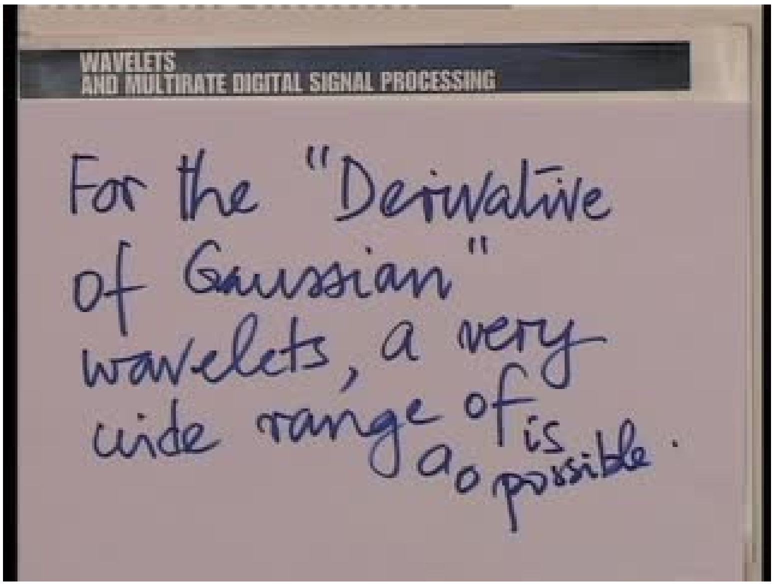

The lower bound is a little more tricky and that I leave as an exercise for the class to do, and of course, this also holds for the Gaussian wavelet, for example, so, I have also put down the following exercise. For the Gaussian wavelet of the derivative Gaussian, one can once again show a very wide range of a naught is possible. So, one can show that the sum of dilated spectra in the context of the derivative of Gaussian kind of wavelets is always between two bounds for a very wide range of a naught, and I encourage as an exercise to find out what that range of a naught is.

Now, one more remark - incase the sum of dilated spectra is one is constant for all omega. So, in other words, c one becomes equal to c two. Then we have the situation where psi and psi tilde are the same, and that is essentially the situation of orthogonal filter banks, the kind of filter banks that we have been talking about all this while.

So, in other words, the specific situation where analysis and reconstruction proceeds from the same wavelet is the case where the sum of dilated spectra. Well, it, you know, please remember, here we are talking about a continuous translation parameter. So, we have not yet discretize the translation parameter.

So, it does not follow that the haar wavelet has the sum of dilated spectra equal to a constant, because we have discretize the translation parameter there, but if you are not to discretize it, then those wavelets where c one is equal to c two give you an orthogonal filter bank; that means the wavelet on the analysis and the synthesis side is the same, and in particular, when a naught equal to two, it gives you a dyadic analysis synthesis. syst With this thing, we conclude the lecture today and we shall build further on the dyadic case in the next lecture. Thank you.