Advanced Digital Signal Processing-Wavelets and Multirate

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Lecture No.#23

Admissibility in Detail Discretization of Scale

A warm welcome to the twenty third lecture on the subject of wavelets and multirate digital signal processing. Let us spend a couple of minutes in recalling the discussion in the previous lecture. We had in the previous lecture - built up the idea of decomposition and reconstruction in the short time Fourier Transform and in the continuous wavelet transform.

The central theme in decomposition and reconstruction was to project in the sense of projection on a vector, project on the basis vectors. The function is to be decomposed, and to reconstruct the function from its components by multiplying each component by vector in that direction.

This simple idea enables us to interpret decomposition and reconstruction both in the short time Fourier Transform as also in the continuous wavelet transform. The short time Fourier Transform was indexed by translation and modulation. The continuous wavelet transform was indexed by translation and scale, and we saw that there was a little bit of asymmetry between translation and scale. Translation could be dealt with easily; scale needed an additional rating factor to deal with it, when we, when we reconstruct it, when reconstructing the function from its continuous wavelet transform.

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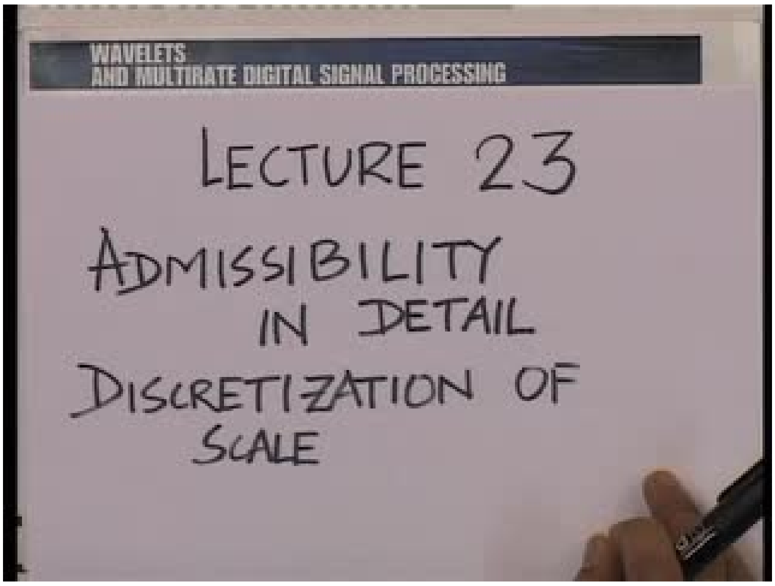

So, with that little recapitulation let me put the discussion in the current lecture in perspective. So, in the current lecture, we are going to talk about admissibility in detail first, and later, we are going to proceed to the discretization of scale, the scale parameter in a particular way.

Now, let us call back to the discussion that let us to admissibility or the idea of being able to accept a function psi t as a wavelet. We had only partly arrived at the answer, and in fact, we had worked rather hard to evaluate the triple integral to arrive there. It also seemed to be getting out of hand, because the triple integral needed to be dealt with one integral at a time, and although, we had almost come to the final step, I think it is worth recapitulating the important steps that took us to the final point where we were.

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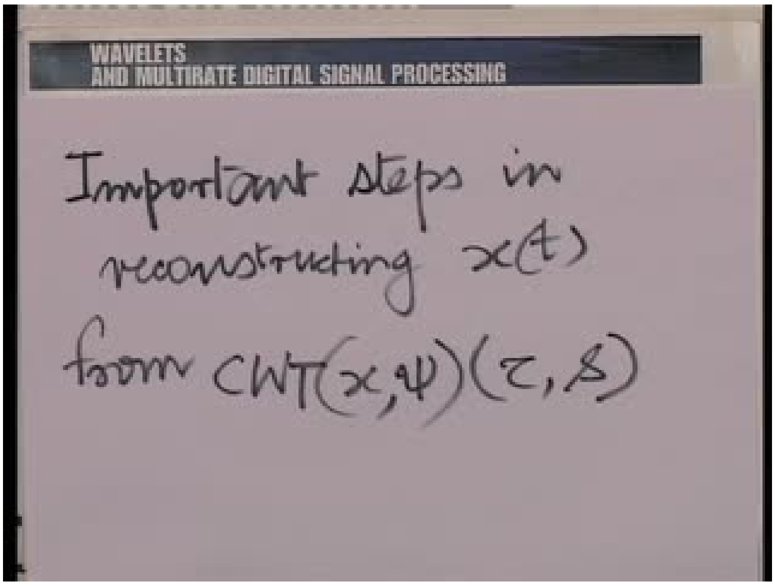

So, let me just put down some important steps before we proceed. Some important steps in reconstructing x t from c w t x psi evaluated at tau and s.

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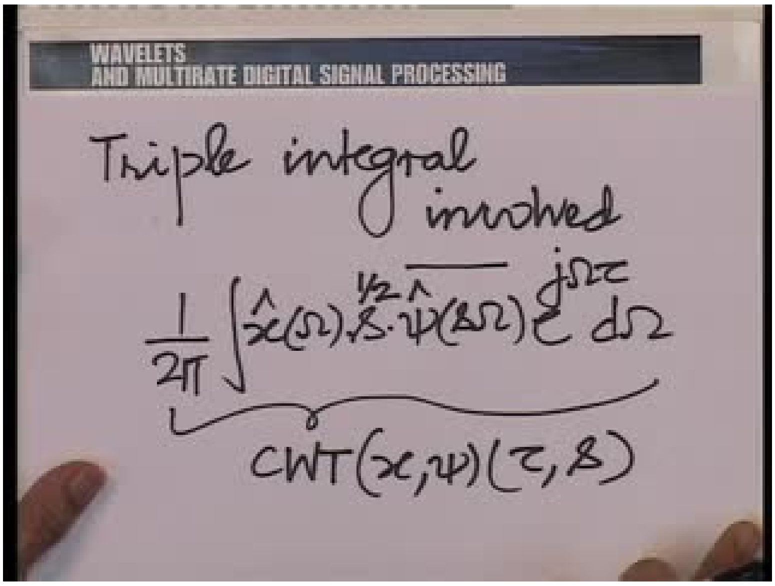

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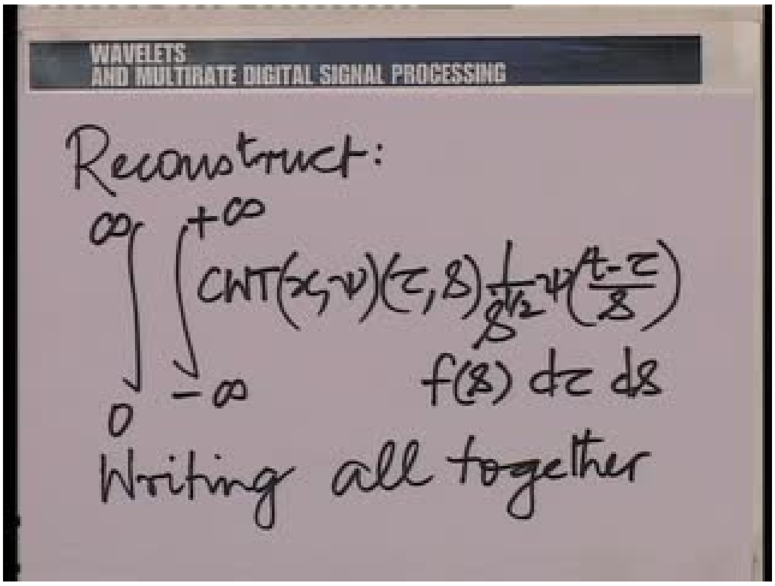

Now, the most important step was the evaluation of this triple integral here. The innermost integral was the so called component, so that was integral x cap omega s to the power half psi cap s omega complex conjugated e rise the power j omega tau d omega. So, this was the innermost integral essentially corresponding to the c w t, and with this, we then had two outer integrals which took care of the translation and the scale parameter, and recall that with the scale parameter, we needed a rating function that translation parameter did not required. So, we said we needed to reconstruct by doing the following: by taking the component as it were, multiplying by a so called unit vector, and integrating overall components. You only catch being that we can just do this. We need a rating function to deal with scale.

So, the integral on tau runs from minus to plus infinity; the integral on s runs from zero to infinity, and now, let us write it all together once again, and repeating, as you note reason repeating a few steps. It is worth doing that, because this was a little formidable as a proof. So, it would be helpful to look at the important steps in the proof again.

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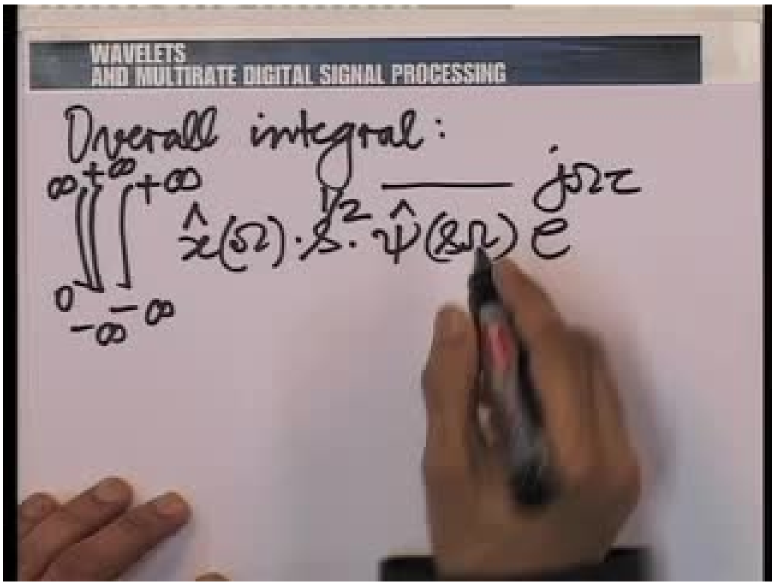

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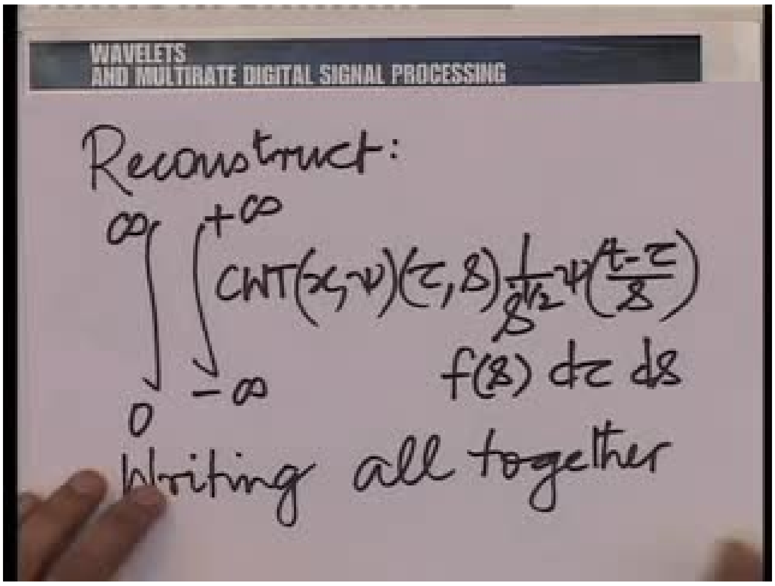

Anyway, so, the overall integral is as follows. So, I will write the triple integral here. The innermost integral is on omega from minus to plus infinity. The next integral is again from minus to plus infinity in the fine integral is from zero to infinity here. So, this is the triple integral. Now, you have the innermost integral taking you with x cap omega s to the power half psi cap s omega complex conjugated e rise the power j omega tau; this takes care of this part. (Refer Slide Time: 07:55)

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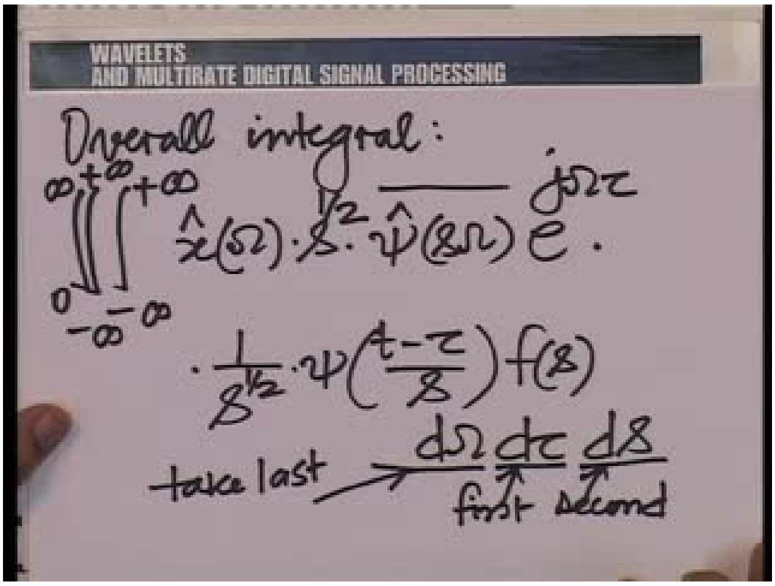

Now, to write the rest times one by s to the power half psi t minus tau by s f s d tau d s, before which there is a d omega there, and we said - we wanted to reverse or you want to change the order, in which, these needed to be dealt with. So, with, we would take this the last; take this first and take this second, this was the approach that we used. (Refer Slide Time: 08:37)

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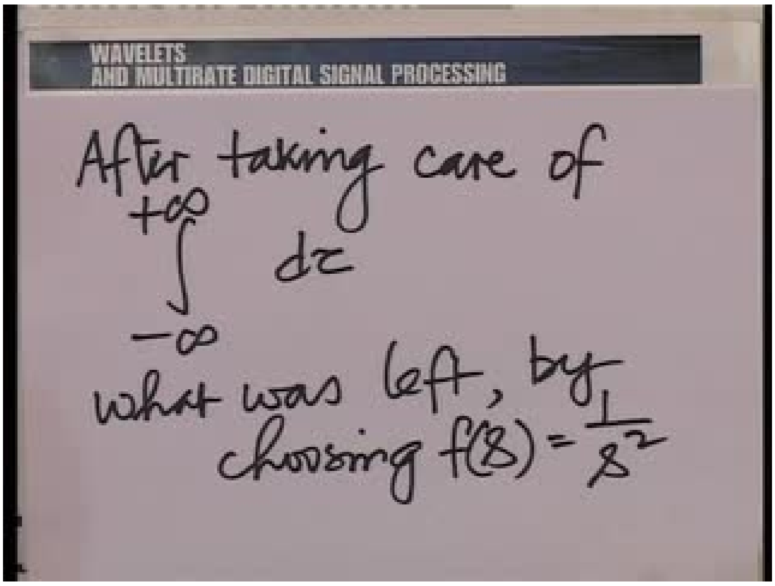

Now, without repeating all the discussion, let me just put down one more important intermediate step. What we saw is that after removing or after taking care of integral d tau first. What was left by choosing f of s to be one by s squared was the following.

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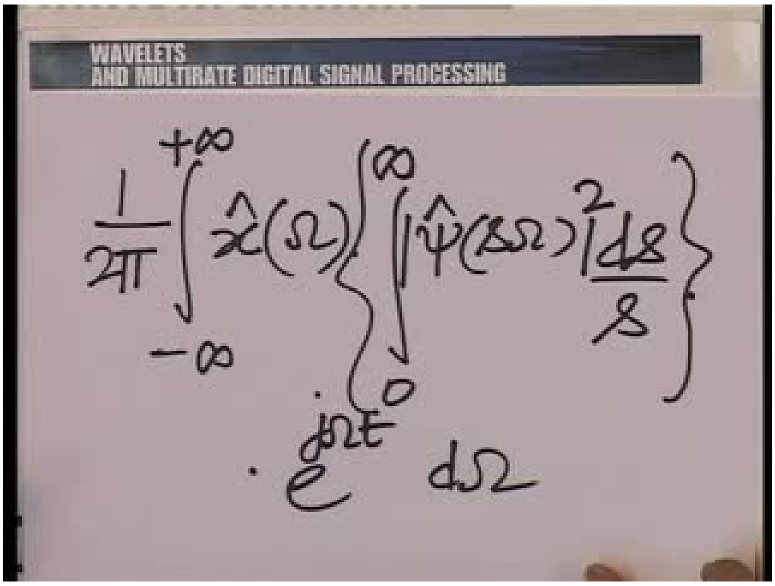

Essentially, it was one by two pi integral from minus infinity to plus infinity x cap omega and integral involving s in a funny way zero to infinity mod psi cap s omega the whole squared d s by s. Seemingly dependent on omega, but that is where our whole discussion ended in the previous lecture and we wanted to build on it today. Seemingly dependent on omega, but we want to do a wave with a dependence on omega here. So, this multiplied by e rise the power j omega t integrated over omega. This is where we were and we noticed that it is essentially this which is causing us trouble at this feeling dependent on omega, at this term, here. Then this would be a constant which could be extracted and what would be left then is just the inverse Fourier Transform of x cap omega which is just x t.

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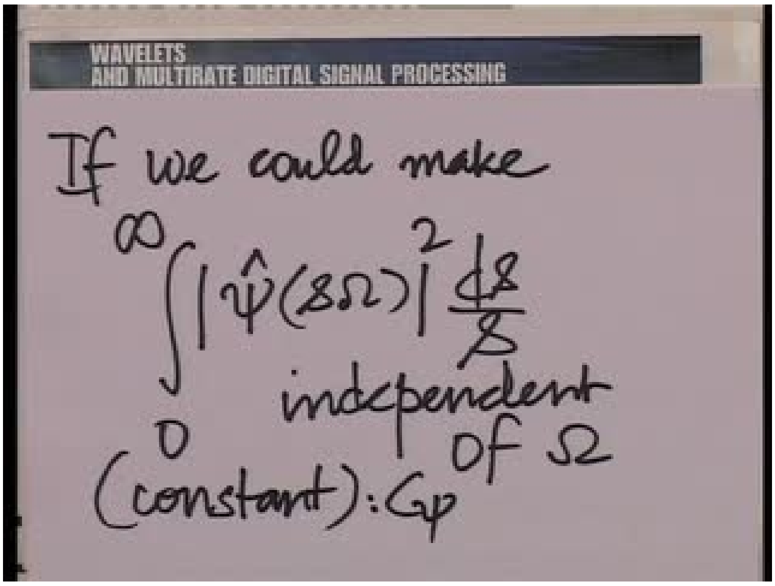

So, let us summarize that step. If we could make the integral from zero to infinity psi cap s omega mod squared d s by s independent of omega, and you see, when it becomes independent of omega, it is dependent on no other parameter, so it would just be a constant.

Let us call that constant c dependent on psi. You see it, of course depends on what psi you use, but beyond the dependence on psi, there is a dependence on nothing else. Once you fix the psi, this should then be independent of omega if we can make it so and that is what admissibility is all about.

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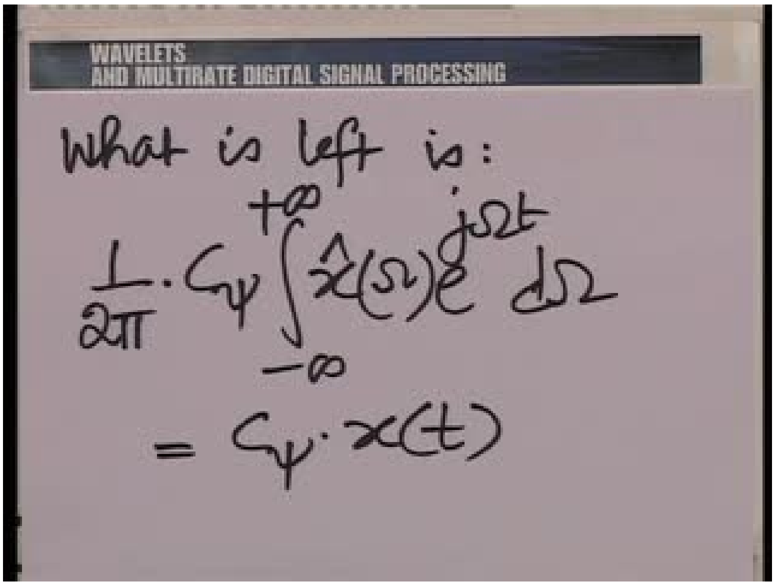

So, anyway, if this could be done, then what is left is: essentially one by two pi times this constant c psi times the integral from minus to plus infinity x cap omega e rise the power j omega t d omega, which is essentially c psi times x t, so simple and so beautiful.

So, in fact, now we also have an interpretation for this constant c psi. The constant c psi actually tells us the factor by which x t has been multiplied in this process of reconstruction. So, we have a meaning to that integral too, but now let us look at that integral which we wish to make independent of omega little more carefully.

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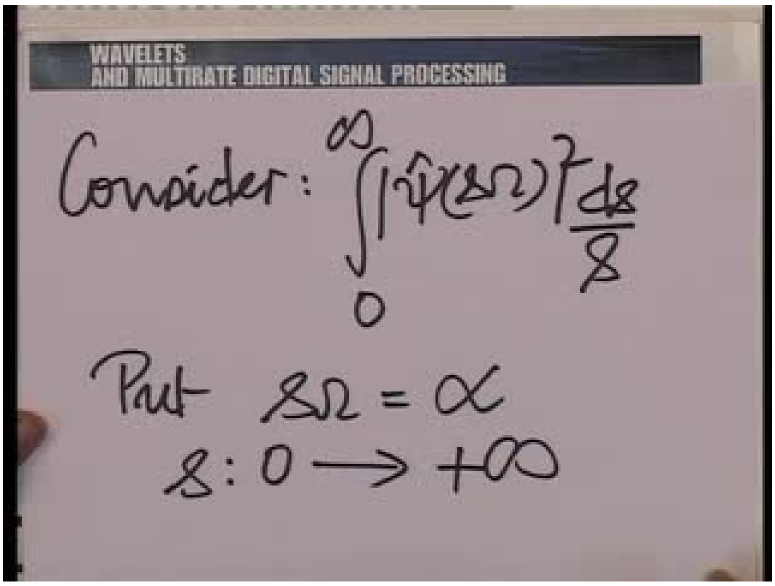

So, consider the integral from zero to infinity psi cap s omega mod squared d s by s and put as usual s equal to a parameter alpha. Now, we must remember that s always runs from zero to plus infinity, and omega is capable of running all the way from minus to plus infinity. So, of course, one thing that we notice is that omega equal to zero is a problem.

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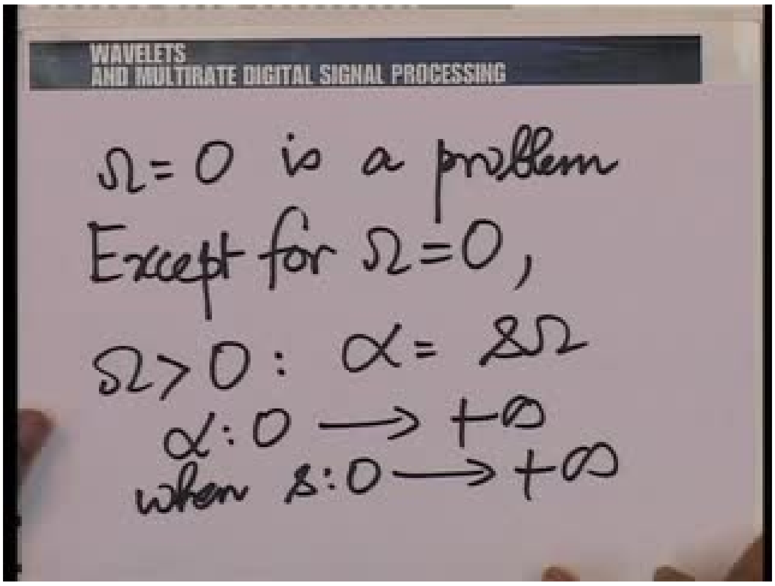

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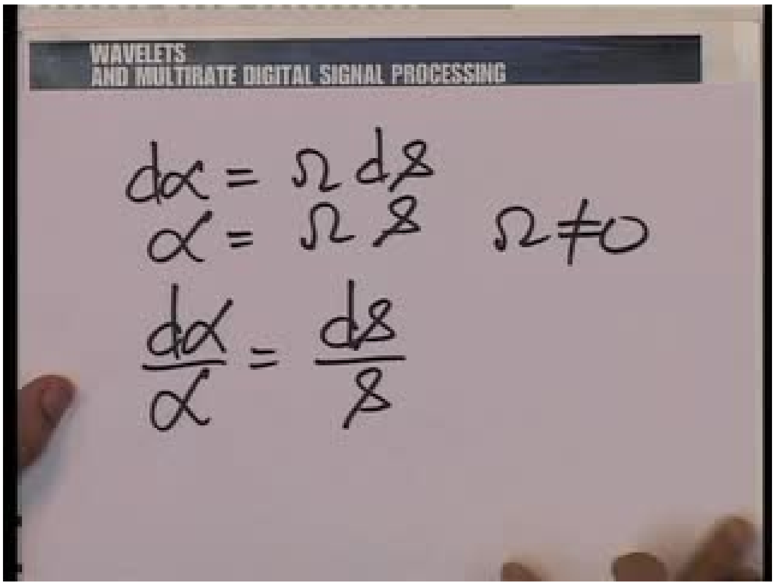

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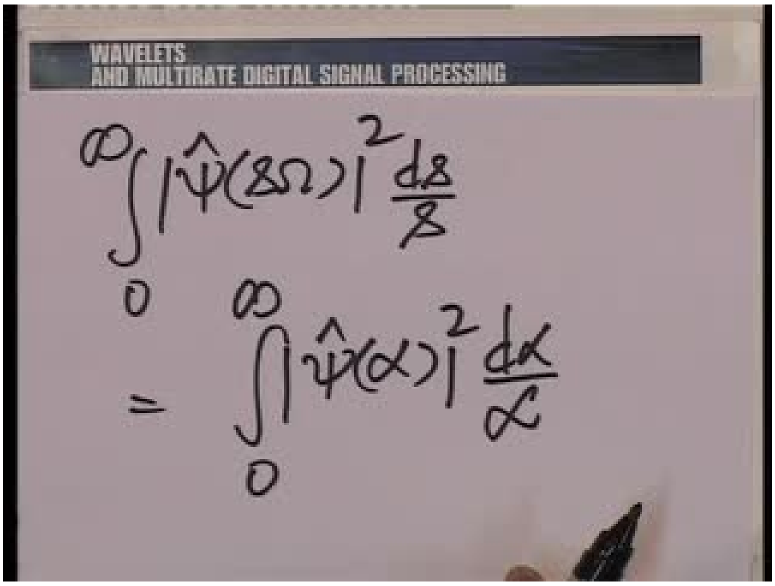

So, except for omega equal to zero, suppose we consider omega greater than zero first, in that case alpha equal to s omega would run from zero to plus infinity when s runs from zero to plus infinity, and of course, if you take the differential on both sides here as we did, we would have d alpha is omega d s, and of course, alpha is omega s, and since omega is not zero, we have d alpha by alpha is d s by s. Where upon this quantity integral from zero to infinity psi cap s omega mod squared d s by s simply becomes integral from zero to infinity psi cap alpha mod squared d alpha by alpha. This is beautiful. Now, this is only based on the function psi; it has nothing to do with capital omega.

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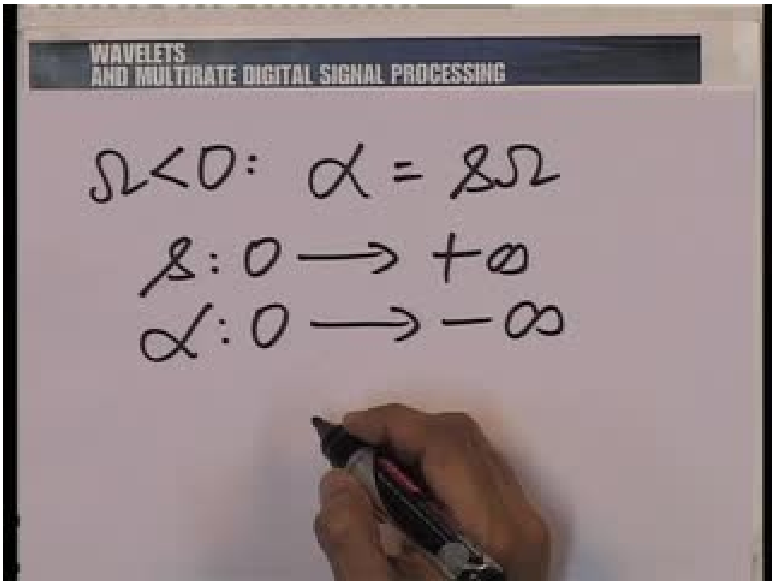

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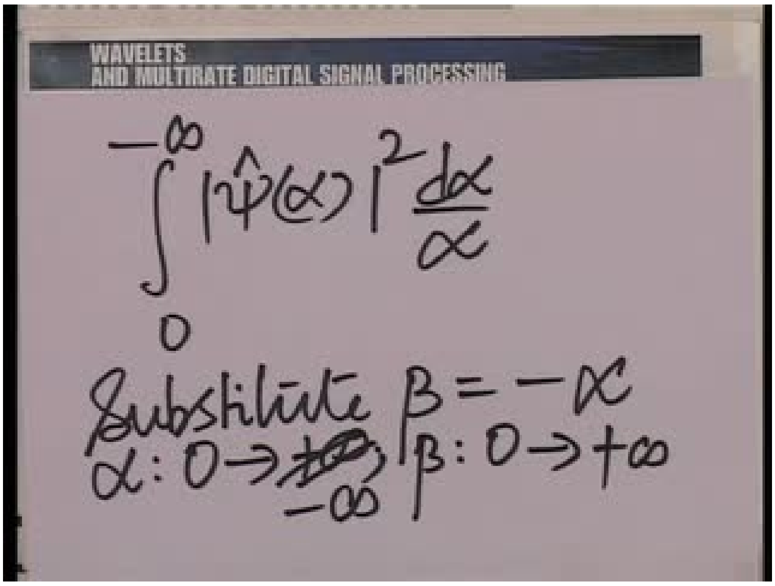

Let us consider the case of capital omega less than zero now; there we have an interesting situation. So, when capital omega is less than zero, again of course we have alpha is s omega. When s runs from zero to plus infinity, alpha runs from zero to minus infinity, and therefore, we would have the integral becoming integral from zero to minus infinity psi cap alpha mod squared d alpha by alpha.

Now, we have a little bit of trouble here. It is not easy to do this integral or related to the previous one. So, let us make a substitution again, substitute beta is minus alpha. Where upon when alpha goes from zero to plus infinity, beta goes from zero to minus infinity or vice versa. So, when alpha goes from zero to minus infinity, beta goes from zero to plus infinity. That is what we wanted.

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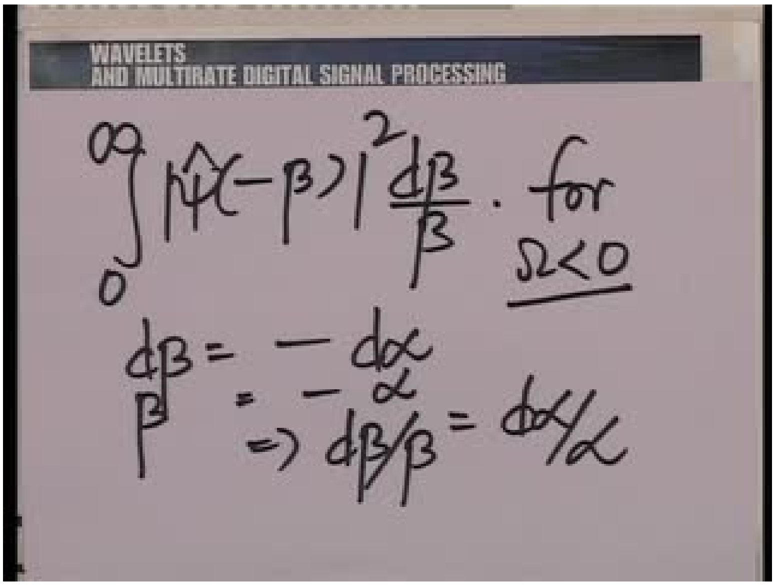

So, let us make the substitution here. Also one can see that d beta is minus d alpha and beta is minus alpha, where upon d beta by beta is equal to d alpha by alpha as before. So, we have a interesting situation; this is also for omega less than zero. We have zero to infinity psi cap minus beta mod squared d beta by beta.

So, we have an interesting situation. You see, we want to make the whole expression independent of capital omega. We seem to have different expressions for omega greater than zero and omega less than zero, and if you want to make this whole expression independent of omega, the two expressions must be equal and must both be finite. The catch is in making them equal and finite.

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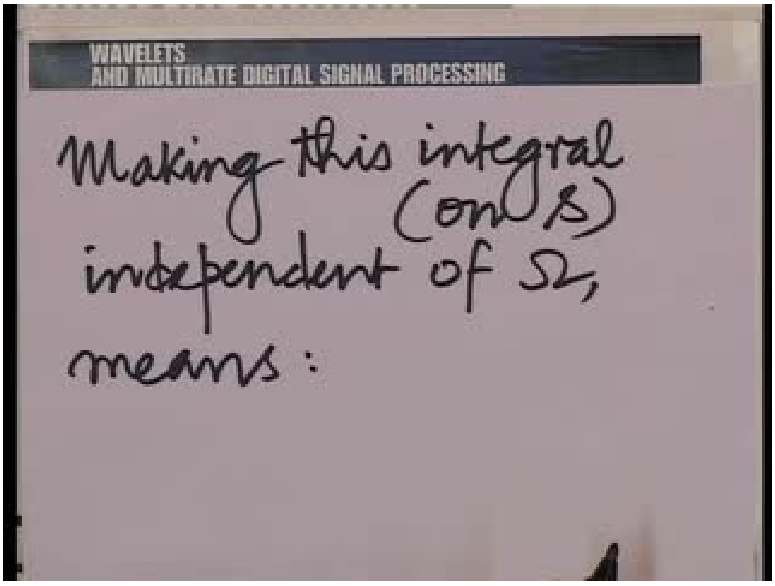

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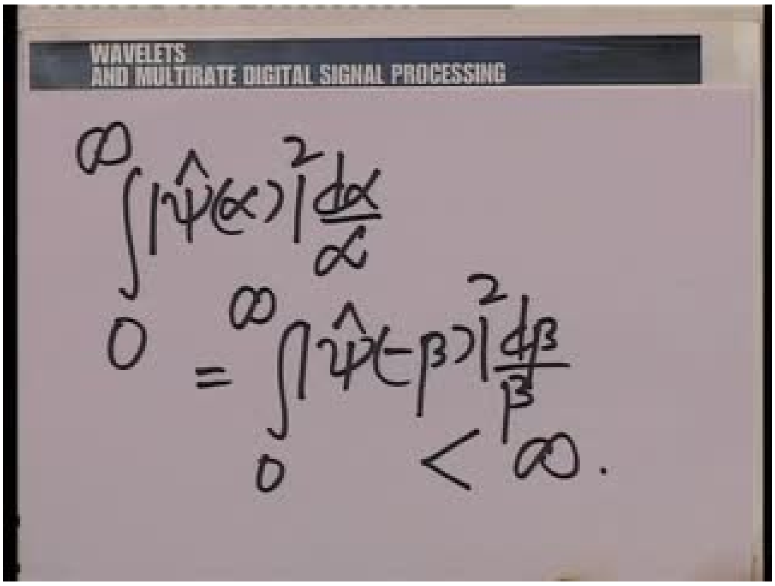

So, making this integral, - on s I mean - independent of capital omega means two things - one integral zero to infinity psi cap alpha mod squared d alpha by alpha is equal to integral zero to infinity psi cap minus beta mod squared d beta by beta, and both of these are less than infinity, and you know I do not need to specify this is positive infinity, because it is obvious that we are talking about positive alpha here. This is all positive, you see you are taking beta over a positive range and this is a non negative quantity mod psi cap minus beta whole squared or mod psi cap alpha squared. So, they are all non negative quantities. So, this is an integral over a non negative integrand, and therefore, it must be non negative. So, it is obvious that this non negative quantity needs to be bounded; there is no question of it is becoming negative.

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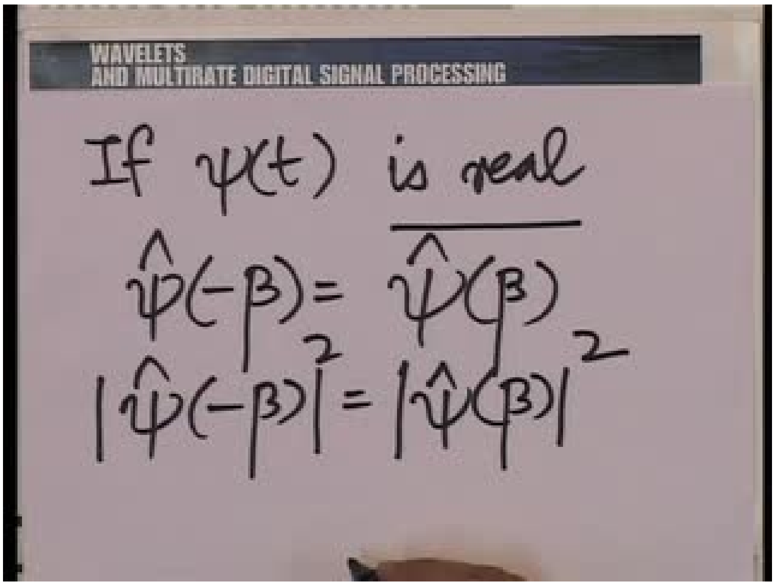

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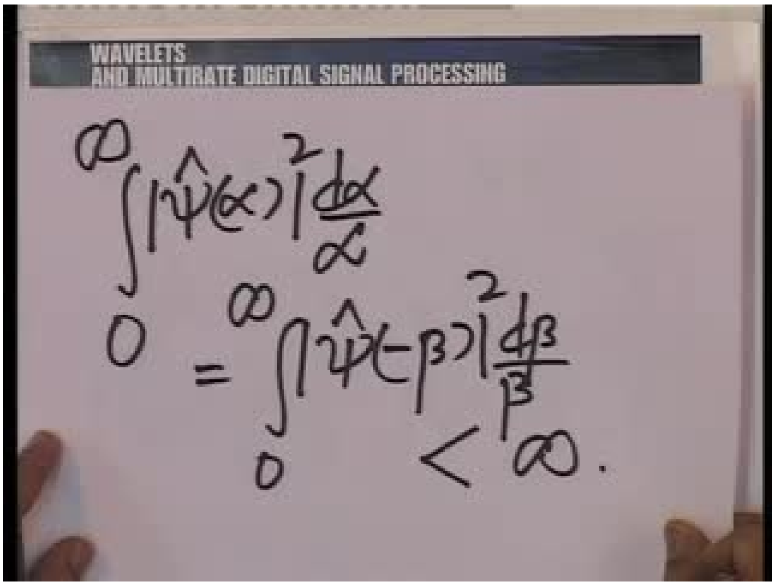

Now, this is not a serious requirement when psi t is real. If psi t is real, then of course we know that psi cap minus beta is equal to psi cap beta complex conjugated, and therefore, mod psi cap minus beta squared is the same as mod psi cap beta squared, and therefore, if I go back to the previous condition, these are not separate condition; these are just the same condition.; the magnitude is symmetric.

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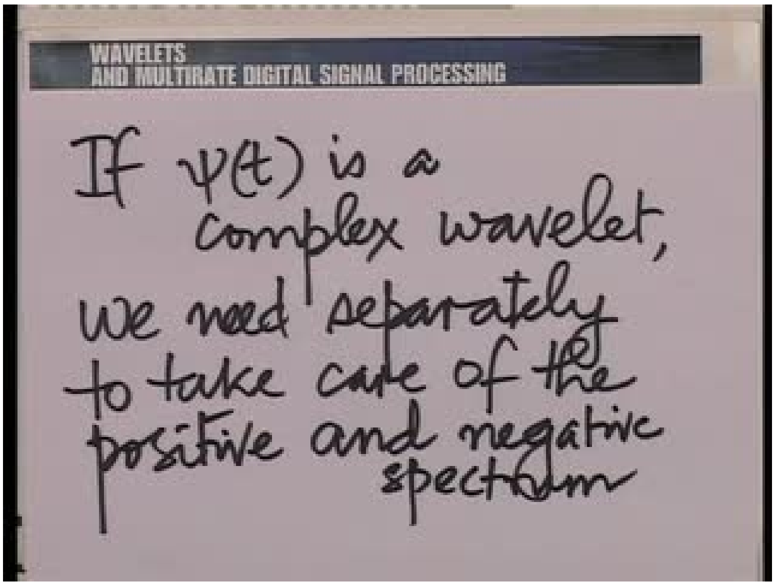

However, when psi t is a complex wavelets which is a distinct possibility, remember. So, in fact, here we are allowing the possibility of complex functions as wavelets; we should not disregard or discount that possibility. If psi t is a complex wavelet, what we are saying is that we need separately to take care of the positive and negative part of the spectrum.

The other way of saying it is - if you using a complex wavelet, and if you insist that the spectrum must be one sided, then make sure that your signal has no component on the other side. In that case, that particular condition can be removed. So, for example, suppose we take a complex wavelet, where, we are not going to take care of the negative part of the spectrum. So, second condition is not obeyed the one which involves psi cap mod of minus alpha minus beta the whole squared. What we are saying in effect is then you may only deal with such x, which have non-zero components, and therefore, x must be complex non -zero components on the positive part of the spectrum for omega greater than zero.

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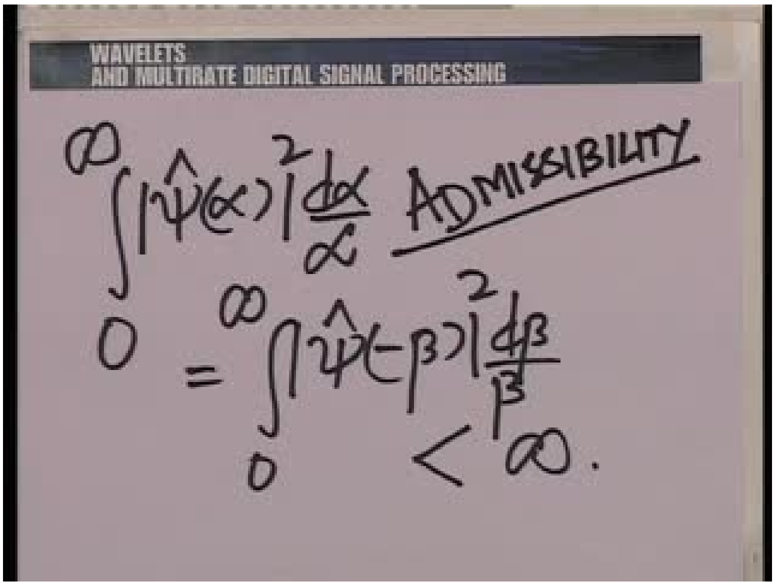

Conversely, if you are only going to satisfy the condition for the negative part of the spectrum mainly for omega less than zero, then make sure that your original signal also does not have any components on the positive part of the spectrum. So, if one is willing to take care of this slightly restricted situation, one can use the complex wavelet. In fact, there is a good reason to use complex wavelets if you are dealing with complex functions. So, this condition, let me put down the condition before you once again, this condition that just you written down here is called the admissibility condition for a wavelet.

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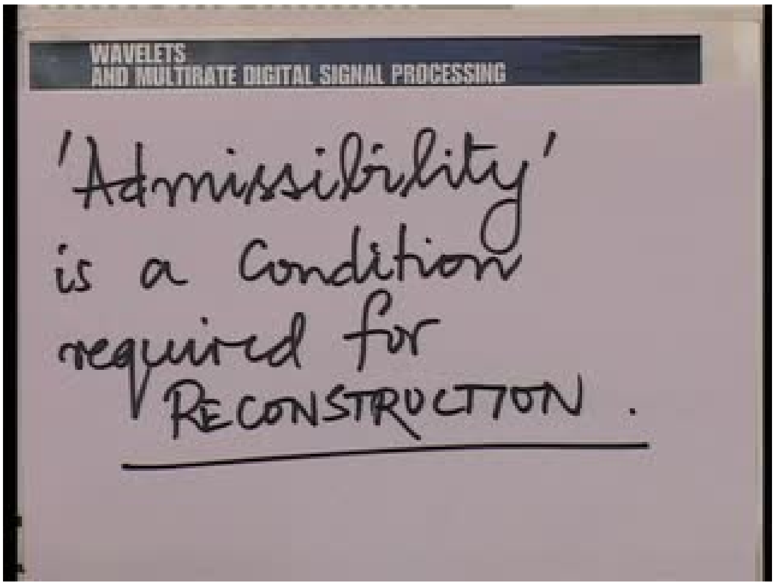

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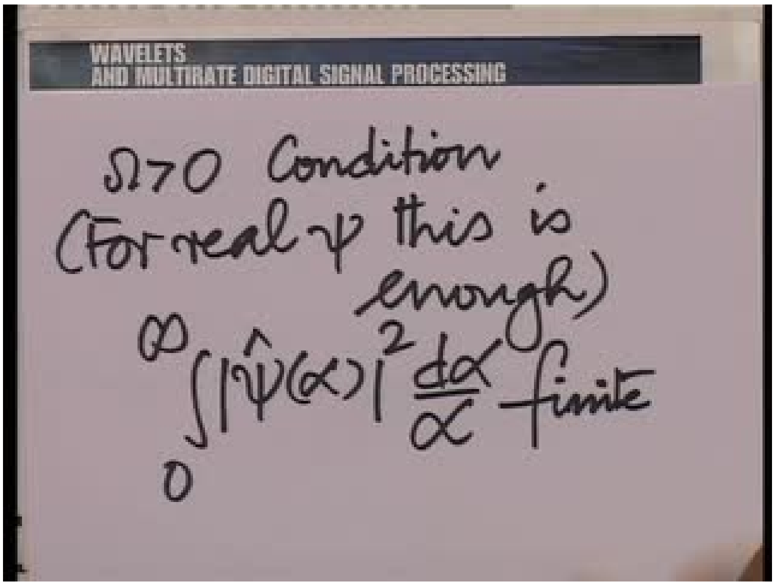

Admissibility essentially refers to allowing that function to be called wavelet; allowing that function to bring out a continuous wavelet transform. In fact, admissibility, if you ask me, is a condition of reconstruction, is the condition required for reconstruction, and if we reflect a little on admissibility, what we are saying now - from a different perspective, makes a lot of sense, as I will just show you in a minute. You see, what is admissibility tell you. Let us take the omega equal to or omega greater than zero condition, you see as I said for real psi, this is enough. So, let us focus on real psi for a moment to make matter simple. It says essentially that integral from zero to infinity, psi cap alpha mod squared d alpha by alpha must be finite. So, where can the trouble come here? You see, as you notice, the trouble can come at the two extremes. Let me for example, put before you a condition on psi or psi cap which yields troubles; the situation in which we shall have trouble.

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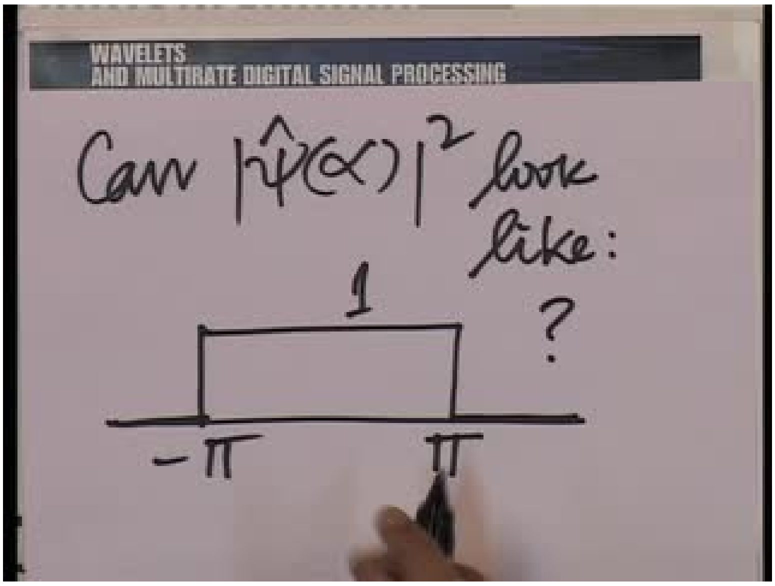

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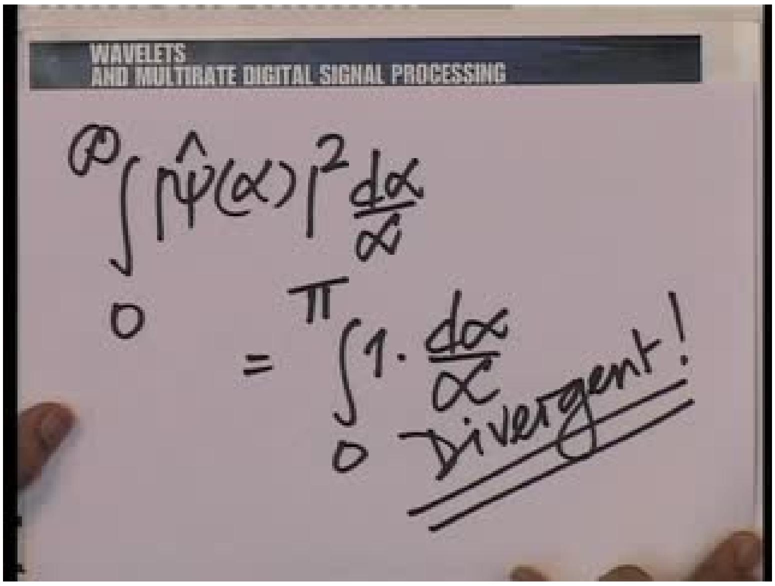

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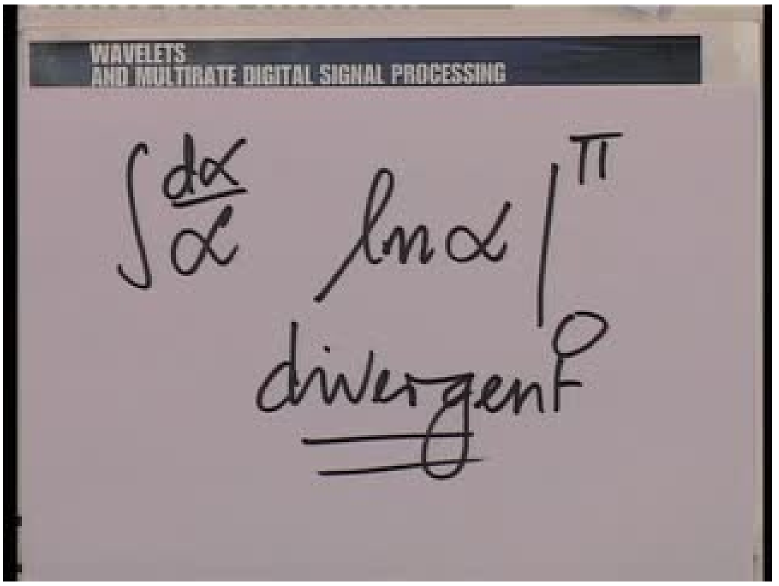

Let us ask the question - can psi cap alpha squared look like this? So, let us take from pi to minus pi; it is one and zero else. Well, it is obvious that it cannot. If we try and construct the integral here, what would it be? It would be essentially the integral from zero to pi rupees one d alpha by alpha, and obviously, this integral is divergent. In fact, as you can see the integral d alpha by alpha, the indefinite integral is essentially of the form log natural of alpha, and if you are trying to make any such substitution, this is divergent.

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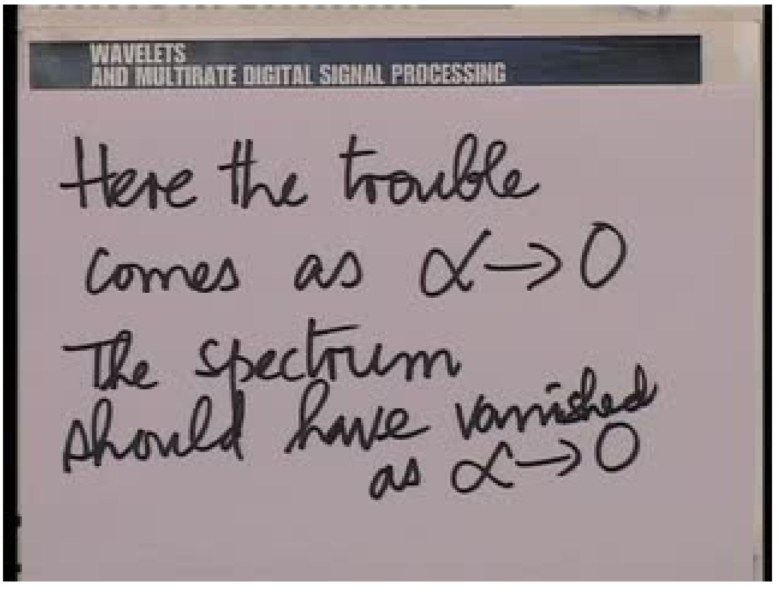

So, therefore, we see where the trouble comes from. The trouble comes from the region around omega equal to zero. Interestingly, one should not have the spectrum giving any significant contribution around the zero frequency. That is what we are saying. So, here, the trouble comes as alpha tends to zero. The spectrum should have vanished as alpha tends to zero.

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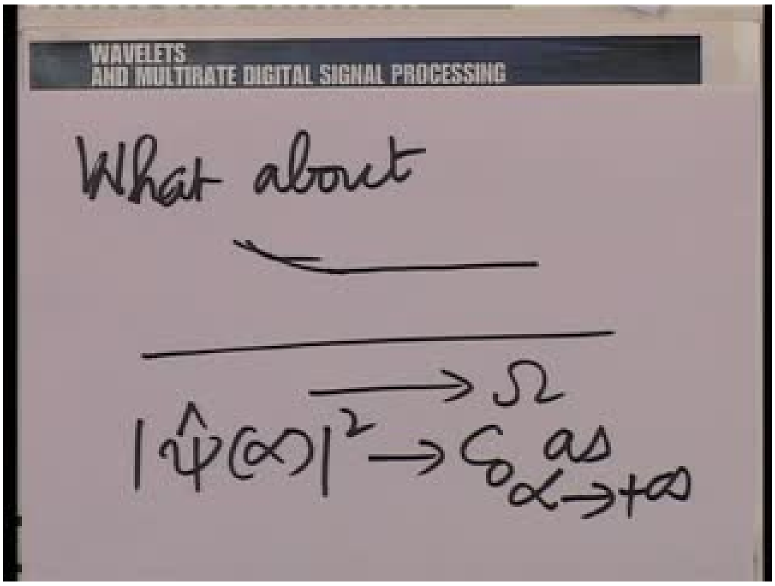

Now, let us see what would happen if we have alpha tending to plus infinity and we still have a spectrum that remains. What about I said - something like this; a spectrum, that is like this. As omega tends to plus infinity, this tends towards the constant. So, psi cap alpha mod squared tends to some constant. Let us say c zero as alpha tends to plus infinity, will this do.

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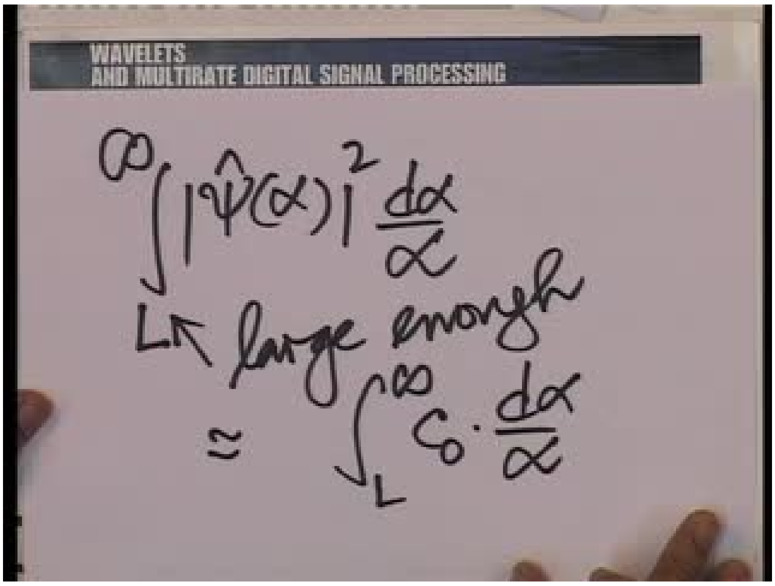

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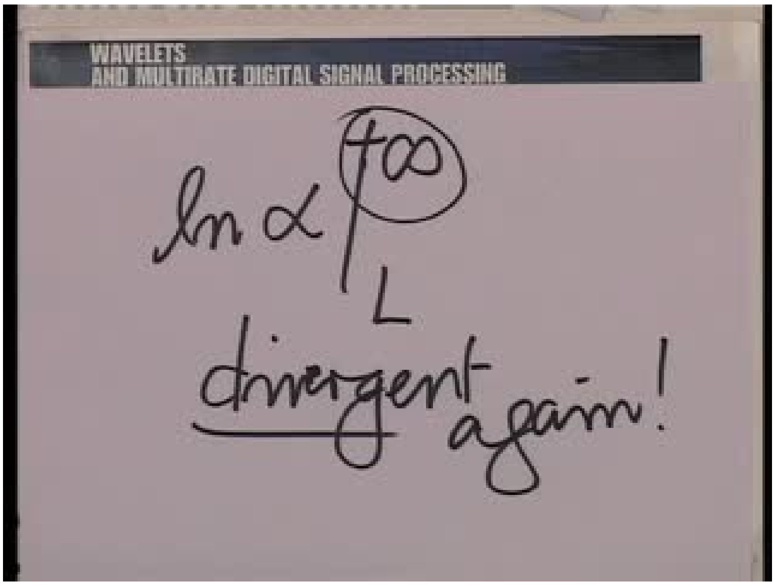

Well, again, let us evaluate that part of the integral. So then, you know, if you take the integral from a large number, let us say l towards plus infinity, psi cap alpha mod squared d alpha by alpha, and if you note that l is large enough asymptotically I mean. So, this is approximately integral from l to plus infinity c zero times d alpha by alpha. Then again we run into trouble. So, this would be essentially log natural of alpha evaluated from l and going towards plus infinity in plus infinity which is also divergent once again; this is the cause of divergence.

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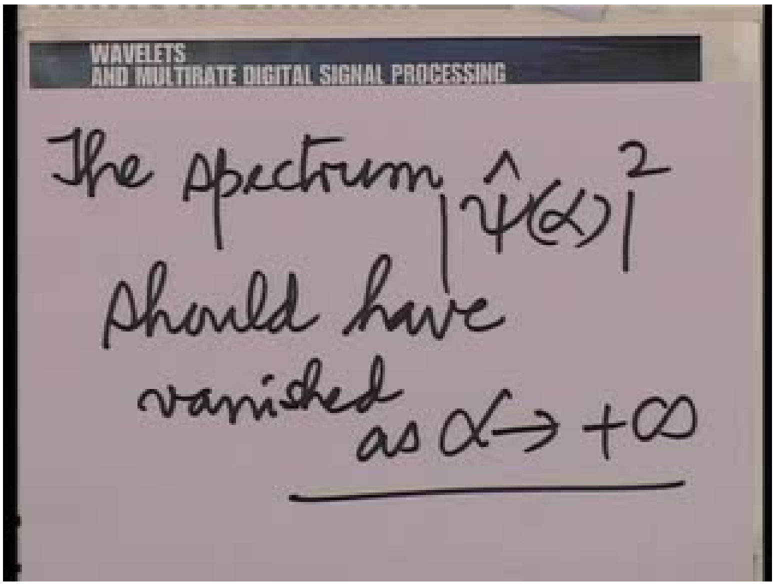

So, again, the spectrum should have vanished. That is what we have concluded should have vanished as alpha tends to plus infinity. So, all in all what are we asking for? We are asking for the spectrum to vanish as alpha tends to zero. We are asking for the spectrum to vanish as alpha tends to infinity. We want the spectrum to vanish at both the extremes. If it does not, if it persists, as you go towards zero or as you go towards infinity you have trouble. That part of the integral is going to divergent.

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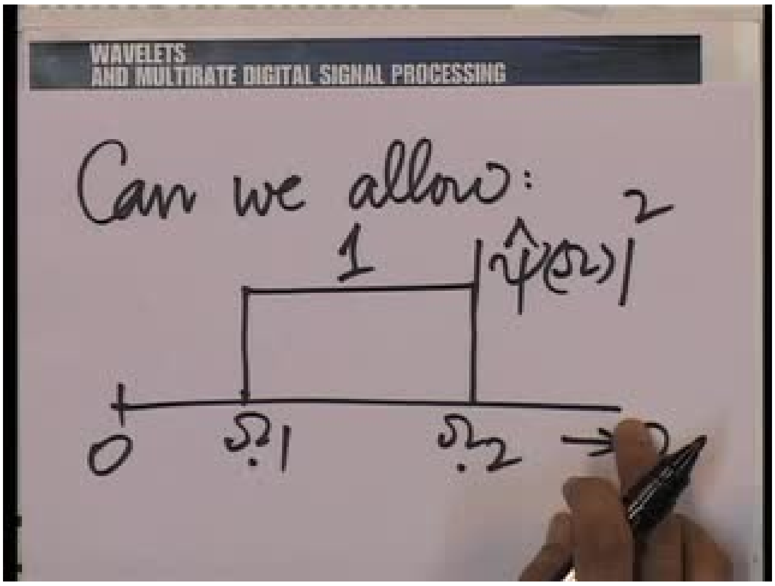

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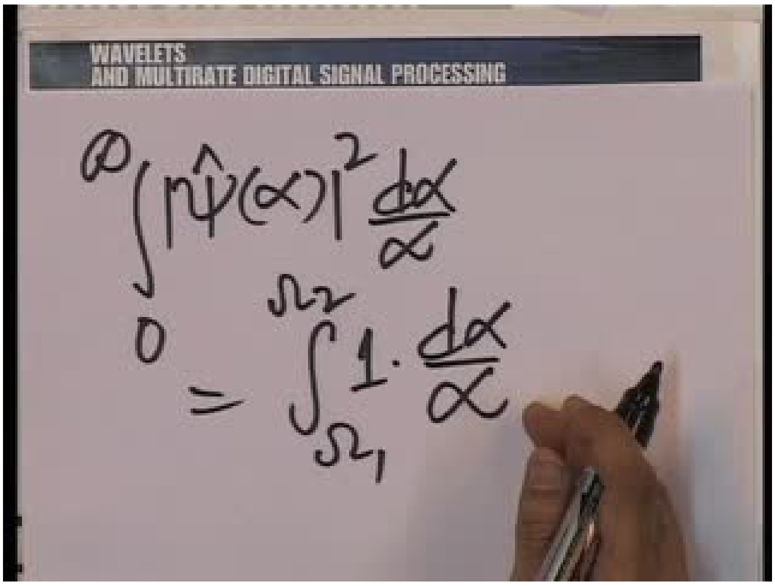

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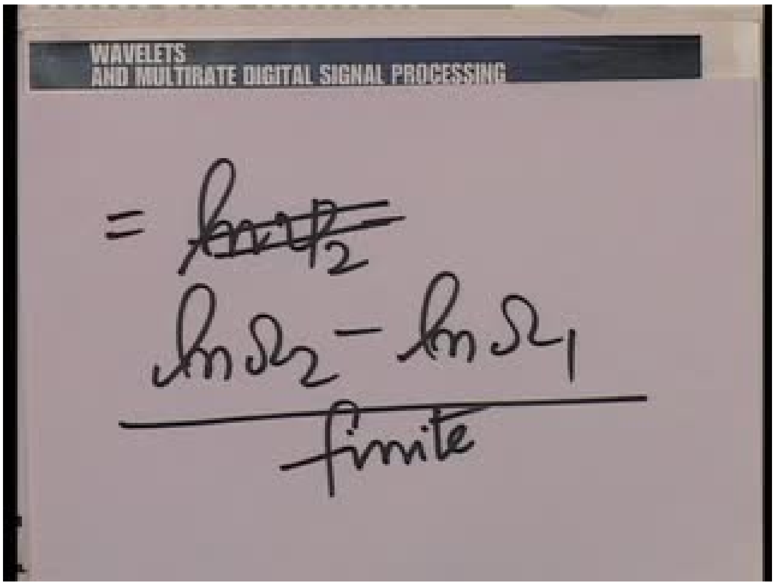

Now, let us take a third case which might just be acceptable and see if it is acceptable. So, can we allow the following- say capital omega one and capital omega two, zero here infinity there, and let us say it is one for simplicity in this region. So, capital omega one, capital omega two are both positive numbers, and the spectrum mod psi cap omega squared is one only between omega one and omega two and zero everywhere else, of course, I am showing only the positive side; the same is mirrored on the negative. So, is this allowed? Well, yes in deems. As we can see, if I were to put down the integral, zero to infinity as it were psi cap alpha mod squared d alpha by alpha. It simply boils down to integral m omega one to omega two one d alpha by alpha. This is very easy to integrate. This is simply log natural omega two minus, I am sorry, log natural capital omega two minus log natural capital omega one which is of course finite and acceptable.

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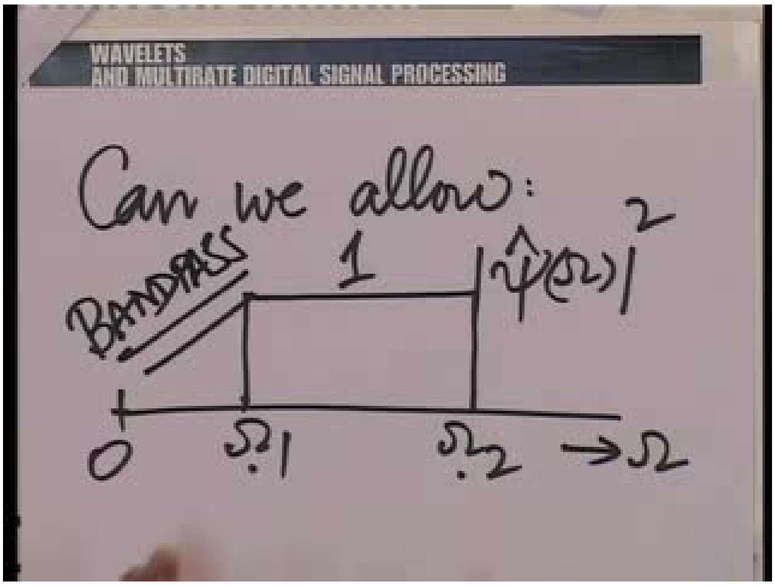

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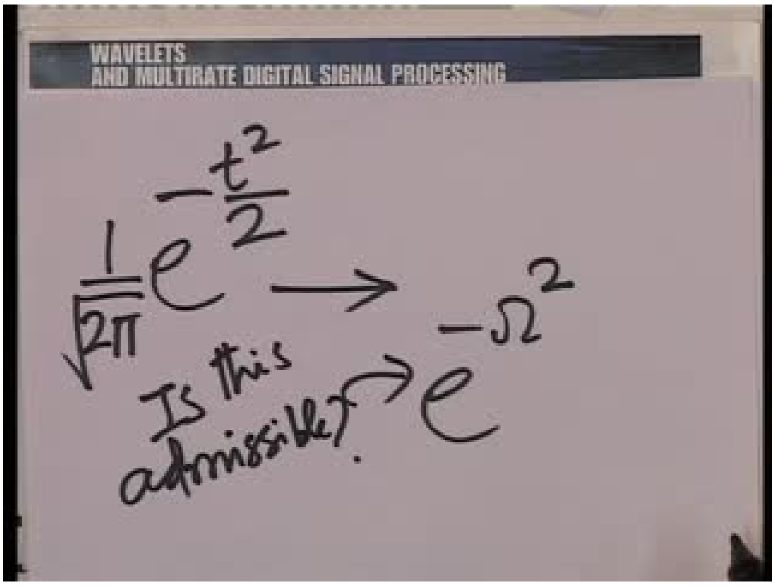

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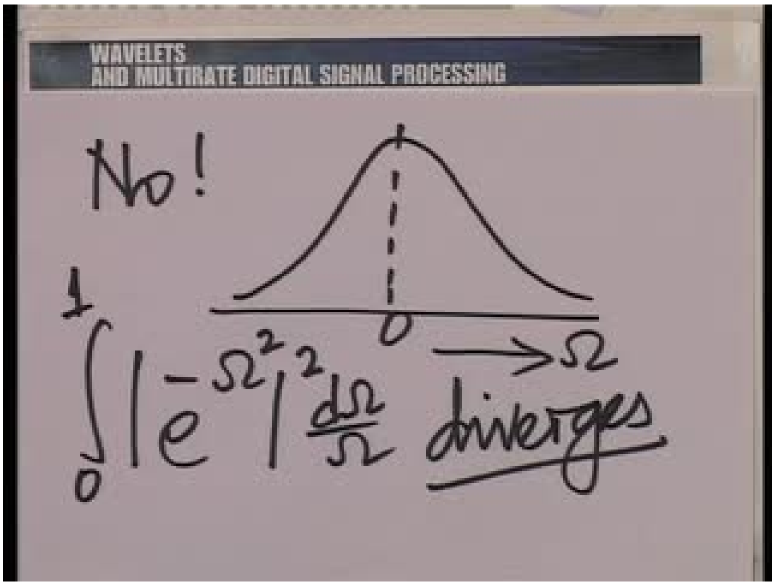

So, it makes it very clear to us what kind of function is acceptable. Let us put back the drawing of that function. This is a kind of function that we can accept. Essentially, a band pass function, that is what we are saying. A band pass function is what we are willing to accept. In fact, here we have considered the ideal case, the extreme case, where it is exactly band pass, but even if it is not exactly band pass, suppose for example, we take the Gaussian function as it were, you know we must ask the question, we found the so called code un-code ideal function in the sense of time frequency product, the Gaussian. If I take the Gaussian in time, it is also Gaussian in frequency; that is the beauty of the Gaussian. What I meant was that if I consider the function e rise the power minus t square by two, and in fact, if I also normalize it properly, so here the variance is one, I put one by square root two pi there. It is Fourier Transform is also going to be of the form e rise the power minus omega squared, and you see a Gaussian creates a Gaussian in the frequency domain. So, question is, is this admissible? And the answer is a very simple, no. In fact, we can sketch it. This is how e rise the power minus omega square would look, and as you can see that not vanish, in fact as you go towards zero, it tends towards one. So, it is very easy to see. Then if I try and take the modulo square of this and start integrating with respect to omega even over a small range between, say zero and one, it is going to diverge; so, the answer is no.

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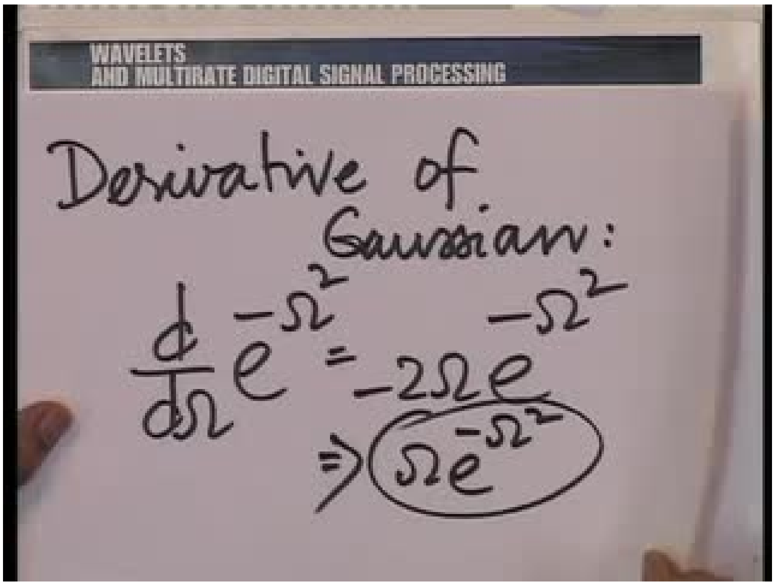

The Gaussian by itself is not admissible. Towards the next best thing that we might explore, make the Gaussian admissible by pulling the spectrum to zero as you go towards zero frequency. Now, how an earth do, you pull the spectrum to zero? Well, the one way to do it is to take a derivative. So, suppose, we have to take the derivative of a Gaussian, that is of the form minus two omega e rise the power minus omega squared, which of course you could just essentially take for consideration as omega times e rise the power minus omega squared. We will just consider this.

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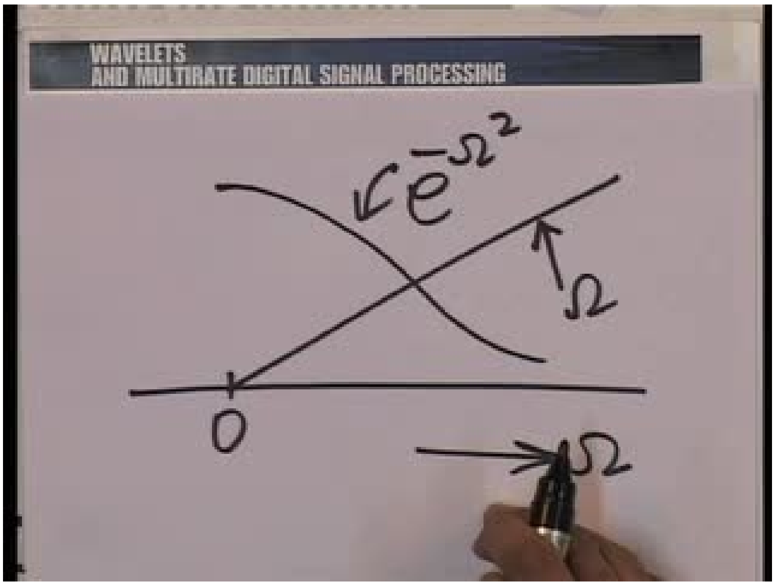

So, how does this derivative - first derivative - of the Gaussian look? The first derivative of the Gaussian is going to be a product. See, I am going to show the positive side of capital omega. So, this was the Gaussian, and this is what capital omega would look like. When you multiply them, you can see that there is going to be a maximum somewhere, and then there is going to a tapering off again, because the Gaussian fall is much stronger than this linear rise. So, their product is ultimately going to be dominated by the Gaussian fall.

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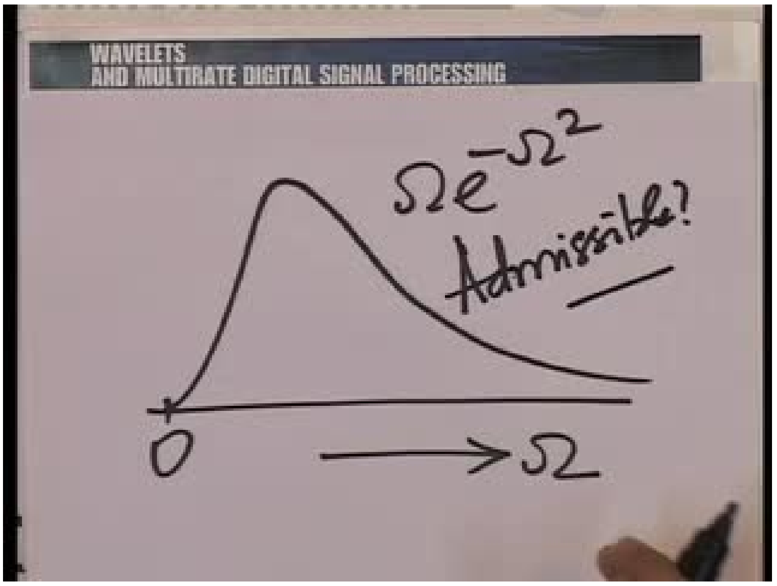

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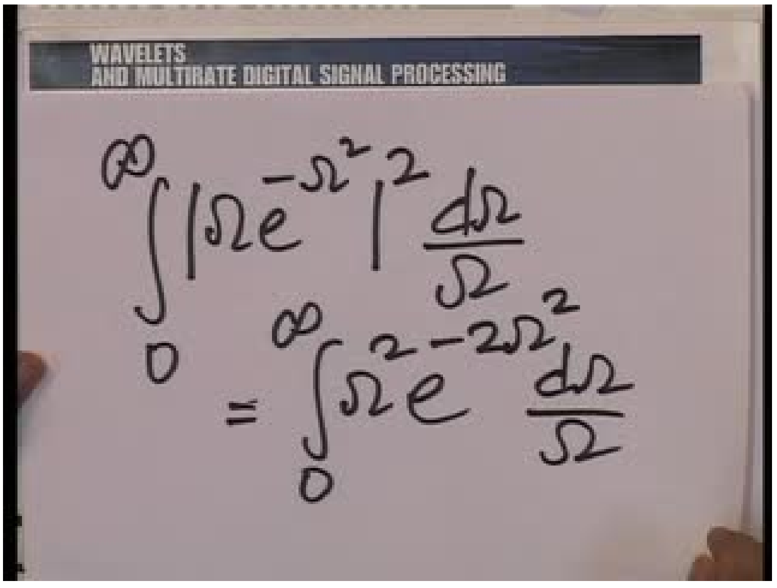

So, we may going to have a spectrum that look something like this, and we might ask that since this seems to prime of x zero x satisfy both our requirements, namely that it vanishes as omega tends to zero and vanishes as omega tends to infinity. Is it admissible? Well, not at all difficult to answer. Indeed, if I put down the admissibility integral, I would have omega squared here e rise the power minus two omega squared there d omega by omega, and this is a very easy integral to evaluate.

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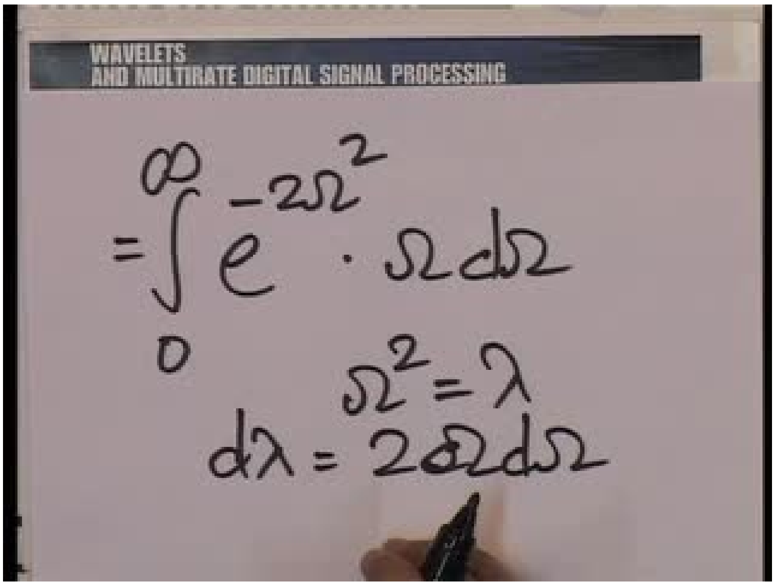

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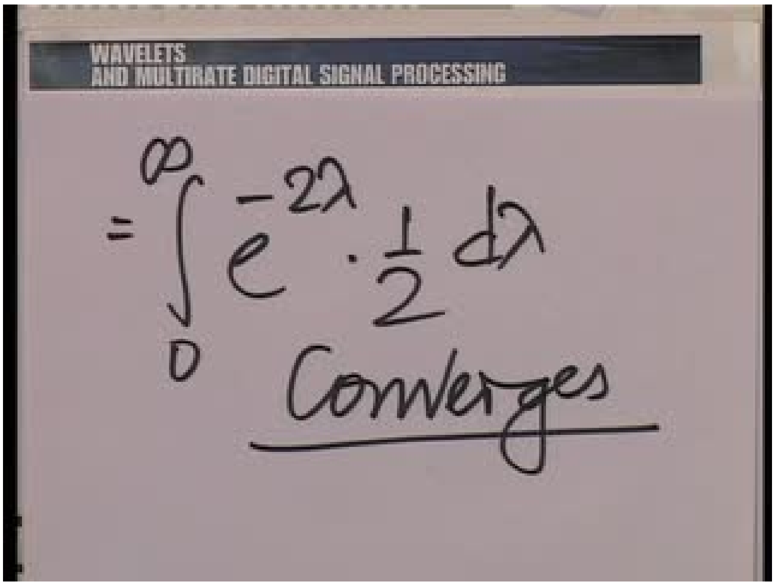

Now, omega d omega is half of d omega squared. What I mean is - if you put capital omega square is lambda, then d lambda is, twice, twice omega d omega, and therefore, you have this integral boiling down to integral zero to infinity e rise the power minus twice lambda half d lambda, and this is a very easy integral to evaluate. In fact, it definitely converges.

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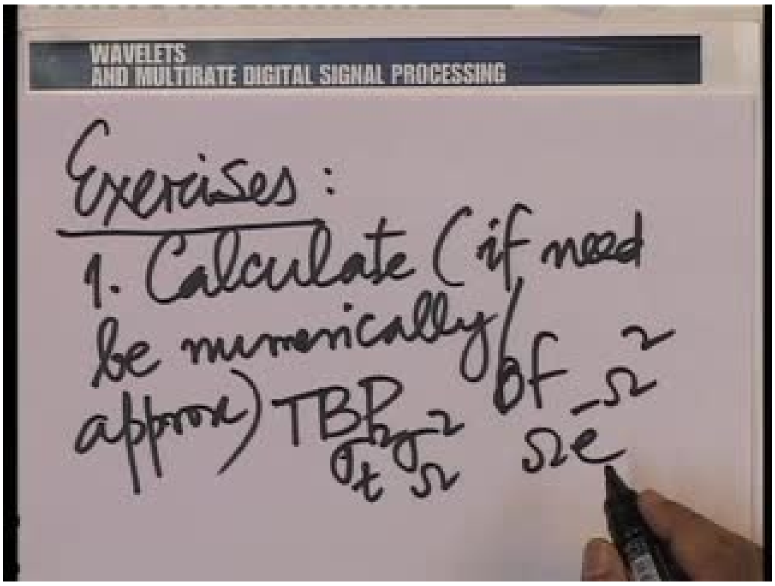

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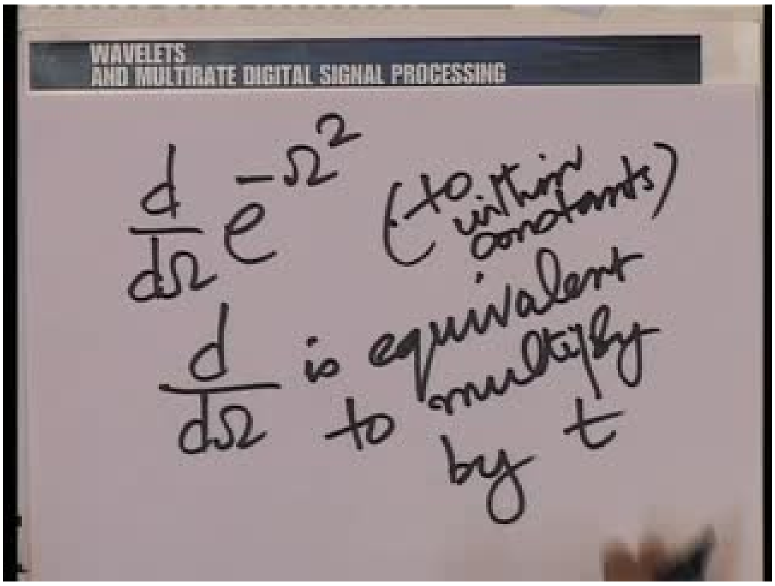

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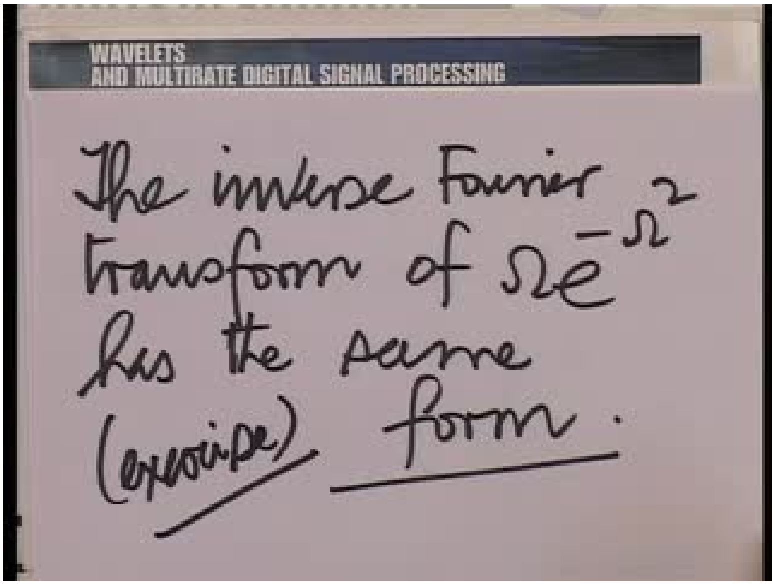

So therefore, the derivative of the Gaussian is admissible, good news; bad news - the derivative of the Gaussian is no longer optimal in the sense of time frequency. So, I in fact now put down a couple of exercises: one - calculate at least approximately numerically or approximately the time band with product. I shall abbreviate time band with product by t b p. That is the sigma t squared sigma omega squared of the derivative of the Gaussian. So, e rise the power minus omega squared in frequency, but you will see in time also it has a similar. So, in fact, let me put down the time expression. You see, when you take the derivative in frequency, we took the derivative in frequency. Now, taking the derivative in frequency is equivalent to multiplying in time by t in time of course, there are constants involved constant of j and so on, but if one does all the book keeping properly, one would see that the time and the frequency expression is very similar. So, the inverse Fourier Transform of omega e rise the power omega squared has to same form, and I leave this to you to show as an exercise.

So, we have the first exercise here. Take this first derivative of the Gaussian, look in its Fourier transform, make an attempt to evaluate it is time frequency product, time bandwidth product approximately if required or numerically if required, and compare it with the time frequency product of the Gaussian. There would be a disappointment. Anyway, that was the first exercise.

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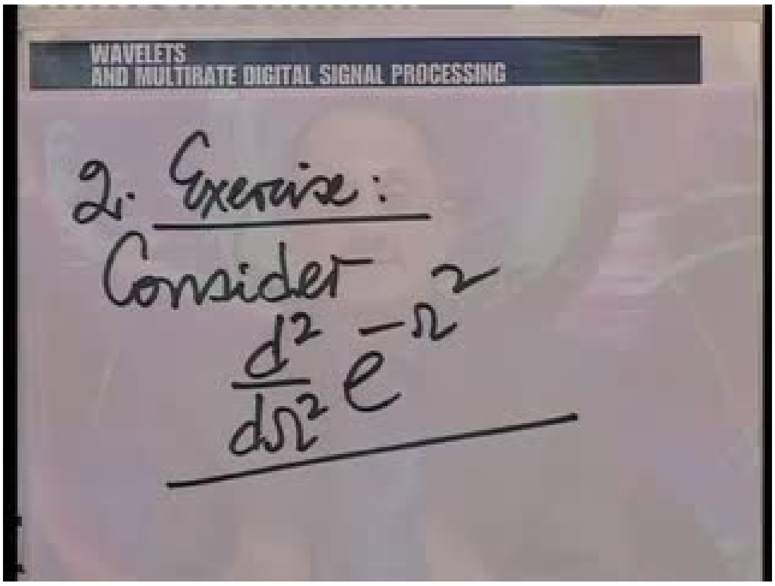

The second exercise is essentially the following: take higher derivatives, so for example, consider the second derivative of the Gaussian and repeat whatever we have done here, that is, finding out the function itself, it is derivative, then checking for admissibility, getting a feel of the time bandwidth product for the second derivative of the Gaussian.

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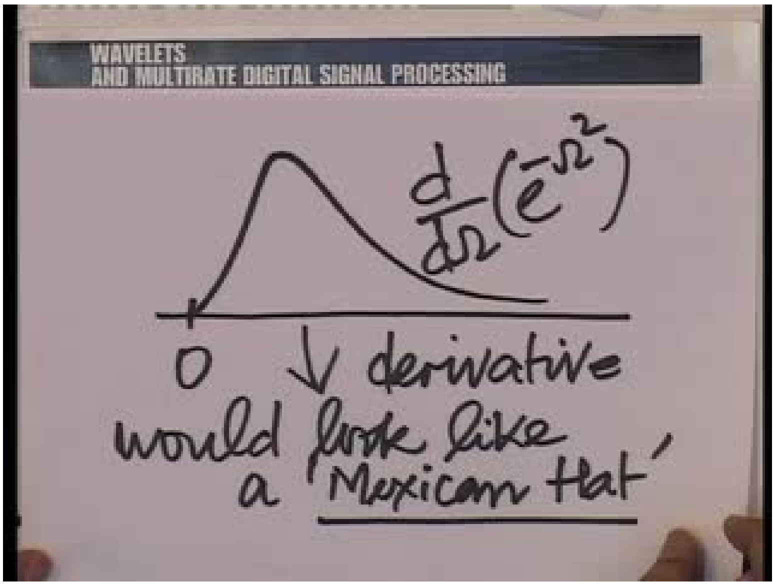

Incidentally, this second derivative of the Gaussian is a celebrated function in the context of wavelets and multirate processes. Some people call it the Mexican hat function. So, you know, I can give you a feel. If you look at it, let us look at the first derivative first.

This is what the first derivative looks like. We can get a feel - a graphical feel - of the second derivative. There would first be a region where that derivative of this derivative is positive, this region. There would be a place where it crosses zero, somewhere here; that be a maximum; that be a zero, and then, the derivative of this is going to be negative. So, you are going to have a region of positivity, and then, a large region of negativity, and even in that region of negativity, you can see very easily that finally that second derivative would also tend towards zero here.

Now, one can also see that this positive segment is large, that derivative is large here, and the derivative falls off slowly here, but does fall towards zero, anyway. So, this looks very much like a hat, so I leave it to you to construct the derivative of this as an exercise, and to verify that, it looks like a hat. So, if you take the derivative of this, would look like a hat, like a Mexican hat; you might one to call it that.

And in fact, it is called the Mexican hat function just for that reason. So, I leave it to you to calculate the admissibility integral for the Mexican, Mexican hat function whether approximately or numerically or exactly also to calculate it is time bandwidth product with a numerically or approximately or exactly. With that little discussion on the admissibility of the Gaussian and the derivatives of the Gaussian, let us see a little more about admissibility here. You see what admissibility essentially requires as we can see in general is that we need to have a sharp enough fall of as you go towards omega equal to zero and a sharp enough fall of as you go towards omega equal to infinity.

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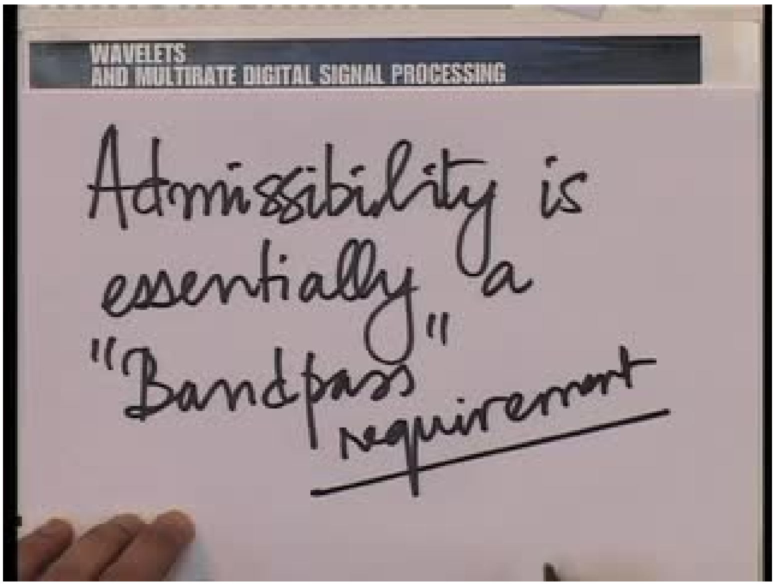

So, we might summarize all this by saying admissibility is essentially a band pass requirement. We want the function to have a band pass character, and this agrees very well with what we have seen so far, whether we take the hour wavelet or whether we take the subsequent the (( )) wavelets or whether we also take the idealize situation that we consider when we brought out the ideal towards which we are moving in the hour situation or in general.

Now, we must remember that here we have still allowed the scale parameter to be continuous. So, admissibility is adequate when you are talking about reconstructing from a continuous wavelet transform, but that is the most difficult thing to do how an earth do you construct this continuum of scale and translation coordinates. Numerically or practically, it is the most silly thing to do. You have one-dimensional function x t and you are trying to construct a two-dimensional continuous pair of parameters tau and s. So, the natural question to ask is can we discretize? In fact, we began with discretizing it. We discretize the scale parameter in powers of two when we looked at the hour multi resolution analysis of that matter any of the direct multi resolution analysis, like for example, the dobash series of m r i's.

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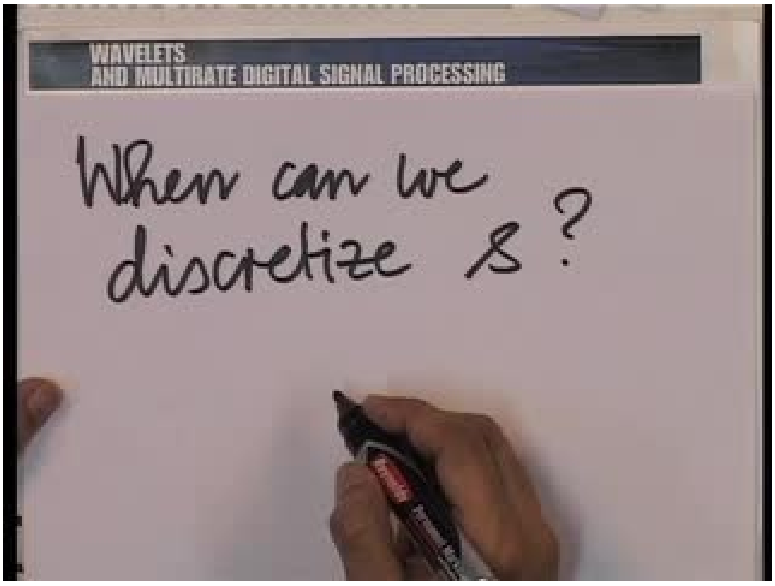

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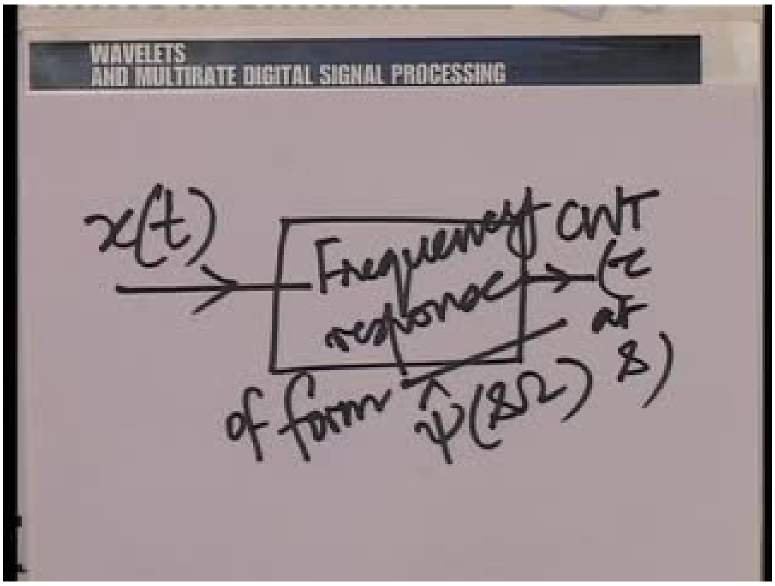

So, the first question that we must now ask is what are the condition when we can discretize the scale parameter? And that is the next question that we shall address, and to answer this question, let us go back to the basic idea that we had when we built the continuous wavelet transform and we try to reconstruct. You see, when we built the continuous wavelet transform, we said that essentially in the continuous wavelet transform, what we are doing is to take the function and put it through a filter with frequency response of the form psi cap s omega complex conjugate here. Of course, there are constants involved, that is not the important thing.

The important thing is that what you get out here indexed by tau. So, suppose, you call the output variable here tau, then this gives you the c w t evaluated as the function of tau at s, at scale parameter s.

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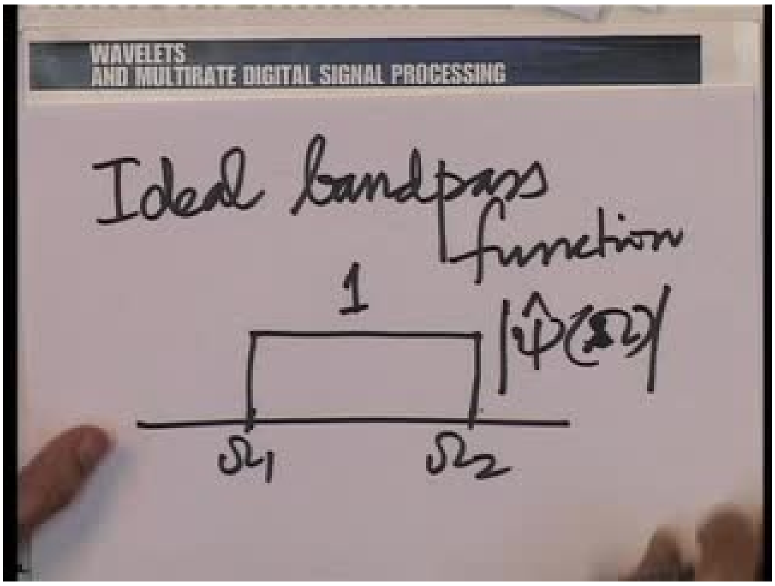

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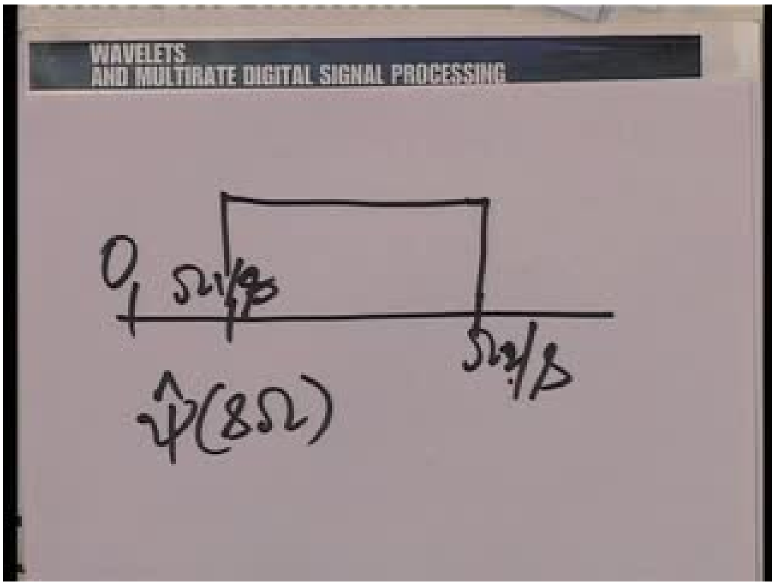

Now, you see, if I want to take a ideal band pass function here, what are we asking for? You see with an ideal band pass function, see between omega one and omega two as we did last time. What happens? Suppose this is psi cap omega, just omega without sin magnitude. How would the magnitude of psi cap of s omegas appear? So, psi cap s omega, you see for example, to fix our ideas, let us take s equal to two; psi cap two omega, which actually go from omega one by two to omega two by two. So, in general, we would go from omega one by s to omega two by s here. All the same omega one and omega two are positive, and therefore, these two will still continue to remain positive. All beat may be lesser or more depending on the value of s, then omega one and omega two respectively, but they will still be in the positive side of the spectrum away from zero.

So, essentially, what is going to happen with the change of s is to move this band of the band pass filter along with the positive part of the spectrum, and the natural condition that we should expect for being able to discretize the scale parameter is to ensure that we are covering the whole spectrum, and there again, we have a natural choice of how it is discretize. You see when we scale by a factor of s, we are also scaling as we see the center frequency and the band. So, there is a logarithmic change. So, natural kind of discretization to consider for the scale parameter is a logarithmic discretization. In fact, we being doing this all this while. We being considering a discretization in powers of two, but now, you do not need to restrict ourselves to power of two.

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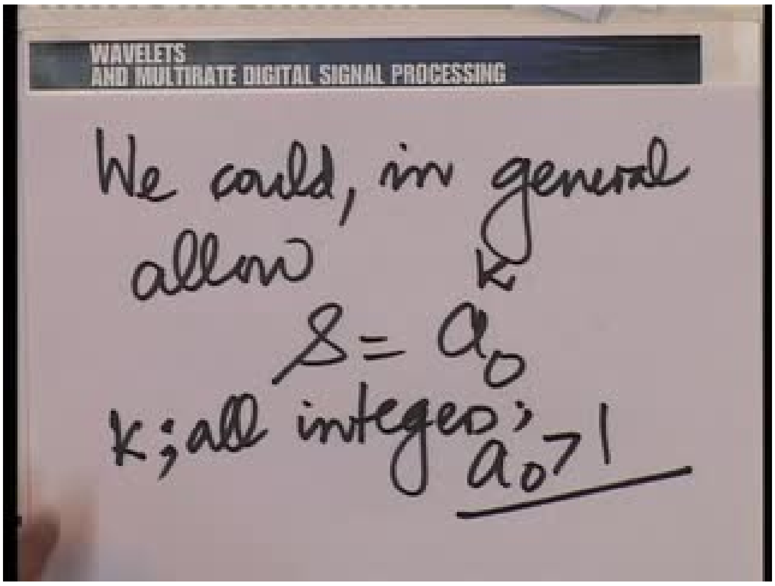

We could, in general allow s to take the form a zero to the power k, where k runs over all integers, and a zero is the quantity greater than one. For example, a zero could be 2 where we consider the dyadic case; a zero could be 3, 1.5 anything more than one. You see the more than one is not a problem. Even if a zero is taken to be less than one but positive, for example, 0.5 will you run k over all the integer. Anyway, you are considering an equivalent a zero which is more than one. What I am saying is if you insist on taking a zero equal to half, and then, run k over all the integers are same as taking a zero equal to two and running over all the integers. So, your mind is well restrict a zero to be greater than one.

So, now, we shall do that; we restrict a zero to be greater than one, and we will consider all the a zero to power k with k integer, and in the next lecture, we shall see when the discretization of s in that form is acceptable for reconstruction. Thank you.