Advanced Digital Signal Processing – Wavelets and Mutlirate
Prof. V. M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture No. # 22
Reconstruction and Admissibility

A warm welcome to the twenty-second lecture on the subject of wavelets and multirate digital signal processing. We continue in this lecture to build upon the theme of the short time Fourier transform and the continuous wavelet transform, from the point of view of reconstruction.

In the previous lecture, we had briefed about the meaning of the short time Fourier transform and the continuous wavelet transform, a generalization of the dyadic wavelet transform, that we had built in the Haar multi resolution analysis or in some of the other examples of the daubechies filter banks and so on.

We saw that in the short time Fourier transform and in the continuous wavelet transform, the two parameters characterizing the transform or indexing the transform were continuous. In the short time Fourier transforms, it was the translation and modulation parameters that were continuous; in the continuous wavelet transform, it was the translation and dilation or scale factors that were continuous. In the short time Fourier transforms, the modulation and translation both extended all over the real line.

In the continuous wavelet transform, the translation extended all over the real line. The scale parameter extended only on the positive real line and with that little recapitulation of what we did in the previous lecture; let us bring out the theme of the lecture today.

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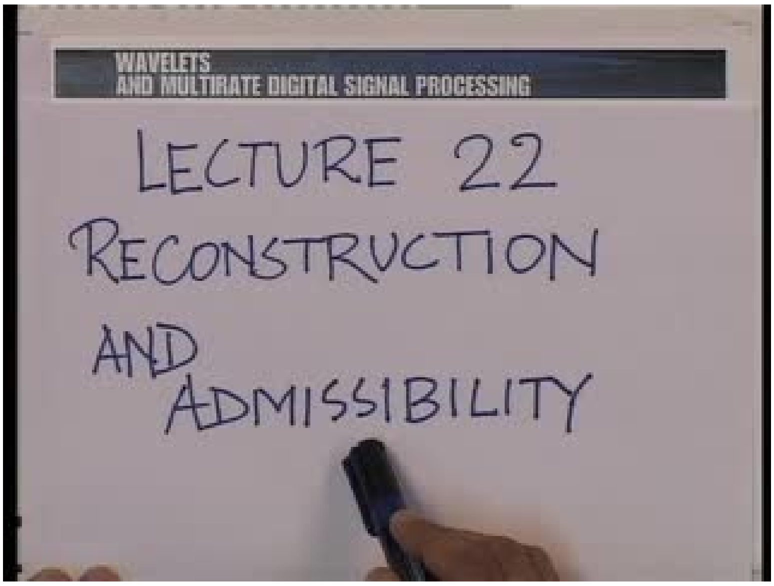

So, in today's lecture, we intend to talk about reconstructing the signal from its transforms, whether the short time Fourier transform or the continuous wavelet transform, and in that process, we wish to bring out the very important notion of what is called admissibility; recall that in the previous lecture, we had raised the question of what qualifies to be a wavelet, we said the Haar function, the Haar wavelet, of course, qualifies to be a wavelet, but what in general characterizes a wavelet function. We had partly given the answer yesterday, we said that a band pass character is essential, but today we need to qualify and build that concept in greater depth and that is what we intent to do.

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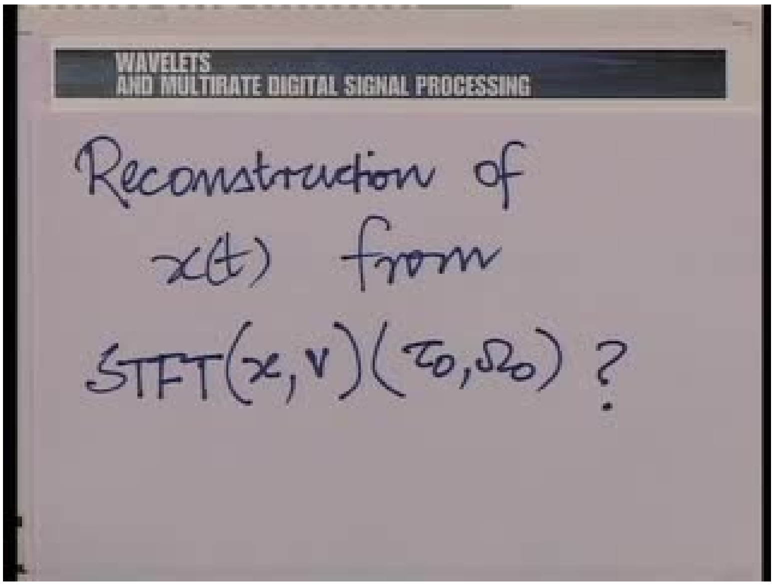

So, with that little background, let us proceed then to ask the first question - how do we reconstruct a signal, from its short time Fourier transform? So, let us recall the notation, that we had used for the short time Fourier transform; the short time Fourier transform of the function x with respect to the window v; these are the secondary arguments. And the primary arguments at the translation and modulation given by tau naught and capital omega naught; so, this is the question how do we reconstruct x t from this.

In order for a transform to be useful, it must be invertible, otherwise we are losing something in the transform and therefore, we need to ask this question and answer it satisfactorily anyway.

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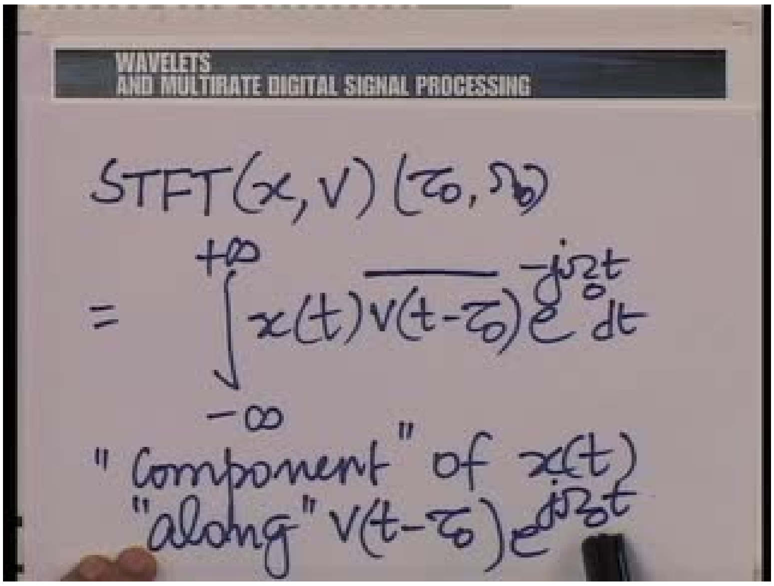

Let us, therefore, recall once again what this quantity was so STFT x, v a secondary and tau naught omega naught has primary arguments was equal essentially to a dot product, the dot product of x t with a suitably translated window and then modulated, we call that v, in general must put a complex conjugate here and that means, that we must make this e raise the power minus j omega naught t.

Now, in a certain sense, we realize that, this is like the component, it is a dot product of x t with a translated version and modulated version of the window; so, it is like a component, component of x t along something you know could be with that way.

So, it is like a component and one simple way to reconstruct from components is to take each component multiply it by I so-called unit vector or if not a unit vector a properly normed vector, in the direction of that component and then some overall such components. We use this idea before, we have used it in understanding the Fourier transform, we have used it in understanding the parseval's theorem and now, we use it also to reconstruct x t from its short time Fourier transform.

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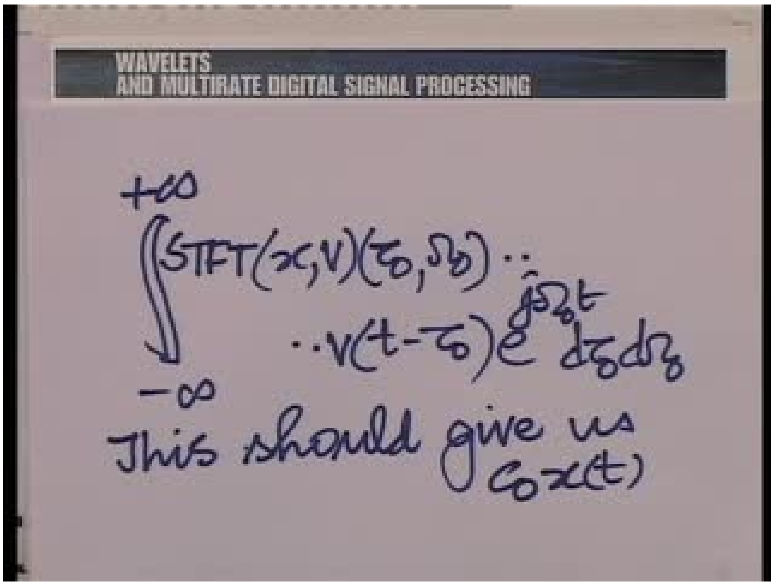

So, suppose we perform that experiment suppose we simply use the same idea of taking this to be a component multiplying it by a unit vector in the direction of that component and summing overall such components, what should we get or in other words, what should we do to get back x t by this process, that is what we should now ask. So, suppose I say take the following take STFT x v tau naught omega naught multiply it by v t minus tau naught e raise to the power j omega naught t and integrate overall tau naught and omega naught, what should we get.

We should ideally to within a constant hopefully get x t; this should give us a multiple of x t let say, c zero times x t and we shall now put down conditions for this to happen; in fact, it is easy all that we need to do is to expand this STFT and then take note of what is happening.

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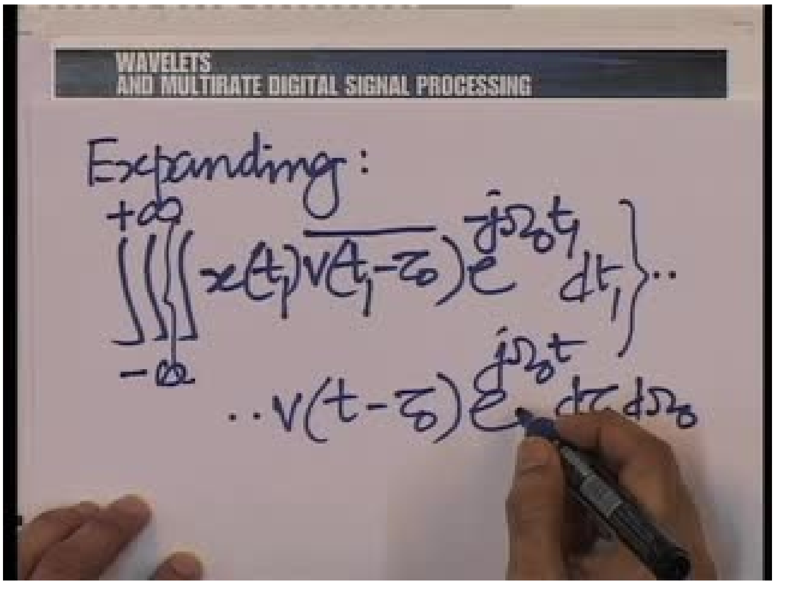

So, expanding we have a triple integral, one should not be frightened of the triple integral it just requires a little bit of rearrangement afterwards; so, the inner most integral is essentially to calculate the STFT and keep in mind, that we have a t outside, we should give a different name to this variable of integration here.

So, this is first an integral with respect to t one and then this whole thing is multiplied by v t minus t zero e raise to the power j omega naught t d tau naught omega naught, this is a kind of integral that we need to construct.

Now, let us make an observation on this, you see, we will first make an observation on this part here; so, let us interchange the order of the integrals incidentally all these integrals are from minus to plus infinity, so I have not written that down explicitly; let us first look at this integral, we, we should take one integral at a time, so it will, it will be easier to evaluate the integral on tau naught on omega naught first and then take up the integral on t 1 and so on.

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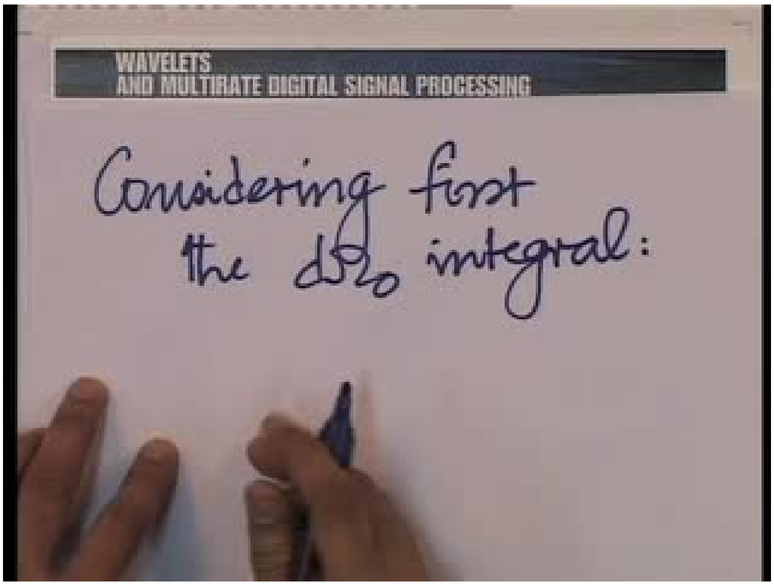

So, here let us consider the integral on omega naught, because that is the easiest one to deal with after all, there are only these two terms are depend upon omega naught and therefore, they would be dispensed off first, considering first the d omega naught integral.

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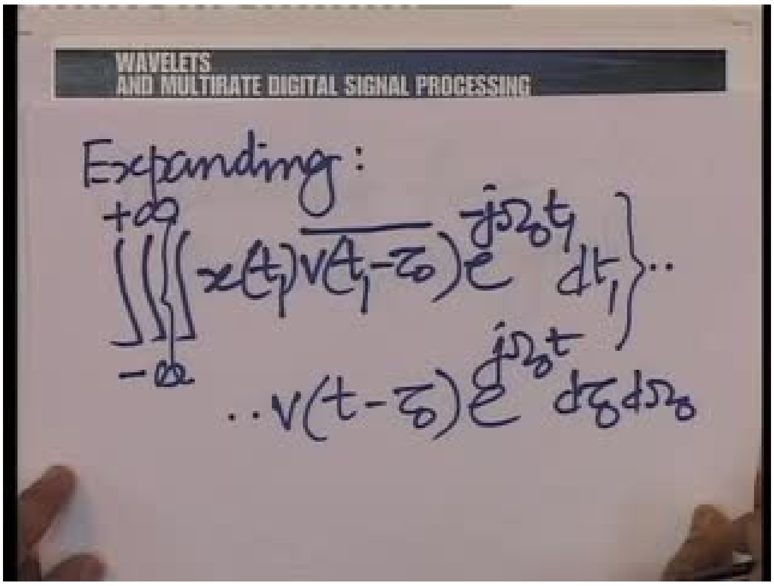

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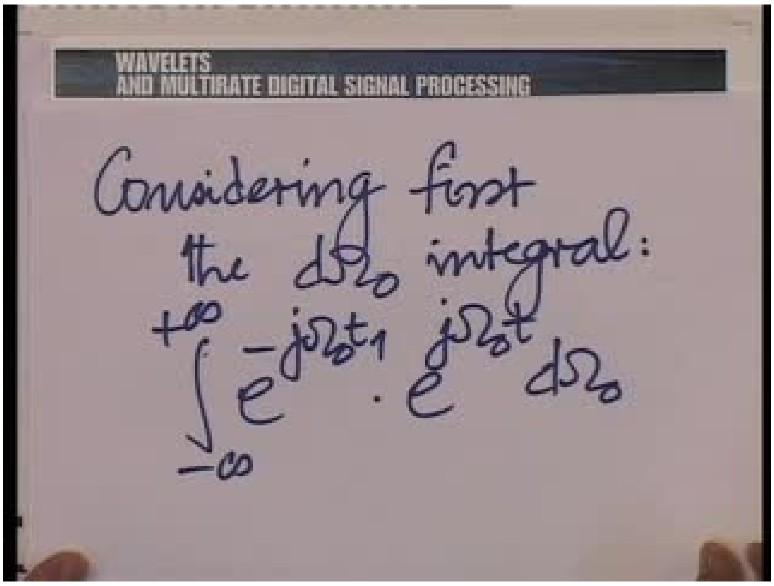

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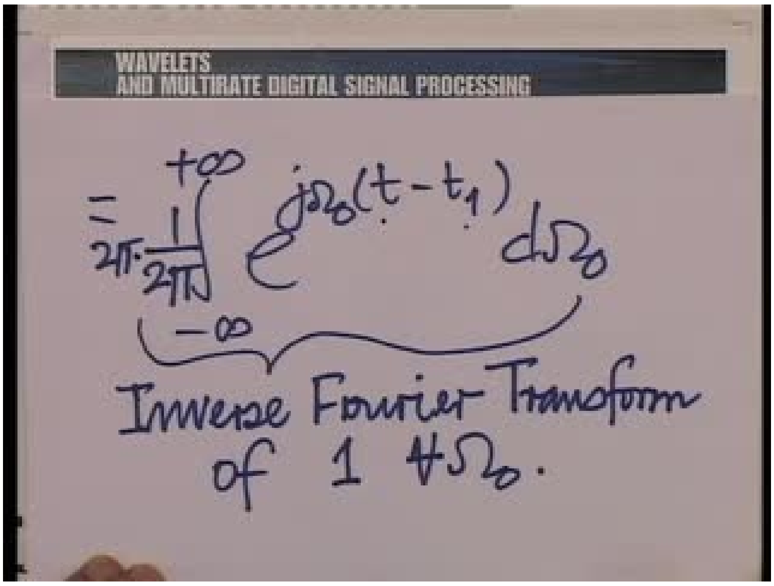

Now, let me put before you again the terms that depend on capital omega naught, so none of these terms are dependent can on capital omega naught, only these two do; so, let us isolate those two terms and dispense off that integral. We have essentially e raise to the power minus j omega naught t 1 times e raise to the power j omega naught t d omega naught from minus to plus infinity and if we go back to our very fundamental principles of the Fourier transform, this integral is very easy to evaluate; after all, when we combine this, what we get is the following, we get e raise to the power j omega naught t minus t one d omega naught

Essentially, if I multiply and divide by 2 pi, this is a very familiar quantity; you know what we are doing here effectively, if you look at it, is taking the inverse Fourier transform; you know if you interpret this carefully, it is the inverse Fourier transform of a quantity of a function which is 1 for all omega naught and that happens to be the Fourier transform of an impulse; so, this part is essentially the valuation of the inverse Fourier transform of a function which is 1 for all omega naught.

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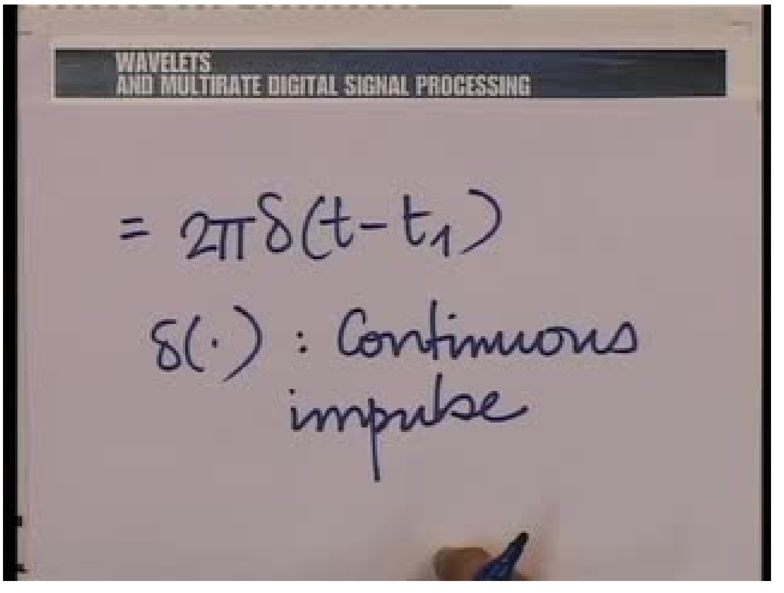

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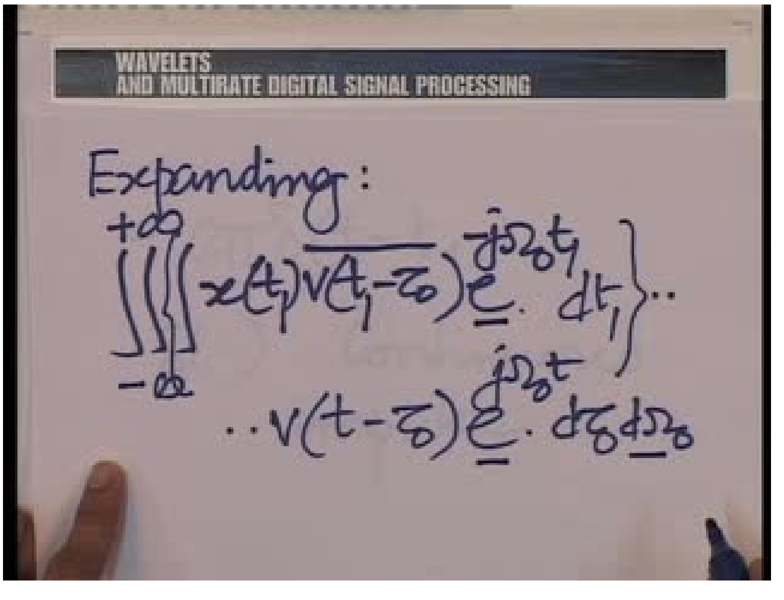

So, that would essentially be the impulse placed at t minus t 1 and of course, this factor of two pi is there, so I can write that down and do a dispense entirely with the integral on omega naught. So, this becomes two pi times delta t minus t 1, where delta is the continuous impulse, unit impulse if you like and now I substitute this, so when I substitute this, one more integral vanishes; let me substitute this back into the original integral that I had the triple integral; as I said the triple integral is no longer so frightening.

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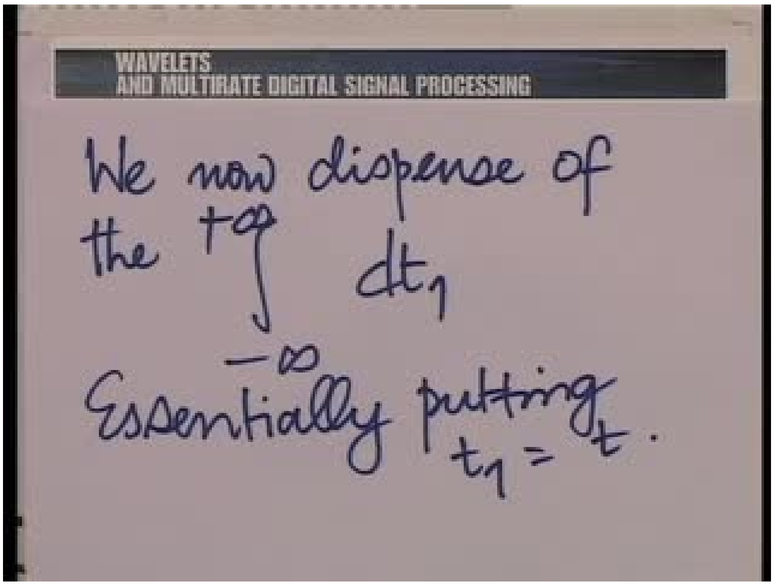

So, all this, this integral on omega naught, let me underline the terms here, this, this and this have been dispensed off with an impulse located at t minus t 1 and now, the natural choice is to dispense of the integral with respect to t 1. How would we dispense off the integral with respect to t 1, we now dispense of the integral with respect to t 1 and doing that is essentially putting t 1 equal to t, that is the way to dispense of it; after all, when we multiplied by an impulse at t minus t 1, what we are saying is simply substitute t 1 by t, not forgetting that the factor of two pi there.

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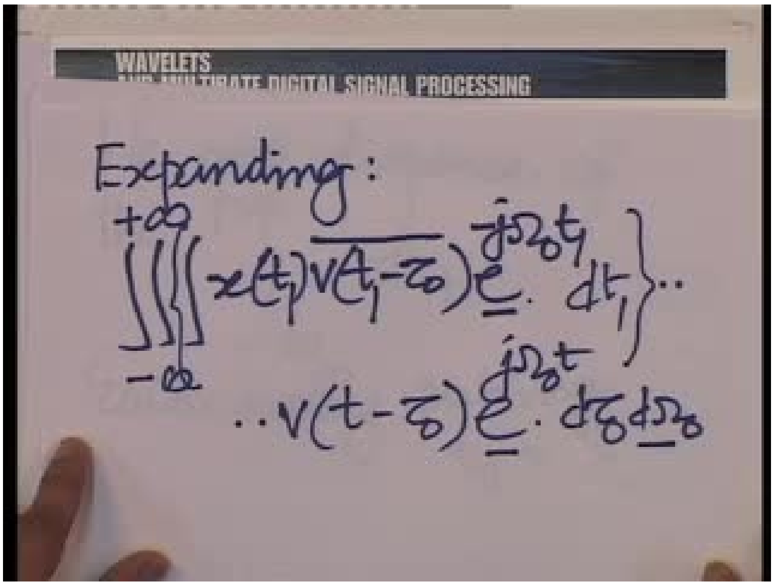

So, in fact without rewriting all of that, let me straight away do that job, let me put before you that triple integral again; now, when we take this integral on t 1 and when, when we note that this part has been dispensed off, these three terms have been dispensed off with an impulse located t minus t 1, what we need to do is to replace all these t 1, these two t ones by t's and the impulse goes away.

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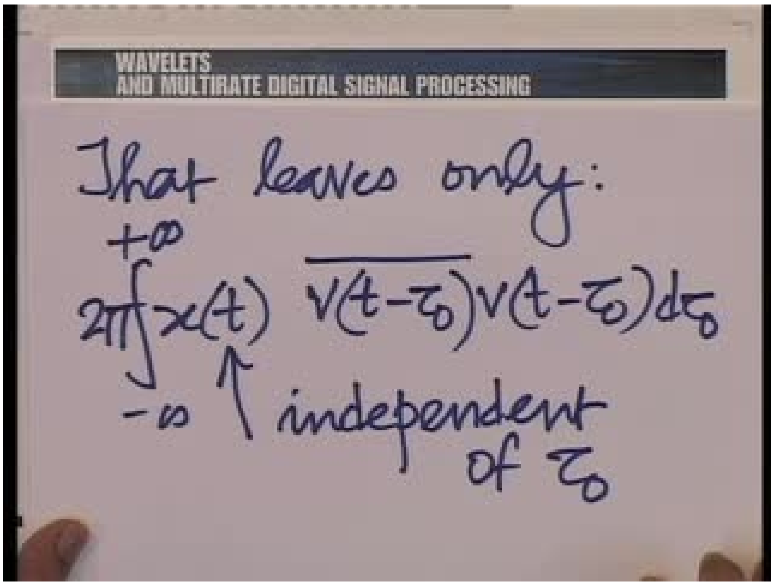

So, I shall now write down what is left, that leaves only one integral now, so that leaves only the integral on tau naught and what is that integral on tau naught x t times v t minus tau naught bar v t minus tau naught d tau naught with the factor of 2 pi.

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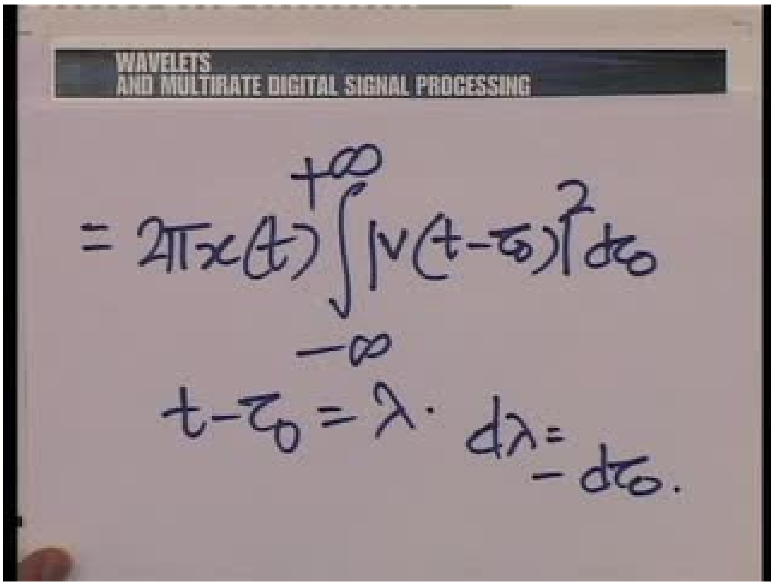

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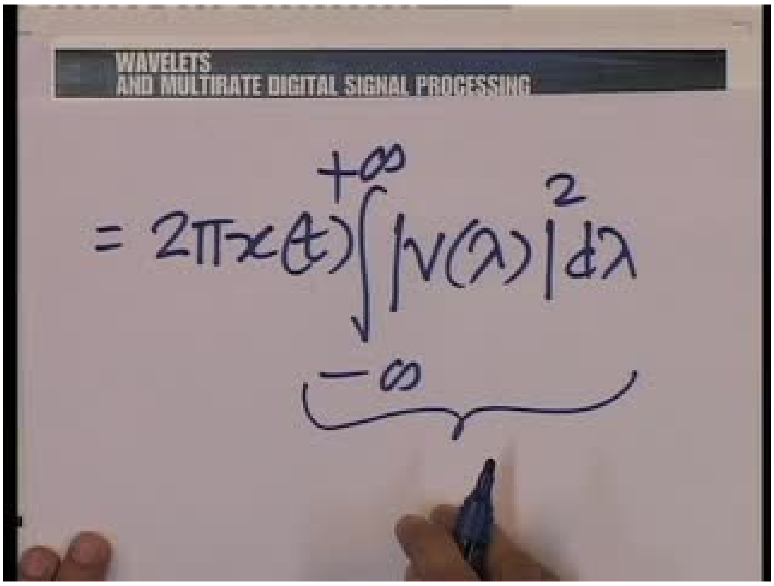

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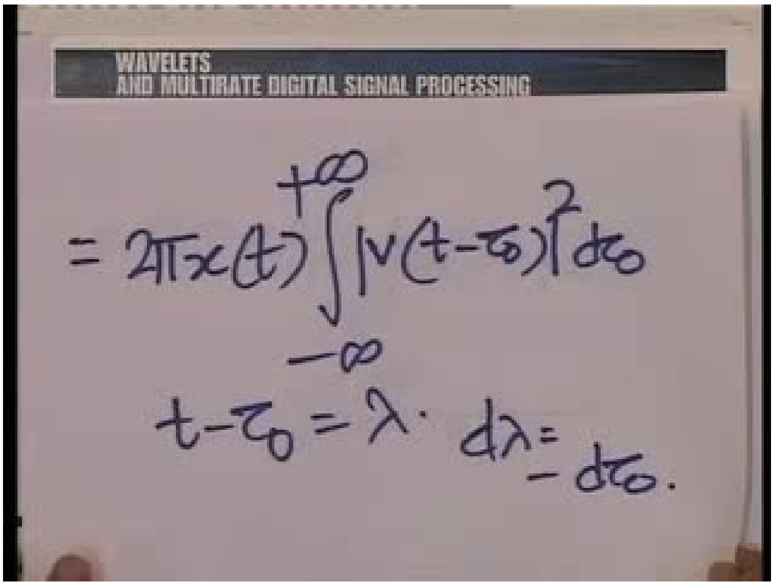

Now, what is easy, after all x t is independent of tau naught, so I can bring it outside the integral and that essentially leaves an integral of this quantity for me, let me write that down explicitly; so, this becomes equal to 2 pi x t times integral minus to plus infinity mod v t minus tau naught the whole squared d tau naught and if you only care to make the replacement t minus tau naught is lambda, where upon d lambda is minus d tau naught and as you note, when tau naught goes from minus to plus infinity, lambda would goes from plus to minus infinity but with a minus sign here; if we observe that minus sign and the order of the integration, we can easily show that, this is equal to 2 pi e x t times integral from minus to plus infinity mod v lambda square d lambda and this is a quantity entirely independent of t, remember that when we made the substitution we did it for a fixed value of t and therefore, this process is justified, essentially this is the l 2 norm squared l 2 norm of v which is finite; so, we are in very good shape.

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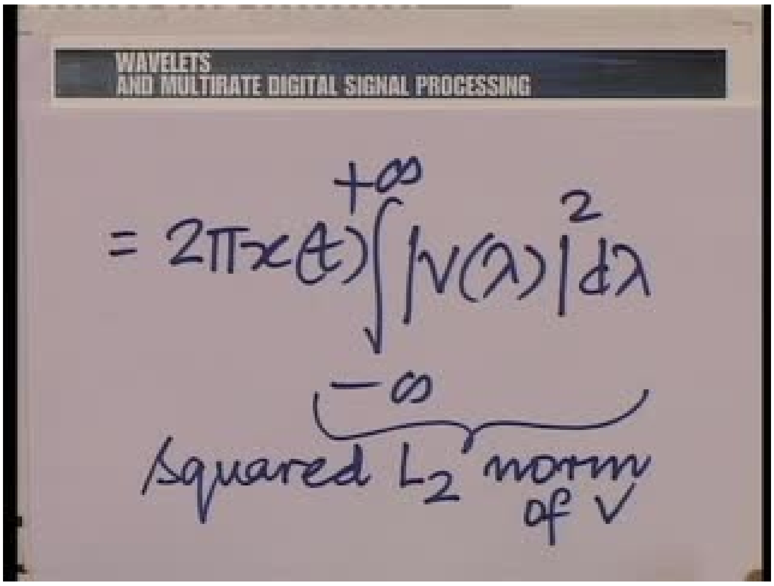

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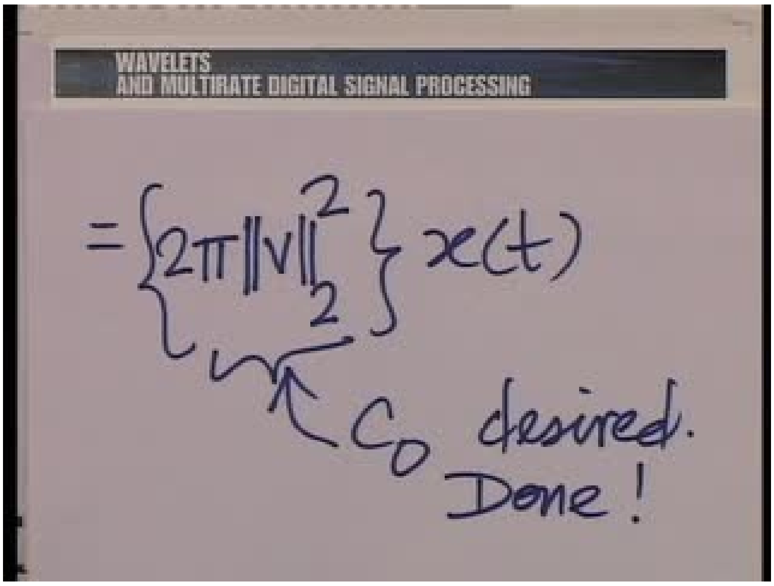

It is very clear that, this reconstructs x t to within a constant; so, this is essentially equal to two pi norm in l 2 of v the whole squared times x t and this is the constant c 0, that we desired, so job is done that is very nice. We reconstructed x t from its short time Fourier transform, in fact, we did not have to work very hard, we did not have to do anything more than just taking each component multiplying by the unit vector in the direction of that component, summing up conceptually, I mean summing is replaced by integration here because tau naught and omega naught are continuous quantities spreading all over the real line; so, when we integrated overall tau naught and omega naught essentially added up all such component multiplied by vector we got back x t to within a constant easy.

Now, of course, we need to ask a few more questions and those questions are little more difficult to answer; we shall not even try to do them all at once. You see here, what you are doing is to take one tile with a fixed shape, a tile which is constructed out of v with the spread given by the time variance of v in the horizontal direction and the frequency variance of v in the vertical direction; so, we are taking the tile and moving it continuously along the time frequency plane and of course, as expected there is a tremendous amount of redundancy, because we have a function in one variable t or time represented by a function in two variables tau naught and omega naught and we are using all possible tau naught and omega naught to reconstruct it. So, we should definitely be able to do that, unless we have chosen are we very badly and as you can see all that you need is that ultimately v b a window function, because otherwise you know its spread in time and spread in frequency are not finite somewhere and that it be an l 2 r function very modest requirements.

So, as you can see the short time Fourier transform was something good to start with, when people started looking at time frequency or joint domain approaches, but it would be more interesting to ask, can we discretize these parameters then it is useful, otherwise this is by itself not so useful by asking for continuous tau naught and omega naught that is quite a demand. So, it would be useful to have discrete tau naught and omega naught, can we take discrete steps that would be a real tiling in the true sets and the answer is yes we can take discrete steps but at this moment, we shall not go into that issue. Safe to say that if you wish to reconstruct x t from discrete points on tau naught and discrete points on omega naught, one can follow an approach similar to what we have just done, namely put down a reconstruction based on multiplying the component by the so-called vector in the direction of that component, summing overall here it is literally summing, because you have discrete tau naught and omega naught literally summing overall these discrete points tau naught and omega naught and then manipulating the behavior of v, so that, when we so sum, we essentially get a constant multiple of x t.

So, that would be the approach to be followed, I shall not dwell further on this issue at the moment, because we wish to go to the continuous wavelet transform now and before we go again to the reconstruction issue in the wavelet transform, we need to answer one question that we had partly discussed yesterday, what kind of tiling is possible with the continuous wavelet transform or what kind of tiling does it give you and let me try again bring that out a little more clearly now.

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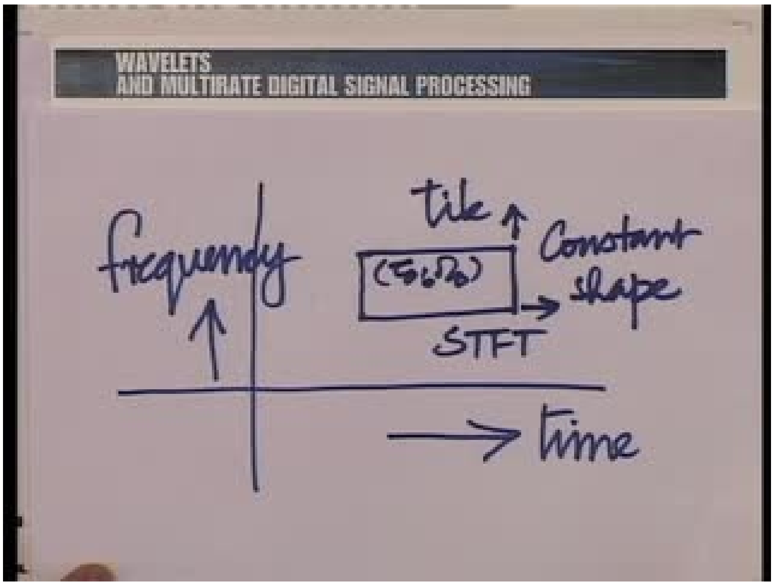

So, yesterday we saw that in the time frequency plane, the short time Fourier transform essentially moves a time of constant shape, constant shape means its height and breadth are fixed as a function of tau naught and omega naught and essentially you should visualize this tau naught and omega naught moving horizontally and vertically; so, it is this very tile just graphically to explains this very tile that moves like this.

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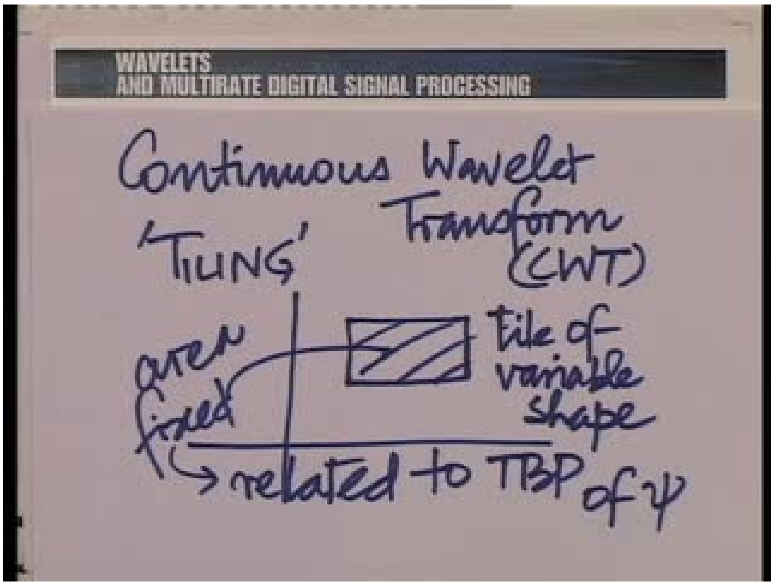

Now, in the continuous wavelet transform, we have a slightly different kind of tile and we understand what we are doing in the c w t, we shall use c w t to abbreviate the continuous Fourier transform. Now, here there is tile of variable shape, the area of the tile is fixed and the area of the tile is related to the uncertainty product or the time bandwidth product; so, abbreviate the time bandwidth product by t b p, the t b p of psi. You know how to calculate the area of this based on the time bandwidth product of psi.

What happens as we change and of course, here we must interpret this tile as moving based on tau naught and s naught. Now, when one increases s naught, one is essentially compressing or expanding, when you know depending on if you increase s naught or decrease s naught, if you increase s naught, you are essentially expanding the function in time and therefore, compressing it in frequency.

Now, of course, s naught always takes on positive real values, so s naught is indicative of frequency in a broad sense and in fact, we also note it taking the example of the Haar wavelet yesterday, that when we make s naught large, that means, we are essentially compressing on the frequency axis; we are compressing both the band or the bandwidth and the center frequency, when we reduce s naught, when s naught goes towards zero notionally, we are going towards higher frequencies; so, you know, for example, if s naught is equal to 2, we have compressed the wavelet by a factor of 2, in the frequency axis but expanded it on the time axis.

On the other hand, if we put s naught equal to half, so taking a value less than 1, then we have expanded the function psi in the frequency sense and compressed it time, so we have gone to higher frequencies; in fact, with s naught equal to half, we were double the center frequency and the band; this should be understood very firmly. So, in fact, let us make a note of this, what we are saying is as follows increasing s naught means expanding in time or compressing in frequency and vice versa.

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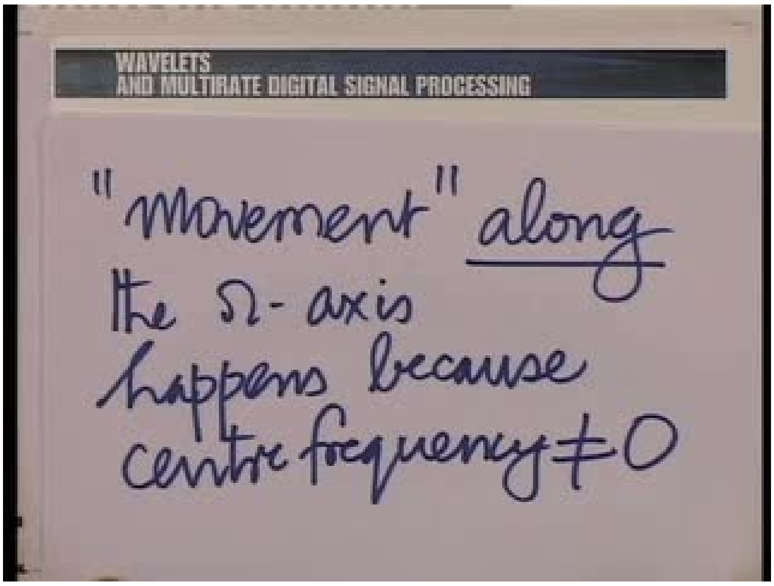

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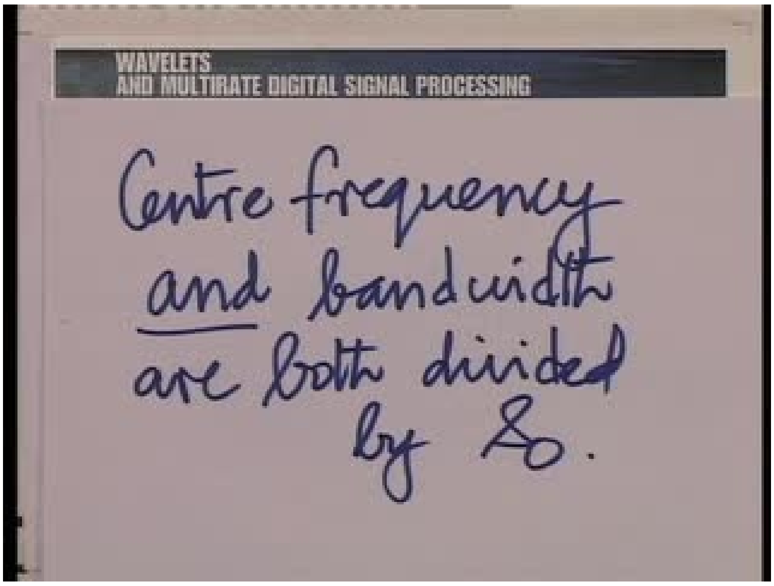

So, I do not need to write that again, decreasing s naught, means, contracting in time and expanding in frequency, I do not need to write down again. So, movement along, so to speak the frequency axis happens, because the center frequency is non zero and the other observation that we have, is that, the center frequency and bandwidth are both divided by s 0; needless to say, the tau naught has no effect on the magnitude of the Fourier transform tau naught is a parameter that controls only the time behavior, it has nothing to do with frequency behavior.

So, here also we have a separation tau naught controls movement on time or the translation parameter; s naught controls movement on frequency, but in an indirect way and that is what we now need to understand, how can we reconstruct x t from its continuous wavelet transform, allowing the entire range of tau naught from minus to plus infinity and the entire range of s naught from zero to infinity.

Now, the only thing is, we will have to allow a concession here, either you can call it a concession or an additional provision; s naught is not directly frequency. So, when we wish to reconstruct x t from its continuous wavelet transform, we shall need to make an allowance for some weighting, differential weighting based on the parameter s naught, what I mean by that is the following.

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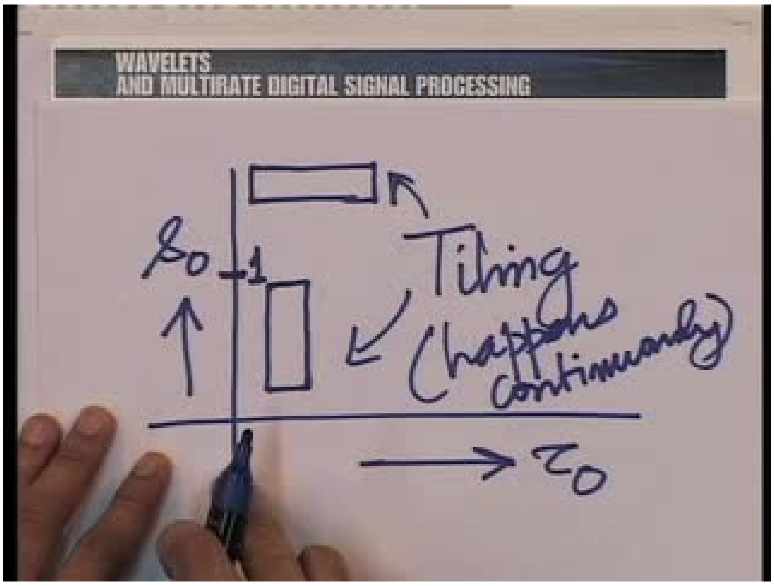

You know now coming back to the tiling what we are saying is the following. In the continuous wavelet transform, larger s naught means smaller frequency and smaller s naught means larger frequency, of course, the distinguishing point is 1; so, when s naught, 1, there is no contraction or expansion, when s naught is less than 1, there is an expansion in time and contraction in well; when s naught is less than 1, there is an expansion in frequency and contraction in time; when s naught is greater than 1, there is a contraction in frequency and expansion in time and in the time frequency plane what we are doing is effectively something like this. You see, so s naught greater than 1, we contract in frequency, but expand correspondingly in time proportionally keeping the area the same and when s naught is less than 1; we have expanded in frequency but contracted in time, keeping the area the same.

So, this is the kind of tiling that we have and we must visualize this as happening continuously. You know what we are saying is, you should think of these tiles as either rubber tiles or elastic tiles. So, when they move along the time axis, there is no change of area; when they move along the s naught axis, then they start expanding vertically, as they come down s naught and they start contracting vertically as they go up s naught and as they contract and expand in this manner, they reverse expand or contract in the tau naught direction in the translation direction; so, in the time direction I mean.

So, of course, what I am saying here is you know you, you must interpret this figure, it is a little require, the little bit of thinking what I mean by this figure, here is that the horizontal is the time axis, the vertical is the frequency axis. And as we increase s naught, we are going towards lower frequency, you know so there is a little trickier, I mean may be what I should do is to redraw, I, I have try to overload this figure; this figure looks overloaded with more than one parameters.

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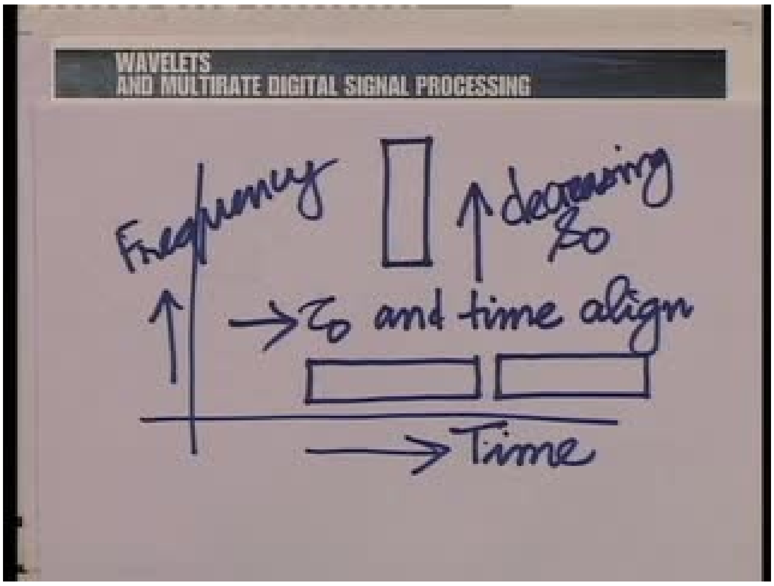

So, perhaps we should redraw it, we will draw it like this; we will say on the time frequency plane. So, if we take this to be the time frequency plane and not tau naught and s naught, this is time and this is frequency. Then, at lower frequencies we have broader functions in time, at higher frequencies we have narrower functions in time and correspondingly broader functions in frequency, this is the situation and of course, what we are saying is, this is decreasing s naught and of course, tau naught follows the same pattern as time; so, tau naught and time align. As one moves along tau naught, one is moving along time, as one move along s naught, for decreasing s naught one is expanding or when one increases s naught one is contracting, so this is the whole situation now.

Now, with this little background, we see there is a bit of asymmetry tau naught is and therefore, we allow tau naught to go as it is, we take a cue from the short time Fourier transform; in the short time Fourier transform, we could just let tau naught, go as it is in reconstruction; so, we would not worry about tau naught here either, but s naught poses a problem.

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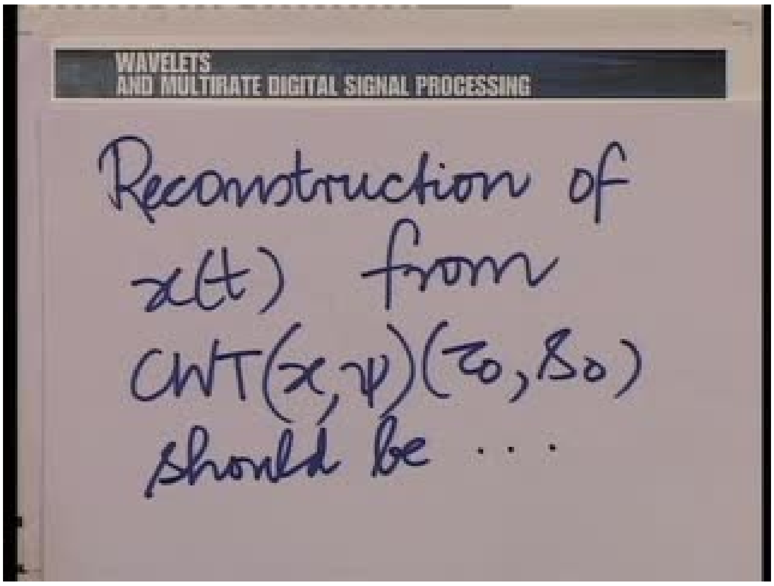

So, we need to allow a weighting factor here, when reconstructing using the same principle of component multiplied by vector and that weighting factor must depend upon s naught here so. In other words, what we are saying is that, the reconstruction of x t from c w t of x with respect to psi, evaluate that tau naught and s naught should be and we have again a triple integral but with the difference.

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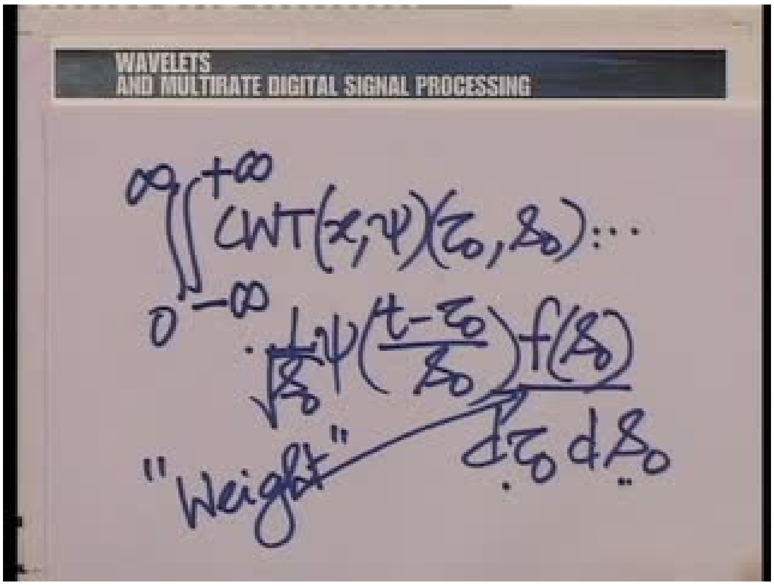

So, before we write the triple integral, we first just write down this c w t x psi tau naught s naught multiply this by the function the unit vectors so to speak, but as I said with a weighting factor; so, the unit vector or the vector in that direction is psi t minus tau naught by s naught and we agreed that we must normalize this, so one by the square root of s naught, a weighting factor, let us call that weighting factor a function; let say f of s naught d tau naught d s naught, we have agreed that we do not need to do anything about tau naught, but we do need to use a weighting factor for s naught which is f of s naught, a weight and this is integrated with respect to tau naught and s naught.

Now, d tau naught is inside tau naught would go from minus to plus infinity, but s naught would go from zero to infinity; there is a little bit of difference. And we need to choose this f of s naught in such a way, that we can get reconstruct, that is what i am trying to say. A little difficult to understand right in the initial phase, but once we go through the working, you will see what I mean.

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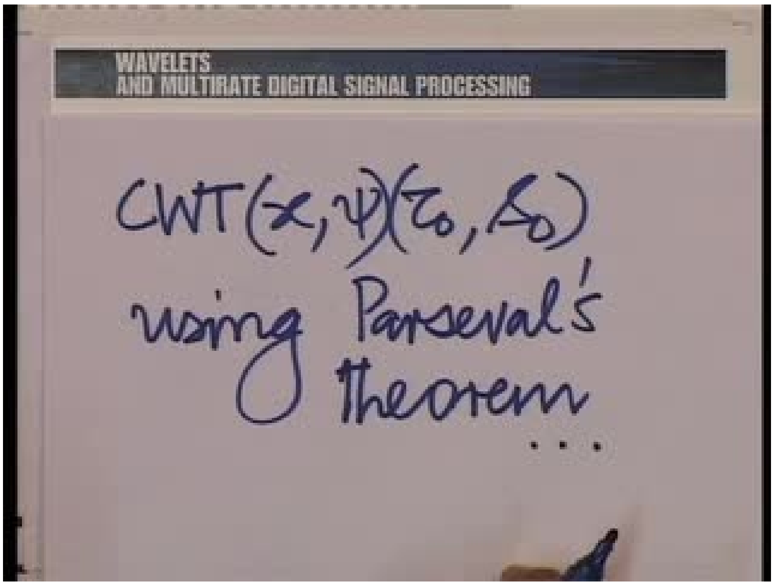

Now, what we shall do, is instead of trying to use the time domain expressions here, we will take recourse to the parseval's theorem; you will recall that we had written down the parseval's theorem for this yesterday and we shall therefore, express c w t using the parseval's theorem in the frequency domain.

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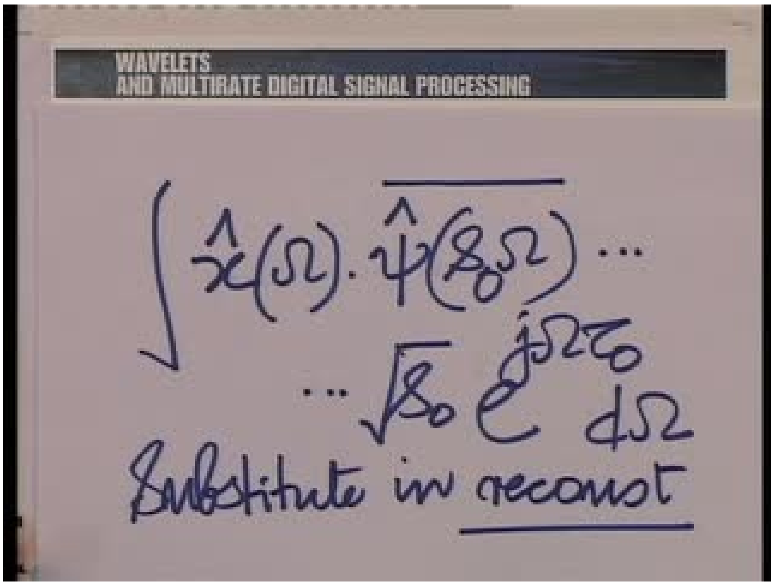

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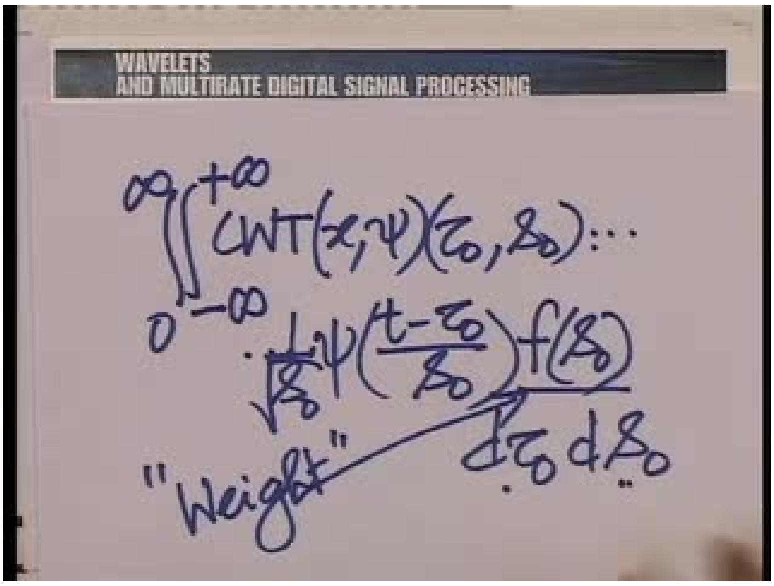

So, the c w t using parseval's theorem is essentially x cap omega psi cap s naught omega complex conjugate multiplied by the square root of s naught and then multiplied by e raise to the power j omega tau naught integrated overall omega, this is what the c w t was and we had also interpreted this yesterday, we have given it a very interpretation based on a filtering operation; we shall be using that interpretation later, but for the movement let us substitute this in the expression of reconstruction. So, substitute in reconstruction and when we do that, let me just put before you, the reconstruction integral once again for convenience, I am going to substitute this by that frequency domain expression, let me do that.

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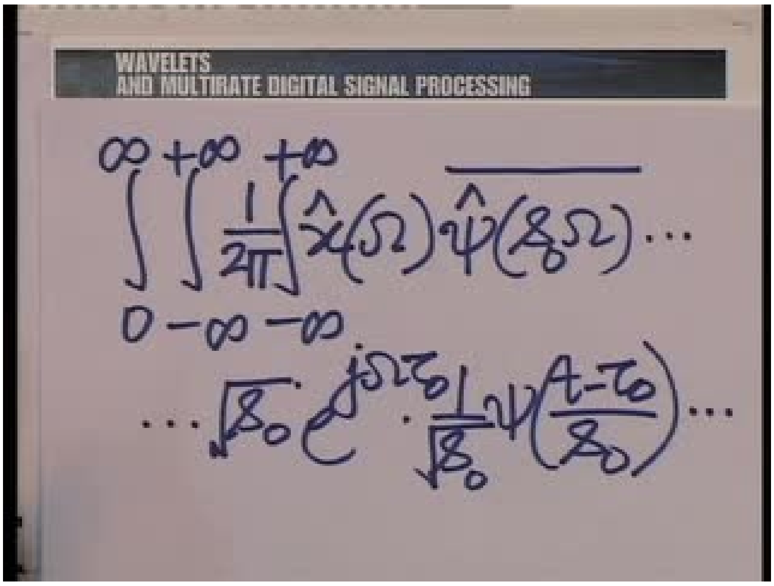

So, I have now again we get you know one, one should not as I said be frightened of a triple integral, because much of it will collapse very soon, but we do get a triple integral and we will write it down bravely, so we have a 1 by 2 pi there x cap omega.

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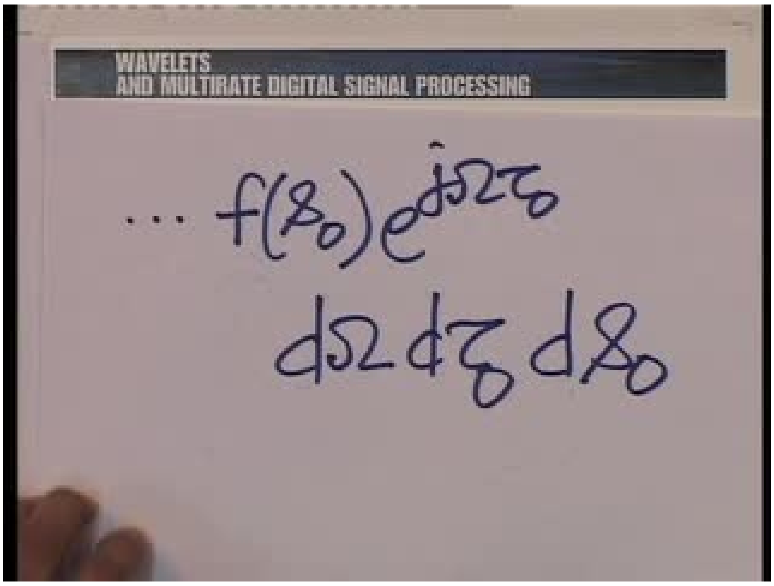

So, I think we have to, it is a big quantity, we will have to continue this so we have all these terms here and then we also have and I do need to write down them separately and we have a triple integral d omega first, then d tau naught d s naught looks formidable, but as we will seen a minute is not all difficult, to evaluate if we start collapsing the integrals one by one as we did in the short time Fourier transform; let us bring out a strategy for this collapse.

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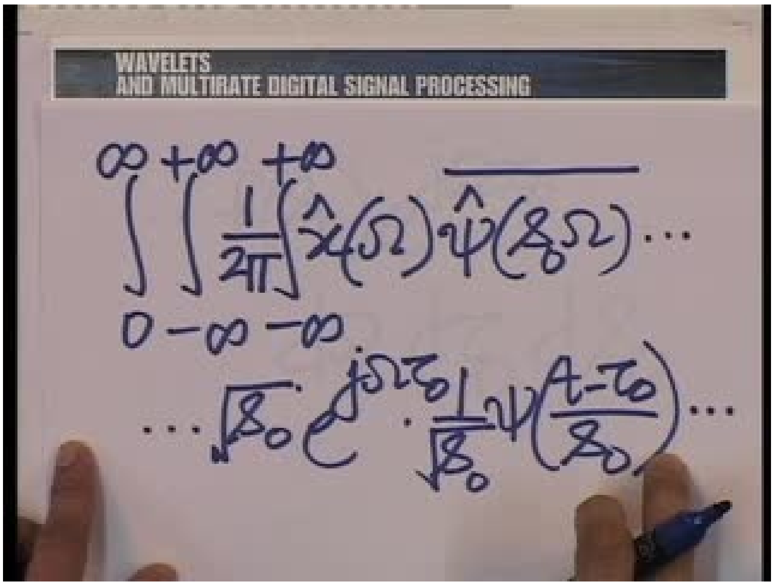

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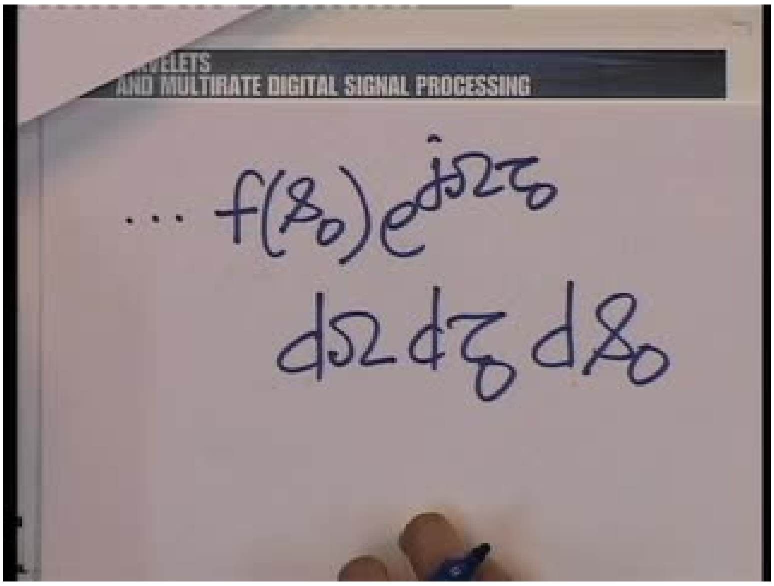

So, let me put flush the whole integral triple integral before you just for a minute; this is the triple integral, an integral on s naught, then the outer most is on s naught, the next on tau naught, the next on omega x cap omega times all this followed by this and these elements of integration.

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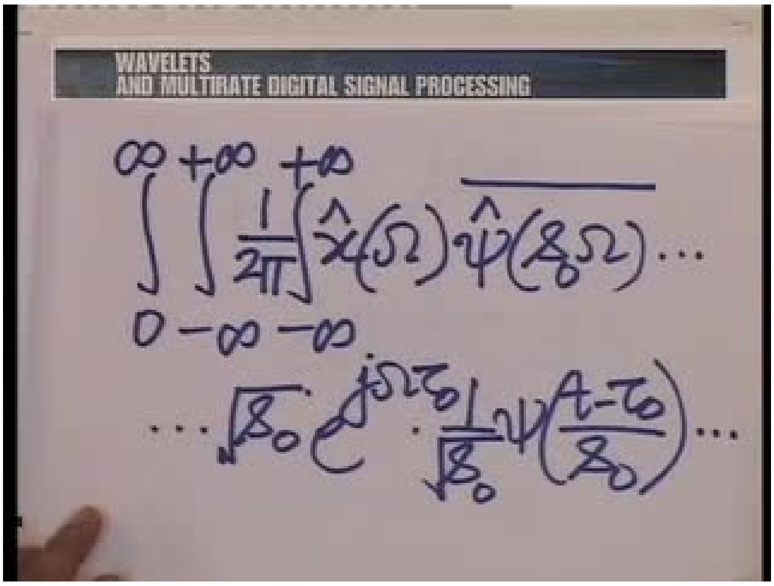

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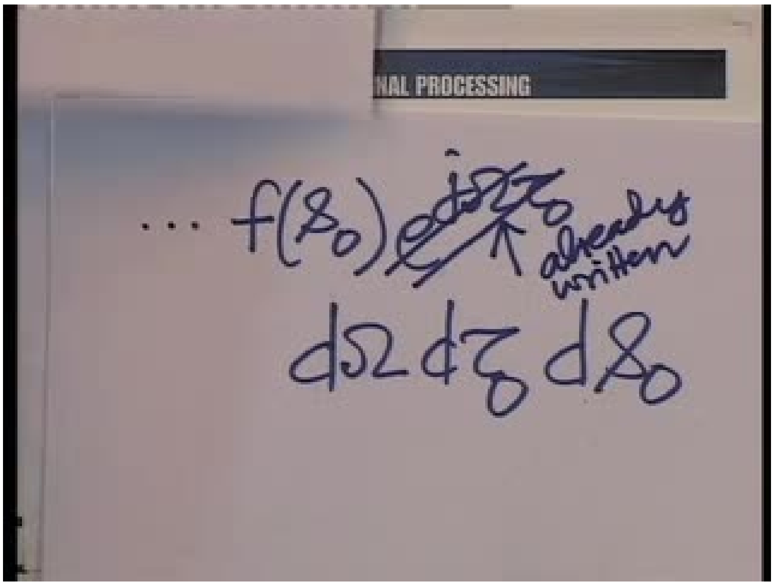

Now, let us observe this carefully, you know as we did in the short time Fourier transform, it will of course, be useful to change the order of integration, because there is a collapse of terms based on the fact, that only a few terms depend on specific parameters; now, for example, if you look carefully at tau naught, after all what we see, is that, it is only this term, this term and this term, that depends on tau naught here.

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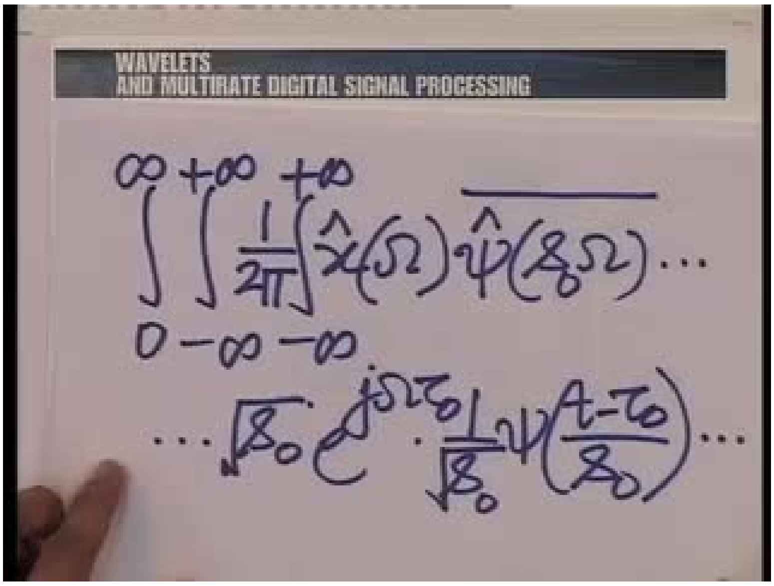

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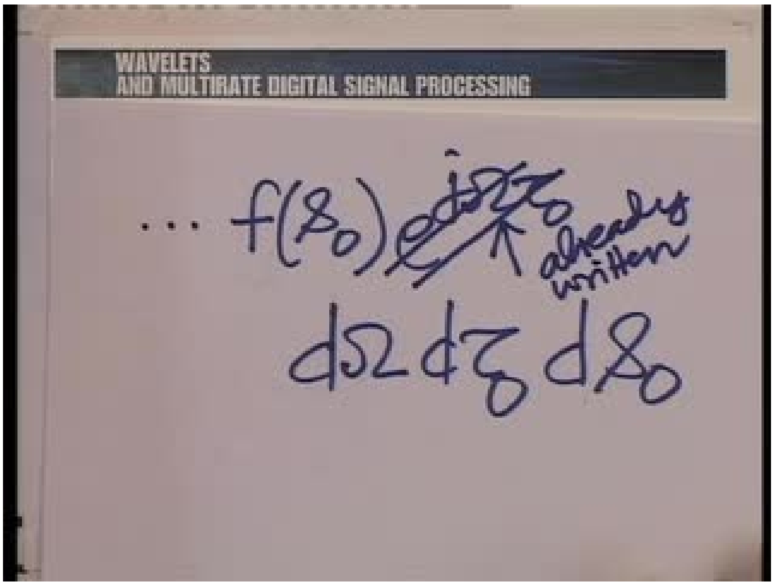

So, in fact well actually there is a little correction, I do not need this e raise to the power, I have repeated this term here, so already written; so, I do not need to write this twice. So, the tau naught part is the easiest to deal with its only this and this that depend on tau naught; so, let us take care of that first.

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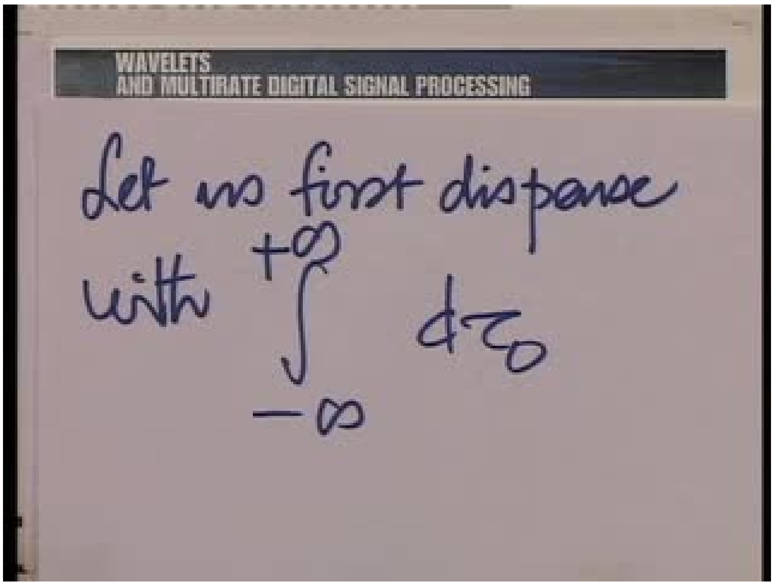

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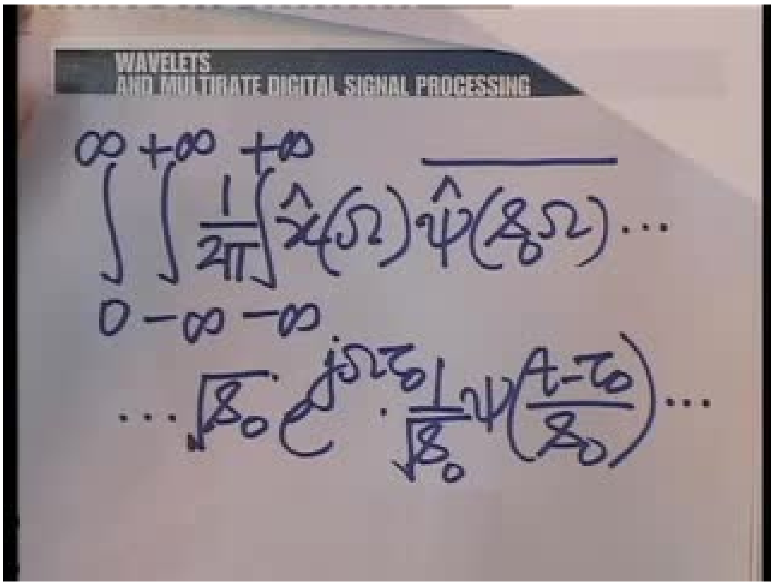

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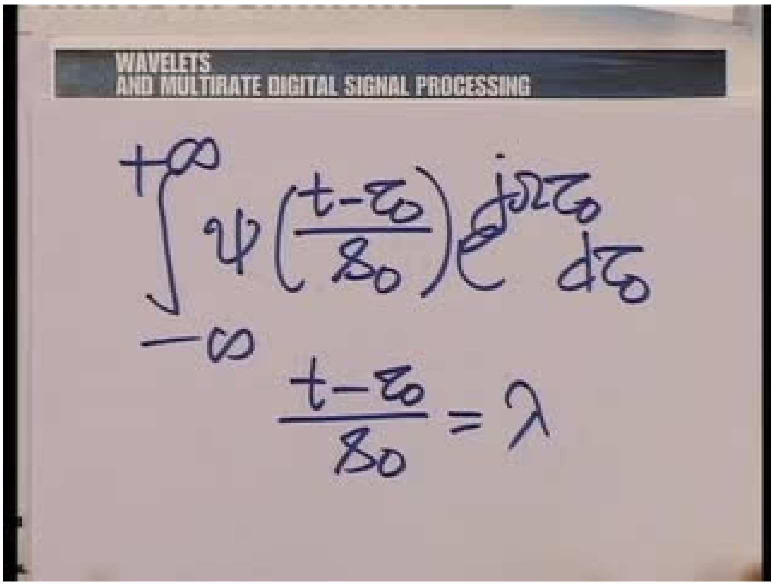

Let us first dispense with the integral on tau naught and therefore, let us identify the parameters that are the functions that depend on tau naught and those are essentially the following; so, let me identify them, they essentially this and this, that is all. And this is a very familiar quantity here, after all this looks very much like a Fourier transform, in fact, if you simply care to put t minus tau naught by s naught equal to lambda, as is the standard trick, you will indeed get exactly that.

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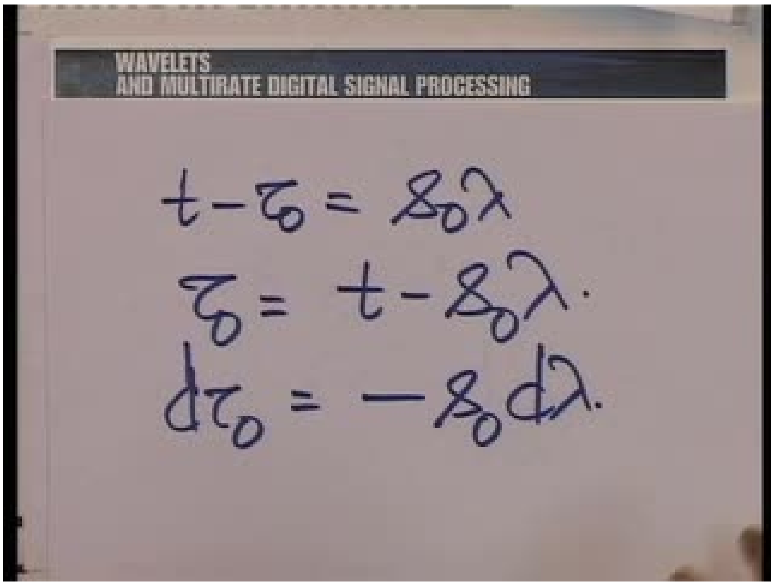

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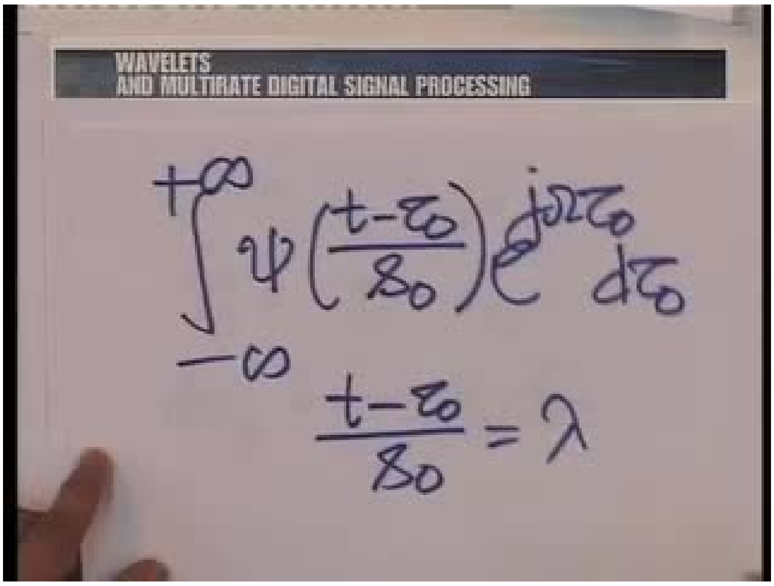

So, when t becomes, well you have t minus tau naught is s naught times lambda and therefore, tau naught taking it to the other side is t minus s naught lambda and therefore, d tau naught, noting that t is a constant for fixed t is minus s naught t lambda and we can substitute that in this expression here.

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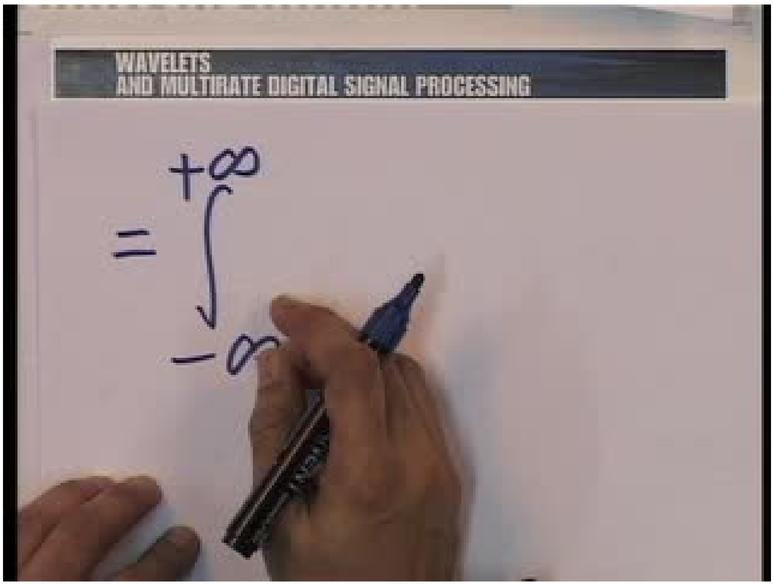

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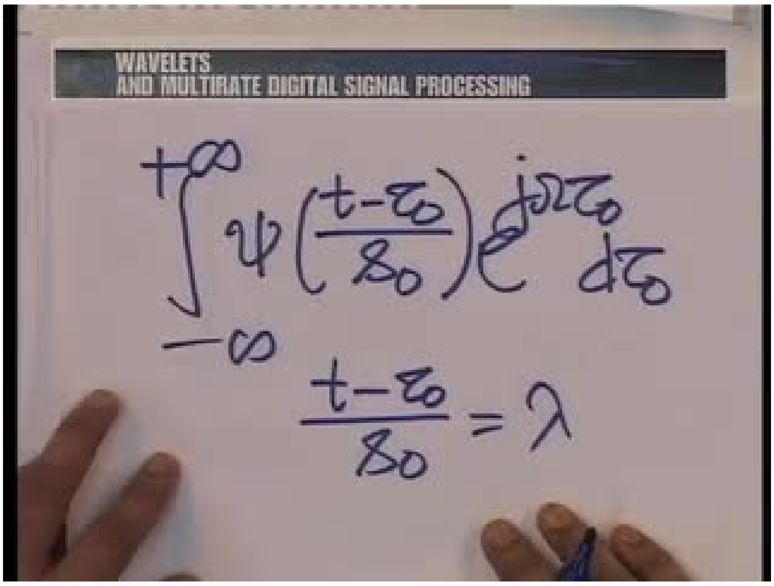

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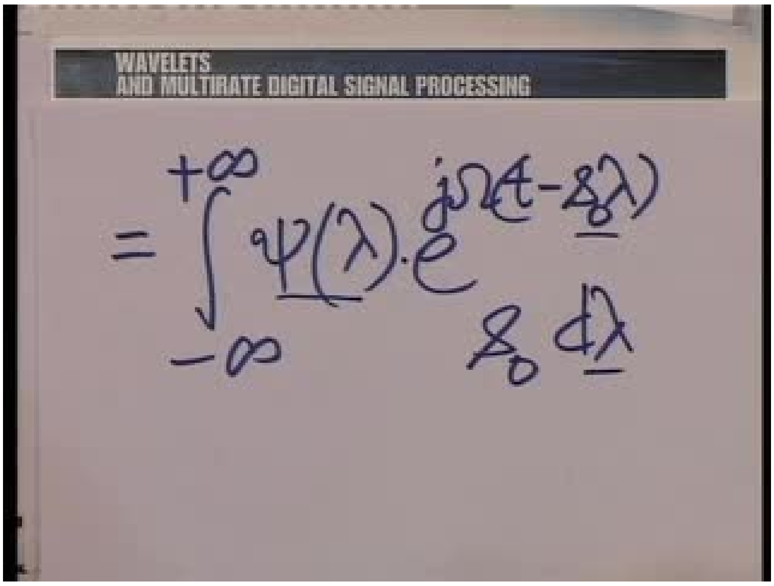

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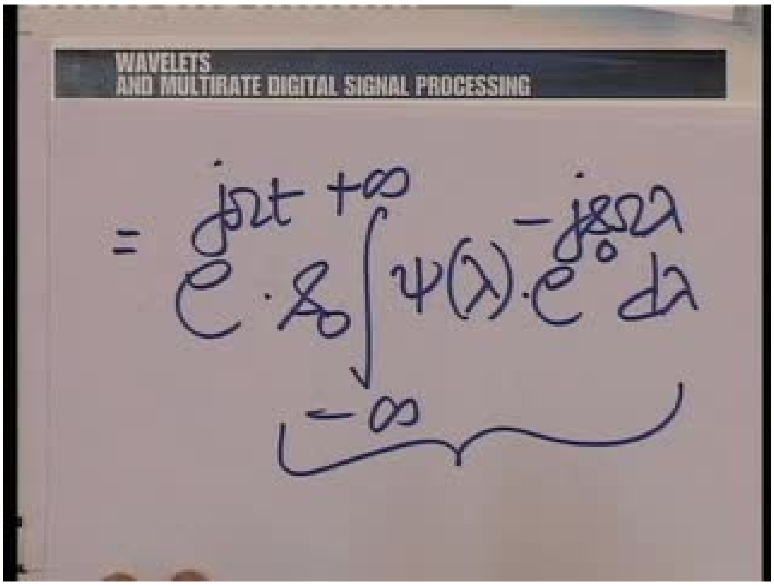

So, we have this becomes equal to and once again, you know if you look at the limits, when tau naught goes from minus to plus infinity, lambda goes from plus to minus infinity, but with that change of sign, again we can rewrite this in the following way. So, I am just skipping those steps, psi of lambda e raise to the power j omega times t minus s naught lambda, minus s naught becomes plus s naught t lambda and then we need only to isolate this part; so, this can be rewritten as e raise to the power j omega t, coming out s naught coming out and then we have minus to plus infinity psi lambda e raise to the power minus j s naught omega times lambda d lambda and this is very familiar here.

This is essentially the Fourier transform of psi evaluated at s naught lambda, instead of just s naught omega, instead of just omega, so we have made a change of this variable, here s naught omega and not just omega and therefore, we can now collapse the integral on tau naught entirely, let me write that down.

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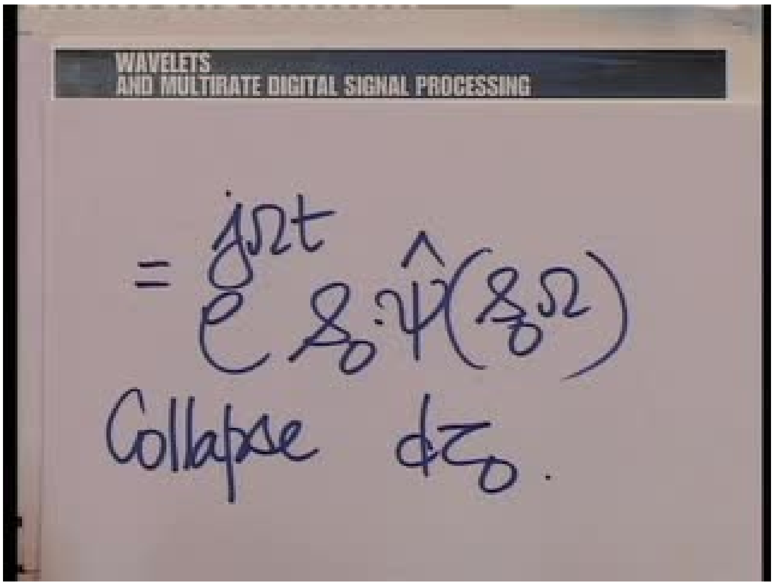

This is essentially e raise to the power j omega t s naught time psi cap evaluated at s naught omega and therefore, we can now collapse the d tau naught part, let me put that down for you once again for reference; I agree that, this is a little bit of work, but it is not so difficult, after all let me put that down, before you once again for reference.

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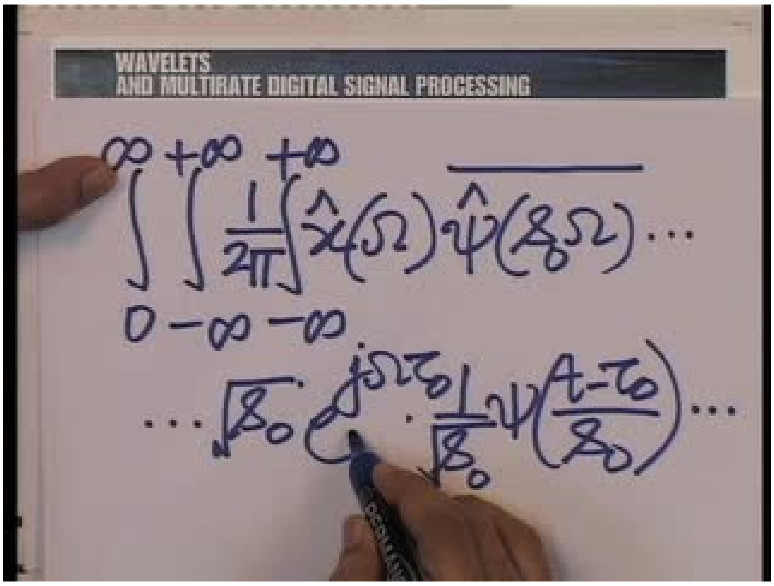

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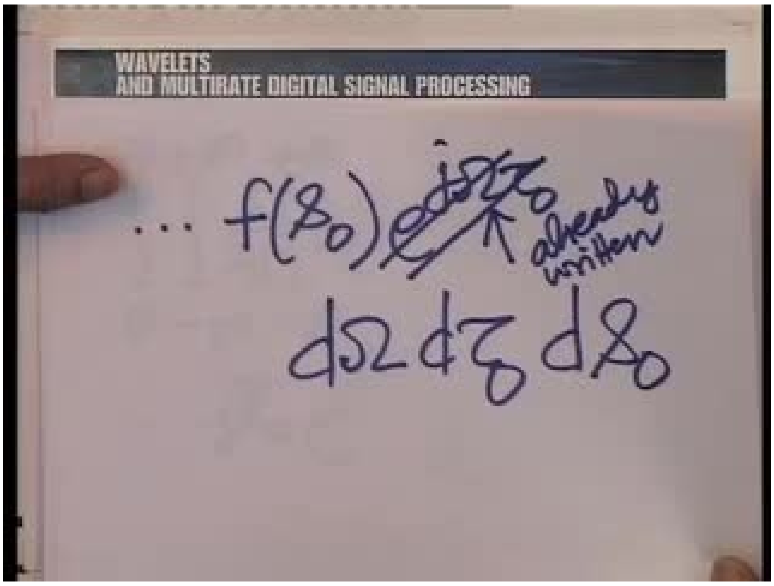

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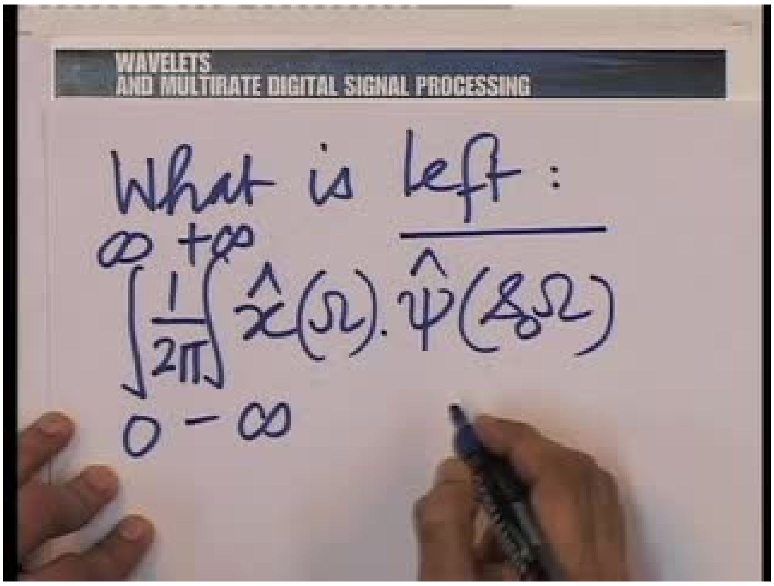

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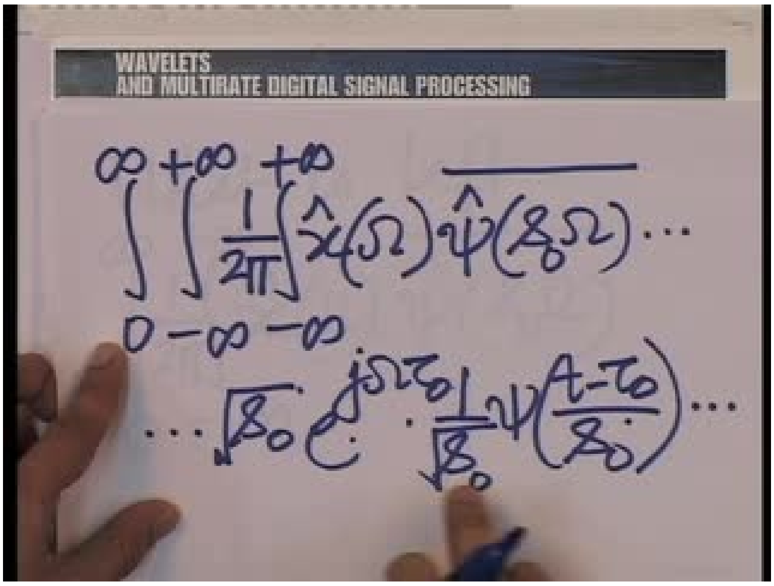

So, what I am saying is this, is all now being collapsed this, this and the d tau naught part here this has been collapsed; what is now left is the d s naught part and the d omega part. Now, let us look at the d s naught part; in fact, let us write down what is left for convenience at the moment, what is left is as follows. The integral on s naught and the integral on omega, the 1 by 2 pi, we keep as it is x cap omega psi cap s naught omega bar, you will note that this square root of s naught and the square root of s naught in the denominator cancel, so this goes away.

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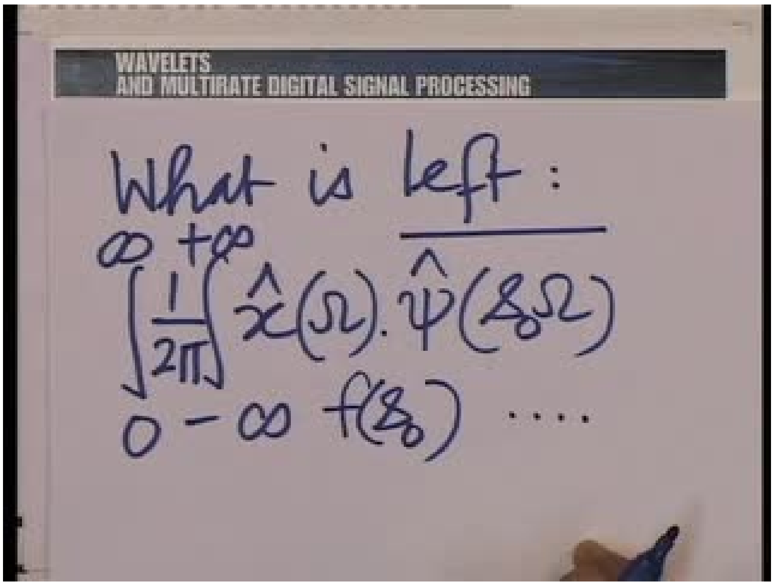

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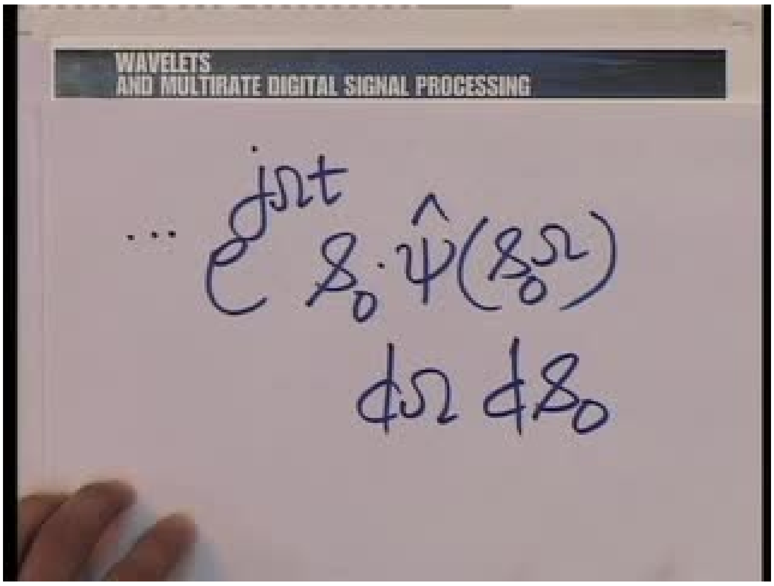

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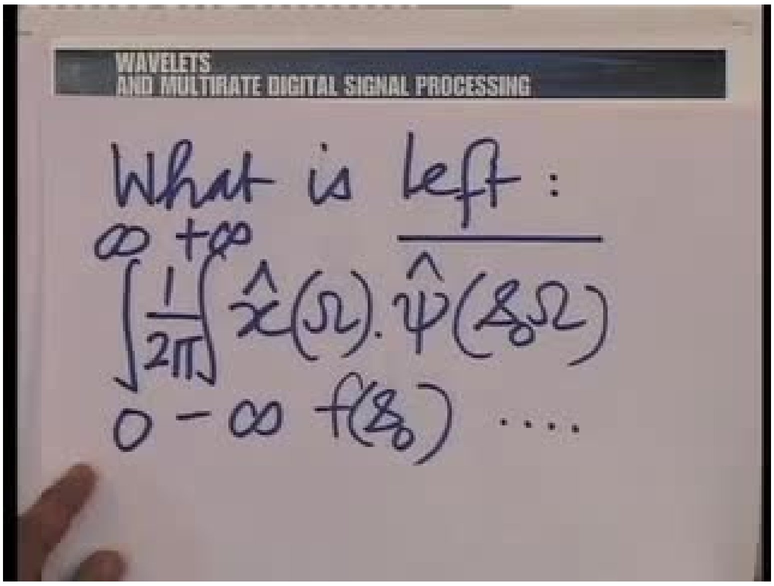

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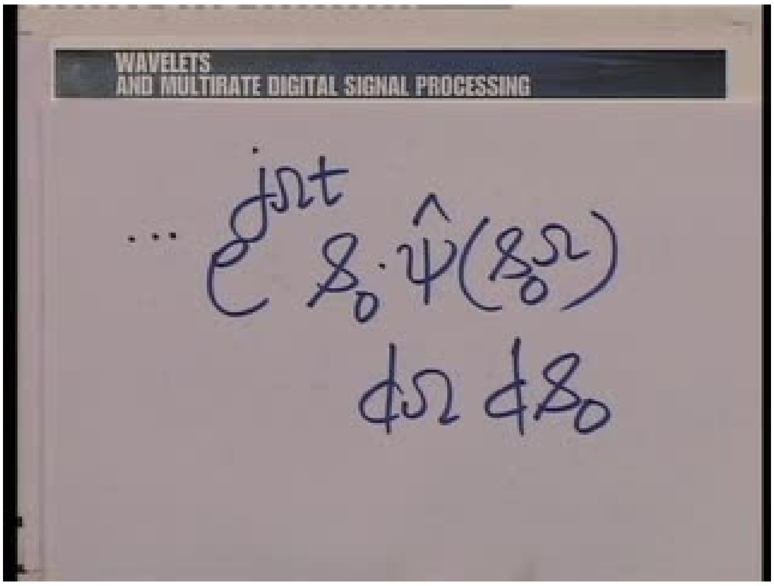

So, I remove them the rest has been collapsed, you have an f of s naught and then you have this part, so I continue that; this is what is left, we need to flash it once again. You have an integral on s naught outside and integral on omega inside, x cap omega psi cap s naught omega complex conjugate a weighting function in s naught and then e raise to the power j omega t naught psi cap s naught omega integrated on omega and then on s naught and now we will use a standard trick again.

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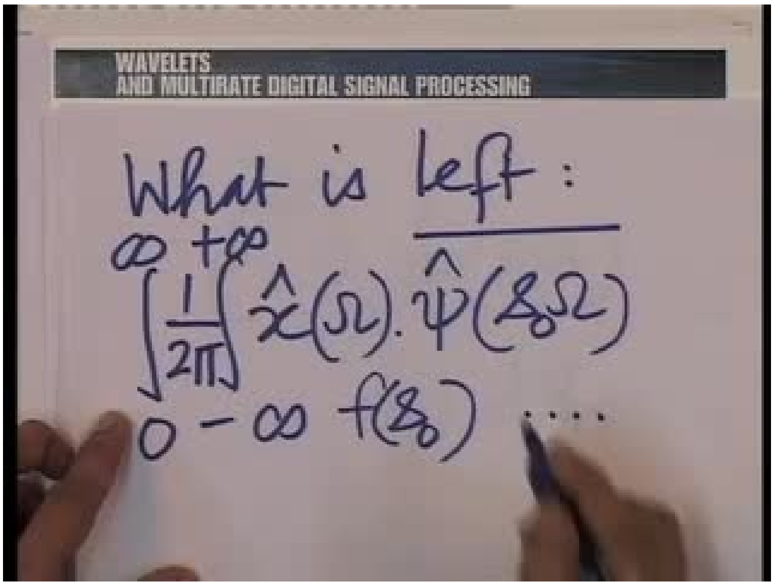

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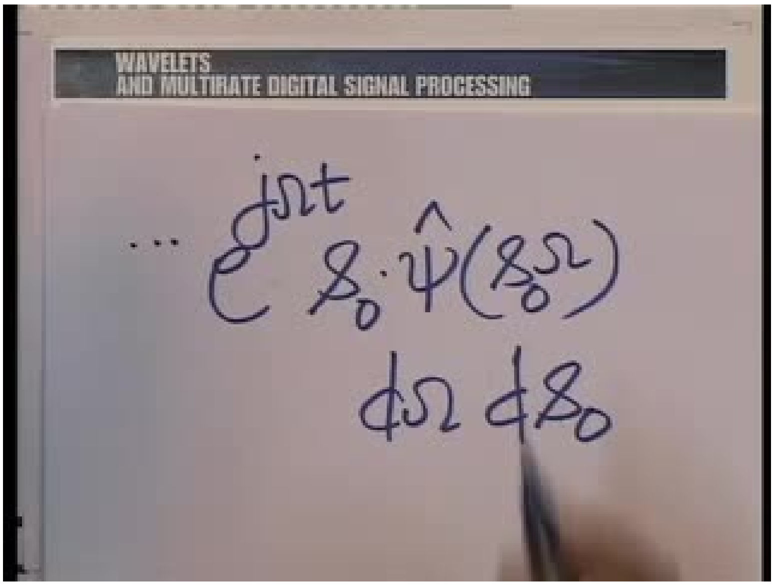

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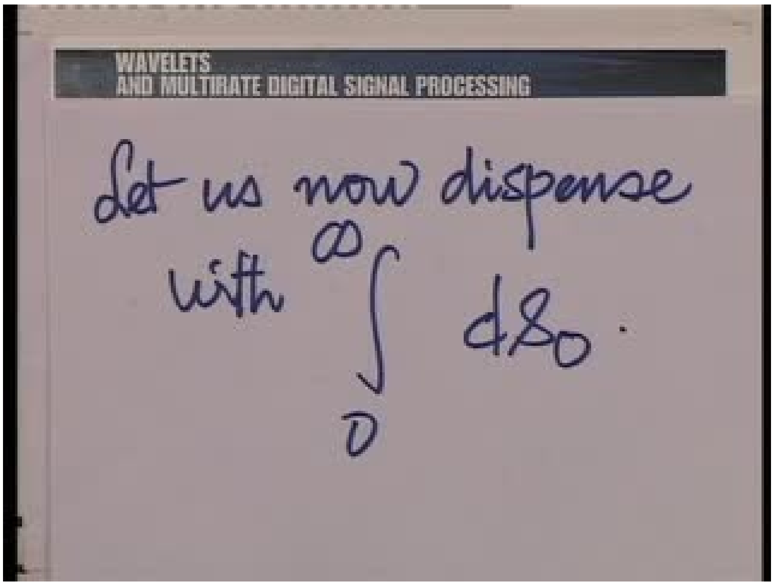

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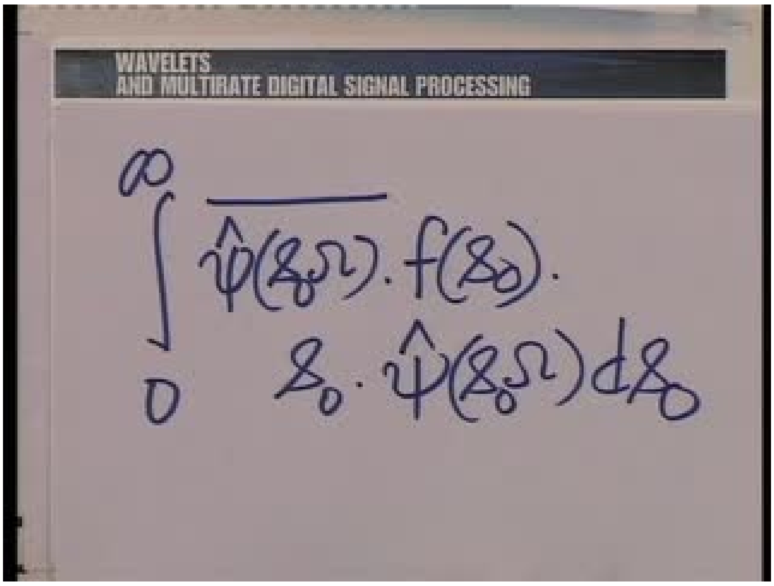

We shall note that, this, this and this depend on s naught and if somehow, we can do away with the integral on s naught, so we will, let us isolate the integral on s naught first, let us now dispense. So, we isolate the terms that depends on s naught and those terms are as follows, integral zero to infinity psi cap s naught omega bar times f s naught times s naught times psi cap s naught omega d s naught

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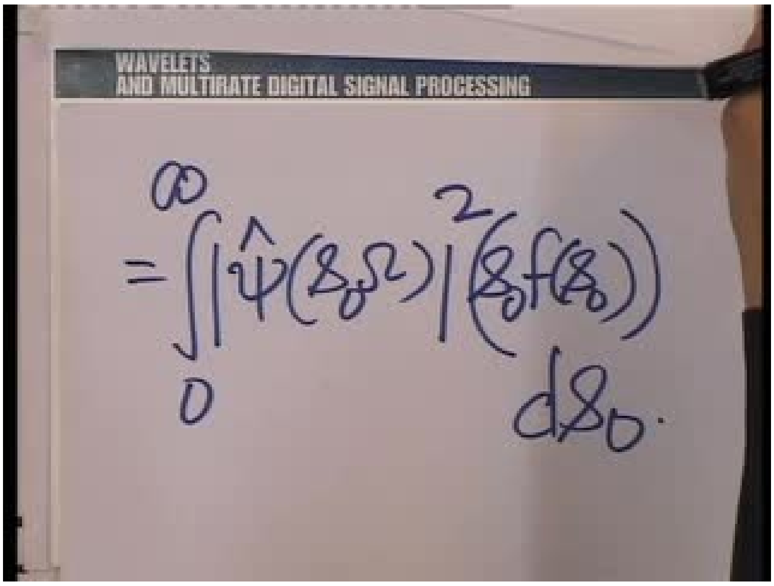

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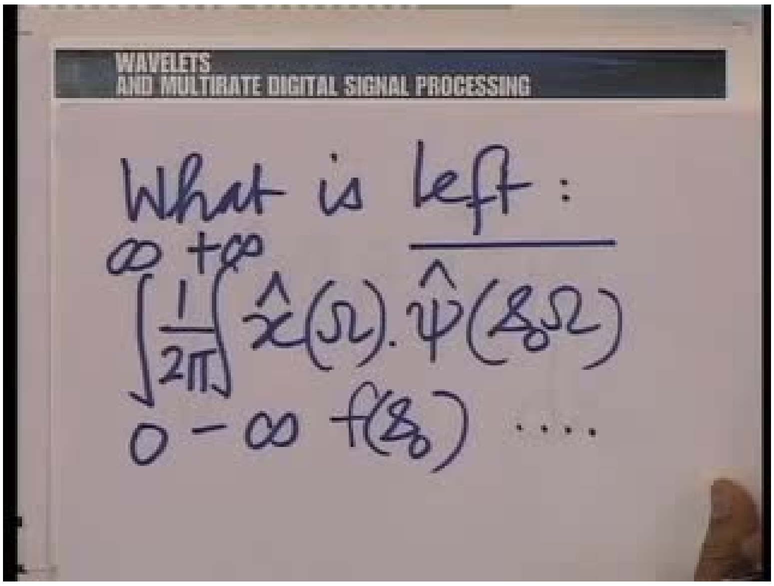

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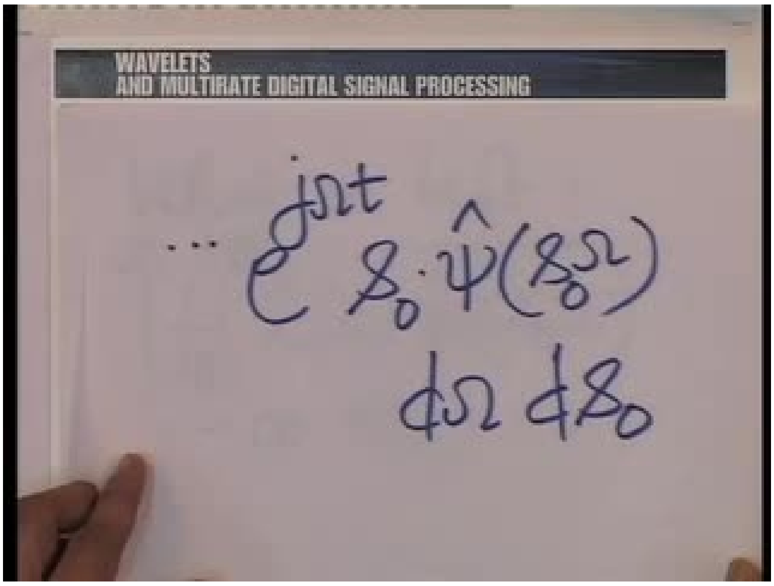

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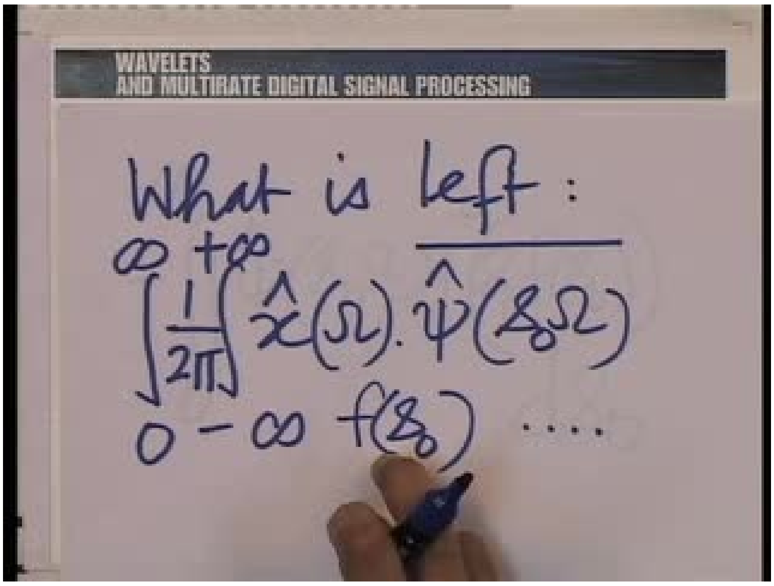

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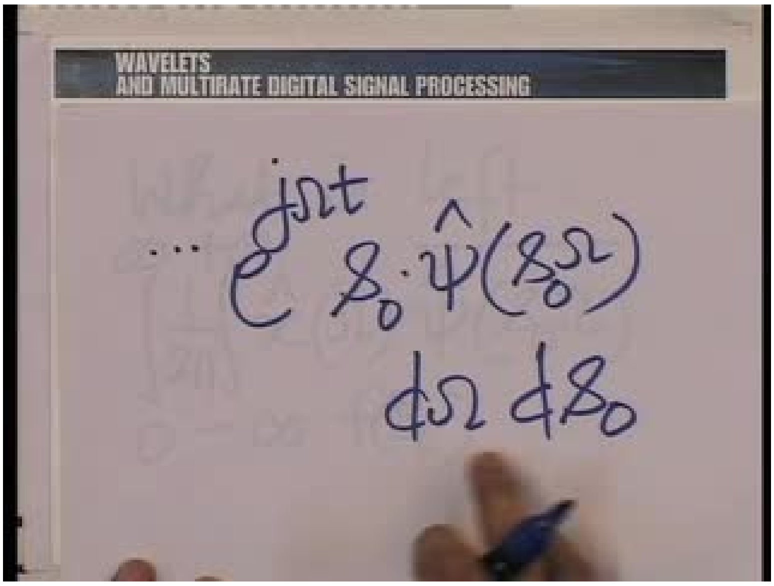

Now, you know if you look at it carefully, of course, this part is easy to deal with its mod psi cap s naught omega the whole squared s naught f s naught d s naught, now we need to interpret what we are trying to do here. You know, if we could make this independent of omega, the whole objective here should be to make this independent of omega, I will explain why. If you look at what was left, we had this term dependent on omega left; this we have taken in the s naught integral, this we have taken in the s naught integral; this is of course a term dependent on omega, we cannot do much about this, we have to keep it and we want it anyway, all this has been taken in the s naught integral, if all this, this,

this and these two terms can be made independent of omega, what will be left essentially a constant in omega, what will be left is x cap omega times e raise to the power j omega t d omega, which is a very familiar quantity; it is essentially the inverse Fourier transform of the Fourier transform of x.

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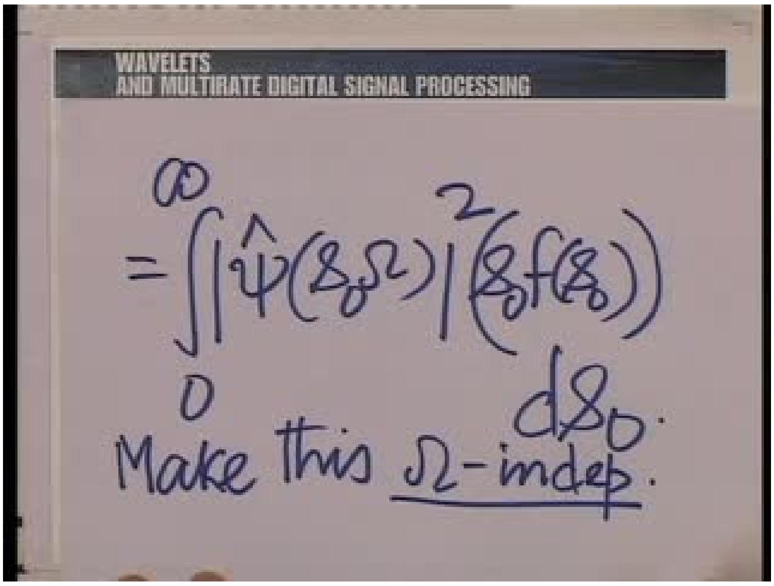

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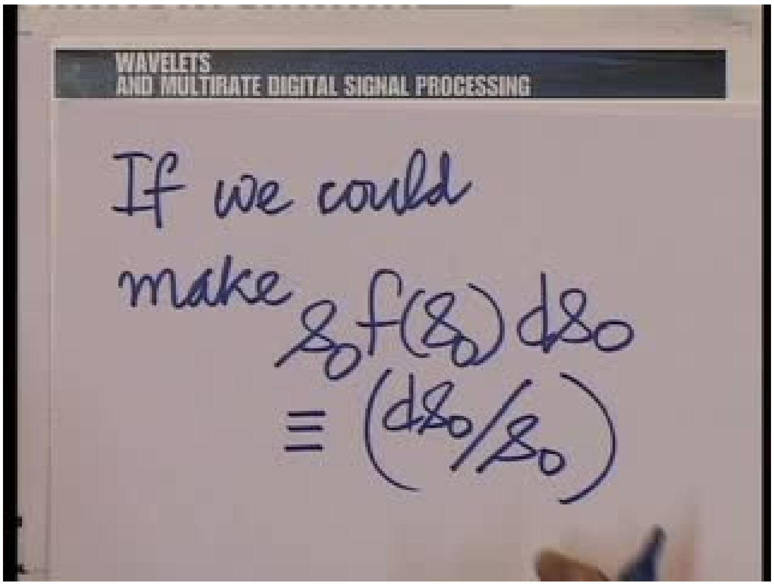

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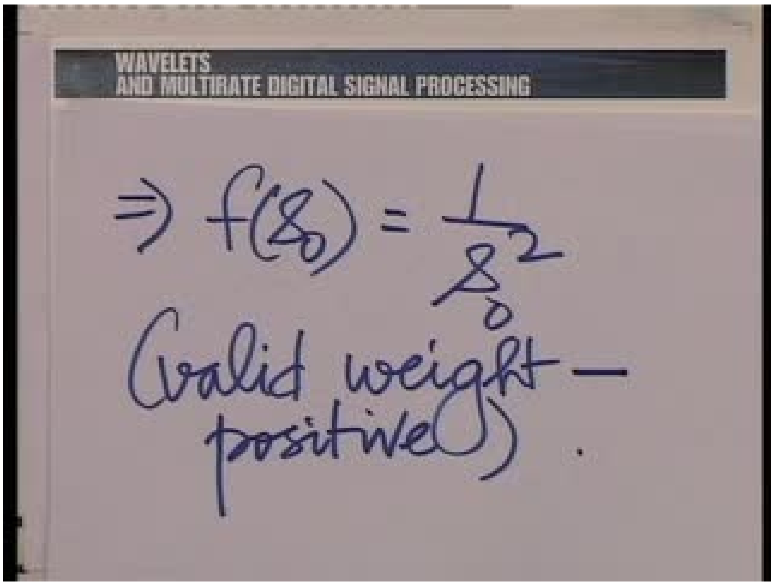

So, we are getting where we know, where our destination is and we know how to get there we shall get there essentially, by making this independent; so, we must make this independent, make this omega independent and that is, where the role of this weight function is, we will employee s naught f s naught d s naught, you know this part can be nicely configured to meet this integral independent of omega and to do that, we use the following lemma. If we could make s naught f s naught d s naught, essentially equivalent to d s naught by s naught; in other words, f s naught is equal to 1 by s naught squared and we note that, this is a valid weight function, it is strictly positive. You know, f f s naught being 1 by s naught squared is a, is, is a valid weight function, why do we want to weight function to correct the so-called magnitudes, when we put the components together and when we put a correction factor for the magnitudes, that correction factor must be non-negative for all values of the parameters in the components.

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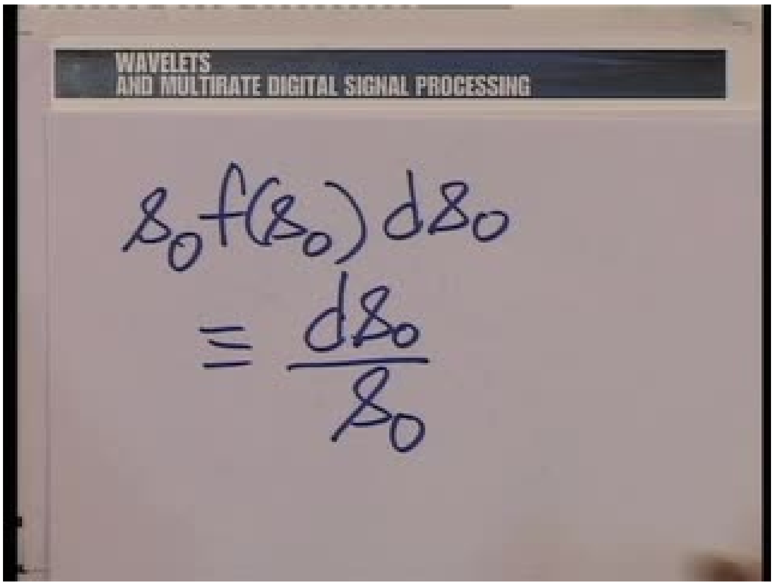

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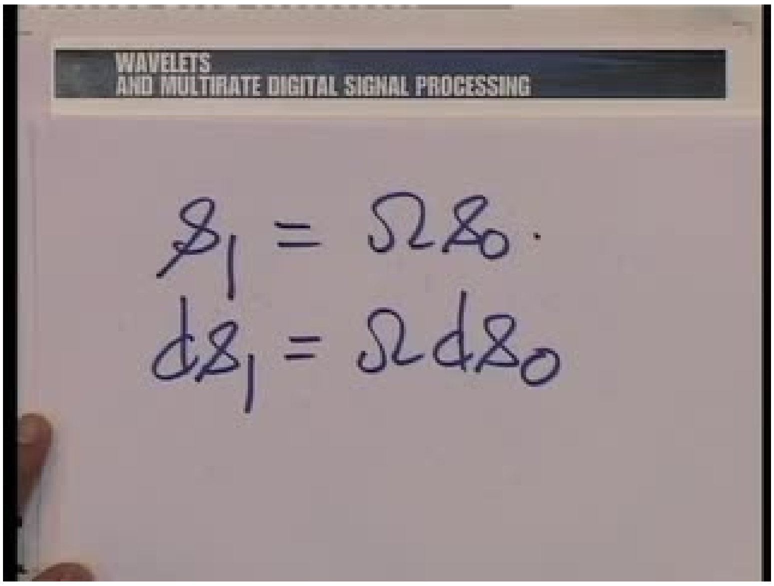

Now, here we have a valid weight function; a weight function which is non-negative for all s naught and therefore, let us choose f of s naught to be equal to 1 by s naught squared as we have done here, where upon, s naught f s naught d s naught essentially becomes d s naught by s naught and now, if we take say s 1 equal to omega times s naught, then d s one is omega d s naught and now, we can divide here.

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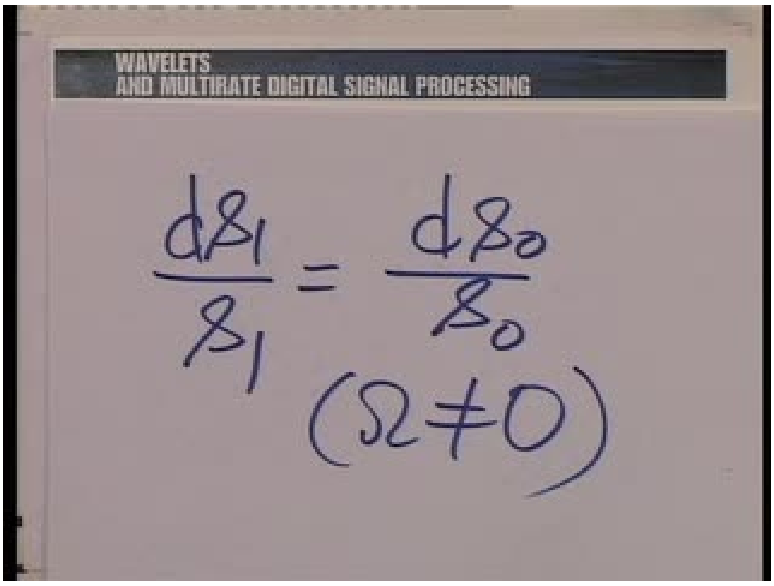

So, d s 1 by s 1 is d s naught by s naught, of course, assuming omega is not equal to 0, the only catch is that when omega is positive the limits are 0 to plus infinity, when omega is negative the limits are 0 to minus infinity and if we take care of that little detail, we have done away with the dependence on omega naught.

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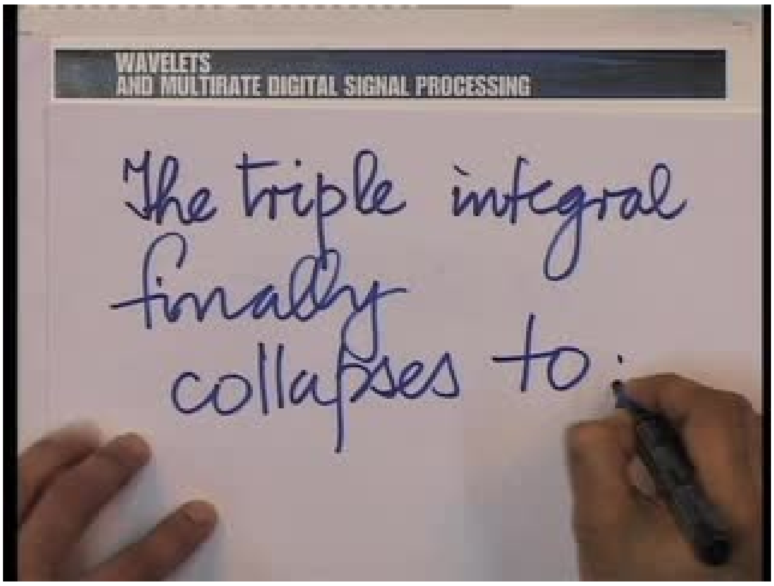

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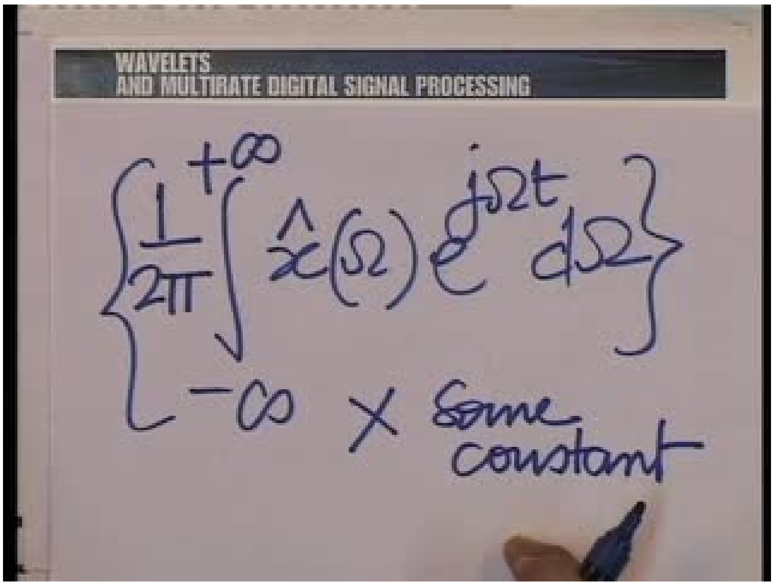

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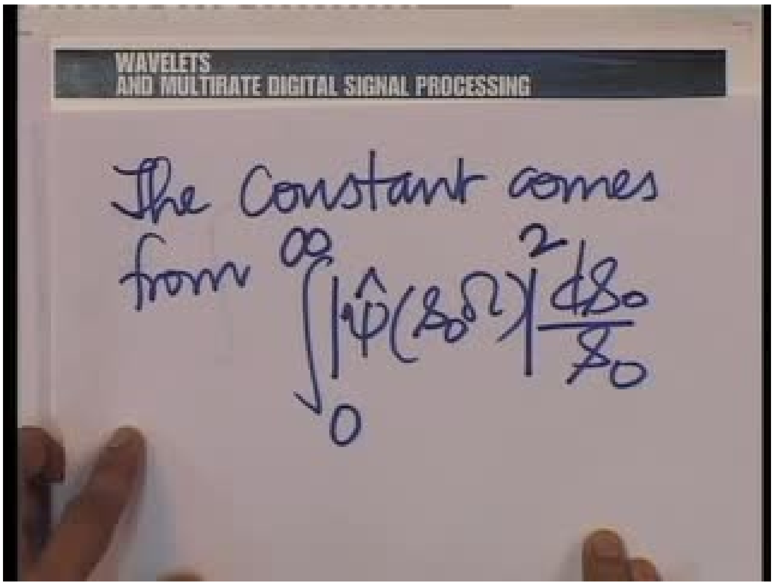

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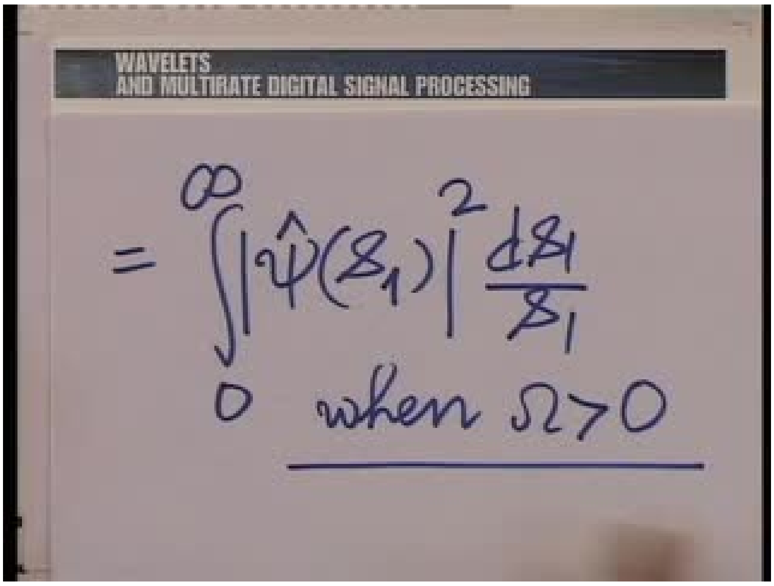

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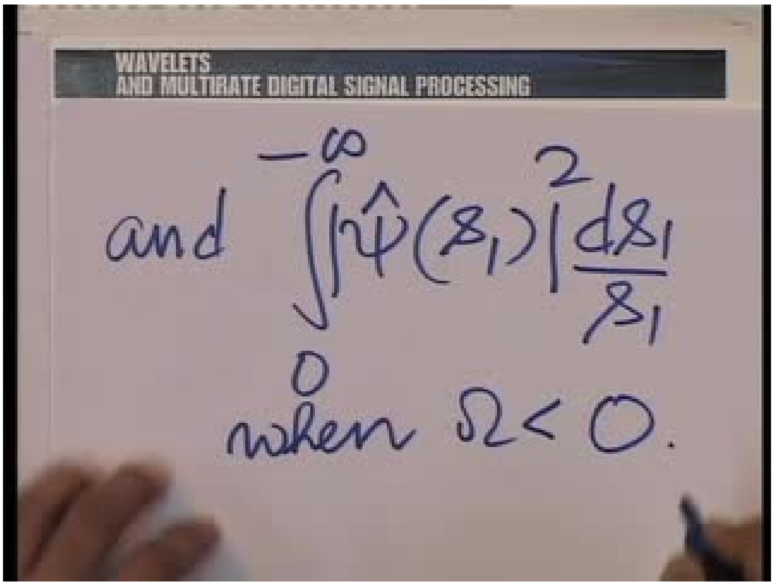

So, what we are saying in effect is the triple integral finally, collapses to some constant multiplied by the following, so this whole thing multiplied by some constant and this constant emerges from integral psi cap s naught omega d s naught by s naught from 0 to infinity which boils down mod squared here, of course, which boils down to essentially mod psi cap s 1 squared d s 1 by s 1, when omega is greater than 0 and integral from 0 to minus infinity psi cap s 1 mod whole squared d s 1 by s ,1 when omega is less than 0.

Now, we have narrow down the evaluation of this integral to essentially an inverse Fourier transform of the Fourier transform of x, but would this condition, these two integrals over s 1 need to be finite and in fact equal and this is where the concept of admissibility of psi would come from. We shall continue discussing this in the next lecture, to complete the idea of admissibility and to interpret it, what do we mean by imposing this condition, that these integrals must be finite. As I said, we shall proceed from this point, in the next lecture. Thank you.