Advanced Digital Signal Processing – Wavelets and Multirate
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Lecture No. # 14
Daubechies' Filter Banks: Conjugate Quadrature Filters

A warm welcome to the fourteenth lecture on the subject of wavelets and multirate digital signal processing. We continue in this lecture to discuss the Daubechies filter bank which we had very briefly introduced in the previous lecture. I would like to put before you the salient points of that filter bank once again, and then, complete the design that I had begun in the previous lecture.

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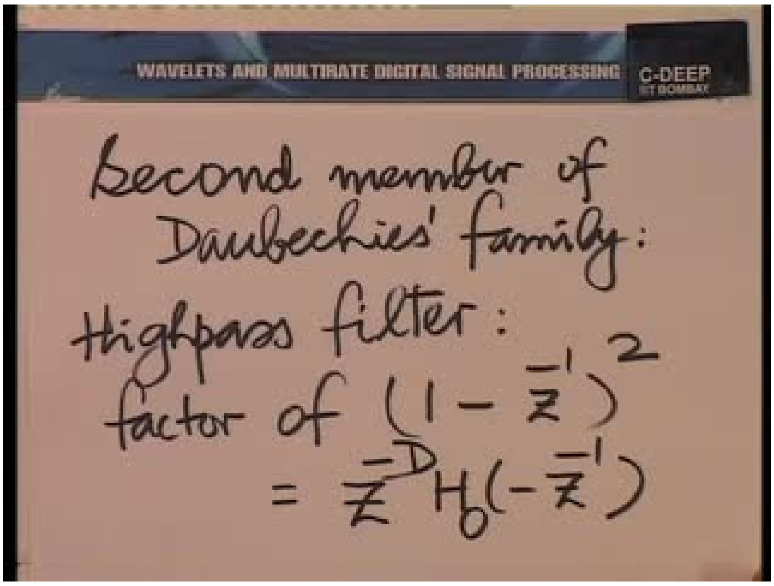

So, recall that we had based the construction of the series of the Daubechies filter banks on the idea of the analying polynomials of higher and higher degree. So, we said that we wanted more and more factors of the form one minus z inverse appearing in the high pass filter. So, for example, let me put down the structure of the next member. In the family after the haar case, where instead of one, you would have two factors. What we said was the second member of the Daubechies family would like this.

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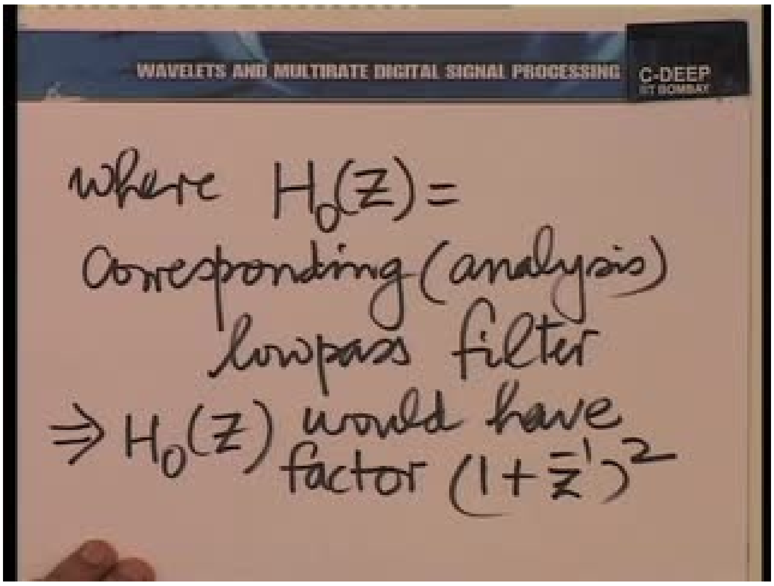

The high pass filter would have a factor of one minus z inverse the whole squared, and therefore, if we look at the low pass filter, remember, the high pass filter was essentially of the form z rise the power minus d h zero minus z inverse, where h zero z is that corresponding low pass filter, of course, I am talking about the analysis sight.

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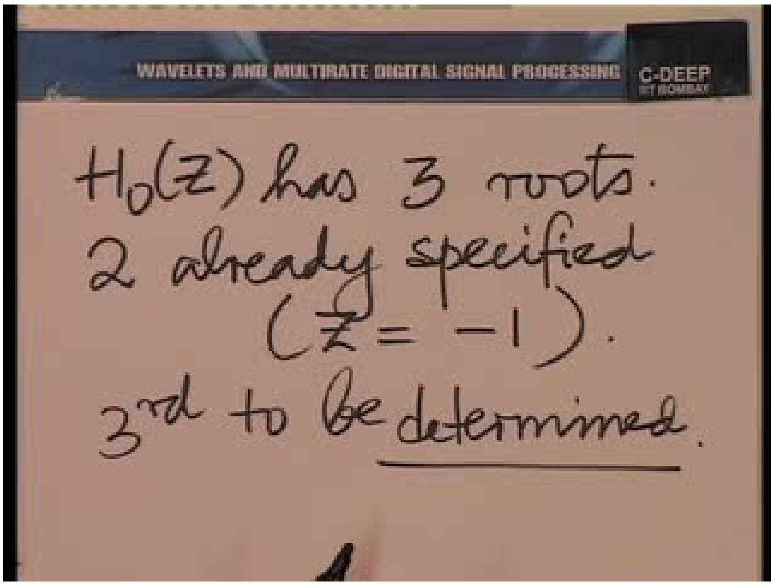

So, essentially, the analysis low pass filter would now have a form like this - h zero z would have the factor one plus z inverse squared, and then, we recall that we had even lengths for the filters, and therefore, we have a situation - h zero z has three zeros, two already specified.

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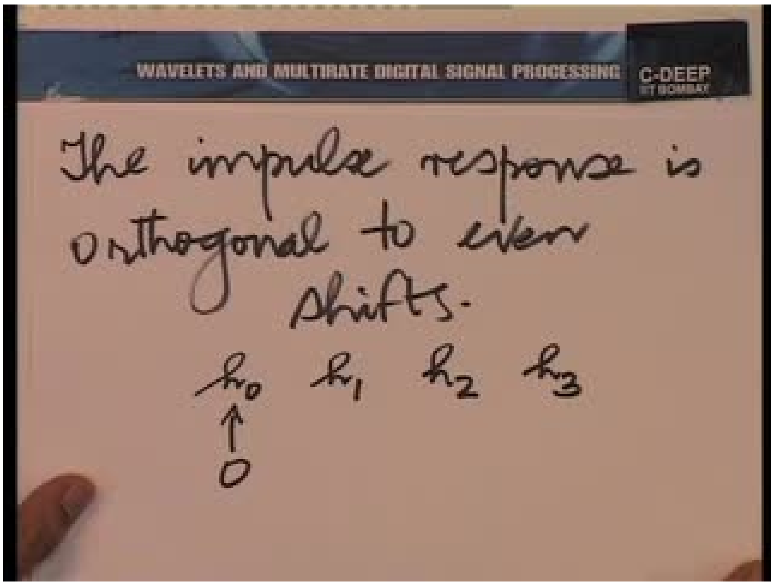

Obviously, they belong to minus one, and therefore, the third to be determined, and where do we determine this third root from? Well, we go back to the requirement of orthogonality to even shifts. So, we call that, we had said that the impulse response is orthogonal to even shifts; that means if I assume that this filter has the impulse response its zero h one h two h three at zero, then this is orthogonal to its shifts by two four and so on.

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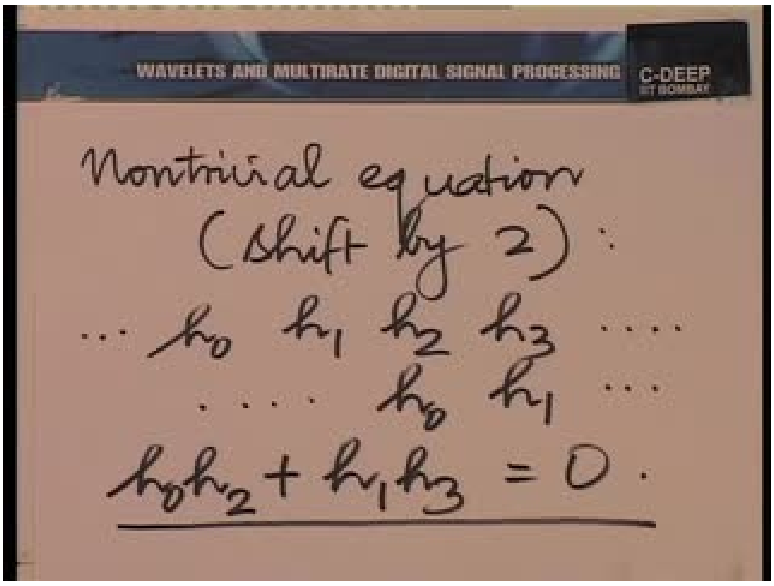

So, the nontrivial case, the only nontrivial equation that we get is the following, it is come from a shift by two: h zero h one h two h three, and when this is shifted by two, you have h zero h one, and then, you know, there all, there all zeros after this; there are zeros before, and there are zeros before.

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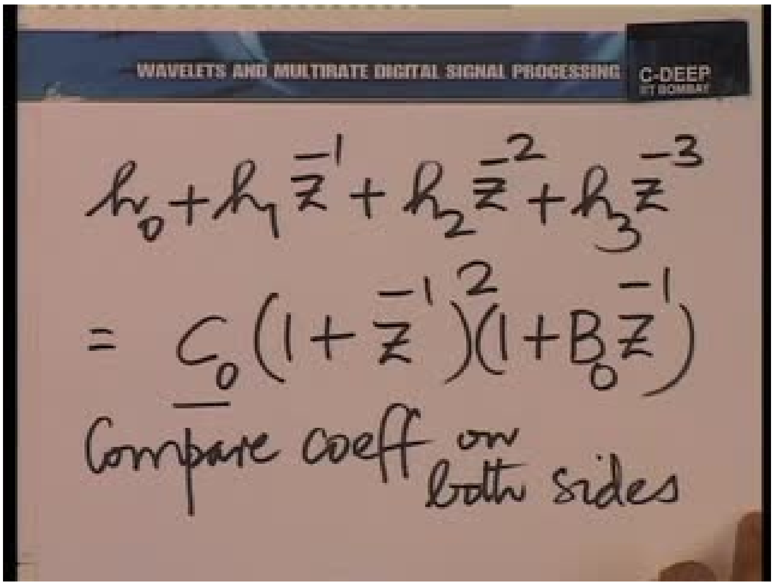

So, all that you have is h zero h two plus h one h three is equal to zero. This is the only nontrivial equation that we get. Now, from the location of the zeros and the free parameter, we need to express h zero through h three in terms of the free parameter. So, what do you have? We have h zero plus h one z rise to the minus one plus h two z rise to the minus two plus h three z rise to the minus three is of the form some constant, let say c or c zero if you please times one plus z inverse the whole squared times one plus b zero z inverse, and we need to compare coefficient on both sides.

Now, the consistency c zero does not affect orthogonality; so, we shall just focus on the rest of the expression, because we first need to satisfy the requirement of the orthogonality, orthogonality to even shifts I mean, and from there, we need to determine the constant c zero. We will see later what helps us determine c zero.

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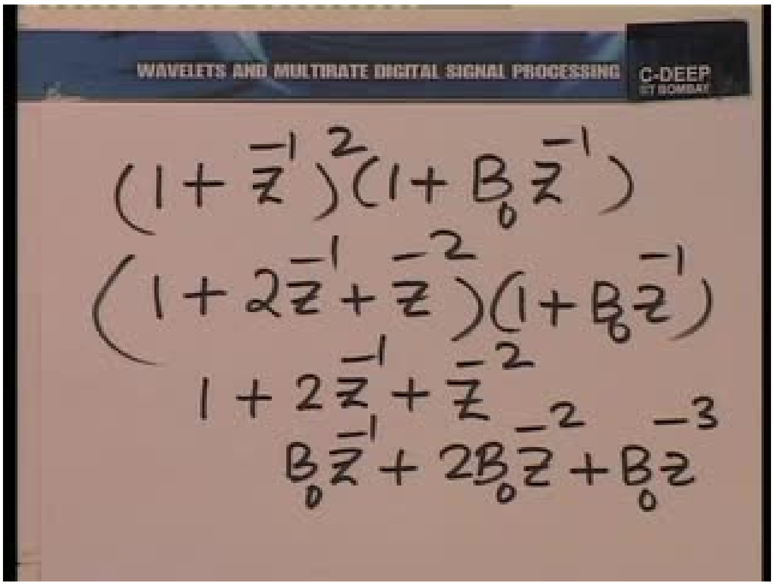

So, in fact, if you look at it, what we are asking for is the following expanded term - one plus z inverse the whole squared times one plus b zero z inverse, where b zero needs to be determined, and this has we said could be expanded as one plus two z inverse plus z rise to the minus two times one plus b zero z inverse, and if we collect terms, it gives us one plus two z inverse plus z rise to the minus two plus b zero z inverse plus two b zero z rise to the minus two plus z rise to the power minus three times b zero. So, we will aggregate terms together here. We will take the coefficients.

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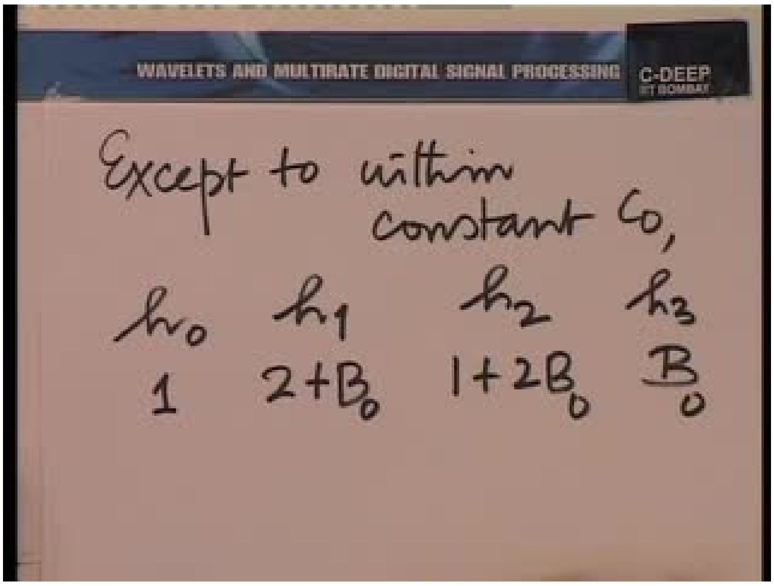

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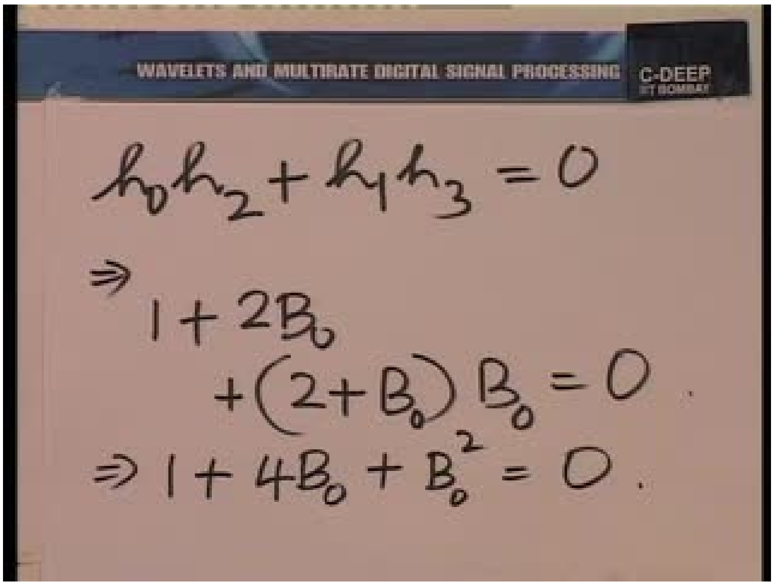

So, essentially, you know, except for the constant, h zero is essentially therefore one h one is of the form two plus b zero h two is of the form one plus two b zero and h three is of the form b zero, and now, we can write down the orthogonality equation that we seek. It says h zero h two plus h one h three must be zero in plain that one plus two b zero plus two plus b zero into b zero is zero where we are effectively saying one plus four b zero plus b zero squared is zero.

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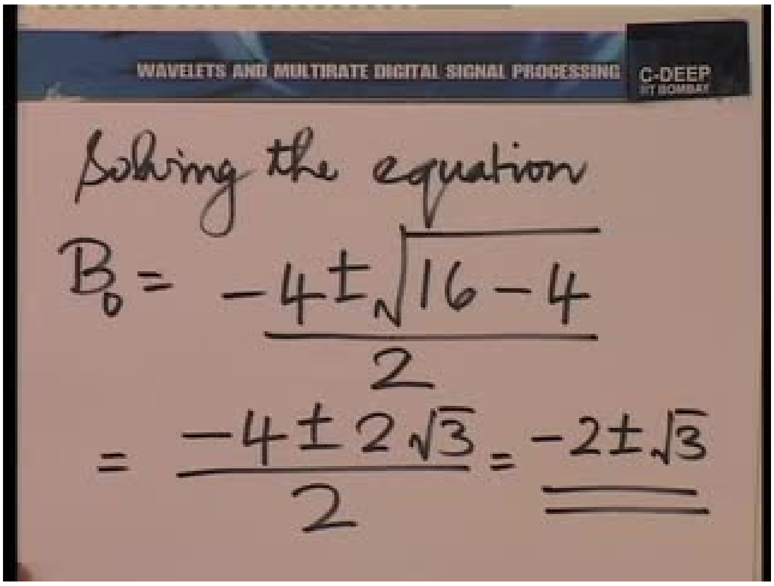

So, there we have an expression for b zero. It is easy to determine. So, b zero has got constrained, as you see, not surprising. There was one free parameter and one nontrivial constraint. Solving the equation, we have b zero is minus b plus minus root b squared minus four a c by two a that gives us minus four plus minus two root three divided by two, and that gives us two solutions minus two plus minus square root of three.

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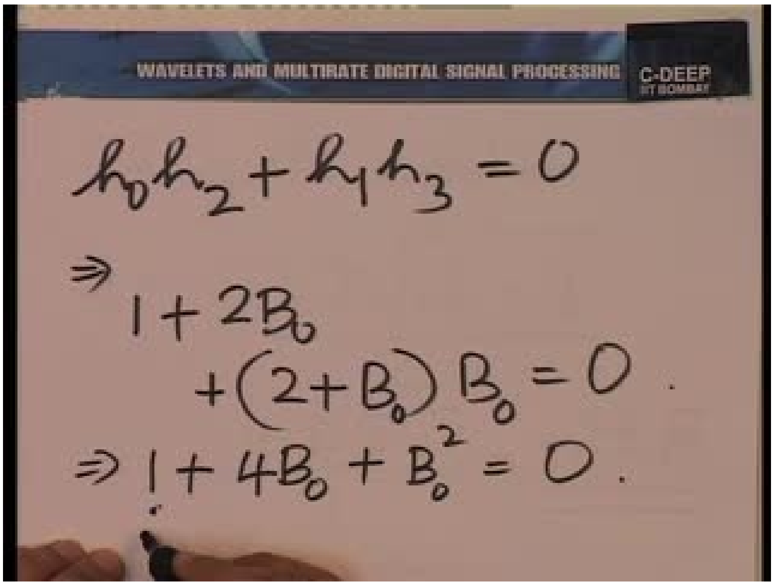

Now, we have two solutions here, which of them should be choose? Well, what distinguishes these two solutions? It is very clear from the quadratic equation and let me go back to it. You see, if we look at this term, this tells us the product of the roots the term one, and the product of the roots clearly has a magnitude of one. So, if one of them lies inside the unit circle, the other must be outside. In fact they cannot lie on the unit circle because none of them has a magnitude of one, and therefore, one must lie inside the unit circle and the other outside. Let us take note of that.

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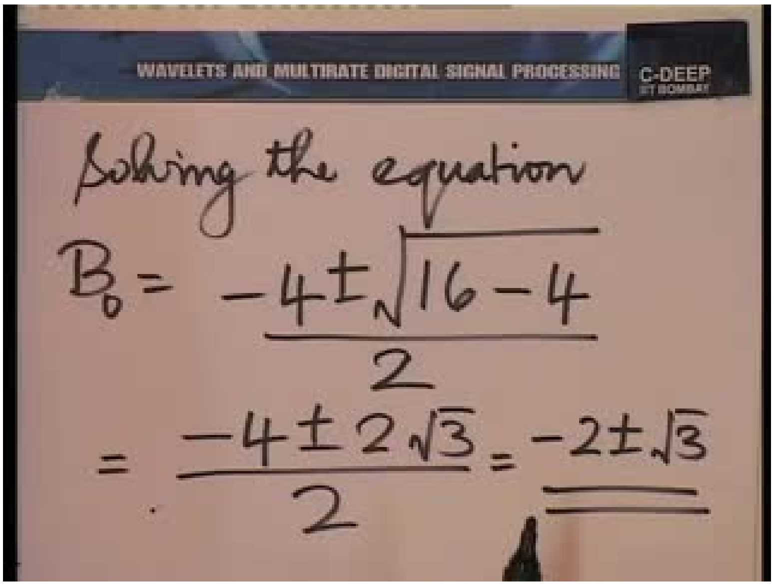

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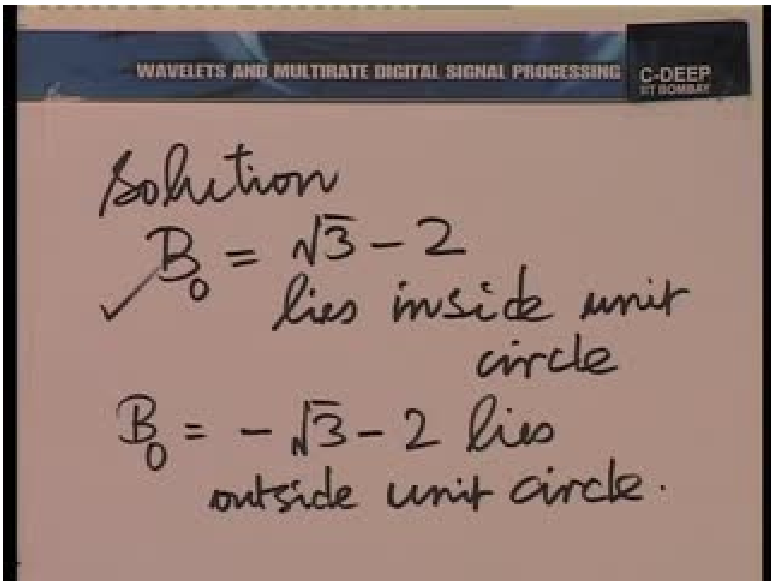

In fact, it is very easy to see which lies inside and which lies outside. If you look at the minus two minus square root of three solution, it clearly has a magnitude greater than one, and the other one has a magnitude less than one. So, among the solutions, b zero is square root three minus two lies inside the unit circle, and b zero is minus square root three minus two lies outside the unit circle. Now, we have a choice; we could either choose this or this. We shall choose this, and there is a reason for it. You see, very often, we like to chose what is called the minimum phase solution. There is this idea of minimum phase. I shall not dwell too much on it at this moment. Suffice it to say that minimum phase essentially means choosing all zeros inside the unit circle wherever possible.

In fact, the idea of minimum phase comes from what is called minimum phase delay. So, when we, you know, after all one interesting thing is that whether we choose the root inside the unit circle or outside. The magnitude of the frequency response would be the same; magnitude would not be different. What would be the different is the phase and when we put the root outside the unit circle, there is going to be an increased phase delay, and very often that would also point to an increased group delay, increased phase delay, increased group delay in the filter.

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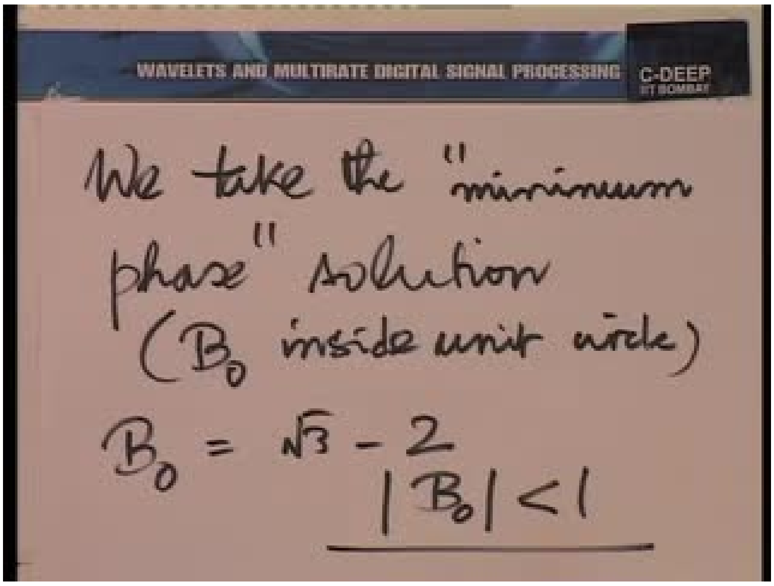

So, when we take the solution inside the unit circle, we are reducing the phase or the group delay as much as we can in the filter. So, it is essentially a question of choosing the better solution in terms of phase. So, let us summarize that. We take the minimum phase solution, that is, b zero inside unit circle and we have b zero is square root three minus two. Obviously, mod b zero is less than one here, and one can also evaluate this approximately. If you recall square root of three is about one point seven three, so this could be one point seven three minus two and one can come up with an approximate value. That is not really miss you.

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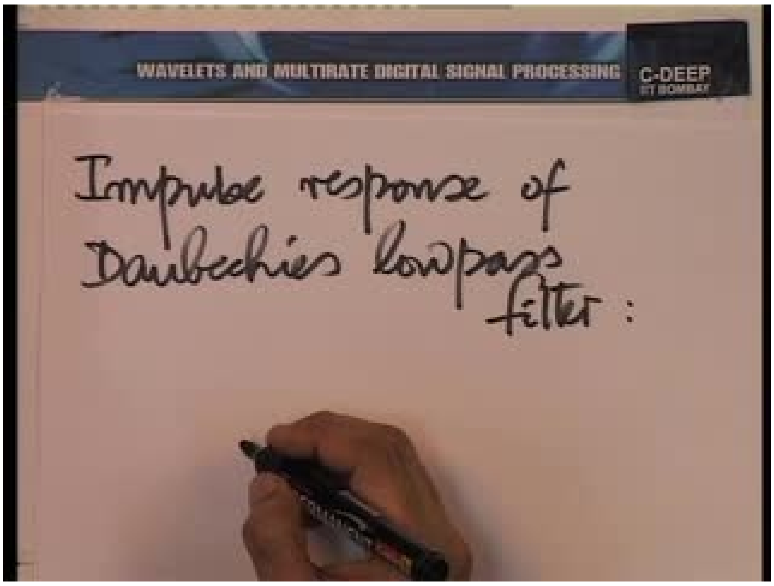

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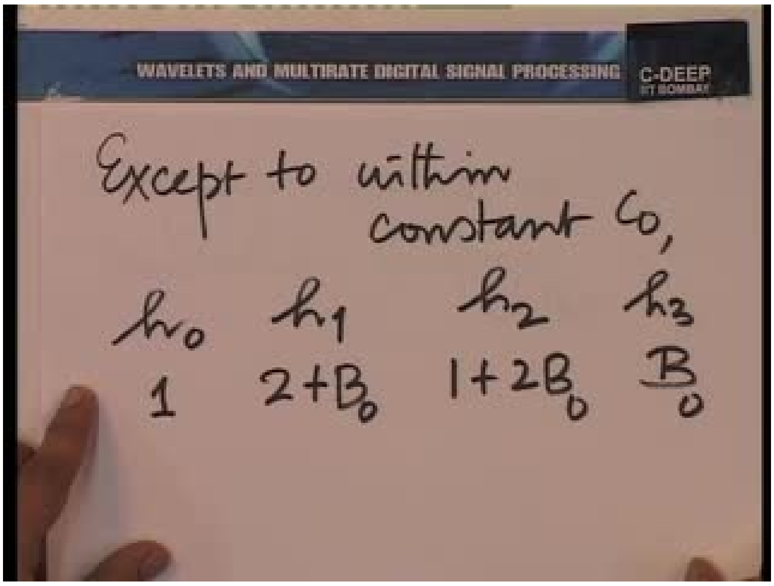

Anyway, we can now put down the impulse response of the daubechies low pass filter. Therefore, it is essentially, well, let me read of the impulse response from here. Let me flash the impulse response before you once again, and then, will substitute. This was the impulse response, now, we substitute for b zero.

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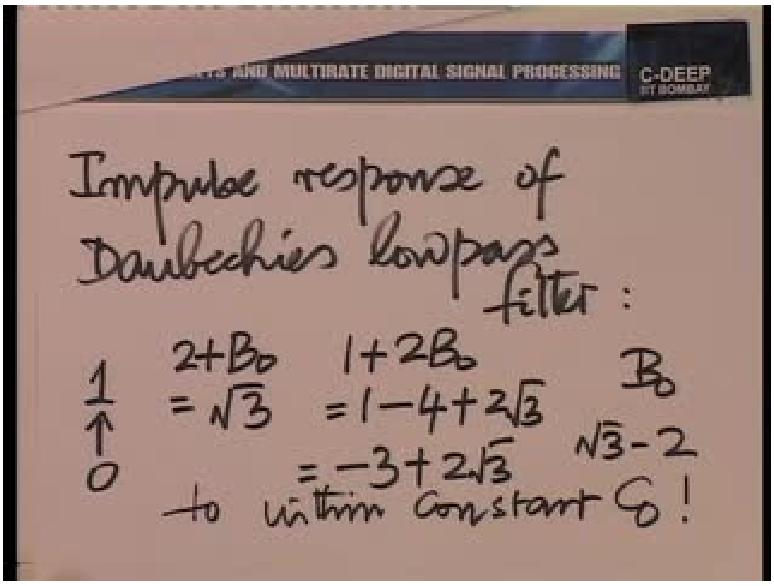

So, one two plus b zero which is essentially square root three. and then, you have one plus twice b zero, which is one minus four plus two root three or minus three plus two square root three, and finally, b zero which is just square root three minus two. So, this is h zero of course, well, please remember this is to within a constant. So, they still that constant c zero that needs to be determined. How do we determine the constant c zero? Well, we go back to the original equation of kappa zero z that we had.

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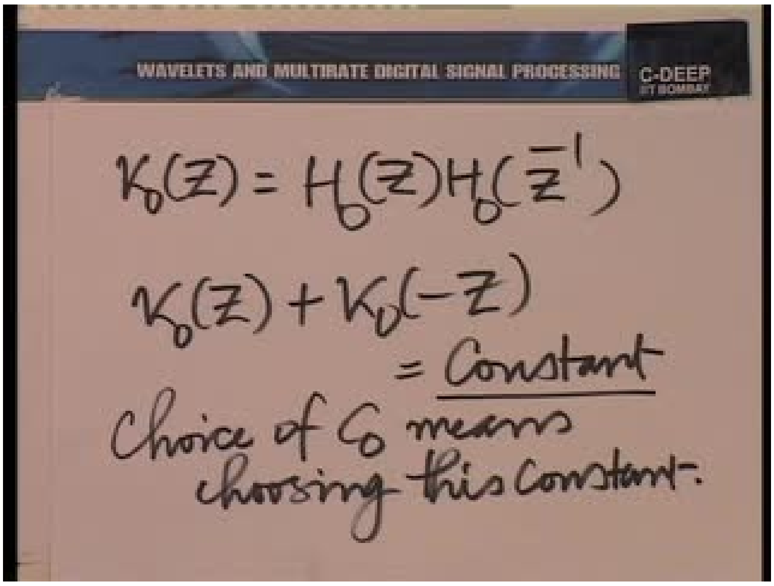

So, we want it kappa zero z defined by x zero z times h zero z inverse to obey the following. Kappa zero z plus kappa zero minus z is a constant. In fact, what it really means is we now need to choose this constant choice of c zero means choosing this constant.

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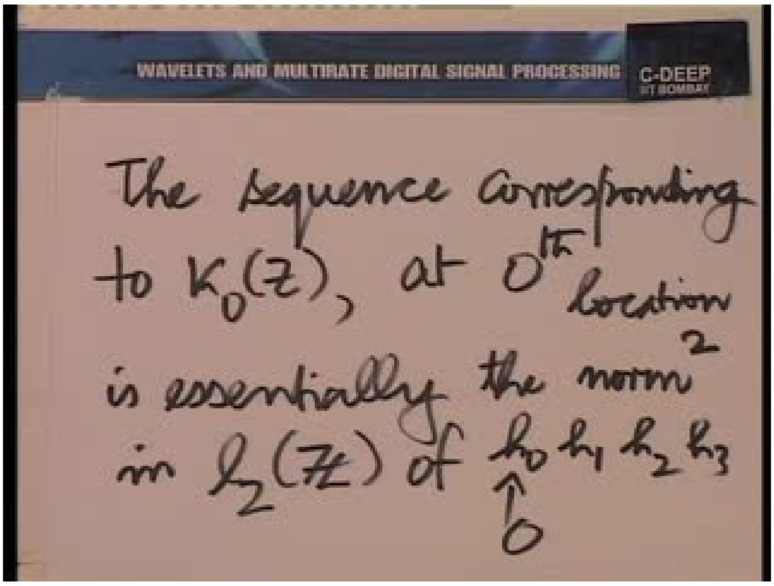

The easy thing to do is to ensure that the impulse response has a unit norm in the sense of l two, because if you look at it, the dot product of the impulse response with itself. You know, what we are saying essentially is kappa zero z, you know, the, the, the sequence corresponding to kappa zero z at the zeroth location is essentially the norm in l two z, the norm squared actually in l two z of the impulse response. It is a dot product of the impulse response with itself, and we could as well make that one for convenience.

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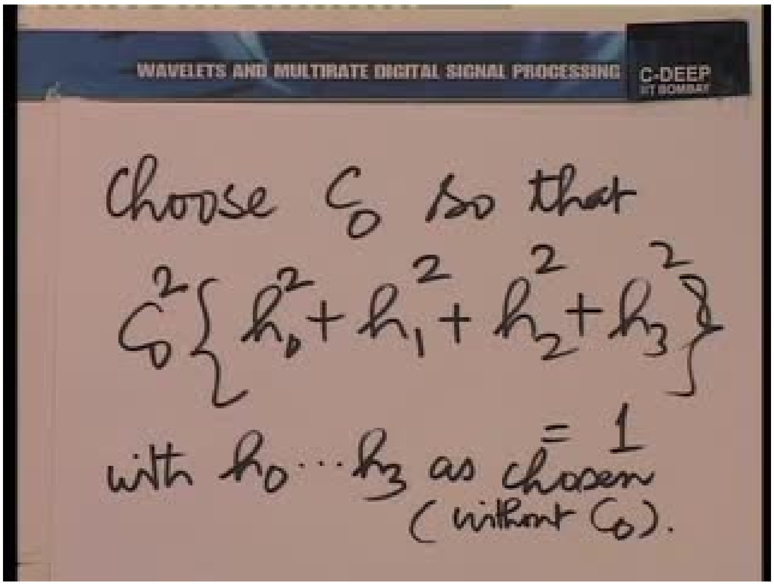

So, now, we shall choose c zero in order that this becomes one, and in fact, I no need to carry out the computations. Essentially what I am saying is choose c zero so that c zero squared times h zero squared plus h one squared plus h two squared plus h three squared is equal to one with h zero through h three as chosen before without c zero.

So, I leave that little calculation for you to do, and I would strongly recommend that students look at various texts that list the Daubechies filter responses and verify that the Daubechies filter response for a filter length of four exactly coincides with what we calculate from here; so much so far the Daubechies filter bank. Now, the next step is to build a phi t or psi t. How would we do that? So, there again, we go back to the iterative convolution that we talked about.

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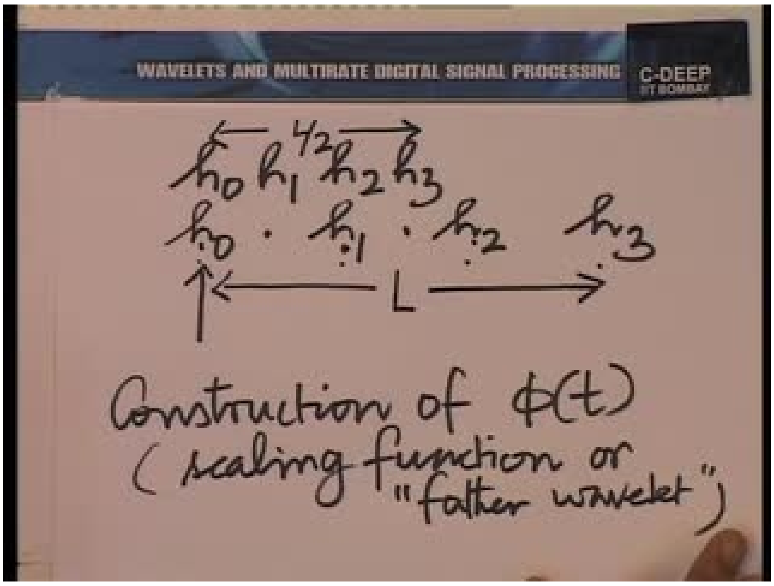

So, essentially you would have a situation where you first construct h zero h one h two h three lying on, you know, so the situation is that you need to compress and convolve, compress and convolve, as we did in the haar cases well. So, you would start essentially by putting h zero h one h two h three like this. Then now, now remember, what we are doing here is to construct pi t the scaling function or the so called father wavelet.

If you recall the basic step every time was first to take the sequence and then to take the sequence squeezed on the time access by a factor of two. So, you know, if you put sequence at these locations, now put the sequence at the following locations h zero h one h two h three. Here, these two location are mid way between these two locations here. So, mid way between this is this, and mid way between these is this and put h zero h one h two h three here. Visualize these to be impulses of strength h zero h one h two h three, and here, again impulses located at these places again which strengths h zero h one h two h three and convolve this with this essentially.

So, you know it is an iterative convolution and you can visualize that you would get impulses in one step of this convolution from four, you would get eight, and in fact, this would stretch a little beyond as you can see. In the next one, you would again get. This is squeezed by a factor of two. Now, in this case, it is much more difficult to visualize where this convolution is leading. What I have given you is the mechanism, and in fact, it is very important to exercise, a simple but a very effective computer exercise in this course to actually carry out this convolution. What kind of a pi t would result? The answer is not trivial.

So, it is unlike the haar case where you had a very neat answer. You got a nice beautiful rectangular pulse of height one. That is not the case here. The impulse response coefficients are somewhat complicated, and when we start convolving them, you have four of them, you convolving four with four squeezed by a factor of two and this is going to go on. What you can see of course is that ultimately all these impulses are going to come over a finite area; that is a point of observation. I want to explain this point to you. So, certain point but not very difficult to understand. You see, let me go back to this drawing.

The first time you carry out this convolution, this with this. (Refer Slide Time: 25:03) You are going to have an extension of the length over which these impulses lie. So, after this step, you can visualize h zero coming here, so the spread would be up to three of the samples going beyond h three. The next time you would have half of this interval getting added on there. So, if I call this interval l, then you have the interval l by two here. Let me draw the situation carefully. Let me draw it for two or three steps.

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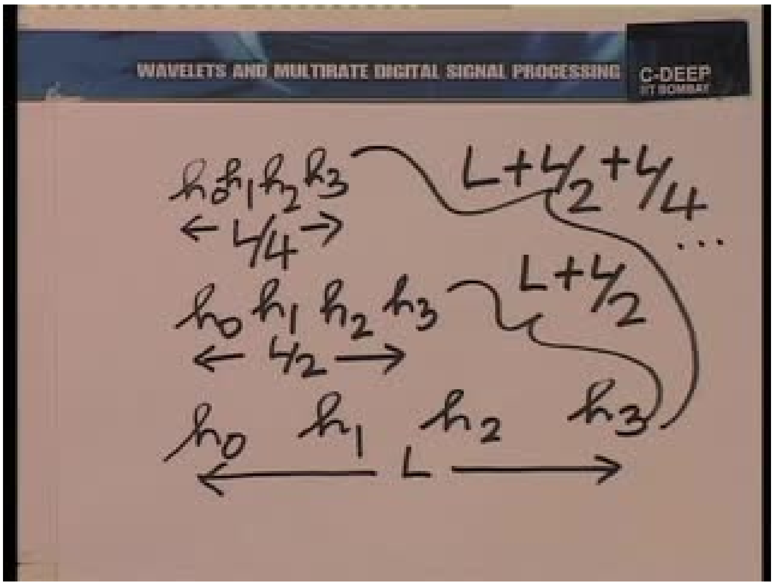

So, what I am saying is it is a convolution something like this. You start with a train of impulses which is spread over a length of l. The next time, the same train is squeezed by a factor of two. So, its spreads over a interval of l by two. The next time, it is squeezed again by a factor of two. So, you know I draw them squeeze together like this, and this length is l by four, and when you convolve this length l with length l by two, you get at this stage length l plus l by two, and when you convolve this with this, you would get l plus l by two plus l by four and so and so forth.

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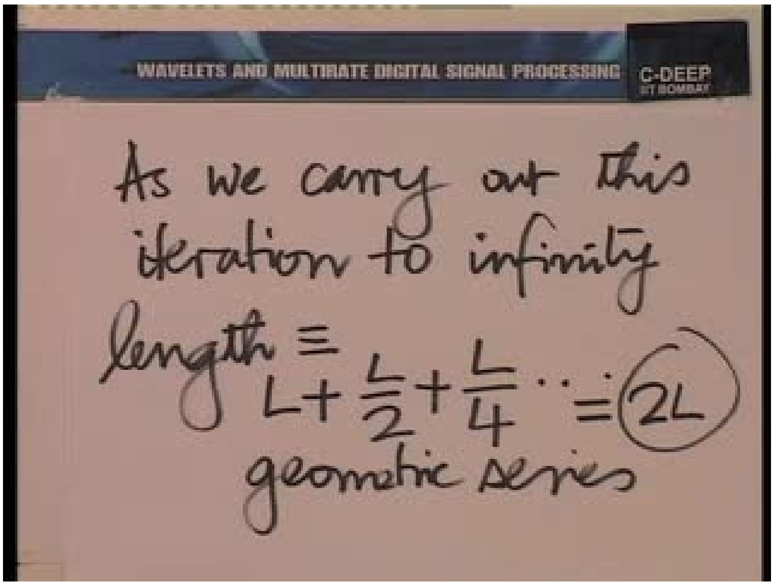

So, you can see that the length is not going to go to infinity but it is going to converge, and what is it going to converge to very simple. As we carry out this iteration to infinity, the length is going to converge to l plus l by two plus l by four and so on. It is a geometric series and the sum is very easy to calculate its two l.

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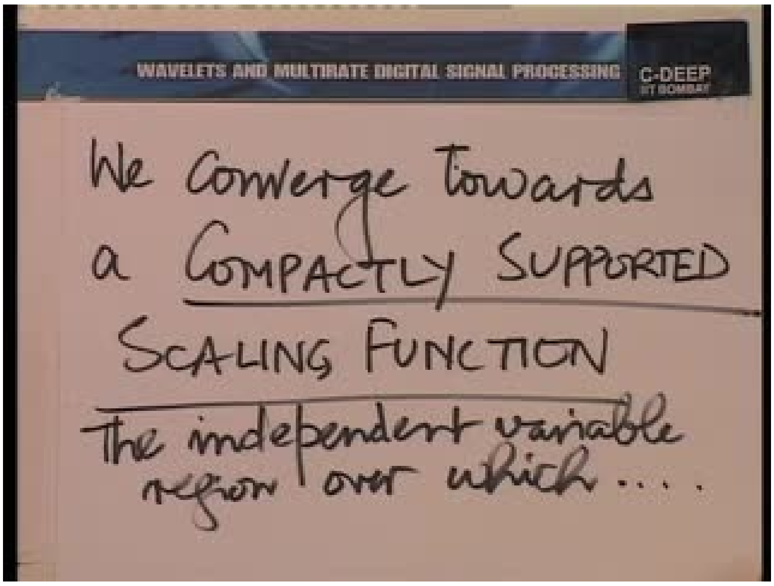

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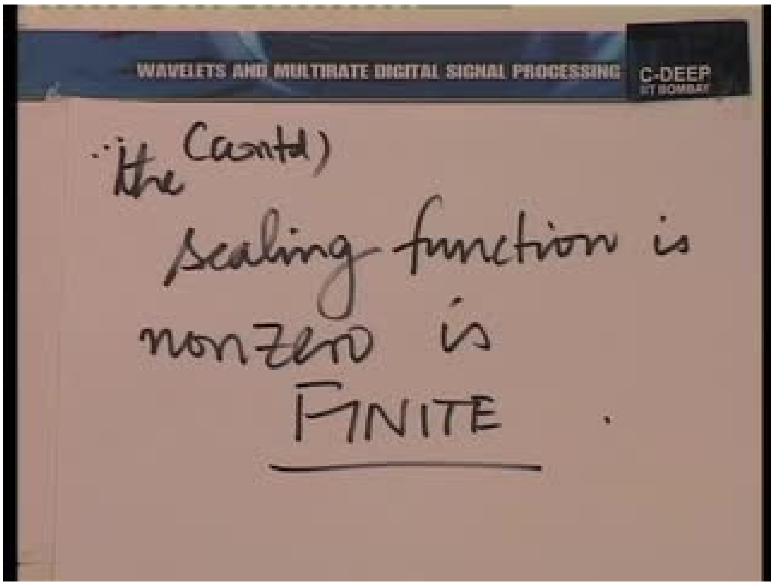

So, what we can see for sure is that this leads to what is called a compactly supported scaling function. When we iterate these impulses with the contraction every time, we ultimately get a function lying on a compactly supported region or more accurate way of saying, that is the function, the scaling function is compactly supported. It has a finite part of the time or the independent variable axis on which it is non zero. This is such an important thing that we should write it down and we should emphasis it. We converge towards a compactly supported scaling function. What does that mean? We are essentially saying, the region, the independent variable region over which, it is non zero. So, I will continue this, over which, the scaling function is non zero is finite.

Now, you know, this is the most important contribution that Daubechies made. Before Duabechies introduced this set of filter banks, the idea of neatly constructing a family of compactly supported multi resolution analysis was not existent in the literature. I would put this as a very very powerful contribution. That is not mean that the scenes were not there. In fact, the subjects of wavelets and, filter banks, filter banks as the multirate signal processing paradigm had almost developed in parallel, but using filter banks in an effective way to construct compactly supported scaling function and wavelets is an important contribution emerging out of daubechies work the work of Daubechies. Not only just compact support, successive scaling functions here have some additional property in terms of their behavior with respect to polynomial as you can see.

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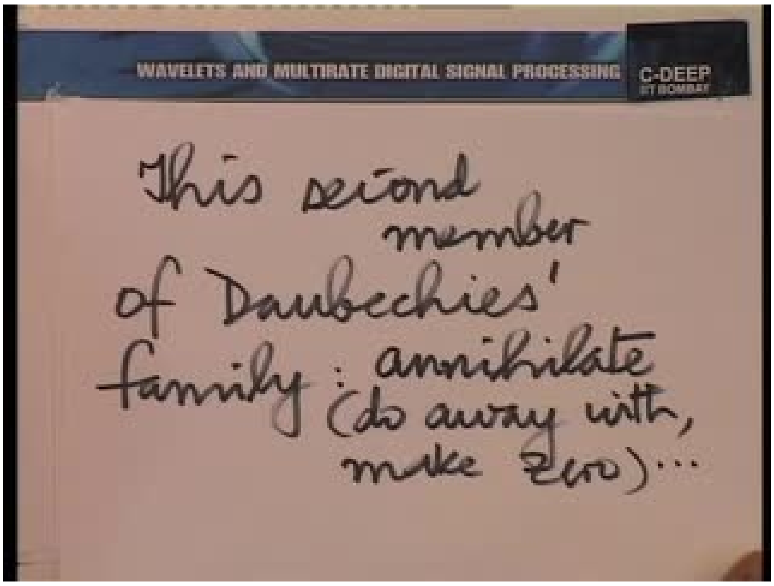

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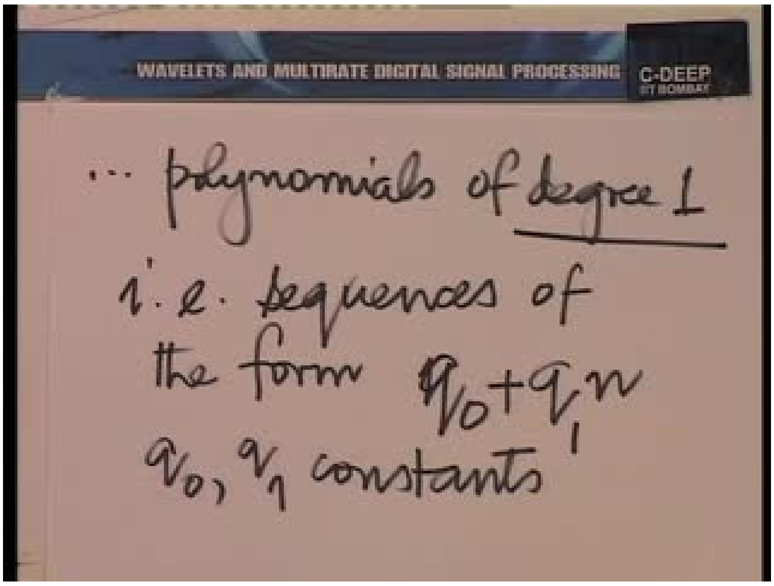

For example, what would happen in the second member of the Daubechies family? Let us reason it out a little better. The second member, this second member would annihilate or kill or do away with or make zero, whatever you wish to call that, annihilate polynomials of degree one. Essentially, sequences of the form, some say q zero plus q one n, q zero and q one are constants. What I mean by that is the following.

If you look at the overall sequence being given to the filter bank at the input, and if you would think of that sequence as possessing one component of this kind and the remaining essentially a residual component, only the residual component would come out on the high pass branch. This polynomial component would only be present in the low pass branch. So, other way of saying it is a few more smoother terms are retained on the low pass branch and remove from the high pass branch. Another way of saying it is – well, the high pass branch becomes even more and more high pass, you know. So, what I am saying effectively is that it really behaves more as a high pass filter than does the case of haar of length two. Daubechies of length two is the haar and that does not behave as well as a high pass filter as does this.

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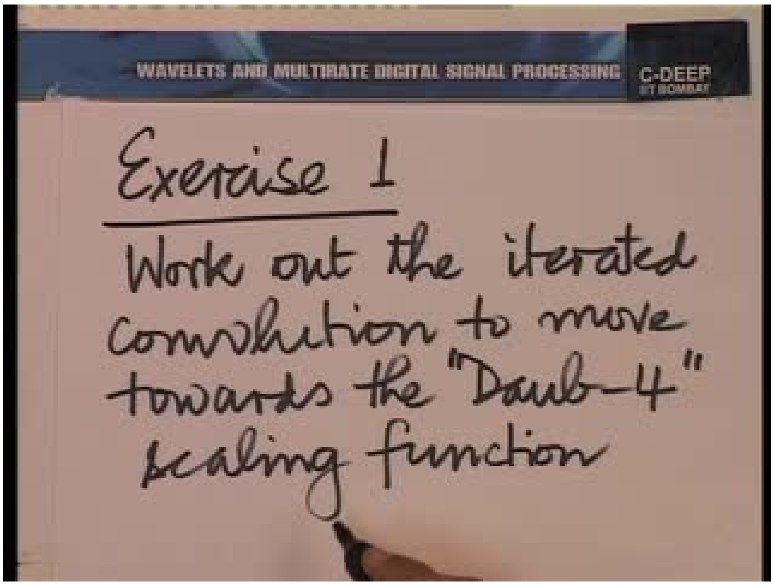

In fact, I would like to put down two exercises for the class now with this discussion that we have had, and I strongly recommend as a part of this course that students workout both of these exercise to understand what i am saying. Working out these exercises does mean using a computer. It would help to use a computer to evaluate the expression finely to get a good feel. It cannot be done by hand but it is worth doing. So, the exercises is as follows. I shall explain the exercise here. Exercise one - workout the iteration to move towards the daub four as they call it.

You know, this is a nomenclature that we would like to introduce here - daub four means the Daubechies filter bank with the filters of length four. So, here what we have just been talking about is the daub four set of filters or daub four filter bank. So, we were already explained how to carry out the iterated convolution and it would be worth actually carrying it out to move towards daub daub four scaling function.

One would notice that the function that emerges is is a continuous function but, it would not be expressible in what is called closed form. So, one would not be able to express it as some sin t or sin e rise the power t or something of that kind, but it would be a continuous function; it would converge to a continuous function, and you know, this is something important here.

We have just setup some kind of a filter bank and we have started iteratively convolving the impulse response with its own compressed versions. What is it that guarantees that when you carry this iteration to infinity, there is going to be some semblance of converges in that process, nothing inherently. So, if you take an arbitrary filter bank like this with an h zero h one g zero and g one, and if you want to take the h zero and carry out an iterated convolution like this, you might learn that with what is called a fractal function.

In fact, when I say fractal function, the word function is a misnomer. It would mean that that iterated convolution process would not converge to a function at all or at least definitely not to a continuous function that could very well happen. In the haar case, we had a neat beautiful rectangular pul pulse pulse to which it converged. In the Daubechies four case again, we are going to converge towards a continuous function. I am assuring you even before you carry out this exercise. We will soon see that for the higher order members of the Daubechies family, you would again converge to continuous functions. However, if you just picked some arbitrary low pass filter and started iterating it like this, may be even a low pass filter which satisfies that orthogonality to even translate that we have asked for.

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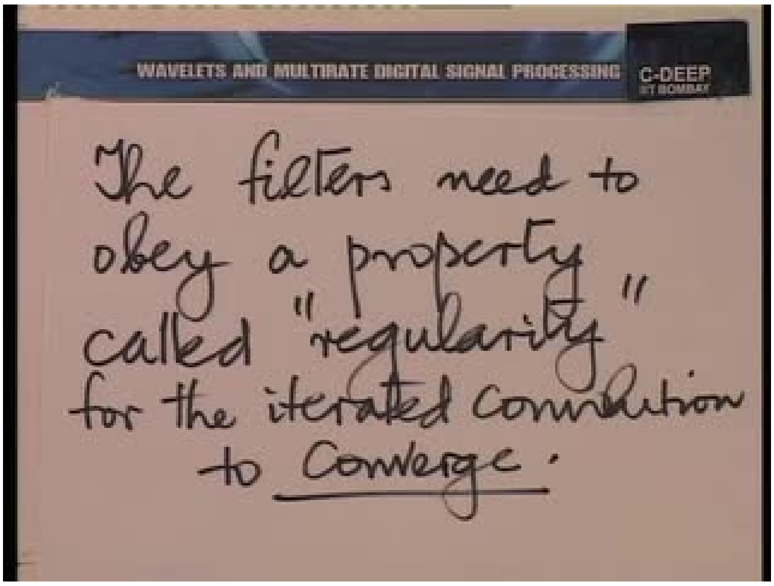

There is no such guarantee that any such arbitrary filter would converge in this iterated convolution. So, what is it about this filter which allows convergence? In the literature on wavelets, they speak of this as a property called regularity. So, we say that the filters need to obey for the iterated convolution to converge. Converge to what? Well, converge to a function which is either continuous or at least continuous almost everywhere except for an isolated finite number of points.

So, you know, if you really want to look at that way, the haar scaling function is not continuous, but it is only two points at which it is discontinuous not an infinite number of points, and we would not want that situation of this iteration taking us to us a quote-unquote function or object which has infinite points of discontinuous. That is what we are trying to say.

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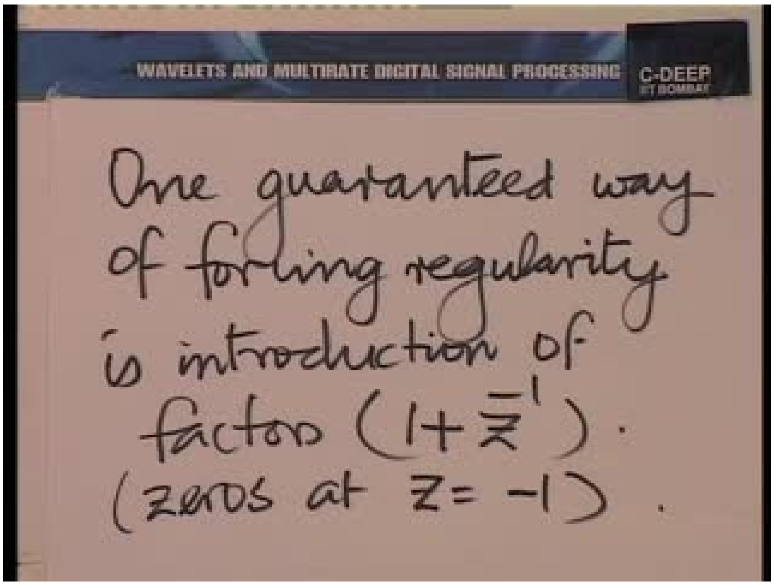

So, whatever it be, this regularity in this case comes from the presence of zeros, so one guaranteed way of forcing regularity is introduction of factors. So, the more zeros you have at z equal to minus one, of course in the low pass filter, and therefore, correspondingly you would have zeros at plus one in the high pass filter. Again, to give a physical significance, when you put z equal to minus one, you are talking about e rise the power, j omega being equal to e rise the power j j plus minus pi. So, omega being equal to pi is the extreme frequency, extreme high frequency.

So, in the low pass filter, we are saying put zeros at the extreme high frequency, and correspondingly, therefore in the high pass filter, we put zeros at the extremely low frequency, namely: zero, omega equal to zero. Omega equal to zero corresponds to z equal to plus one; e rise the power j zero is one, simple. So, one way to force regularity is to put zeros at minus one, and that is what we are doing in the Daubechies family. Haar one zero, daub four two zeros, daub six - that is the next member of the family, length six would have three zeros and so on, so forth, and you know, what we say? We say the higher you go in the Daubechies family in terms of length, the more regular your filters are, the more regular.

What that means is the functions to which we converge on iterated convolution becomes smoother and smoother. They have more and more derivatives that are continuous. So, if you look at daub four, its differentiability in the traditional sense is under scanner. It is continuous, but as far as differentiability goes there are issues, but when you go to higher order Daubechies, that is also taken care of. So, the function becomes smoother and smoother, and now, you can see how to do this for, higher order, higher order Daubechies filters. I mean whether its length six or length eight or length ten, we know exactly how to carry out an iterate convolution. Put all those impulses together, the impulse response coefficients as impulses uniformly spaced, squeeze that set of impulses by a factor of two convolve it. With the first set again, squeeze by a factor of two convolve it again and this can continue and continue and continue.

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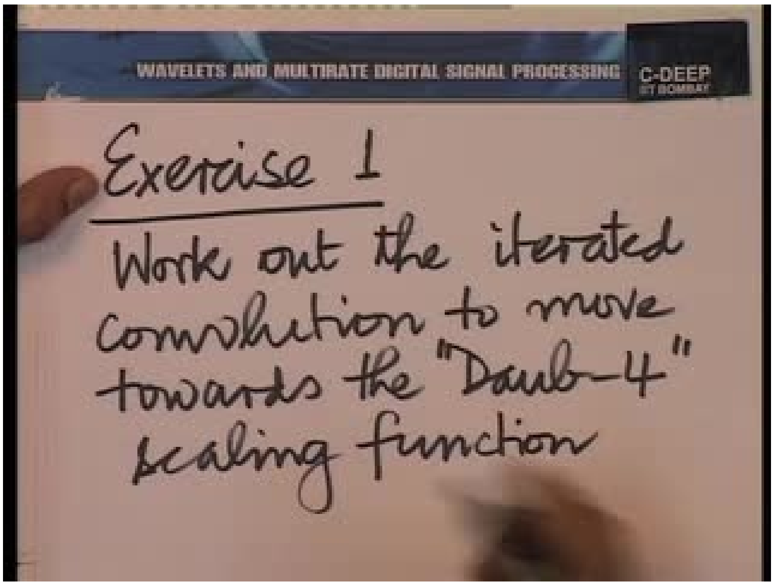

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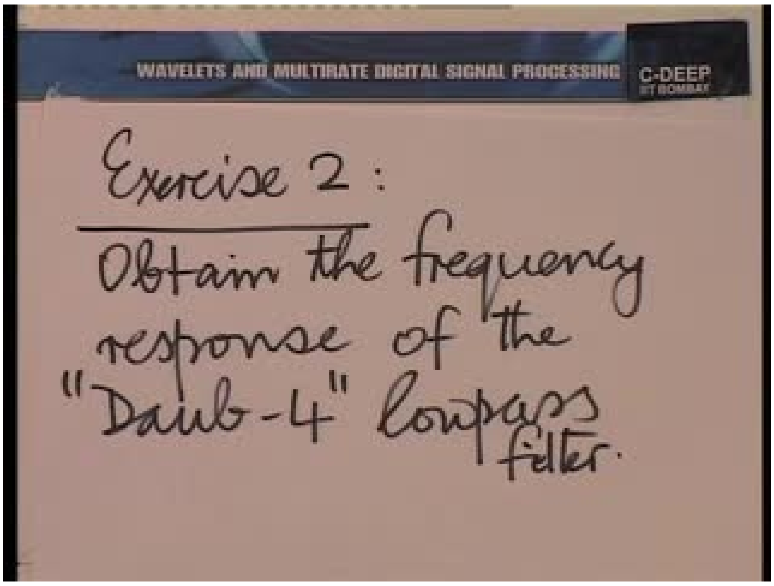

So, I leave it as I said as an exercise to complete this iterative convolution. I repeat the exercise which all of us must do work out the iterated convolution to move towards the daub four scaling function. The second exercise which I would like to ask the class to do is the following: obtain the frequency response of daub-four low pass filter, and of course, therefore, also of the high pass.

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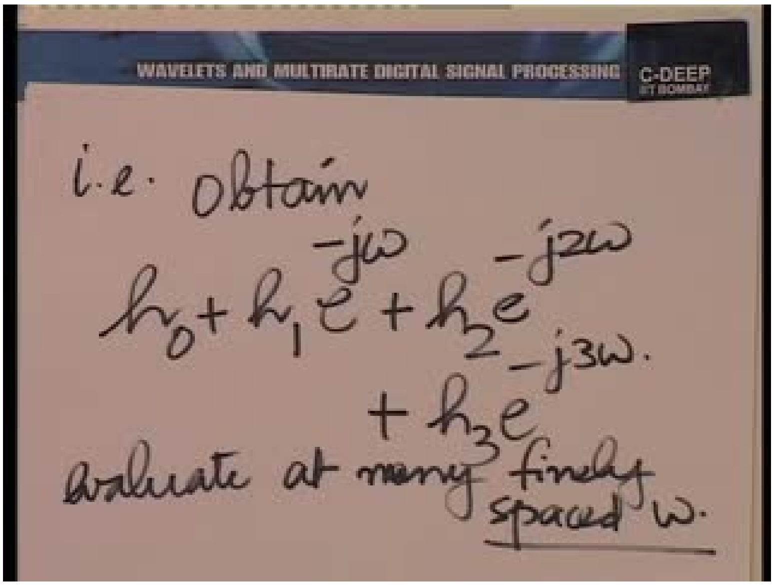

So, just for completeness, let me write down the expression for the frequency response. What I am saying is obtain h zero plus h one e rise the power j omega plus h two, well e rise the power minus j omega h two e rise the power minus j two omega plus h three e rise the power minus j three omega. Evaluate at many finely spaced omega. So, where we, you know, between zero and pi, one could take one thousand points and evaluate this expression to get a feel of the frequency response, and the idea is to compare it with the frequency response of the haar.

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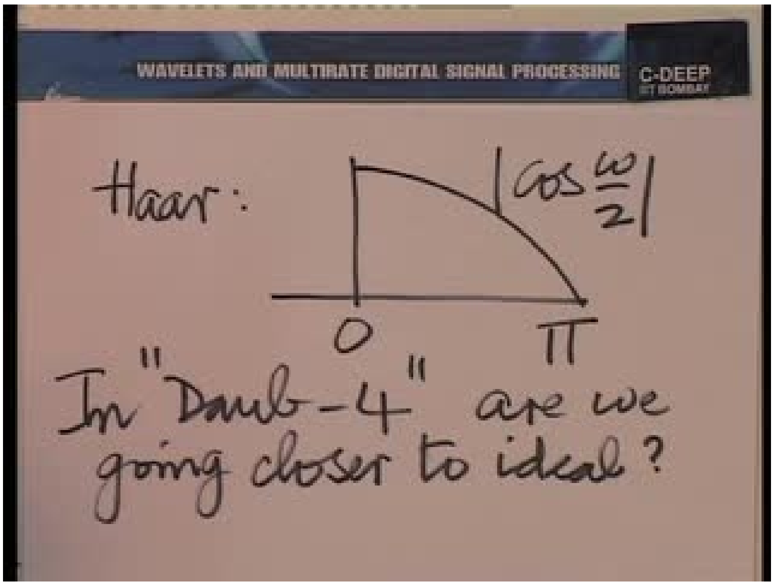

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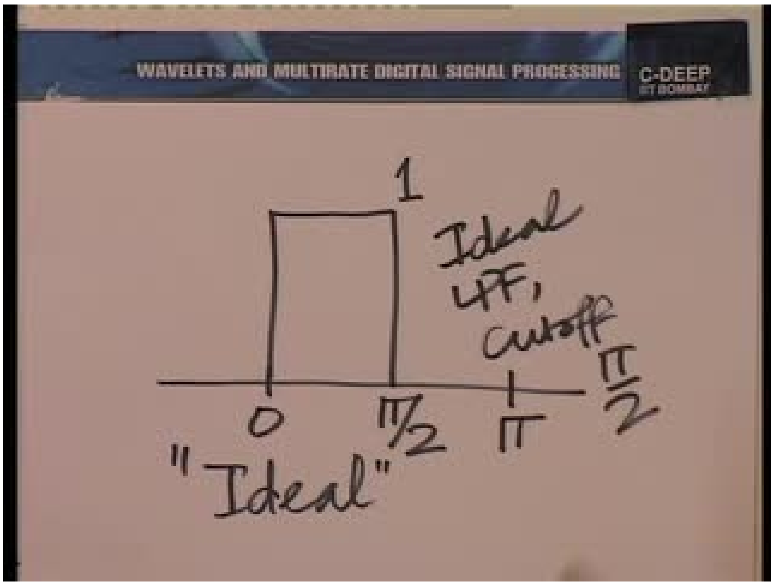

So, what we are specifically looking for is this - haar give us essentially this cos omega by two kind of response between zero and pi. In daub-four, are we going closer to ideal?

What ideal are we talking about? Remember, what the ideal was. The ideal is essentially this. This is the ideal, an ideal low pass filter with the cutoff of pi by two. Now, let me tell you how to built the next member of the Daubechies family. To do that, then we would put one more zero at z equal to minus one in the low pass filter.

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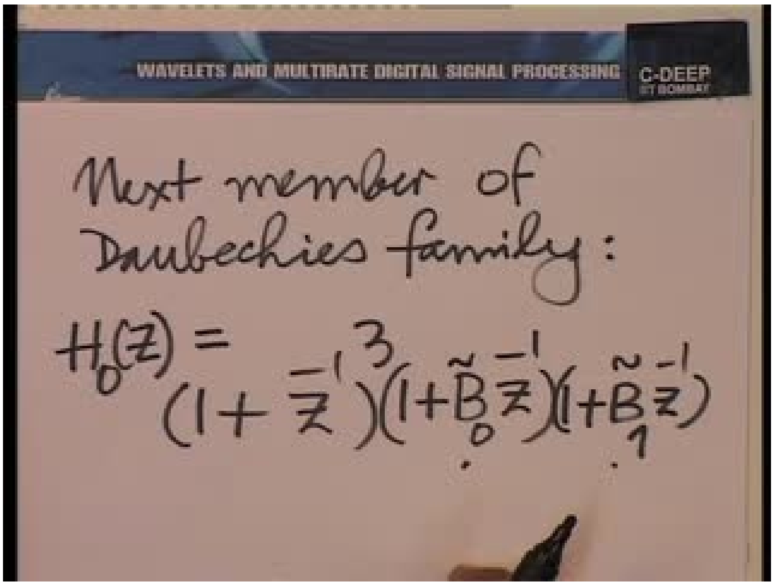

So, next member of Daubechies family: essentially, take h zero z to be of the following form - one plus z inverse the whole cubed and then would be two more degrees of freedom now or you could call b zero tilde if you like just to distinguish it from b zero that we have calculated here b one tilde z inverse. Remember that this member would have a low pass filter of length five, I mean, sorry, of degree five, and therefore, length six, and since its degree five or length six, three of the zeros are constrained, two of them are free. So, you have two free parameters here - b zero tilde and b one tilde.

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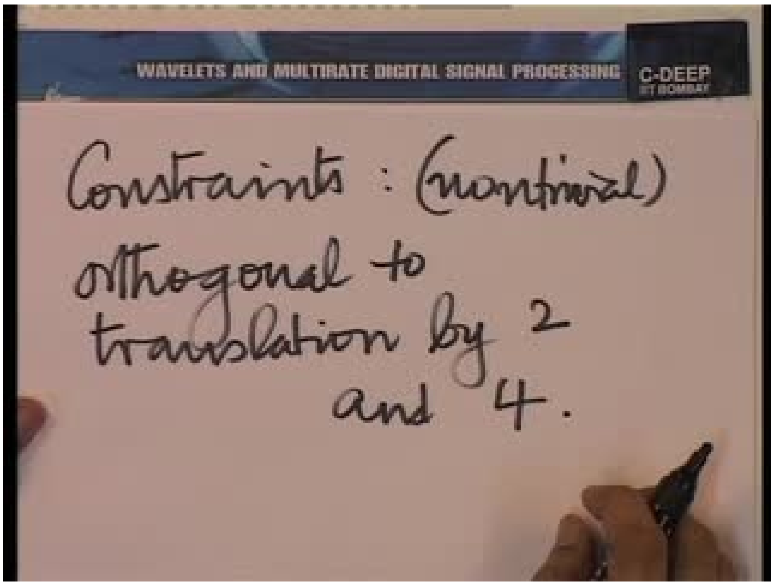

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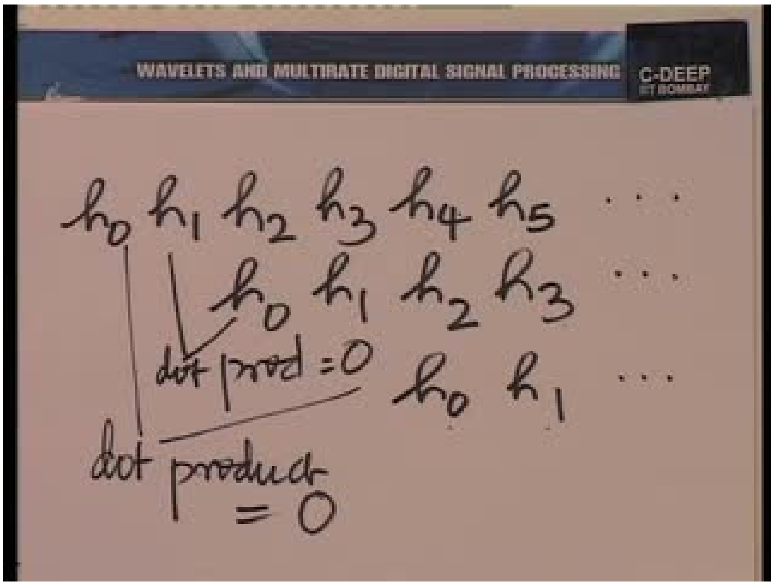

And what are the constrains? In this case, the constrains are and in the non trivial one timing orthogonality to translation by two and four. In other words, let me put it down explicitly. You have h zero h one h two three four five. Translation by two would mean this, the rest of them are zeros. We do not need to bother translation by four would mean this. So, take the dot product of these and put it equal to zero; take the dot product of these two and put it equal to zero. (Refer Slide Time: 48:50) These are two constraints. So, I will read them of - h zero times h two plus h one times h three plus h two times h four plus h three times h five is zero in this and h zero times h four plus h one times h five is zero in this and this the only two non trivial constrains that we have. Two constrains, two free parameters, one can determine them. Simple quadratic equations this time they are simultaneous quadratic equations involved little more work but do able, and one can keep doing this for higher and higher order members. Now, by the way, there are different ways of building this family of Daubechies filter; this is one way. They are more convenient was two or what might be seen as more convenient by sum.

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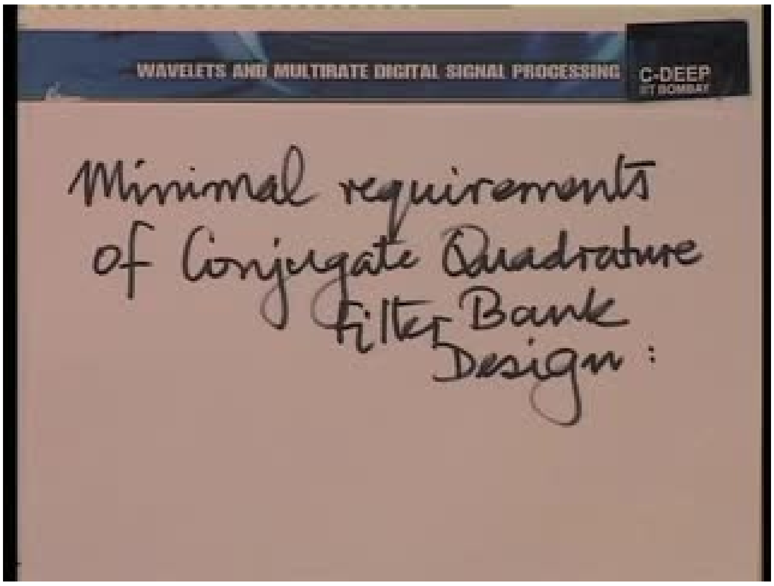

It is not our objective to dwell on those methods in this lecture, but rather to make a more general remark now about this class of filter banks that we are talking about, namely: the conjugate Quadrature of filters. So, we wish to push down, what we call the minimal requirements of design in a conjugate quadrature filter bank.

Incidentally, you might wonder where this name comes from conjugate Quadrature. Actually it is the quadrature word which is important there. The quadrature word comes in some sense from the idea of a ninety degree's shift. So, you know, in a certain sense, what we have done is to relate the high pass filter and the low pass filter frequency responses by a shift of pi on the frequency access. So, notionally what we are saying is a low pass filter with cut off pi by two as pi has to become a high pass filter with cut off pi by two in this relationship replacing z by minus z essentially. So, this replacement of z by minus z to relate the low and high pass filter brings what is called as a quadrature relationship; that about the name, but anyway.

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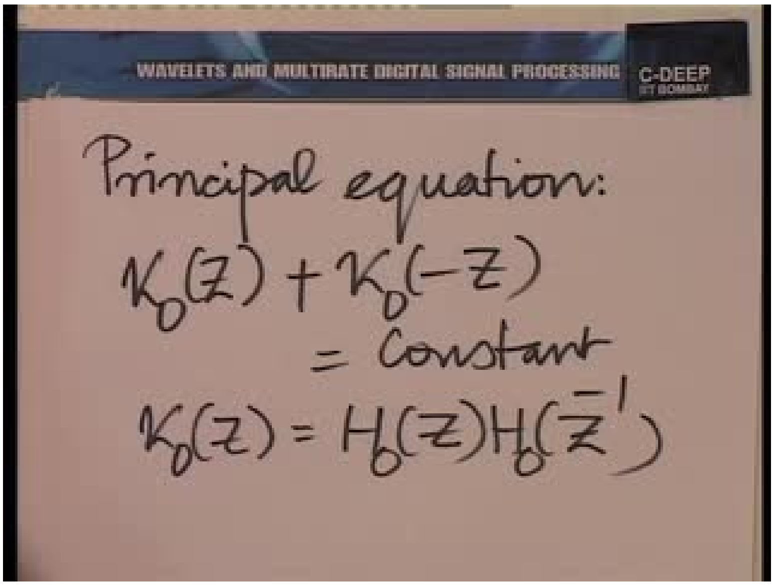

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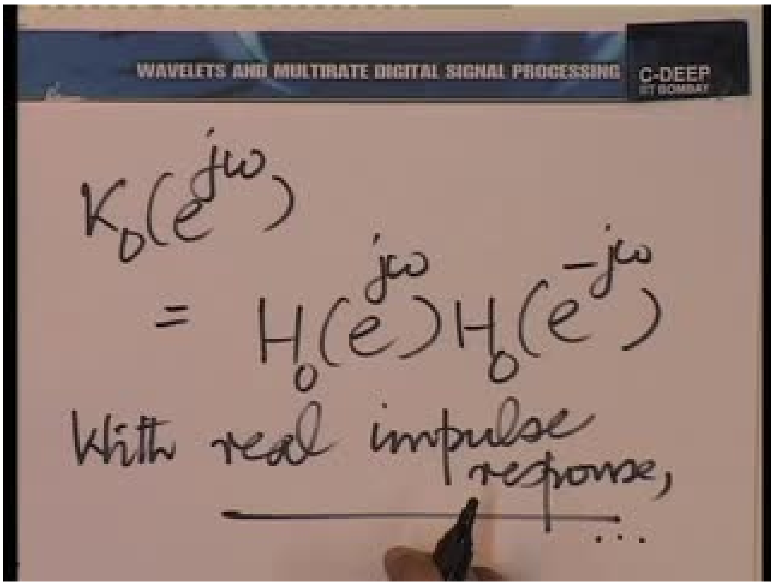

So, what we have is essentially the following equation. The principle equation governing the quadrature filter bank is this - kappa zero z plus kappa zero minus z is a constant, where kappa zero z is h zero z h zero z inverse, and therefore, if you look at it the frequency domain says kappa zero e rise the power j omega is h zero e rise the power j omega h zero e rise the power minus j omega essentially.

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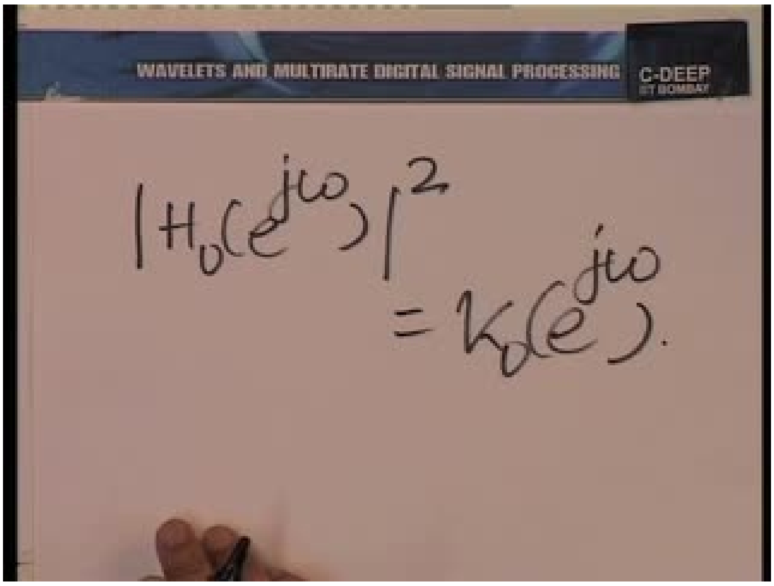

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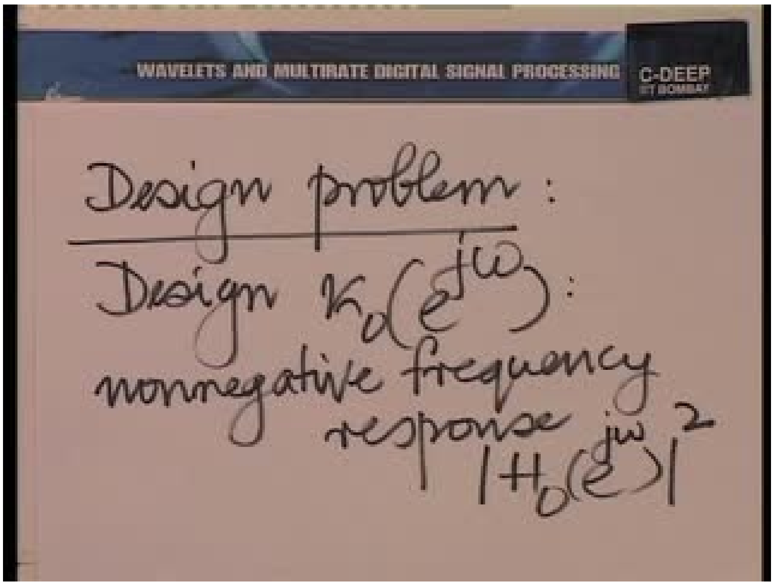

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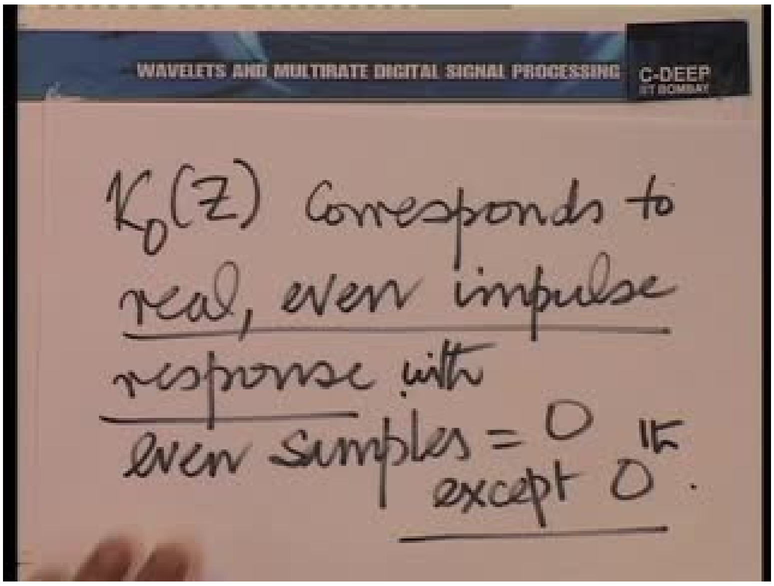

And with the real impulse response, what do we have? With the real impulse response, we have mod h zero e rise the power j omega squared is kappa zero e rise the power j omega. So, in other words, then we have a very clear design problem before us. Designing a conjugate quadrature filter bank is equivalent to designing essentially one filter kappa zero e rise the power j omega. So, the design problem is - design kappa zero e rise the power j omega, and if you look at kappa zero e rise the power j omega, it is a non negative frequency response as you can see, mod h zero e rise the power j omega the whole squared, and this non negativity can only come from an even, real and even response. So, we are saying kappa zero z corresponds to real and even impulse response. With that constrain that, even samples must be zero, except zeroth.

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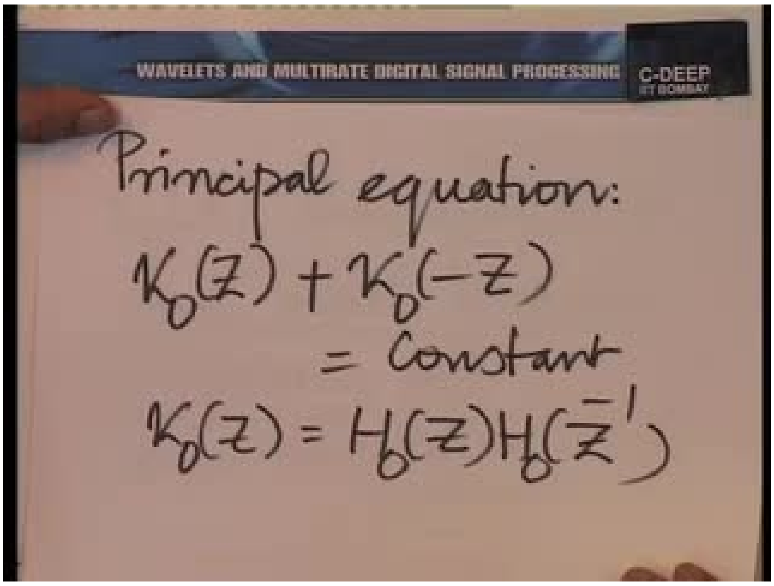

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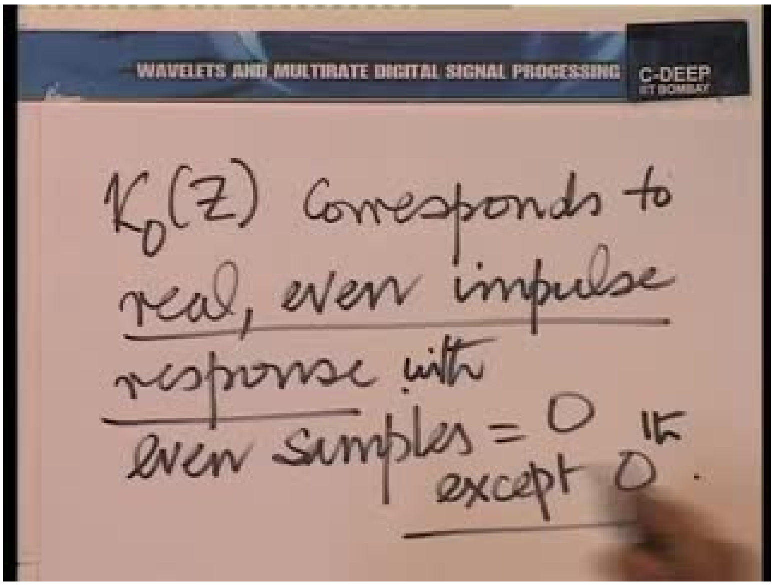

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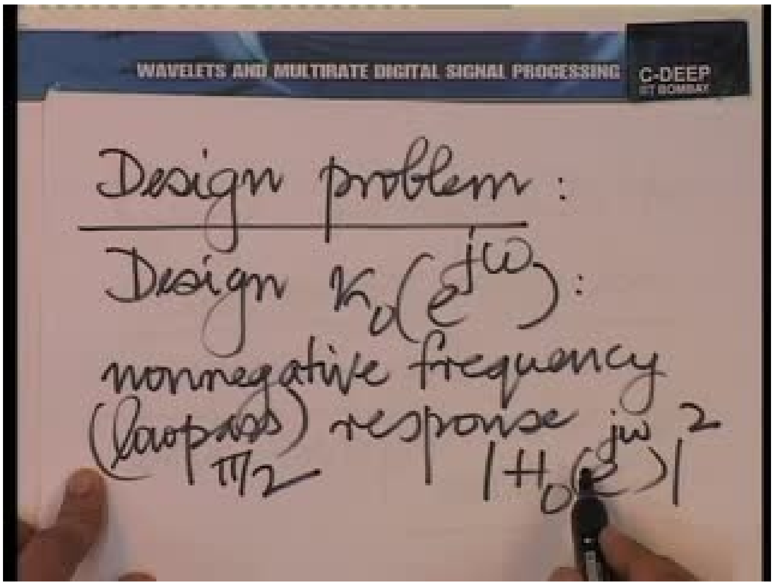

So, you know, this equation that we had here kappa zero z plus kappa zero minus z is a constant. Essentially says the even samples, other than the zeroth sample is all zero. So, you are trying to design a low pass filter. So, in fact, we should qualified this further non negative low pass frequency response, and what kind of a low pass frequency response with a cut off pi by two. Now, we have the design problem very clear.

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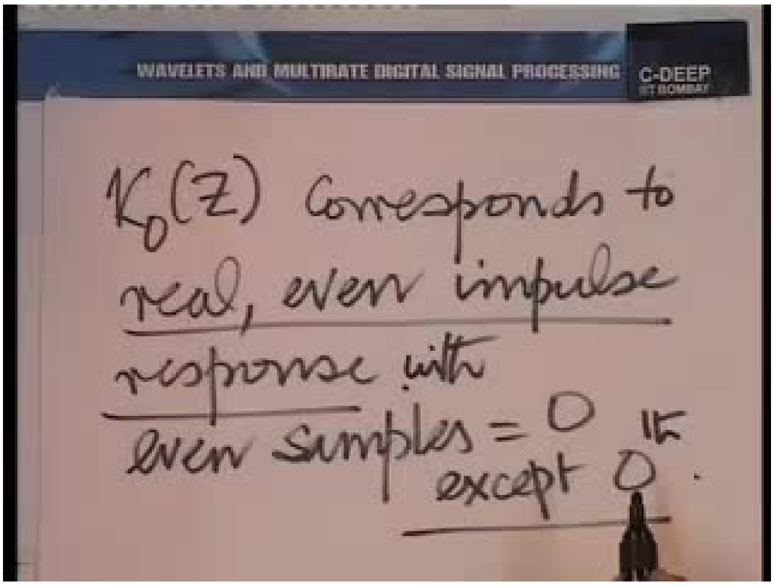

Design an even impulse response a low pass as powering to be a low pass filter with cut off pi by two non-negative with the constrain that the even samples of the impulse response are zero except for the zeroth sample. There are many different ways in which one can design finite impulse response filters and any optimization which allows us to design kappa zero with these constraints is acceptable to design kappa zero, and once we have kappa zero, then you look at its routes. For each route, we have pairs of reciprocal routes h zero z h zero z inverse. Out of each pair of reciprocal routes, put one in h zero and the other one automatically goes to h zero z inverse.

So, this is the general strategy to design conjugate quadrature filters, and the Daubechies family is just one of many such families. So, with this then, we have put down a whole family of multi resolution analysis of filter banks for you. In the next lecture, we shall ask what is it that we are for looking in these families. In other words, is there some fundamental limit? Is there some fundamental two domain requirement that we are trying to seek and fulfill?

Thank you.