Advanced Digital Signal Processing – Wavelets and Multirate
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Module No. # 01
Lecture No. # 13
Conjugate Quadrature Filters: Daubechies' Family of MRA

A warm welcome to the thirteenth lecture, on the subject of wavelets and multirate digital signal processing, we continue in this lecture to build upon the particular class of filter banks, which we yet introduced in the previous lecture; namely the conjugate quadrature filter bank, a number of issues related to that filter bank were left unanswered in the previous lecture, to some extent our introduction of the filter bank seemed adhoc at points, what I mean by that is we had suddenly made little twists in the nature of the filters, where a proper justification had not been given simply because there was a bit of a chicken and egg problem.

The justification was best seen after we went through the discussion, and that is what I had promised that, after we complete an understanding of this filter bank many things will be a little more clear. So, let us impact that upon that filter bank once again.

Let us look at that conjugate quadrature structure once again, first in total and then in specifics.

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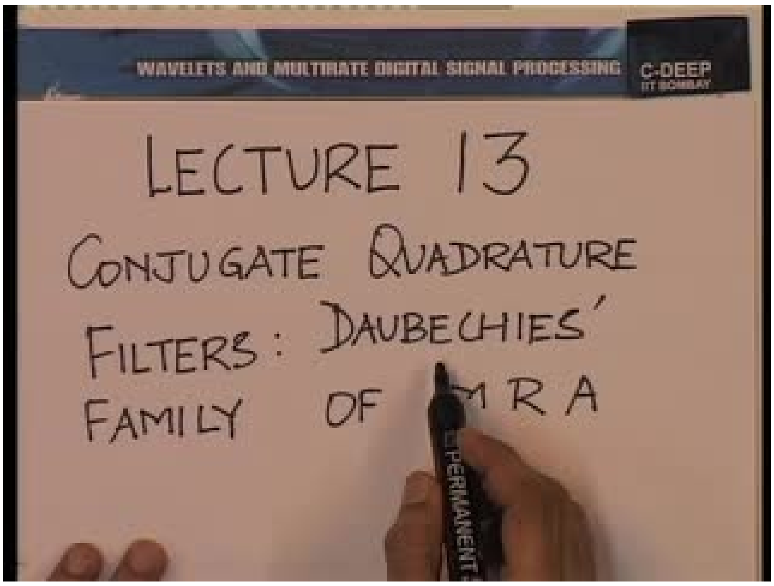

So, in today's theme, we shall look at conjugate quadrature filters in depths, and we shall again consider one specific class of those conjugate quadrature filter banks, namely the family of filter banks and family of multi-resolution analysis that emerge from Daubechies' filters.

Incidentally, as I mentioned Daubechies' or sometimes it is pronounced as doubechies' has been a mathematician, scientist, engineer whatever you want to call her of repute, her important contribution in this field has been to propose a family of compactly supported wavelengths, which also have some other interesting properties.

It turns out that the haar wavelet is the baby of the Daubechies' family, the simplest of the Daubechies wavelets, and there are further and further once of which we shall give an introduction today. In fact, the central idea in the Daubechies' family is to build upon what we had briefly mentioned in the previous lecture, namely the idea of keeping and annihilating polynomials of higher and higher degree, on one of the two branches of a filter bank.

Anyway, we shall look at specifics as we go along, but this is to put the lecture in perspective. So, we shall talk today about the conjugate quadrature filter bank and we shall look specifically at the Daubechies' family of MRA.

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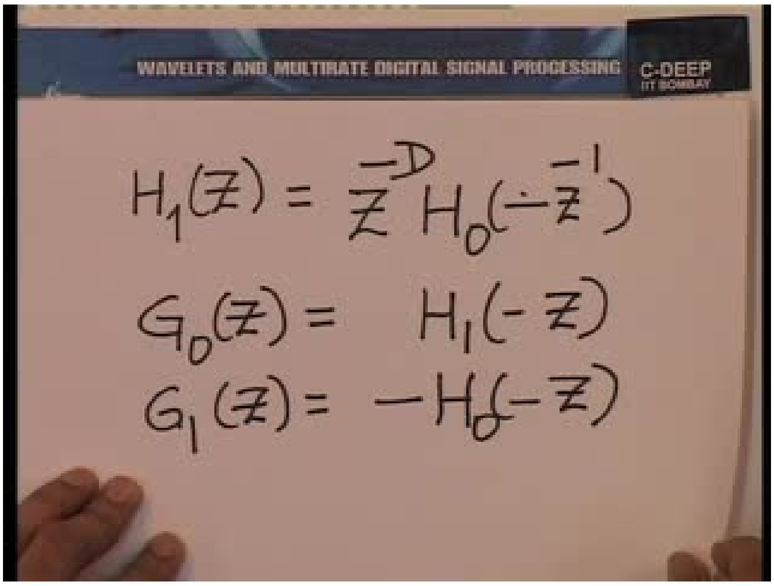

Now, you see the conjugate quadrature filter structure as we understood it at the following relationships between the filters, so we had the analysis high pass filter was related to the analysis low pass filter by the following relationship, and we had promised that we shall understand this, a little better today. Of course the synthesis filters were related very easily to the analysis filters, so you had G 0 Z being H 1 of minus Z and G 1 Z being minus H 0 of minus Z, this of course was essentially alias cancellation for you, these two conditions, but now let us focus on this relationship of H 1 to H 0.

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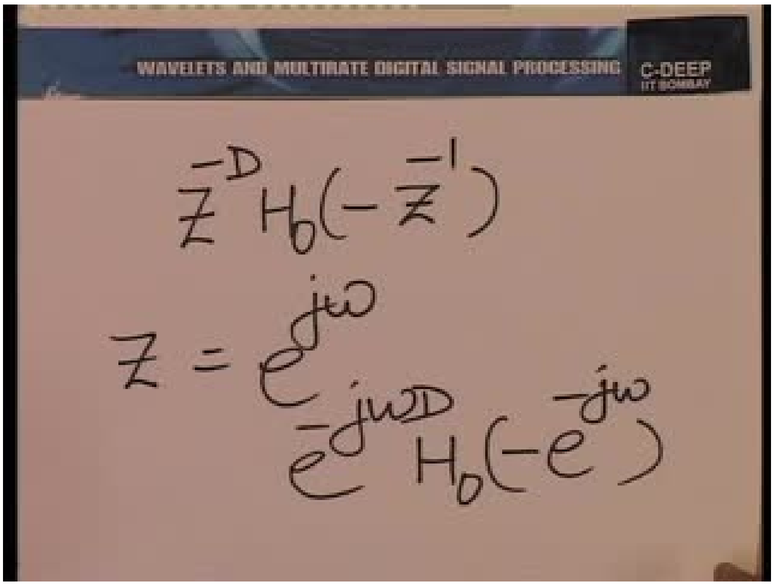

So, first let us justify why it is a high pass filter, so let us consider this expression Z raise the power minus D, H 0 minus Z inverse, and let us put Z equal to e raise the power j omega as we do to obtain a frequency response, where upon we will have e raise the power minus j omega D, H 0 minus e raise the power minus j omega. Now, if we take the magnitude of this, as it is normally what we are interested in.

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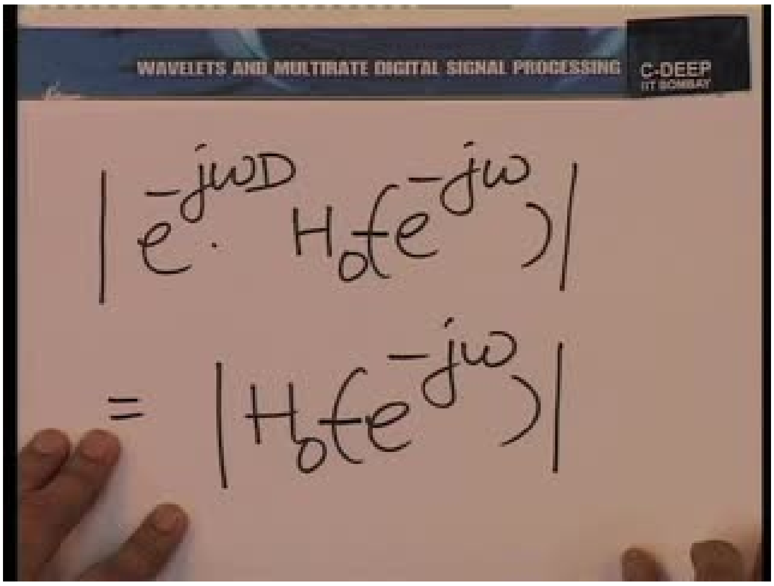

We have the magnitude of e raise the power minus j omega D, H 0 e raise the power minus j omega, is the same as the magnitude of H 0 e raise the power minus j omega, well minus of this, and that is, because the magnitude of this is 1, and now let us look at this quantity, the magnitude of H 0 minus e raise the power minus j omega.

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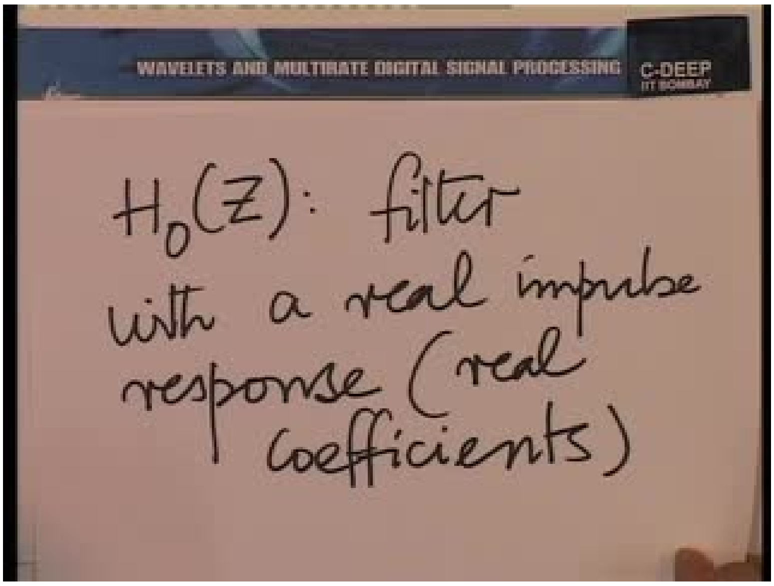

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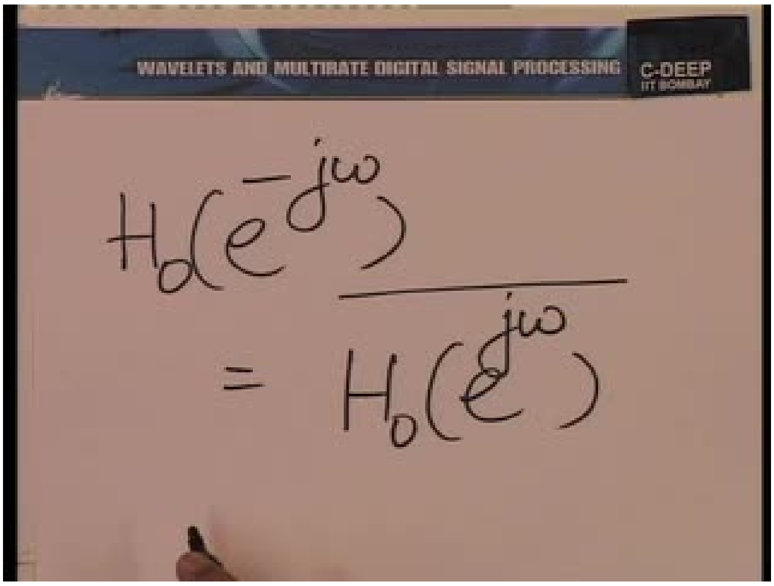

You see if H 0 is a filter with a real coefficients, so if H 0 Z corresponds to a filter with a real impulse response, and that is the class in which we are most interested, in that case H 0 e raise the power minus j omega is going to be H 0 e raise the power j omega complex conjugated, that follows in a straight forward way from some basic properties of the discrete time Fourier transform. What we are saying essentially is that the magnitude response of a filter with real impulse response is symmetric in omega and the phase is anti-symmetric.

Now, if we now replace e raise the power minus j omega by minus e raise the power minus j omega, what are we really doing.

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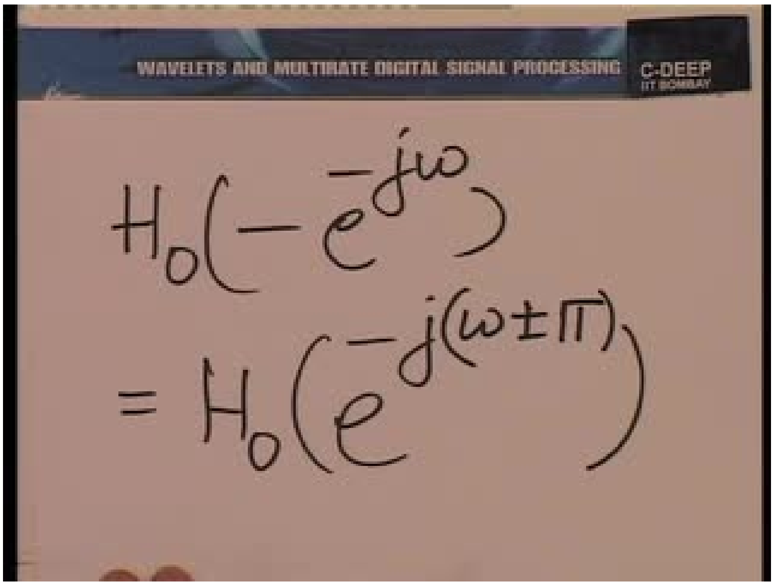

So, H 0 minus e raise the power minus j omega is essentially H 0 e raise the power minus j omega plus minus pi, we have done this before.

We have noted that minus 1 is essentially e raise the power plus minus j pi, and therefore what we have done here, is essentially to shift this by pi, either forward or backward it does not make any difference, because there is a periodicity with a period of 2 pi.

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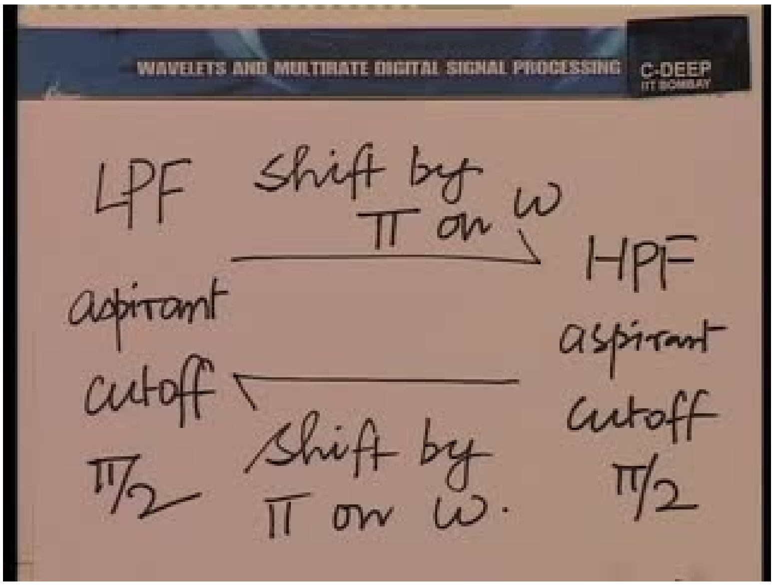

Anyway, what we do now is that a low pass filter, I think we have seen this quite frequently now, a low pass filter when shifted by pi on the frequency axis, becomes a high pass filter, of course a low pass filter aspiring to be a low pass filter with a cutoff of pi by 2.

It becomes an aspirant for a high pass filter with a cutoff of pi by 2 again, and similarly when a high pass filter is shifted by pi on the omega axis, it becomes a low pass aspirant with a cutoff of pi by 2; we have seen this petty much before. Anyway recognizing this then, we have an interpretation for what we just did.

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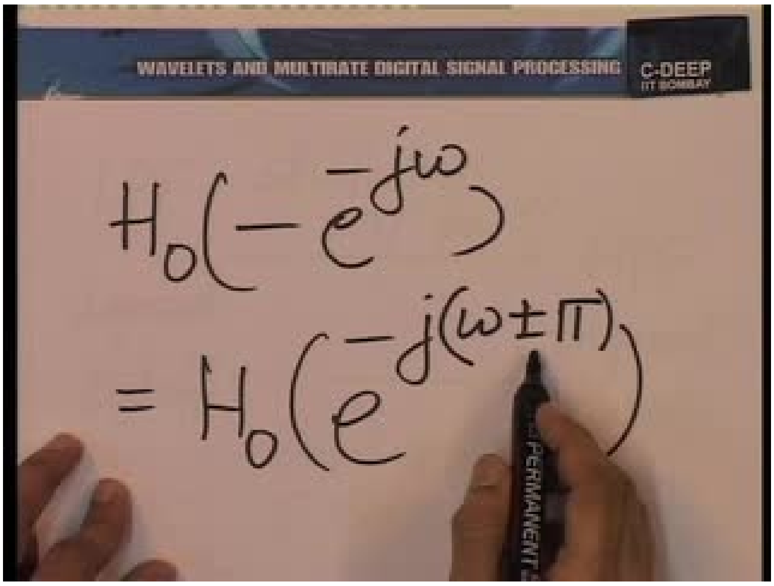

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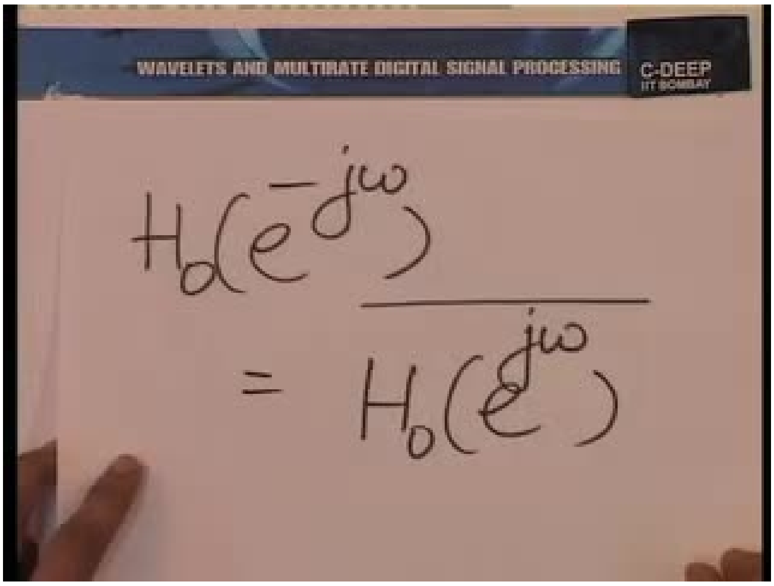

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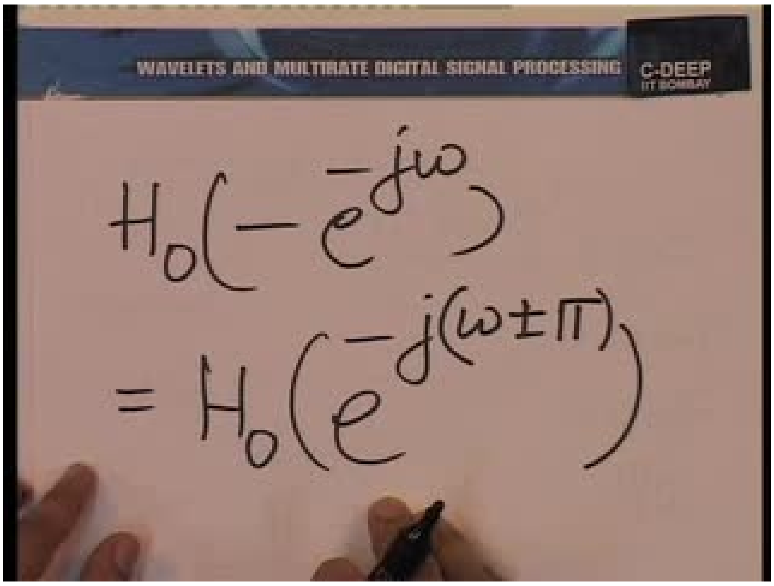

So, we said this minus essentially shifts by pi, and therefore H 0 e raise the power minus j omega without the minus sign would have been a low pass filter as it is, because of this conjugate symmetry that we have here, and now with the introduction of a minus sign, it becomes a high pass filter. So we have a convincing argument now that H 1 Z, the way we have constructed it.

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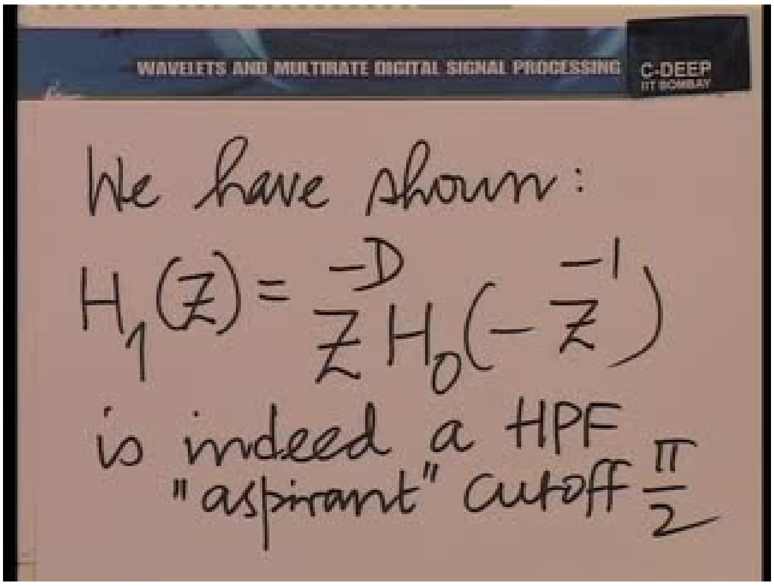

So, we have convinced ourselves, we have shown H 1 Z in the way that we have constructed it, namely Z raise the power minus D, H 0 minus Z inverse is indeed high pass or a high pass aspirant, aspires to be an ideal high pass filter with cutoff pi by 2.

For my read of course H 0 Z is an aspirant to be a low pass filter with cutoff pi by 2, so now things have fallen into place, the only issue is, why have we taken this peculiar expression, not so peculiar really, now we do not see it is so peculiar, but why that Z inverse and so on, so we will understand that in a minute.

I will just give you a trailer for the reason, the trailer is said this automatically brings condition on the magnitude, we will see that shortly. Anyway, now let us put down the alias cancellation condition is anyway put down, we need to put down the perfect reconstruction condition, so let us put down the perfect reconstruction condition.

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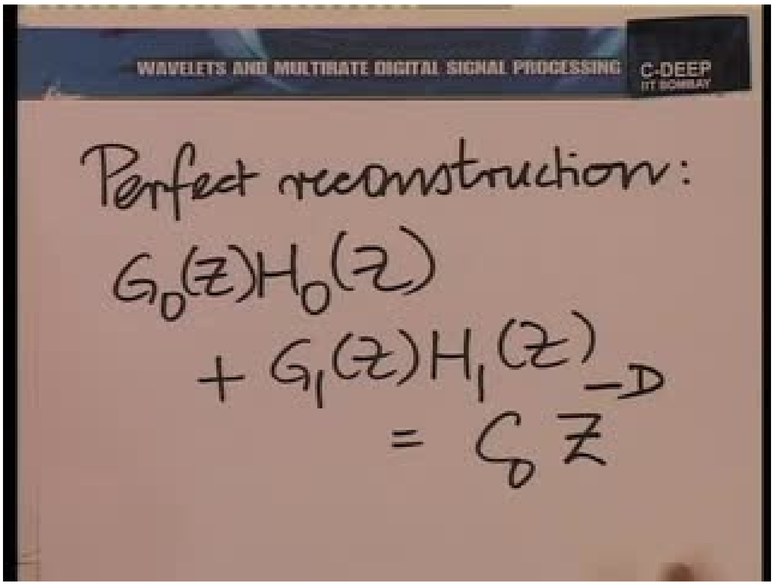

We did that yesterday, but we will do it a little more carefully. The perfect reconstruction condition, essentially says that you would have G 0 Z, H 0 Z plus G 1 Z, H 1 Z must be some constant, we call it C 0 times Z raise the power minus D, and what are G 0 G 1 H 0 H 1 here.

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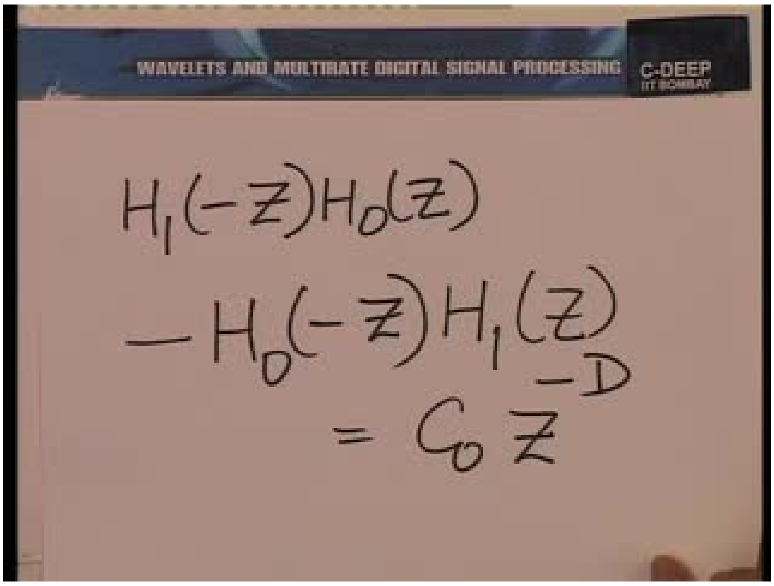

And, you agreed that G 0 Z is essentially H 1 of minus Z, so we had H 1 minus Z, H 0 Z plus now G 1 Z we had agreed to make minus H 0, minus Z and H 1 Z of course we agreed to make it Z raise the power minus D and so on, but let me write H 1 Z for the moment, and we want this whole thing to be C 0 Z raise the power minus D.

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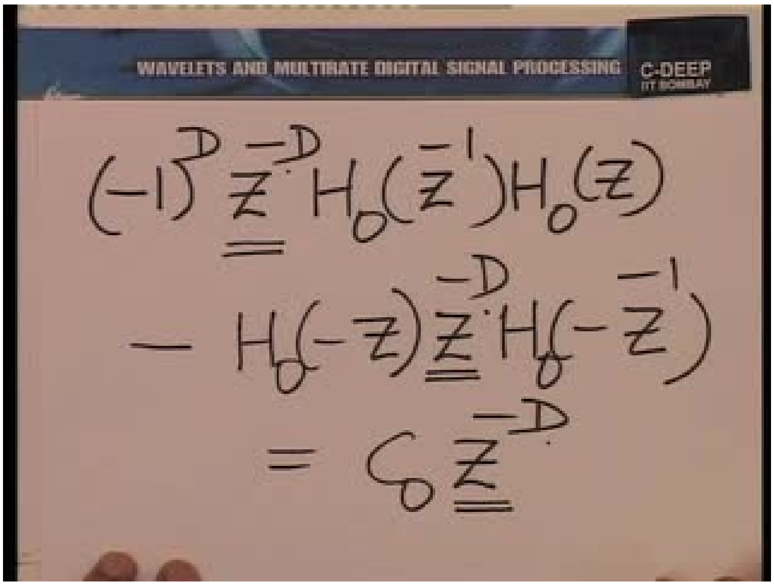

Now, we will substitute H 1 Z in this equation, and we have minus 1 to the power D Z raise the power minus D, H 0 Z inverse times H 0 Z, minus, now again you have H 0 minus Z here, and H 1 Z becomes Z raise the power minus D, H 0 minus Z inverse.

This you desire should be C 0 Z raise the power minus D. Now, you know this Z raise the power minus D that we have here, in fact we should not quite have written it like this, though what we have written now happens to be correct, we should have started by giving a different value for the delay here, and the delay on this side, but now again through serendipity or through convenience we can actually make them the same, the purpose of putting this Z raise the power minus D here was actually to take care of this term here.

So, it is not coincident till that we have written the same D on both sides, so that should not have been done. Initially, we are doing it right away to emphasis that this Z raise the minus power D term that we introduced in H 1 was meant to take care of this. So, what we are saying in effect is that we want the rest of it to match as well.

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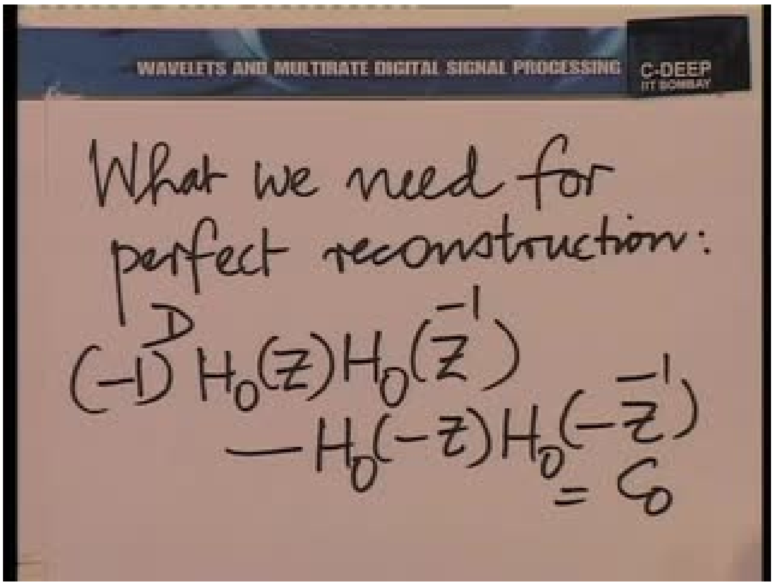

So, what we desire for perfect reconstruction, is essentially this; minus 1 raise to the power D, H 0 Z, H 0 Z inverse, minus H 0, minus Z H 0, minus Z inverse is a constant.

Now, again we have the freedom to choose the value of capital D here, again the main issue is whether capital D is odd or even, if capital D is odd then we have a minus in both places for both the terms, if it is even then this is a plus and this is a minus.

Let, us choose to make capital D odd, and in fact again there is a reason for that, it is not arbitrary, we have just looked at the haar filter bank, where we have a filter of even length, a length 2 filter actually; all of them you know 1 plus Z inverse, 1 minus Z inverse, on both sides are of length 2, when we replace Z by Z inverse.

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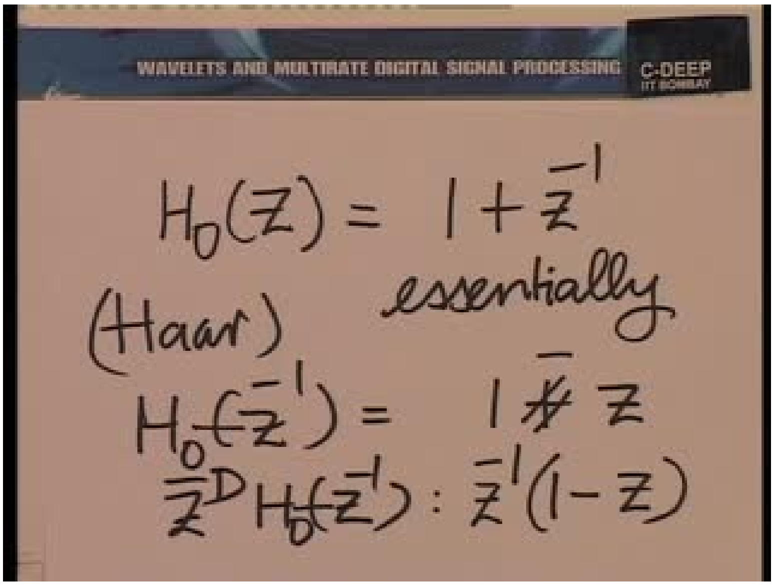

So, let us take the haar case once again, you had H 0 Z of the form 1 plus Z inverse, whatever forget the by 2 here, in the haar case H 0 Z inverse would have been 1 plus Z and D Z to the power minus D H 0 Z inverse or if you choose, you can write minus Z inverse as we do and this would then become minus here, Z raise the power minus D H 0 minus Z inverse is actually intended to make this causal, this filter is non-causal, so you need to introduce Z raise the power minus 1 here to make this causal.

Therefore D becomes 1 in this case, so you see the roll of D, I had hinted at this yesterday, we said that the reason why we cannot avoid a delay is, because you want the filters to be causal, now you see what we need.

The Z raise the power minus D term has been put there to retain causality, and you just put as much of a D as is needed to allowable causality, and so here the D required is 1. Now, the Daubechies' family, we keep augmenting the filter length by 2 in every round of the family ladder.

So, when we go from the baby of the family, namely the haar MRA to the next member of the family, we augment the length by 2, so we have a length of 4. When we go to a length 6, it gives us the third member and so on, Length 8 the fourth member and so on.

So, successive even lengths of filters give us successive members of the family, in the Daubechies' family. Now, what we are going to do is slowly move towards building, the second member of the Daubechies' family, and therefore the next case would be capital D equal to 3, so you would have a length of 4 and you would have a maximum power of Z equal to 3, Z cubed when you write H 0 minus Z inverse, that is the roll of Z raise the power minus D here. Therefore, it is justified for us to begin by assuming that D is odd. So, let me put that down once again for you.

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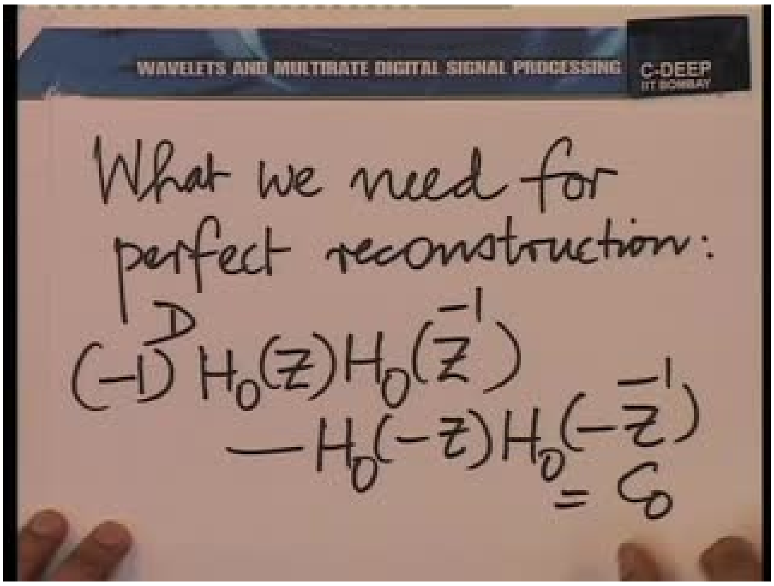

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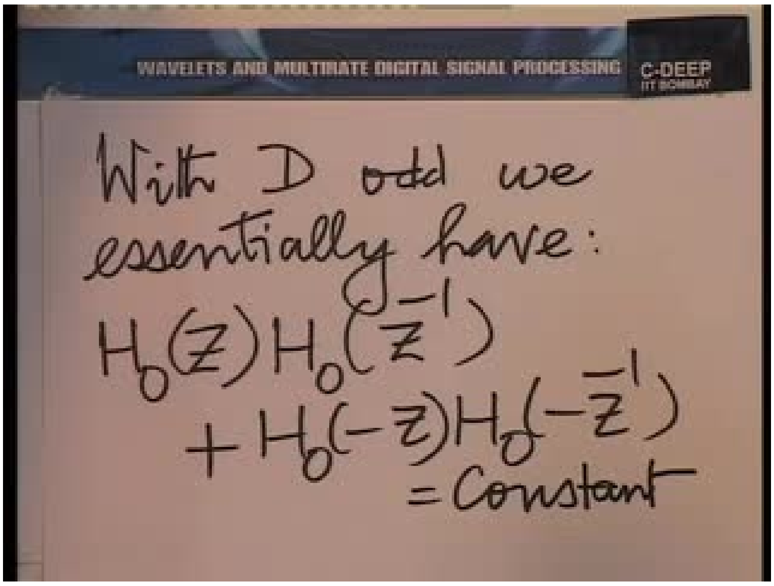

In this relationship that we have here, we shall now assume D to be odd, with D odd we essentially have for perfect reconstruction, H 0 Z, H 0 Z inverse plus H 0 minus Z, H 0 minus Z inverse is a constant.

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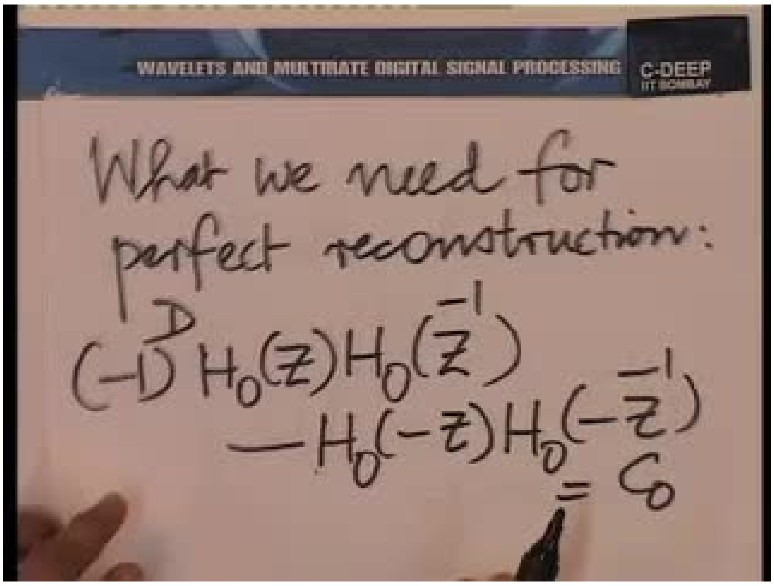

Let, me explain you see when D is odd then both of these are minus signs, so you can take away the minus sign from the left hand side and put it on the right, and this is any way a constant, so negative of a constant is also a constant, so there we are.

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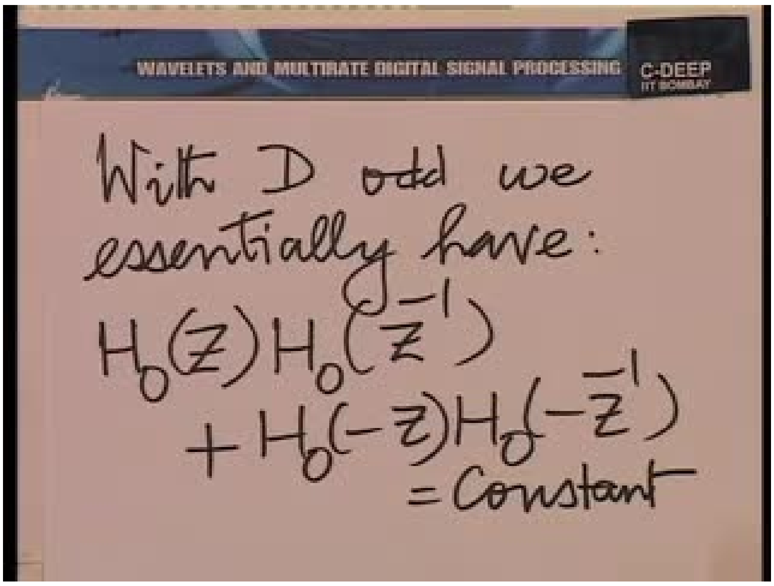

Now, what is this mean we need to reflect on it a little, we first reflect on it in the frequency domain, so when they put Z equal to e raise the power j omega what do we have here.

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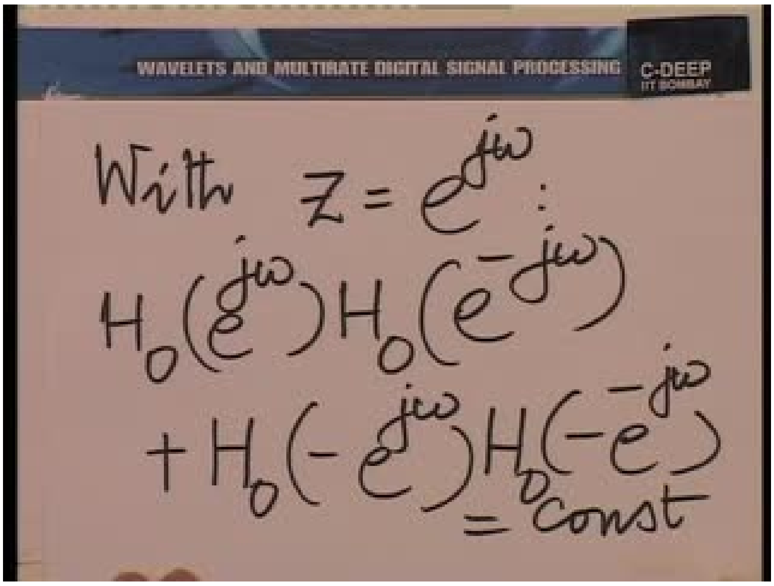

H 0 Z or rather H 0 e raise the power j omega times H 0 e raise the power minus j omega, plus H 0 minus e raise the power j omega, H 0 minus e raise the power minus j omega is a constant. Now, once again we shall remove the minus sign here and shift omega by pi, and we shall also note that if you have a filter with a real impulse response, then H 0 e raise the power minus j omega is essentially the complex conjugate of H 0 e raise the power j omega, the same holds here, when you have omega replaced by minus omega here, you can get a complex conjugate of this.

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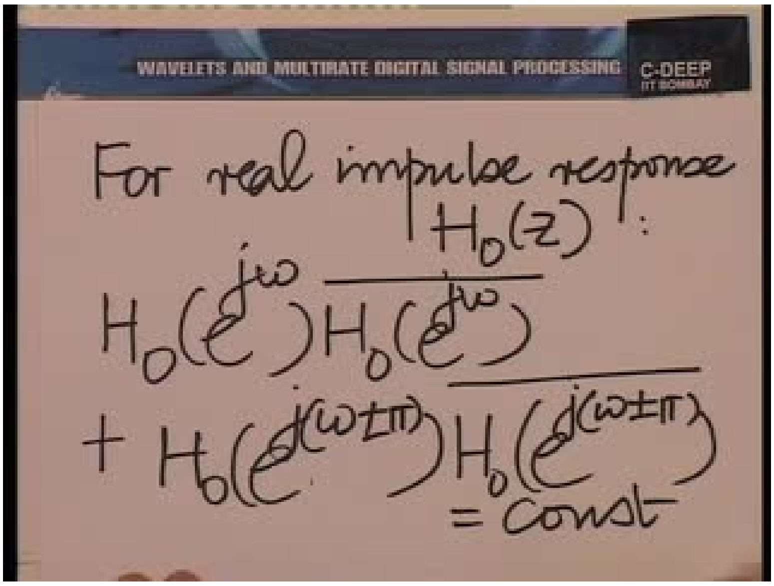

So, all in all for real filters, we have H 0 e raise the power j omega, H 0 e raise the power j omega complex conjugate, plus H 0 e raise the power j omega plus pi plus minus, if you please H 0 e raise the power j omega plus minus pi, complex conjugate is a constant.

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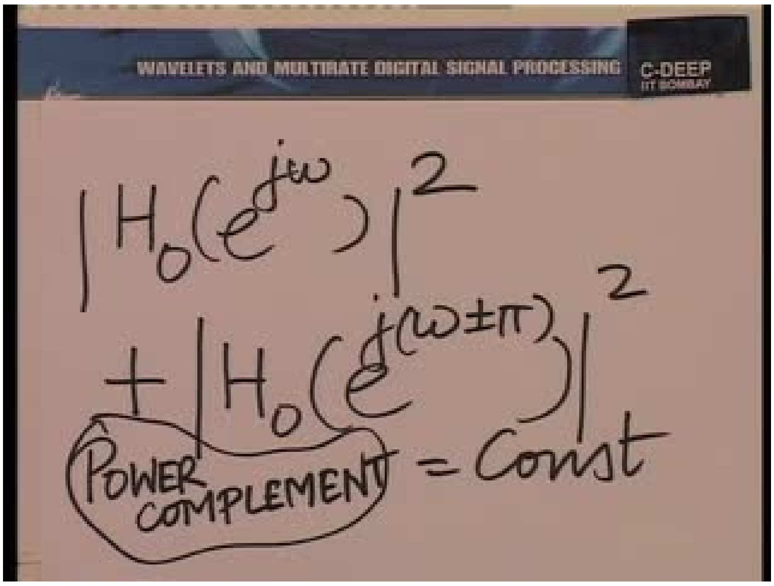

Now, we have a very beautiful conclusion here, you see this is the magnitude square and this is again a magnitude squared, so there we are. What we are saying an effect is mod H 0 e raise the power j omega squared, plus mod H 0 e raise the power j omega plus minus pi, the whole square is a constant.

Now, this is very interesting, this is exactly one of the properties that we had introduced in the context of the haar system, namely the property of what is called power complementarity, here it is clear now that by this construction we have achieved power complementarity in the high pass and low pass filters of the analysis side, and in fact it is a simple consequence that if we look at the synthesis side, there also power complementary. In fact I leave it to you as an exercise, by using the relation between G 0 G 1 and H 0 to show that the synthesis side is also power complementary.

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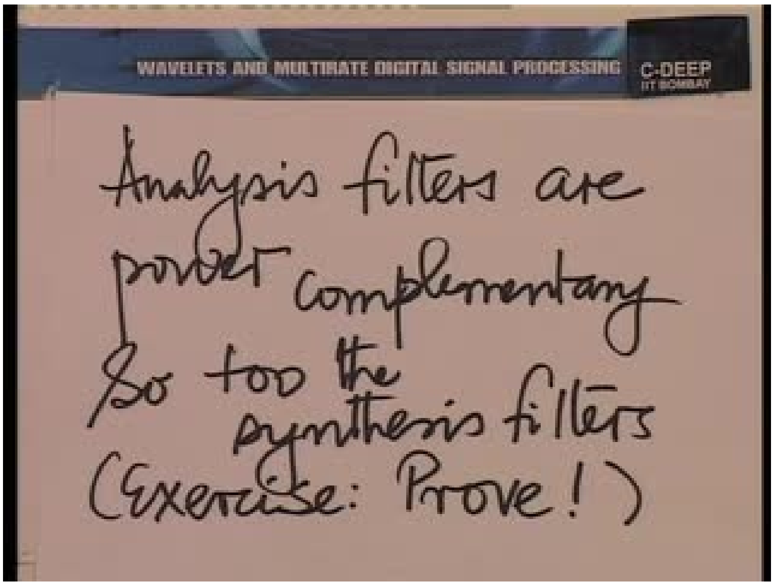

So, what do we have here it is very interesting, the analysis filters are power complementary and so to the synthesis filters, so I said exercise show this, we have already proved it more or less, it is just a little bit of as I say taunting your eyes and crossing your knees, you need to write down, need prove, I think that is the good thing to do we must leave a couple of exercises for the class to do, and this is the very simple exercise with which we begin, use the discussion that we just had over the last couple of minutes to work out the details to show that the analysis filters and the synthesis filters are both a cop, a power complementary pair.

Anyway, this is the motivation for that so called quote unquote peculiar choice of H 1, now we see things falling in place, the Z inverse was require to bring this complex conjugation, replace omega by minus omega and of course as you see for a real impulse response, it had no effect on the magnitude, but we could remove the phase.

So, it is a strategic choice of analysis high pass, you could have chosen H 0 minus Z or something like that, but you choose H 0 minus Z inverse, because you wanted that complex conjugation, and then you put a Z raise the power minus D, because you wanted to make it causal.

So a Z raise the power minus D is to introduce causality, the Z replace by Z inverse is to bring in this complex conjugation to bring in power complementarily, and finally the minus, I mean minus Z inverse instead of just Z inverse is to convert the low pass to a high pass, so now it all falls in place, and we have justified our choice.

And now we also know what we demand of H 0 Z, so that we get perfect reconstruction. Let us look at that condition once again, that condition tells us and let me write it slightly differently.

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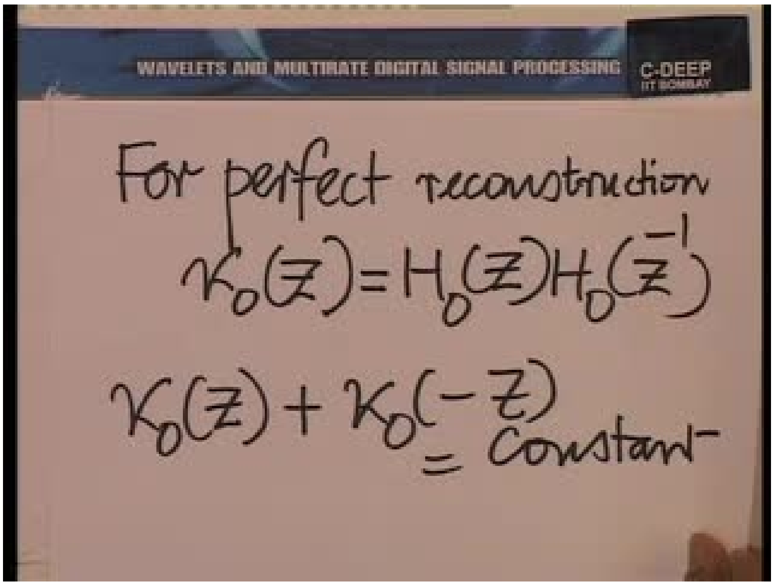

That condition tells us for perfect reconstruction, some interesting intermediate filter which we shall define by kappa 0 Z, so let us define kappa 0 Z as H 0 Z, H 0 Z inverse, what we are saying is that for perfect reconstruction, we require kappa 0 Z plus kappa 0 minus Z to be a constant. Now, things are beginning to make even more sense, if we know the sequence that gives us H 0 Z, what is the sequence that gives us H 0 Z inverse, let us reflect a minute on this.

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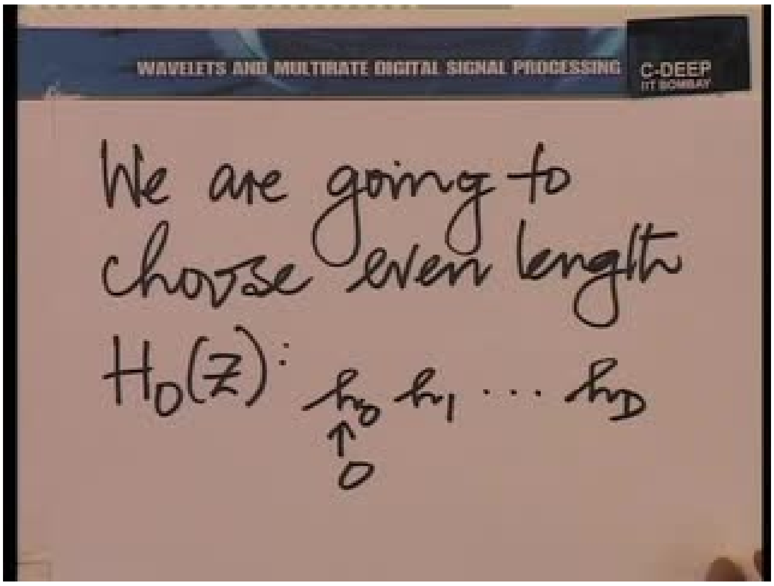

So, what I am trying to say, is we have agreed that we are going to choose even length H 0 Z, something like an impulse response of the following form, h0 h 1 and so on, h 0 lies at 0 up to h D, remember D was odd.

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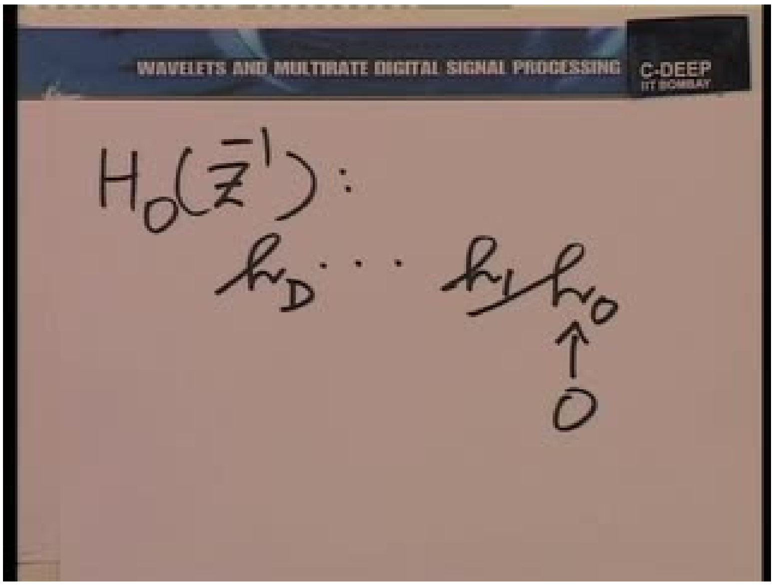

Therefore, H 0 Z inverse would then correspond to the following, quite clear when you replace Z by Z inverse, you are essentially reflecting the sequence about the point n equal to 0.

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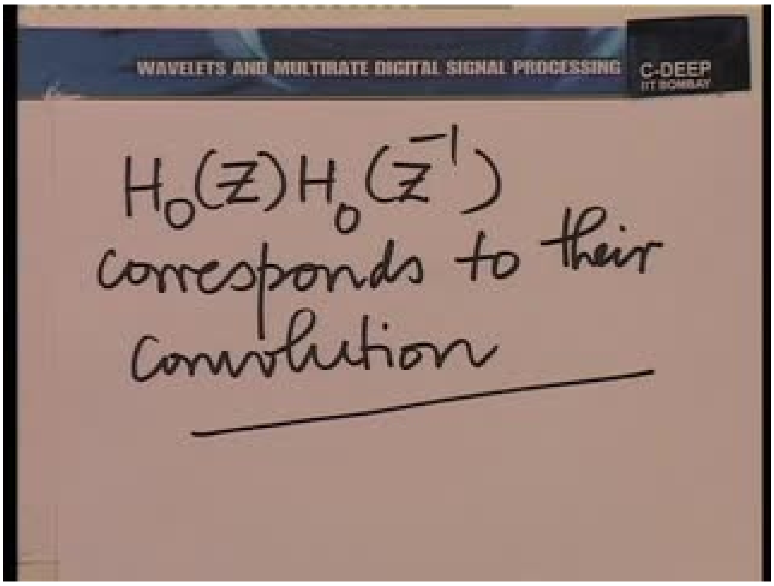

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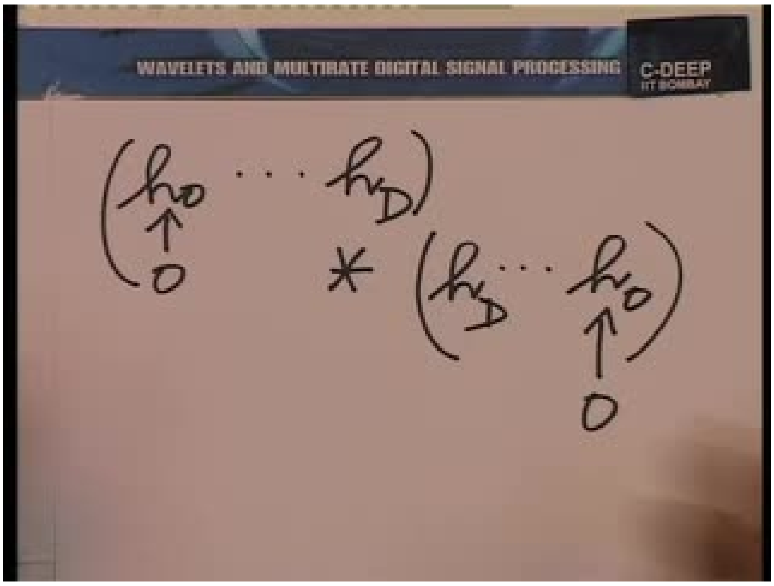

Now, H 0 Z times H 0 Z inverse corresponds to their convolution, you know when you multiply to Z transform, the corresponding sequences are convolved, and therefore we have this convolved with this, maybe I should put parenthesis here, and indicate the 0 clearly there.

Now, how do you convolve, well these are all of equal lengths, so I could choose either of them as the static one and the other one as the moving one. So, just for convenience what I will do is, the sequence which we started with, the one corresponding to H 0 we shall keep as the static sequence and the one corresponding to H 0 that is inverse we shall make it move.

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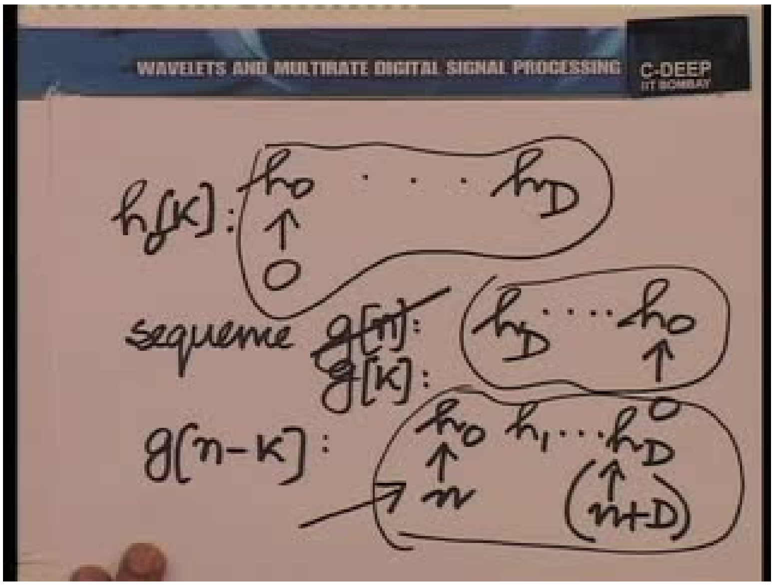

Now, what we are saying essential is keep this static, so you have and make this move, so when you make the other one move, you are doing two things, you are bringing, you see you want to, essentially you have sequence one, let us say sequence, it is called as sequence g n, just for the time being the sequence g n is this or g K if you like.

In which case, the sequence g n minus K, this is of course a function of K, so K equal to 0; it is h 0 and so on. So, g of n minus K is going to look like this, this 0 would go to n and whatever comes before 0 would go after n there, so you have h 1 and so on up to h D.

So, this reaches the point n plus D here, this is the sequence g n minus K, and this you may of course call the sequence h 0 of K if you like. So, we are trying to convolve this sequence, essentially with this sequence, but in that convolution you are going to move around this at different locations here.

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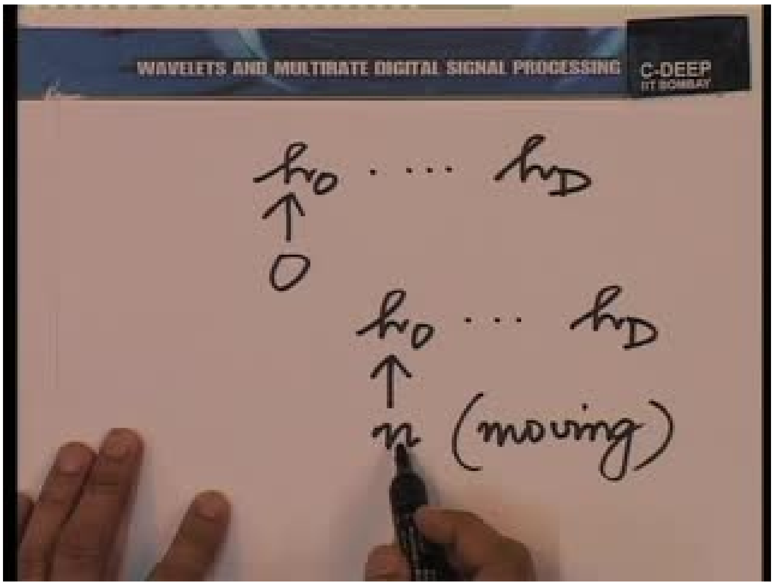

Now, visualize let me put that down clearly once again for you; we are saying, we have this so called static sequence, and this is going to move around, n is moving, so you can visualize the situation. For different values of n, this lies at different locations with respect to the static sequence.

For example when n is equal to 0, the samples actually coincide, when n is equal to 1 then h 0 clashes with h 1, and of course h D has gone out of range, so it has gone to a 0 sample here, when n is equal to minus 1 you are here, and then of course h 1 clashes with h 0 h D with h of D minus 1 here and so on so forth. So, you see what we have is actually the dot product of the sequence and it is own shifted versions, this is very interesting.

What we are saying is that the samples of kappa 0 are actually dot products of the original filter impulse response, shifted by different amounts of shift; let us write that down, that is the very important conclusion.

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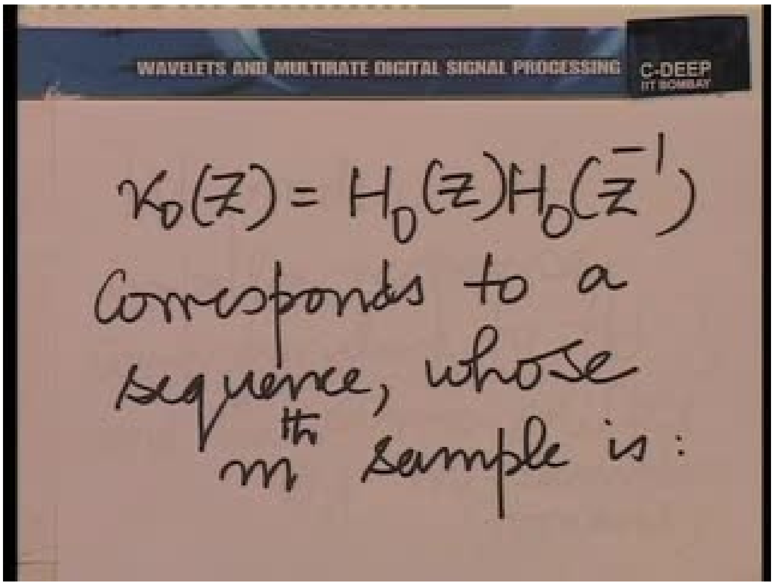

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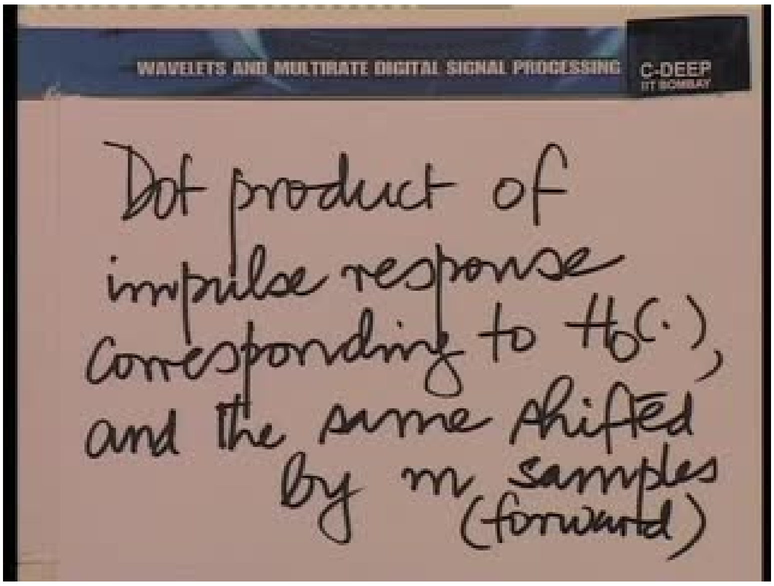

Kappa 0, kappa 0 Z, which is H 0 Z times H 0 Z inverse corresponds to a sequence whose m th sample is as follows; the dot product of the impulse response corresponding to H 0, and the same shifted by m sample, if you want to be very specific, you should say m samples forward, but that does not really matter.

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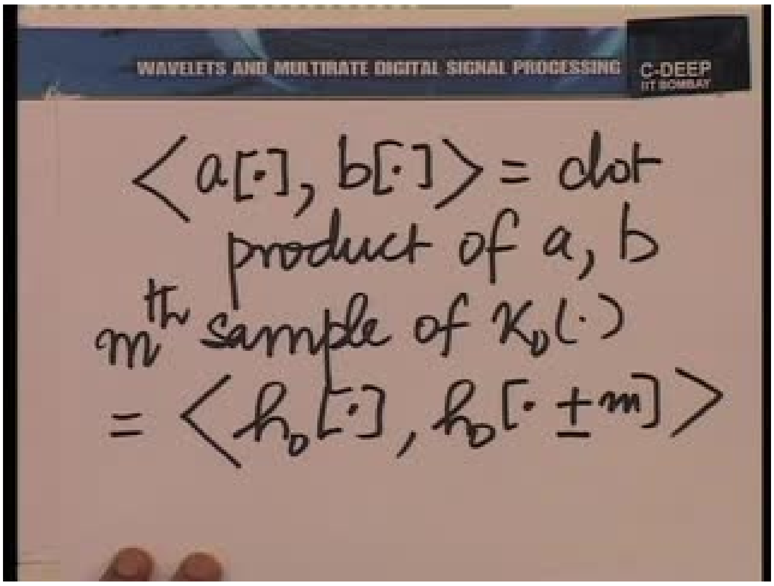

So, if you want to write it down in the notation of dot products, what we are saying, is that this denotes the dot product of sequences a and b, so a with an argument, integer argument, b with an integer argument, this is the dot product of a and b, and we are saying the m th sample of the filter kappa 0 is essentially the dot product h 0 and h 0 shifted by m, plus or minus is not really an issue, if you like you can make this minus, there is a symmetry, you can visualize that. If you shift backward by 2 or forward by 2, it is a same let us verify that for a length 4for example you will see what I mean.

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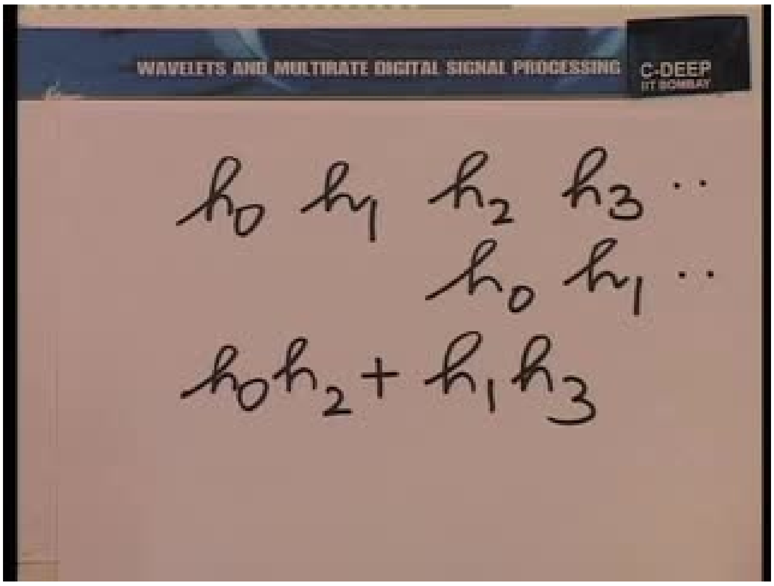

So, if you had a length 4 for example, you would have h 0 h 1 h 2 h 3, and if you took this and the same thing shifted by two, you are talking about this dot product, the rest of it is 0 of course, so here again you get zeroes and you do not need to write that, so the dot product is essentially h 0 h 2 plus h 1 h 3.

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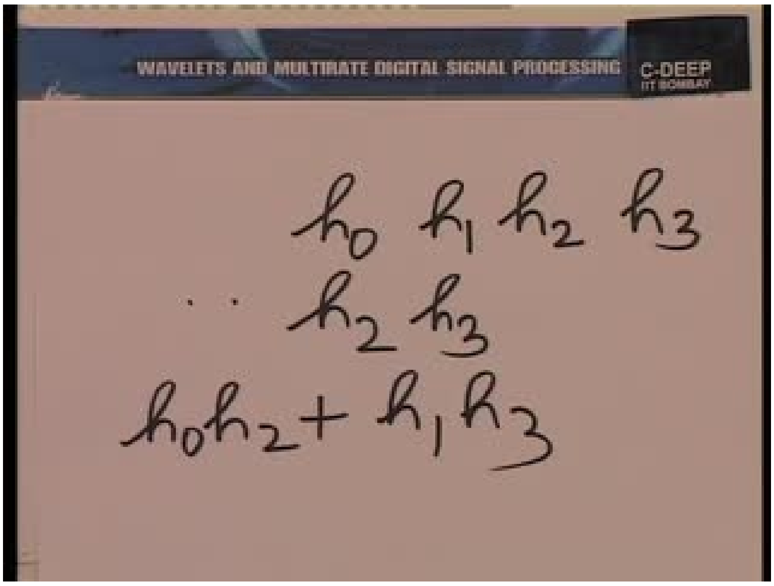

Now, if you were to shift it backwards, so you had h 0 h 1 h 2 h 3 there and you shifted it backwards, and of course this is all 0. So, again the dot product would be h 0 h 2 plus h 1 h 3, so as you can see shifting backward or forward by m is not an issue. However, what we are saying here is something very interesting; we are saying that with this understanding of the samples corresponding to kappa 0 Z.

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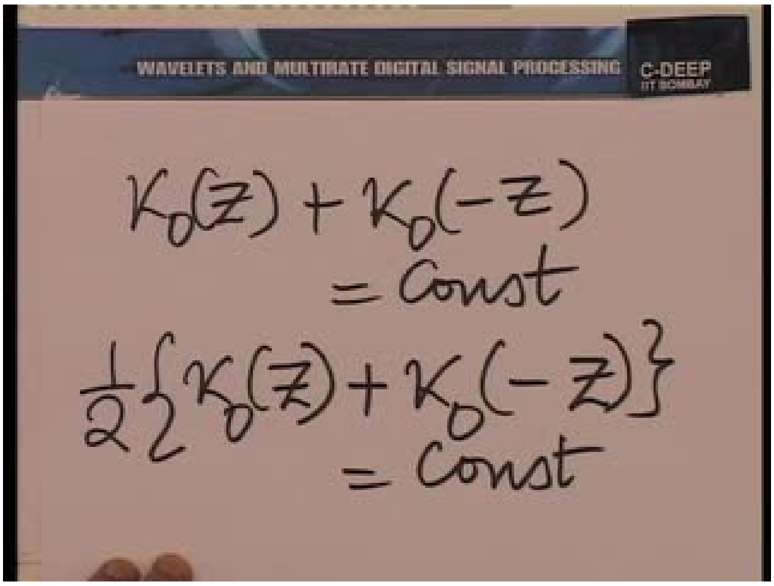

Kappa 0 Z plus kappa 0 minus Z is a constant, and if you take the inverse Z transformer now, and if you only care to multiply by half on both sides. This also a constant obviously, and this is something very familiar to us, we have encountered this when we did down sampling, so in fact if the original sequence corresponding to kappa 0 Z.

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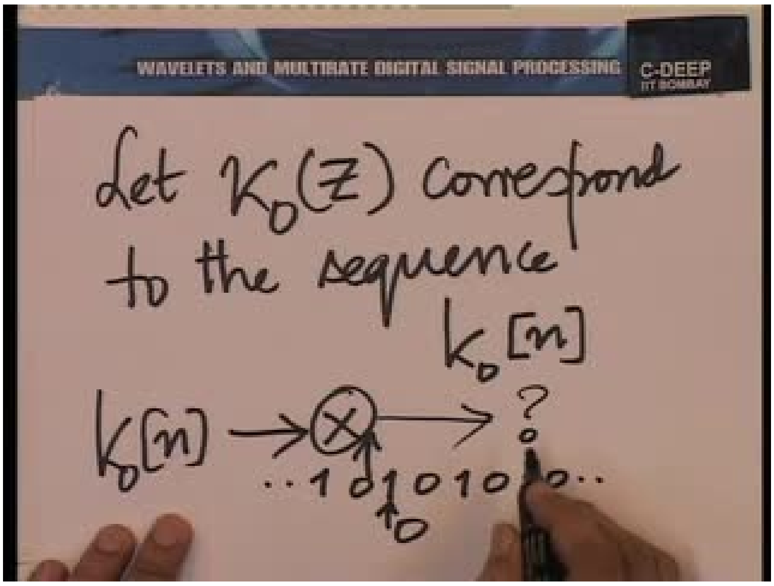

So, let kappa 0 Z correspond to the sequence, let us write small k 0 n. Then what we are saying is that, when this sequence is modulated, by a sequence which is one at the even location and 0 at the odd locations. So, it is something interesting we are doing, we are modulating this kappa 0 n by a sequence which is 1 at the even location and 0 at the odd locations.

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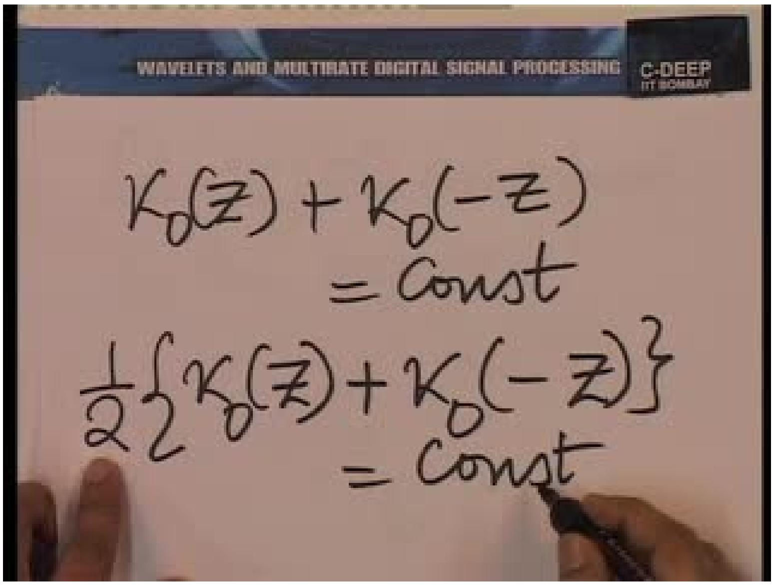

This gives us a sequence corresponding to the inverse Z transform of a constant, which is essentially an impulse. Now, you know this modulation is what we derived when we talked about the Z transform across a down sampler.

So, remember when we go across a down sampler by a factor of 2, it is like first modulating by a sequence which is 1 at the even location and 0 at the odd locations, in general when you go across a down sampler by a factor of capital M, it is like modulating with a sequence which is 1 at all multiples of capital M and 0 elsewhere, followed by an inverse up sampling operation, so remember a down sampling by 2 was modulation by a periodic sequence with period 2, which was 1 at locations equal to multiples of 2 and 0 else, followed by an inverse up sampler by a factor of 2.

Inverse up sampler means a compressor, throw away the zeroes, down sampling by a factor of m was essentially multiplication by a periodic sequence period capital M, 1 at all multiples of m, zero elsewhere followed by an inverse up sampler by a factor of capital M which means throw away the zeroes and compress.

So, you see that throwing away the zeros was what made Z replaced by Z raise the power half, so here in this expression, kappa 0 Z plus kappa 0 minus Z we are not writing Z raise the power of half, so we do not do that inverse up sampling operation.

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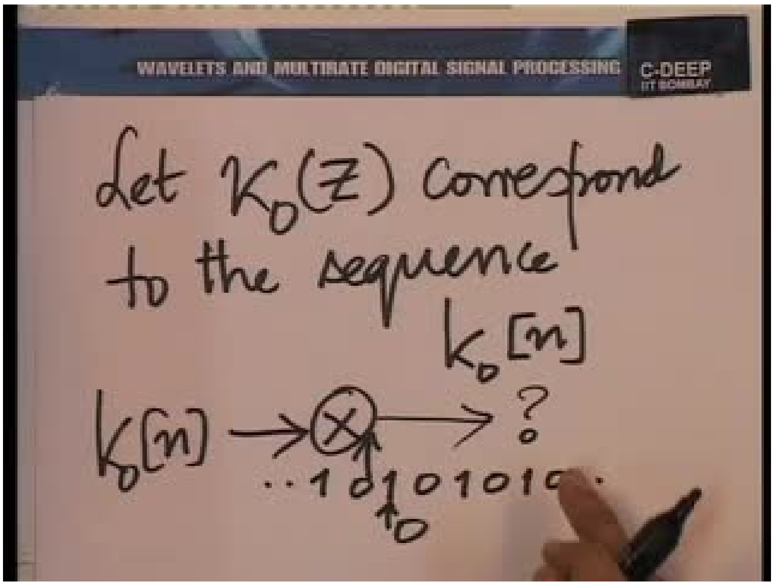

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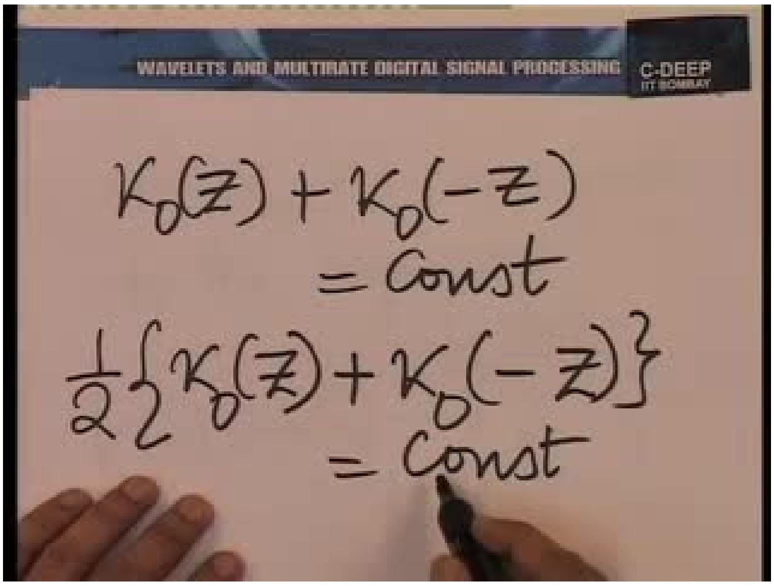

But, the rest of it is there and that is the justification for this step here, modulation with this periodic sequence, and now this is equal to a constant which means if we take the inverse Z transform here.

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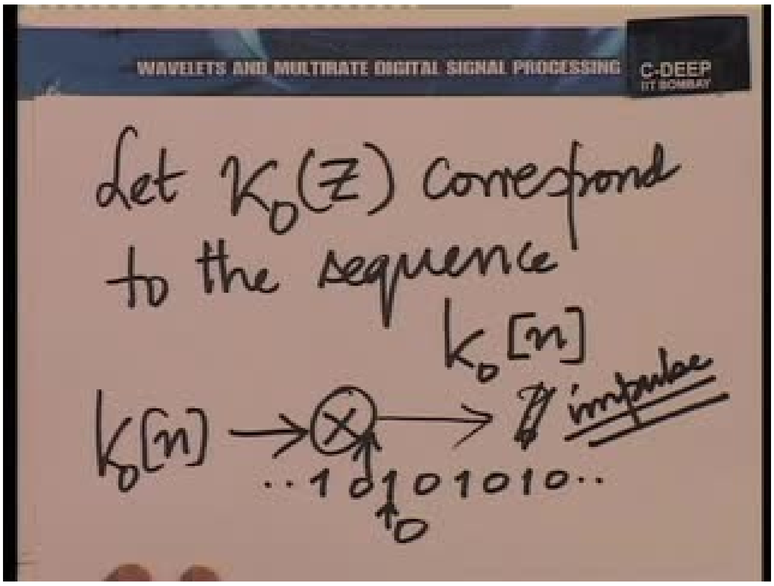

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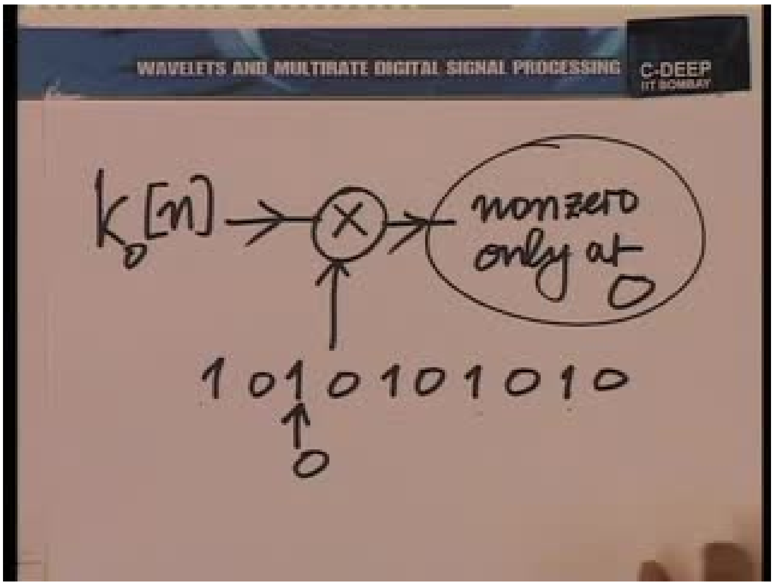

We are saying this is essentially the impulse, which means this has a non-zero value at 0, but 0 everywhere else. So, let us write that down, kappa 0 n when modulated with this periodic sequence with period 2, with the ones at multiples of 2 and 0 elsewhere, results in a sequence which is non-zero only at n equal to 0 that is what we are saying, and obviously at the odd locations anyway it is 0, so it is nothing very surprising here, it is at the even locations that we have a surprising result there.

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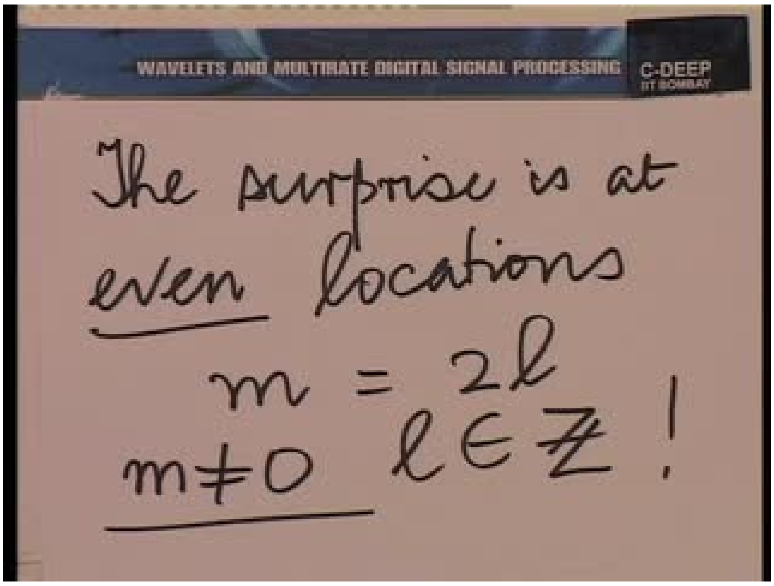

So, the surprise is at the even locations. Of course m not equal to 0, so what we are saying is that if I take the impulse response of the low pass filter on the analysis side, shifted by any even number of samples; 2 4 minus 2 minus 4 6 minus 6 and so on and take the dot product of that shifted impulse response with the original impulse response that dot product is zero.

For, those of us who are familiar with the idea of autocorrelation, what we are saying is that the autocorrelation of the impulse response of the low pass filter is 0 at the even locations other than 0. Now, let us use this to build the first of the family of the Daubechies' filters.

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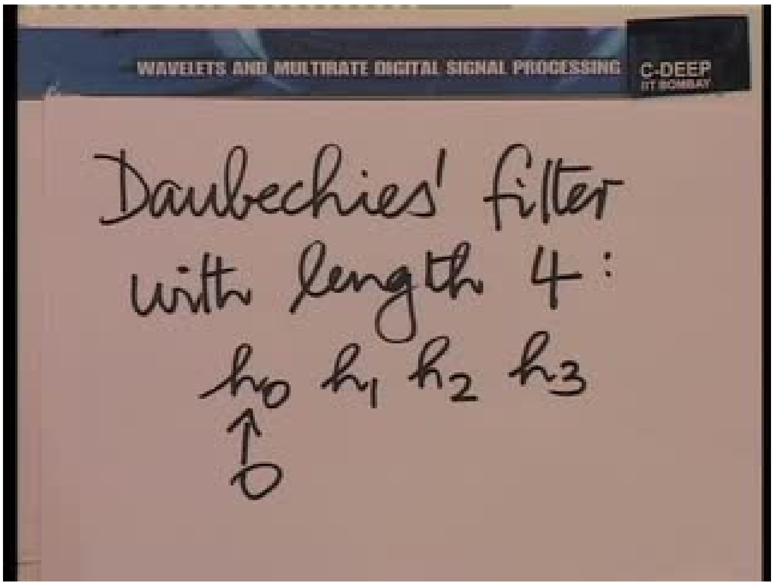

So, well I should say first non-trivial, so it is second in that sense, the first non-baby member, Daubechies' filter with length 4 is going to look something like this, it is going to have an impulse response h 0 h 1 h 2 h 3 and recall what we did yesterday.

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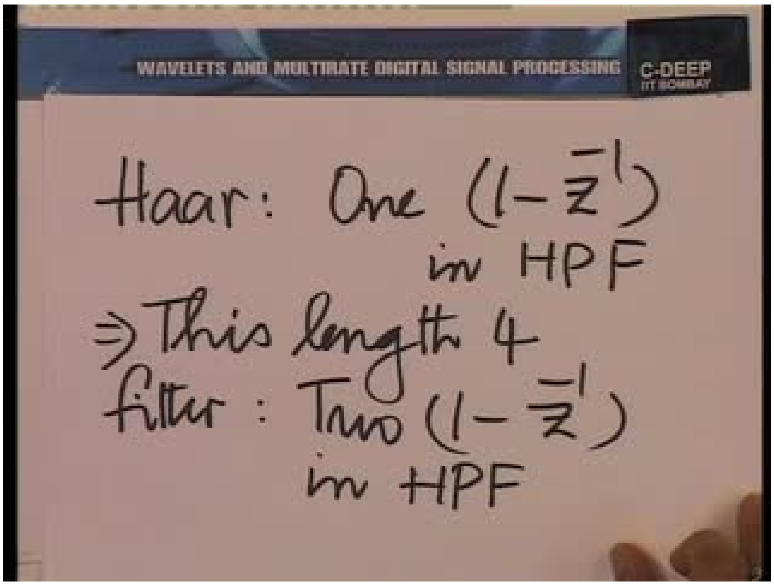

We said that in this filter, we would need to bring in one more factor of the form 1minus Z inverse in the high pass filter. So, haar had one 1 minus Z inverse in the H P F, so this length 4 filter would have two factors, 2 1 minus Z inverse in the high pass filters and that means, you see the high pass filter was obtained by replacing Z by Z inverse and then by minus Z as well, so if the Z inverse part gets taken care of by the delay Z raise the power minus D, but the Z replaced by minus Z needs to be undone to go to the low pass.

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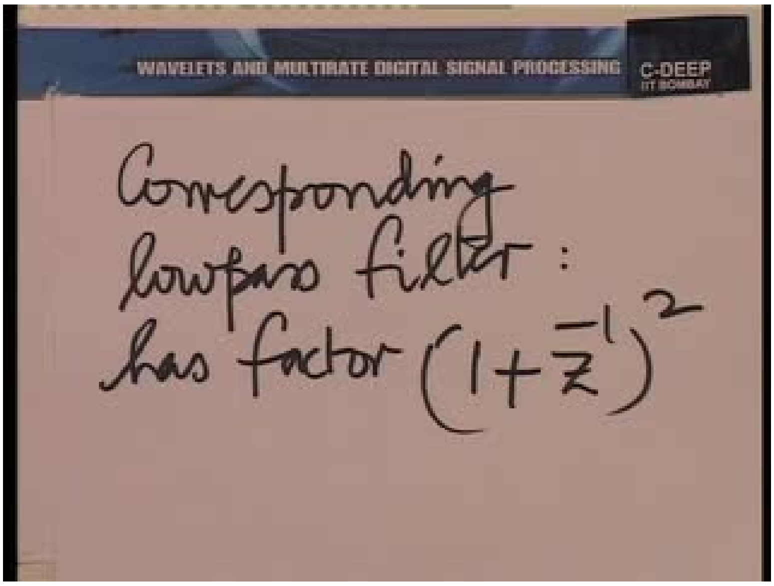

And therefore the low pass filter would have a factor 1 plus Z inverse squared. Now, when you say it has a factor of one plus Z inverse the whole squared, you have already constrained 2 of the 3 zeroes that it has free to be chosen.

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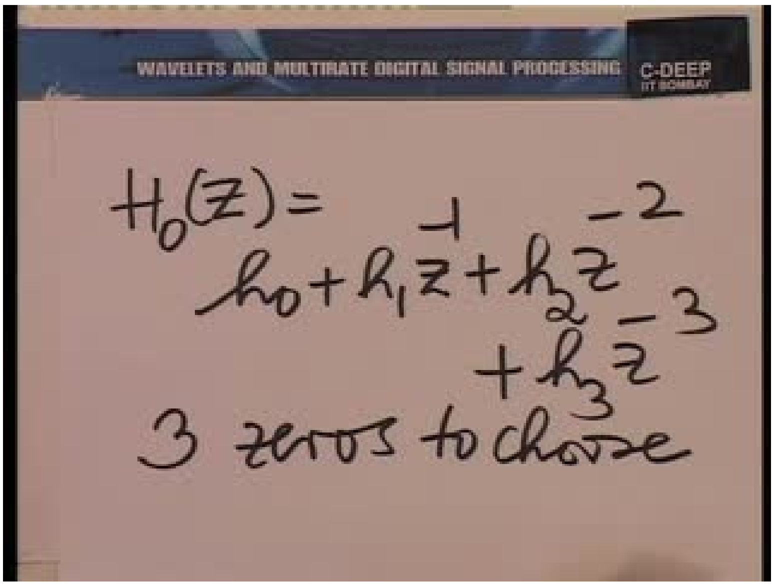

What I mean is if you looked at H 0 Z, it would have been H 0 plus H 1 Z inverse plus h 2 z raise the power minus 2 plus h 3 z raise the power minus 3, so there are 3 zeroes to be chosen, out of them we have already chosen 2.

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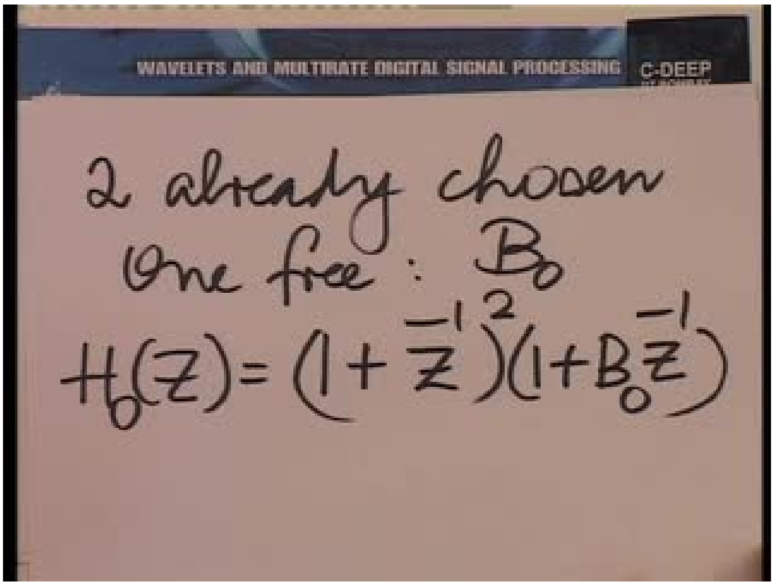

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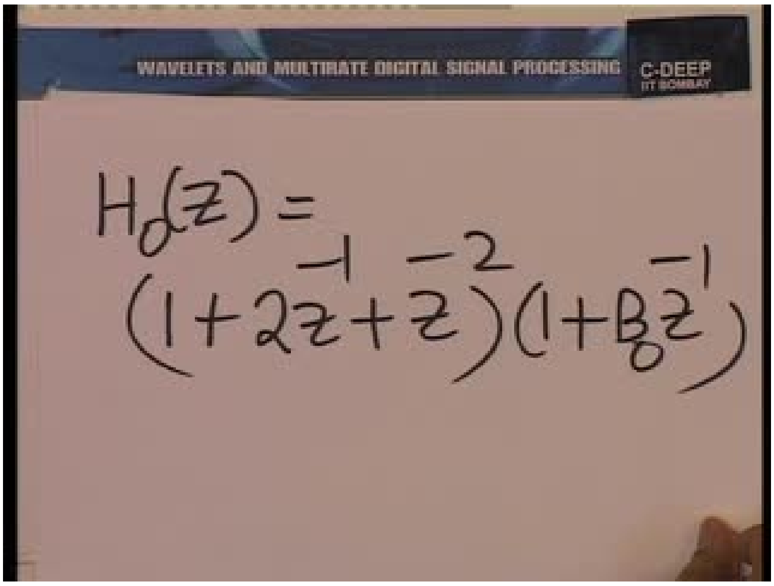

So, we have only 1 free, let that free one be at B 0, so in all it is very simple we can take H 0 Z to be of the form 1 plus Z inverse the whole square times 1 plus B 0 Z inverse, what do we have then, let us expand this, we have H 0 Z is essentially 1 plus 2 Z inverse plus Z raise the power minus 2 times 1 plus B 0 Z inverse, and we can expand this further.

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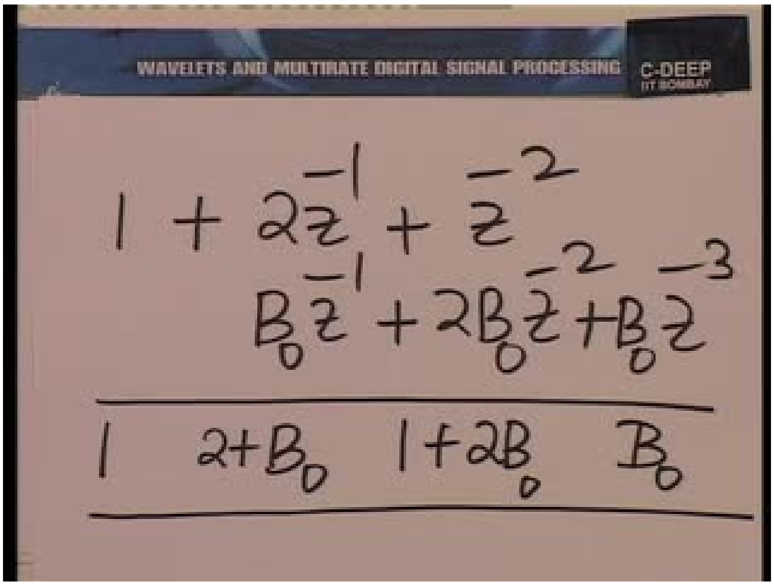

That product will be 1 plus 2 Z inverse, plus Z raise the power minus 2, plus B 0 Z inverse times this, so B 0 Z inverse plus 2 B 0 Z raise the power minus 2. plus B 0 Z raise the power minus 3.

So, in essence we have the following impulse response for the filter; 1, 2 plus B 0, 1 plus 2 B 0 and B 0 here, this is the impulse response. Now, we have set up the low pass filter for the second member in the Daubechies' family.

Where do we go from here, we shall use the constrained that we just derived; namely that the dot product of this impulse response with it is shifts by even shifts must be 0, and we shall see the constrains that emerge on the free parameters, in the next lecture therefore we shall constrain the value of B 0, make a choice for B 0 and derive precisely the impulse response of the Daubechies' second member, and thereby also establish a general procedure for building up the Daubechies' family low pass filters.

Concurrently, we shall explain how this family evolves, and recall again the significance of going from one member to the other, with that then we shall conclude the lecture today.

Thank you.