Science and Technology of Weft and Warp Knitting
Prof. Dr. Bipin Kumar
Department of Textile Technology
Indian Institute of Technology - Delhi

Module - 6
Lecture - 29
Knitting Calculation - Geometrical Modeling

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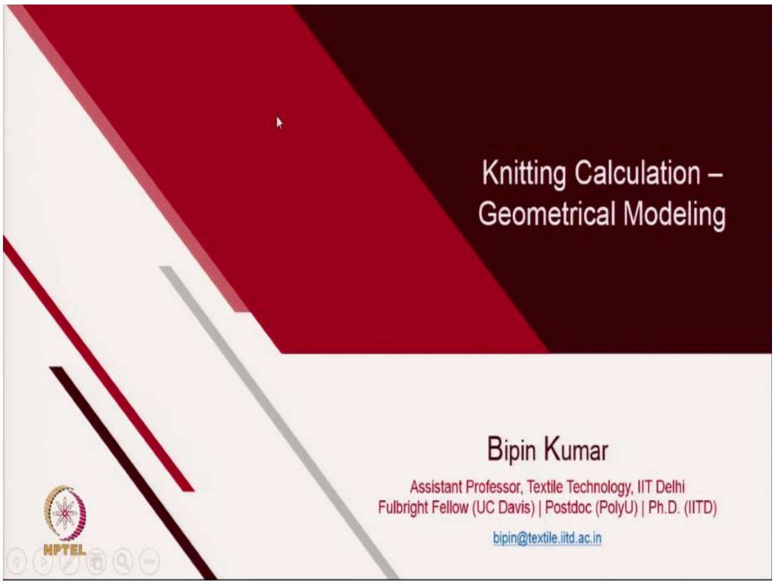

Welcome participants. Now, we are moving to the next section in this week; again, continuing with the knitting calculations. Today we are going to do some modeling, especially structural modeling; how you can relate some of the fabric properties with the loop length. So, that will be the topic of this particular lecture. So, we have seen, for any structure of knitting, loop is the most integral part.

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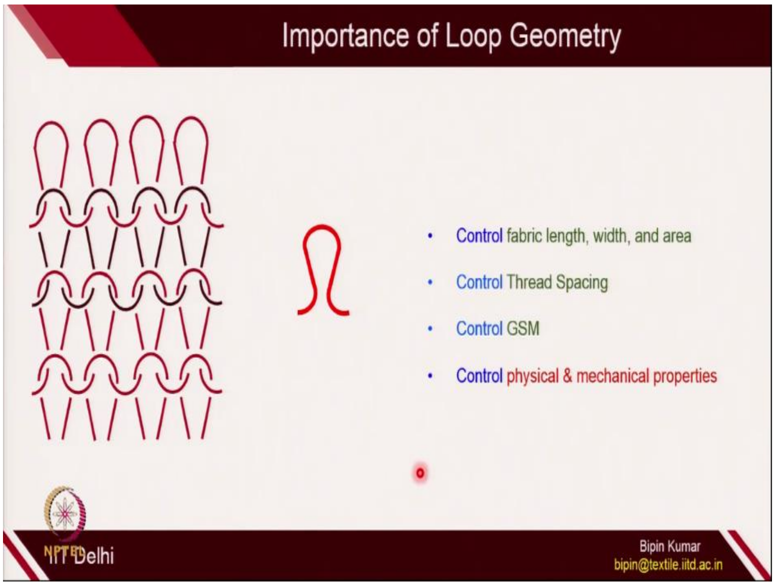

So, if you can connect the property of the fabric with the loop length, then it will be very, very useful, because we have seen how loops control so many properties. So, we have seen how the loop actually control the fabric length, width and the area. We have seen how the loop control thread spacing. We have also seen how the loop control the GSM. Because, in the GSM equations also, you have seen, GSM is related with the loop length.

And also, we can see how it control physical and mechanical properties. Because, if you make bigger loops, the fabric will be more porous, more permeable. If you make bigger loops, the fabric will be more flexible, low modulus. So, this is what we have already seen, that how the fabric can be related with a loop length. So, if there is some model available through which we use just one parameter, which is loop length; and we can find out most of these fabric properties.

So, it will be very useful for us. So, many researchers have made several attempts for connecting the fabric properties with the loop length. And those properties are either GSM, spacing, length, width; or physical or mechanical properties. So, they propose some geometrical model with some constant and they found very good correlation that loop length actually affect these fabric properties. And there are some concrete relationship does exist. So, that we are going to explain in this particular lecture.

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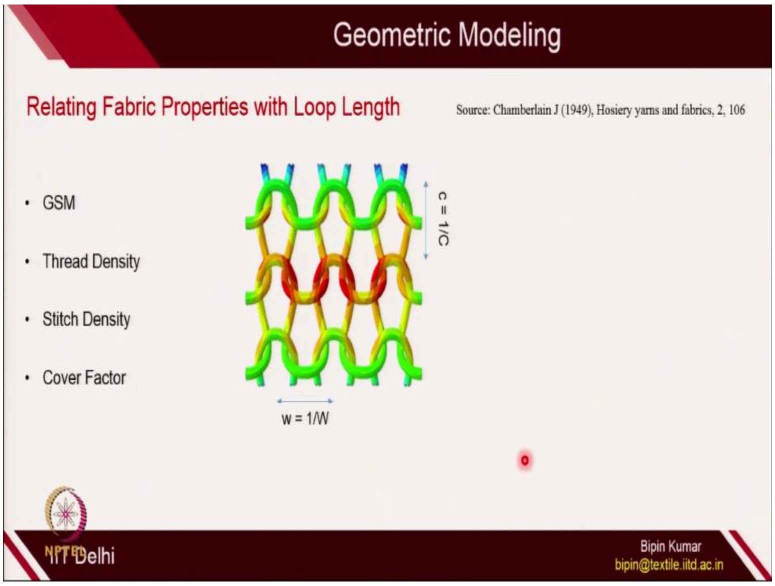

So, for doing the geometrical modeling, naturally we know we need one parameter which is the loop length. And we can connect some of these properties like GSM, tread density, stitch density and cover factor. So, the first scientist who thought to model these fabric properties with loop length was Chamberlain. So, you can imagine like it is almost 60, 70 years old model. And, he found that, once you know the loop length of the fabric, many of the properties can be derived.

So, he proposed some equations that we are going to derive today; relating GSM with loop length and other variables with the loop length. So, for proposing the model, he makes certain assumptions. And, that we need to first understand. So, those assumptions are like; he defined first some parameters, like course spacing, wales spacing, which we already know. So, w is the wales spacing, distance between 2 wales.

W is the wales per inch. c is the distance between 2 courses. So, this c is the distance from head to head or feet to feet, whatever you feel comfortable. So, distance between 2 wale, 2 course is c. C is courses per inch. So, he defined these terms. And because these terms will be very, very useful; we can also get these terms in terms of loop length. So, thread density, stitch density, cover factor, GSM, C, W, all can be related with one unit, which is the loop length.

So, Chamberlain, for proposing this loop length, because it is perfectly a geometrical problem. So, he defined some characteristics of the fabric. He made certain assumptions before deriving these equations.

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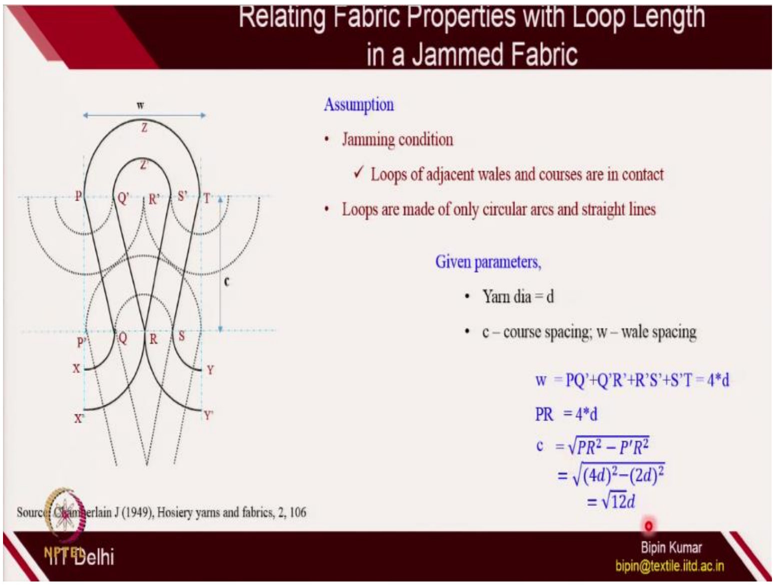

Those assumptions were, that the fabric; we have also seen, that once you take out the fabric from the machine, the fabric shrinks. And usually, they shrink so much that the loops of one column will start touching with the other column. On the bed, no needles are touching each other. But the moment you take out the fabric from the bed, they start touching each other. So, the Chamberlain actually made the assumptions that the fabric, the loops are actually in jammed condition.

What do you mean by jammed condition? Jammed condition means, the loops of adjacent wales, so, this loop, you can see it here, is actually touching with the other loop. So, you can see here, the sinker loop is touching with the other sinker loop in the series. Okay. Similarly, the head of old loop, you can see it here, which is the head, it is touching with the feet of new loop.

So, this is the maximum shrinkage possibility. You cannot expect the fabric to shrink even more. Because, the yarn cannot overlap with the sinker region. The head cannot go over the top of feet region. So, this is the maximum shrinkage you can expect from the fabric. So, this is the perfect jamming conditions. The Chamberlain assumed that the fabric should be in perfect jammed conditions; which means, loops of adjacent wales and courses are in perfect contact.

The second assumptions there were made, that loops are actually made from a either circular arc or a straight line. So, whatever distance you are observing on the loop, it is either made up of straight segments. So, here the loops are actually straight segments. And the head and the feet are actually curved sections. So, he defined these assumptions before deriving the relationship.

So, once you define this, this is the, that the fabric is perfectly jamming. And either you can observe the circular arc or you can observe the straight segment. Then he start deriving those relations. So, and we have also already defined the distance between 2 wales, which is w. And distance between 2 courses, which is c. So, w is wales spacing and c is the course spacing. And he wanted to find the geometrical relationship of w and d with the loop length.

So, to connect loop length with w and c, the other variables which is very, very important is the thickness of the yarn. So, thickness by meaning of thickness of the yarn is nothing but the diameter of the yarn. So, if you see, the entire loop is actually very thick. And this thick is, this is thick because it is made for, from a yarn which has certain diameter. So, that variables is also known to you.

And now, you can express w and c with the diameter of the yarn. And also, you can express w and c with the loop length. So, to finding the loop length, you need to be very careful, like which path of the yarn will you follow. So, if you see this loop length, the yarn of the path is, if you start from; let's number it some A B C D. So, if you start from X, then you are moving to Q, then P, Z, T, X, S and Y.

So, this is one path. The other path is X, R, Q dash, Z dash, W dash, R and Y dash. This is the other part of the. And if you measure the distance, geometrically you can observe, either you may start from X dash or finish on Y dash; or if you start from X and you finish on Y; both the length will remain same, because of the geometrical nature of this loop. So, we can measure the length X, Q, P, Z, T, S and Y.

And that will be actually equals to loop length. So, Chamberlain proposed that, once you start finding these loop length, then you will realize that loop length is actually related with w and c. So, from that understanding, he started proposing many useful relationships. So, let's see how he made those relation. So, if you see w, which is nothing but the wale spacing. So, if you move from P to T, that distance is equals to w.

So, your w is nothing but, it is the distance from P to T, which means, you are moving from P to Q´; Q´ to R´; R´ to S´; and then S´ to T. So, there are 4 distance, PQ´; QR; Q´R´; R´S´; and S´T. And each of these distance is nothing but equals to yarn diameter. Because you know how thicker the yarn is. So, PQ is nothing but the yarn diameter.

Q´R´ is nothing but again second the yarn diameter. So, all these distance are equals. So, wales spacing is automatically become 4d, keeping this condition. Now, let's see course spacing. So, what is this course spacing. So, distance from this point to this point. So, you have to measure P´and P. So, how you can measure it? So, if you know P´and P, to measure that we need to find out distance PR.

So, this is R point, this is P point. So, if you move from P to R, actually you are crossing 4 times yarn diameter. Because, if you see, this is a circular arc, this is a semicircular arc. So, from P to first arc point, this is R. This distance will be again equals to yarn diameter. From here, then you are moving to the head of old loop. Again, this distance will be R. Then, again you are moving to the inner circle which is again R.

And then, you are reaching to again R. So, PR is nothing but 4 times yarn diameter. Because you are, if you are moving from point P to point R, you are moving 4 times yarn diameter. So, in this right angle triangle, you know P´ and R; and you know PR. So, using the hypotenuse formulas, you can find out the course spacing. So, the
course spacing = PR^2 -$P'R'^2$.

So, **PR^2**, we have already find out **$(4d)^2$**. And **$P'R^2$**; so, distance between this point and this point = 2d. Because, if you see P´ to Q which will be equals to d. And Q to R is again 2d. So, P´R is 2d. So, **$((4d)^2 - (2d)^2)^{1/2}$**. Automatically, this will become **$(12d)^{1/2}$**. So, you can see, w and c is related with simply yarn diameter. There is no nothing, there is no need for defining which machines we are using.

If you know just yarn diameter, thread density is known to you. So, this is the beauty of this relation. If we assume that the fabric is jamming and loops are straight and circular arcs, your w and c is dependent on yarn diameter.
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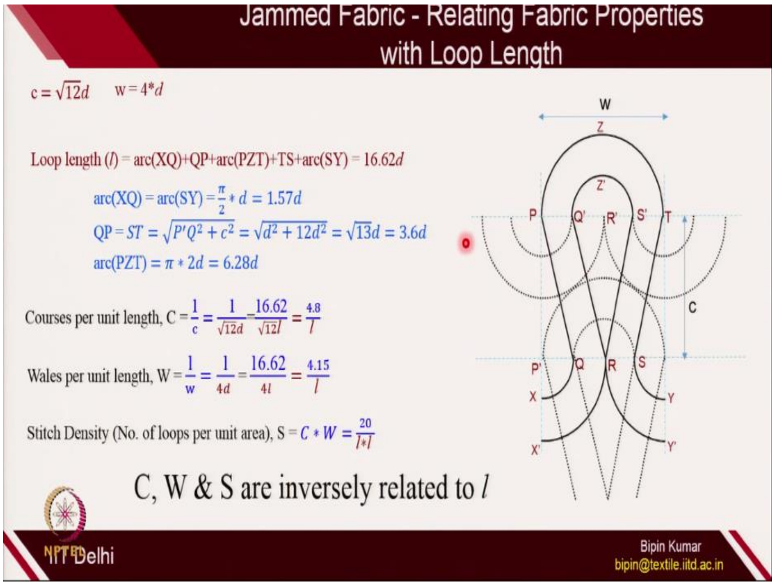

Now, let's relate the yarn diameter with the loop length. So, your

loop length = arc(XQ) + QP + arc(PZT) + TS + arc(SY). So, this five length we have to measure. And which = 16.62d. Again, loop length will be simply related with yarn diameter. Okay. So, you can measure this arc(XQ). If you see, this is your, as we have defined, this is a circle.

And the radius is the yarn diameter. So, arc(XQ) and arc(SY); this length and **XQ=1/4*circumference**. Because, this is a quarter of the circle. So, and the radius of the circle is d. So, 1.57d is the length of XQ. Now, if you want to find out the length of PQ and ST. Again, we know that c, value of **c = PP´**. And we know P´Q. So, we can find out QP and ST as **$P'Q^2 + c^2$**, because this is a hypotenuse.

So, PQ is a hypotenuse; P´Q is the one distance and P´P is the other distance. So, **$(d^2 + 12d^2)^{1/2} = 3.6d$**. So, you know QP; you know ST. Now, the last length is PZT. So, in PZT, it is a bigger circle. And the radius of this bigger circle is **P´ + R=2d**. So, the circumference of the full circle will be **π*d**. If you divide by half, you will get the circumference of half circle.

So, arc PZT is **π*2d**. So, 2d is the radius of this bigger circle. So, we simply multiply **π***R, where R is 2d. So, you know these 3 values. If you simply put it here, you will get 16.62d. So, now you can see, the loop length is also been connected with d; course spacing is connected with d; w is connected with d. So, we can replace d with loop length. And you can get the relationship of c.

So, course per unit, which is C. We generally calculate this in **courses/inch = 2/c**. So, **$1/(12d^2)^{1/2}=16.62/(12l^2)^{1/2}$**. So, you will get **4.8/l**. So, you can see how the courses per unit length is inversely related with l. And you just need one constant parameters, which is 4.8. Similarly, wales per unit, you replace d with loop length.

So, **1/w, 1/4d**. And d you can replace by **l/16.62**, from this relation. You will get wales per inch again is related with loop length. And the constant is 4.15. If you multiply **c*w**, you will get stitch density, number of loops per unit area. This will be **= 20/(l*l)**. So, this is how this geometrical relationship is very, very important. Because of, simply you need to measure the loop length and you can use this formulas for jammed fabric; and you can find out the courses per inch, wales per inch or stitch density.

So, the bottom line of this slide is, both the fabric structural parameters C and W as well as S are inversely related to loop length. Okay. So, Chamberlain gives a very powerful and useful equations, giving the importance of loop length in fabric properties. So, later after 10 years, Munden, there was a another scientist who tried to observe the behavior of non-jammed fabric.

So, we have seen that, whatever relations we are deriving here, all relationship, we are assuming that fabric are in jammed condition and the loops are either circular or straight. But in reality, the fabric does not follow these assumptions.

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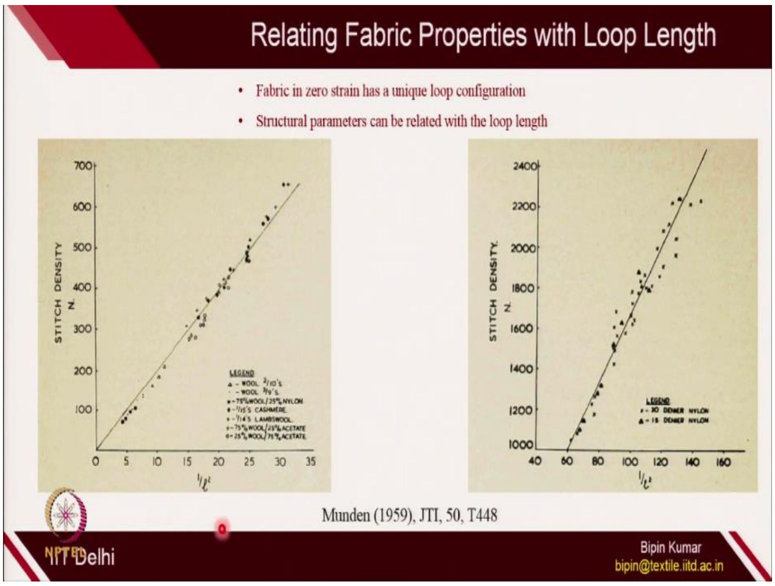

So, what the Munden did; Munden made lot of fabrics on different technologies, especially single jersey fabrics and he wanted to relate stitch density with **$1/l^2$**. So, he again found a very good correlation of a stitch density and l^2. And the equations is relating with some constant.

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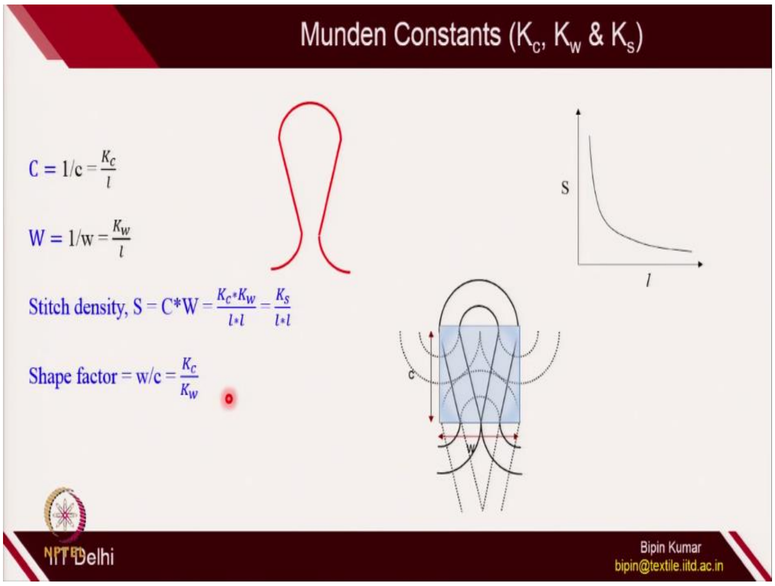

So, Munden actually proposed 3 constant relating fabric structural parameters with loop length. So, where the first constant is Kc, which relates courses per inch with loop length. The second constant is Kw, which relate wales per inch with loop length. And the third constant is relating stitch density with loop length square. So, if you know Kc and Kw, this Ks can be find out.

So, these 3 Munden constant now; then, once they found this relationship, it become very, very useful in knitting industries, because you do not need to measure many things. You simply measure the loop length. And if you know the constant, you can find out many structural parameters. So, this is how these constants are useful. The another constant is **shape factor = wales spacing / course spacing.**

Not that much useful, but you can see again, if you want to find out the shape factor of the fabric, you simply need to have these 2 constant. If you take the ratio, you will get it. So, the beauty of this particular equations are; if you see, these equations are free from cotton or wool. It is just a geometrical relationship. So, if you simply know the constant and loop length, you can find out many structural parameters.

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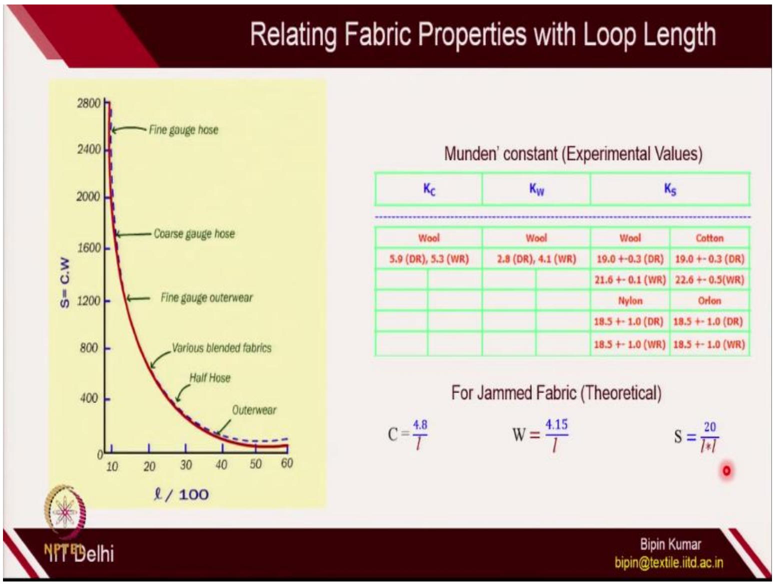

So, later on, many other scientists tried to find out the Kc, Kw and Ks for different types of fabrics. So, they tried with wool; they tried with cotton; in dry relaxed state and wetted relaxed state. And they found, the value of constant Kc is around 5.9 or 5.3, depending on what are the relaxation methods you are following. Kw is 2.8 and 4.1. Ks for wool is different and cotton it is different.

Nylon, Orlon also, the constant has been. So, once you know the constants, it has become very, very useful, because you do not need to do the complicated fabric analysis. You simply use this formula and you will get the tentative idea of fabric properties. For jammed fabric, these constants are of not dependent on what type of yarn we are using and what type of relaxation methods we are using.

The Kc is always 4.8; Kw is always 4.15; and Ks is always 20. So, these values are fixed in case of jammed fabric, which we derived the relation. But if you, if the fabric is not jammed, you need to have these values in your hand to find out this parameters.

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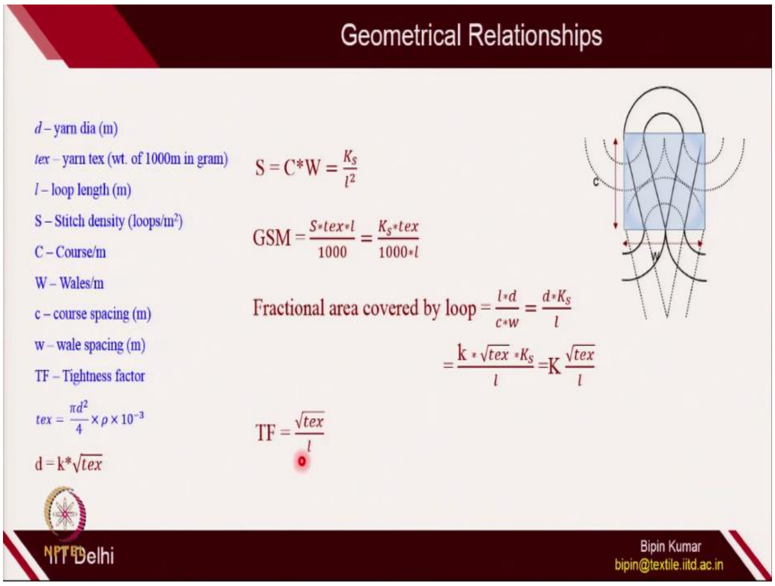

So, using this relations the other variables can also be found. And many scientists propose these type of relationship. And you can simply use this relationship in daily practice. So, the first one, **S = Ks/ l²**. This, we already know. The second one is GSM. In the last class, I derived, **GSM = (S*tex*l)/1000**. So, you can put **S = Ks/l²** in this equations.

And you will get, the GSM is also only dependent on loop length. Because Ks is a constant. And yarn tex and loop length. So, GSM, if you see this equation, GSM is directly proportional to length. But here, in reality, GSM is inversely proportional to loop length. It means, when you increase the loop length, definitely, the GSM of the fabric will go down. The other thing is fractional area covered by the loop.

It means, if you see this shaded region, in this shaded region, the certain yarns has been placed. And rest other area is free for air. So, fractional area covered by the loop is, how much area is actually covered by the yarn divided by total area. So, **the total area=c/w**. And the area covered by the yarn, if you see, this is the two legs. In the shaded region, you can find also the head of old loop and feet of the new loop.

So, in total, if you would see the length of the yarn which is used in this shaded area, is equals to a loop length. And the width of the yarn is equals to d. So, **l * d = area occupied** by the yarn in this shaded region. So, if you take the ratio **(l * d) / (c * w),** you can simply replace c and w, with respect to l. And you will get **d * Ks / l**. So, d is the yarn diameter.

And further in one of the lecture, I have shown you that the tex is actually dependent on the yarn diameter. So, **tex α d^2**. So, the yarn diameter is dependent on the yarn density, which is **k * $(tex)^{1/2}$**. So, you can put it here in d. So, you will get **k * $(tex)^{1/2}$ *Ks/l**. So, Ks is the Munden constant, Ks. And small k is actually the constant relating yarn diameter and yarn tex.

So, **k * Ks**; you can another; if you multiply these 2 constant, one new constant will come, which is **$K(tex)^{1/2}/l$**. So, if you know, this fractional area covered by loop, it is depend, it **=$K(tex)^{1/2}/l$**, become a very significant value, because it is deciding how much area is occupied by the yarn inside the fabric.

Because, if more area is occupied by the yarn, then obviously less space for the air. So, the porosity, permeability, water permeability, everything will be dependent on fractional area covered by the loop, which is actually dependent on **$(tex)^{1/2}/l$**. So, this parameter, **$(tex)^{1/2}/l$**, is actually called as tightness factor. So, tightness factor means, if you feel **$(tex)^{1/2}/l$** is more, then automatically fabric will become very, very tight.

And less area will be available for the air inside the fabric. This relationship, although started with the jammed fabric, but nowadays, these relationships are very, very useful. Although these relationships are very empirical in natures, but very significant in practice. Let's do a very simple example. And then we finish this particular lecture. So, the example is:

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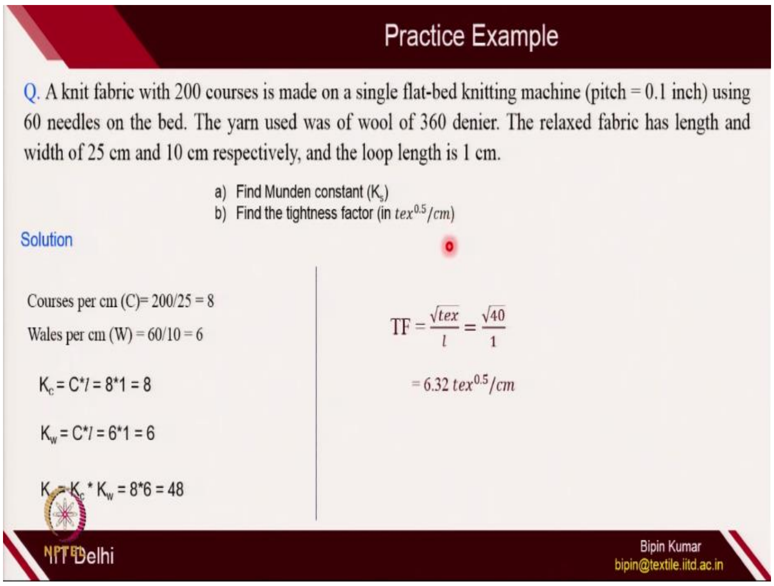

Let's see this question. So, a knit fabric with 200 courses is made on a single flat bed machine. The pitch is 0.1 inch, using 60 needles on the bed. The yarn used was 360 denier. And the relaxed fabric length is 25 centimeter. And width is 10 centimeter. And the loop length is given 1 centimeter. So, we need to find out the Munden constant and the tightness factor. So, to find the Munden constant Ks, we need to find the Kc and Kw.

So, for finding the Kc, we need to find out the loop length and the course per inch. So, course per centimeter, because the fabric length is known. And 200 courses has been formed. So, courses per centimeter, **200/25 = 8 courses/centimeter**. Similarly, the width is known and number of needles is known to you. So, 60/10. So, which is wales per. So, you know the course per centimeter, wales per centimeter; loop length is known to you.

So, you can find out the Kc, which is courses per centimeter into loop length, **8*1**. **Kw=C*l= 6*1**. Please remember, this constant has no unit. So, whatever you will be taking the unit of l, automatically that will be canceled by C, because C is courses per unit length. So, l is the, whatever the unit of length you will take, automatically it will cancel out. So, there is no unit for Kc and Kw.

So, 8 and 6. So, **Ks=Kc*Kw = 8*6 = 48**. So, the Munden constant Ks is now known to you. Tightness factor: the tightness factor, the relation is **$(tex)^{1/2}/l$**. So, in some of the fabric examples also, I have shown you. So, **$(40)^{1/2}/1$**. So, simply **6.32 $(tex)^{0.5}$/centimeter**. So, the tightness factor has the unit, **$(tex)^{1/2}$/unit length.**
So, here the length is expressed in centimeter. So, obviously, the unit will remain same. So, this is how you can see, we do not have to measure anything, we can simply use these type of relations. And we can say a lot of, we can define the fabric behavior with these relationships. So, geometrical modeling is also very, very important. There are many other types of geometrical models.

They are available in plenty in the literature, but this type of geometrical modeling is widely used till today. So, you can follow some very significant literature from the journals and you can get more informations. So, with this, I am ending this particular topic. In next class, I will be covering another small calculation related to knitting; very useful. So, thank you very much. Stay tuned.