Theory of Yarn Structure
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Lecture – 12
Mass Irregularity of Yarns

Welcome to this MOOCs online video course theory of yarn structure. Today we will start module 5. Module 5 discusses about mass irregularity of yarns. Any longitudinal fibrous assembly like sliver roving yarn, they are produced by certain technological processes, therefore it is evident that those technological processes will impart certain natural irregularity in the yarn or in the sliver or in the roving.

This irregularity is natural so what are the reasons of this irregularity or how do we discuss about this irregularity. This irregularity can be associated with any attribute of the assemblies, for example, this irregularity can be related to mass or this irregularity can be related to diameter or this irregularity can be relate to volume or this irregularity can be related to packing density that means let us take sliver.

If we divide the sliver into some equal parts then we measure the mass of those parts, we will see that the masses will be different. The same situation will happen in case of volume of sliver or packing density of sliver, diameter of sliver like that. So irregularity can be associated with any attribute of the material. In this particular module we will focus on mass irregularity, that means the mass or the weight of slivers at different positions along the sliver is different and we will talk about the natural causes of irregularity.

Say for example irregularity can be generated by also say poor fiber material or it can be generated due to some faults in the machines, they are preventable irregularity. We will not discuss them in this module; however, we will talk about the natural mass irregularity in yarn. Probably the first theoretical concept about mass irregularity of sliver or yarn was introduced by Martindale in the year of 1945.

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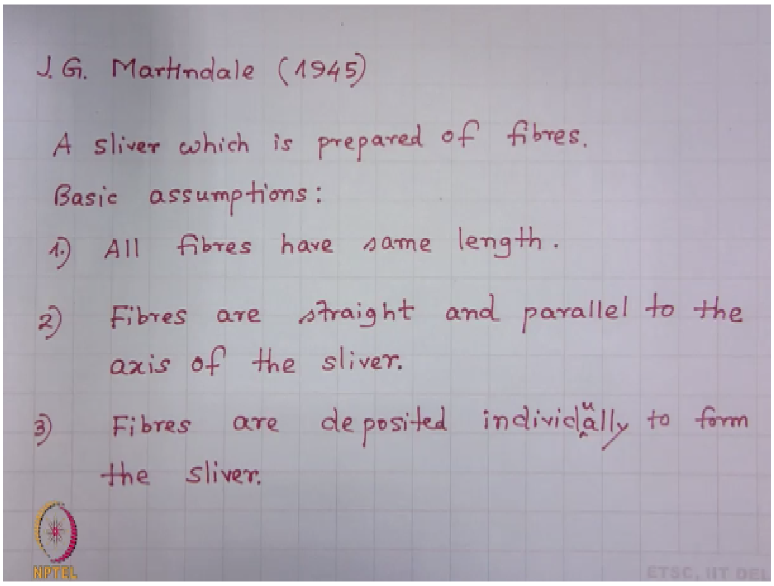

So Martindale tried to analyze the mass irregularity of sliver and Martindale’s model is also applicable to yarns. Martindale thought about a sliver which is prepared of fibers. Then he introduced certain basic assumptions. First assumption what he thought is all fibers have same length. This was his first assumption. Second assumption, fibers are straight and parallel to the axis of the sliver.

Third assumption was fibers are deposited individually to form the sliver. What does this mean? You have seen a drafting system, so the third assumption indicates that the fibers move individually in the drafting system to form a sliver that means no 2 fibers are together or no 3 fibers are together that means fibers move individually or they are deposited individually to form the sliver.

So these 3 were Martindale's basic assumptions apart from that he also introduced one important assumption that we will discuss later on, before that we would like to see how he imagined about the formation of sliver.

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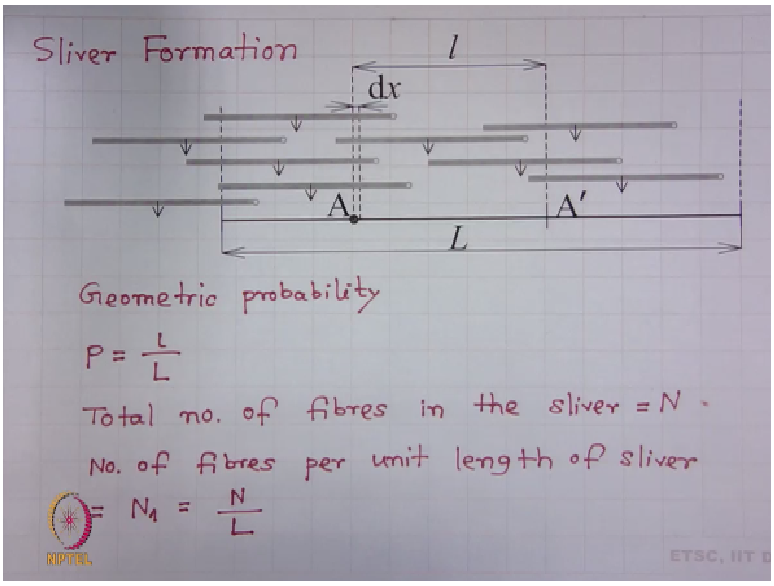

So this picture depicts the formation of sliver according to Martindale's model. What you see in this diagram is, there is a length capital L, we will consider this as sliver length in this length capital L, there are many fibers present. These are the fibers. Each fiber has length small l, all fibers have same length, so all fibers have length equal that is small l. The downward arrow indicates the deposition of sliver.

So here imaginatively slivers will fall at the bottom and they will form the sliver. We consider a small point A, on this point we draw a line, dash line, what is the probability that a fiber is passing this line at A. Now if your fiber is passing this line at A that means the right end small circle of the fiber must be lying within a length small l. If a fiber is passing this line that means the right end must lie within small l.

Otherwise they will not be passing this line. So for example this fiber is passing, so it is right end is lying within this l, this fiber is passing so it is right end small circle is lying within this length right, this fiber is passing, so it is right end is lying in this length small l. This fiber is passing so it is right end is lying within this small length l and what is the total length? Total length is capital L that is the sliver length.

So this probability, the probability that a fiber is passing this line refers to the so-called geometric probability. This probability must be = small l/L. So what is the probability that a fiber is passing the line at A. This probability is = small l/capital L, because the right end if a fiber is passing this line then it is right end must lie within a small length l and what is the total length possible is capital L. So small l/capital L is the probability, right.

Now suppose total number of fibers in the sliver = capital N, right, so number of fibers per unit length of sliver is = say small n1, that is = capital N/capital L right.

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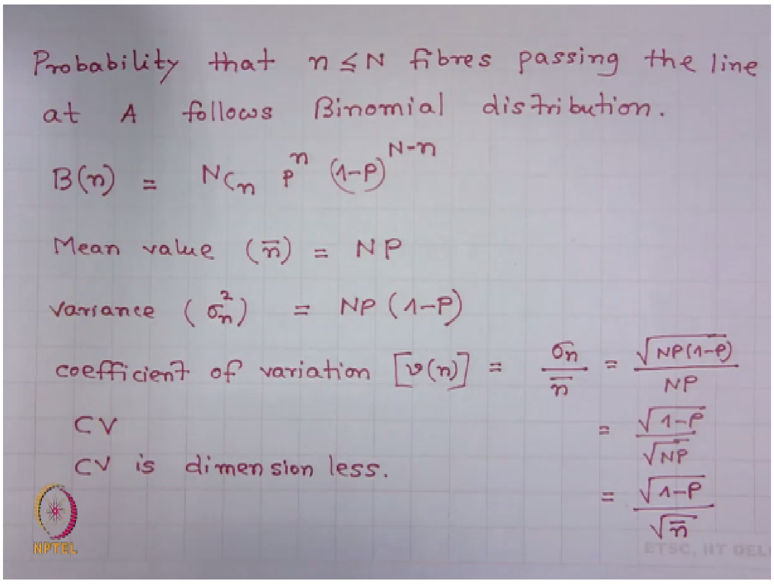

Then the probability that small n, this small n is much < capital N fibers passing the line at A follows binomial distribution. This binomial distribution, the probability of this is capital Ncn probability to the power n failure to the power capital N – n, right. This binomial distribution has certain parameters, for example mean value small n bar is capital N * P, right. Variance is N * P 1- P should be the variance of binomial distribution.

Then coefficient of variation is = square root of variance that is standard deviation/mean. So root over NP * 1-P/NP show root over 1-P/root over NP. Further we can write root over 1-P/NP is here, n bar, right. By the way we often abbreviate coefficient of variation by CV. So CV stands for coefficient of variation. You might have noticed that in this expression CV coefficient of variation is dimensionless.

It can be expressed in percentage if we multiply this by 100 right. So theoretically we deal with dimensionless CV; however, it is possible to deal with CV also in percentage in that case we have to multiply by 100 okay. So this is basically the coefficient of variation of number of fibers present in the cross section of sliver, right. Now we will talk about coefficient of variation of mass irregularity.

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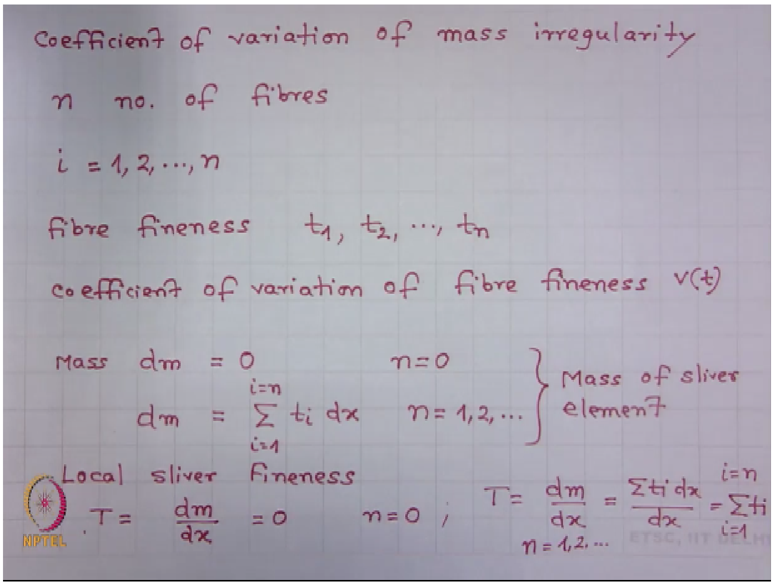

There are small n number of fibers present. We denote each fiber by a subscript I where I will be = till n. Similarly, fiber fineness we will denote by t1 fiber number 1, t2 is the finest of fiber number 2. Similarly, tn is the fineness of fiber number n. Coefficient of variation of fiber fineness we denote by Vt. Again it is dimensionless. Now we will consider a very short length in a sliver.

Say we come back to our sliver formation diagram, here you see a small length dx. So what is the mass of sliver in this small length dx. So the mass will be obviously very small because the length is very small. So let us denote this small mass by dm. So what will be the mass of sliver of this small length dx. So the mass dm can be 0 if there is no fiber when n is = 0, otherwise this dm can be, mass will be summation of all fiber mass.

So what is all fiber mass, that is fiber fineness * length. Mass per unit length * length and the summation i = 1 to i = n, when n = 1, 2 like that so this is the mass of sliver element. Small element of sliver right. Now what will be local sliver fineness. Local sliver fineness means fineness of a small element of sliver, T this is mass/length, dx is the length and dm is the mass. It can be 0, even n = 0.

Fineness is 0 which is little odd to hear that is why we use the word local. If we choose a very small element of a sliver, there can be no fiber. So in that very small element fineness can be 0, that is why we use the word local sliver fineness, otherwise this T = dm/dx, what is dm? dm is your ti * dx/dx = summation ti because dx is constant and the summation will vary from i = 1 to i = n, when n = 1, 2 like that, right.

Now, so what do we see is that sliver fineness can be 0 if there is no fiber, but if there is a fiber then sliver fineness = summation of fiber fineness. Now fineness of sliver is different at different positions along this sliver because the number of fibers will vary in the cross-section. Some parts number of fibers cannot be equal to the number of fibers in other parts because of the technological operations. So sliver fineness capital T is the random variable right and how this is related?

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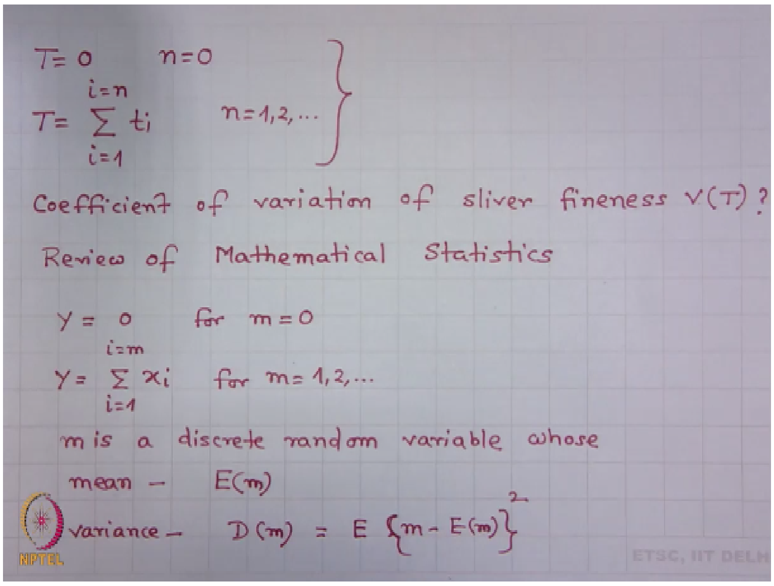

This is related with T can be 0 when n = 0 else T will be summation ti, 1 to n, when n = 1, 2 and others. Now we have to find out the coefficient of variation of sliver fineness that is v of capital T. What is the expression of coefficient of variation of sliver fineness? When these 2 equations are weld.

Now we will now go back to mathematical statistics and review the part of mean variance, standard deviation all those things. So if you take any standard handbook on mathematical statistics you will see this part. Suppose Y is the random variable which is = 0 for m = 0 or y = summation of xi, i = 1 to m for m = 1, 2 and others. Here M is a discrete random variable, is a number, whose mean we denote by expectation of m.

E stands for an operator of expectation and variance we denote by Dm. D stands for the operator for variance and we know variance is related by this. So this is the definition for variance okay.

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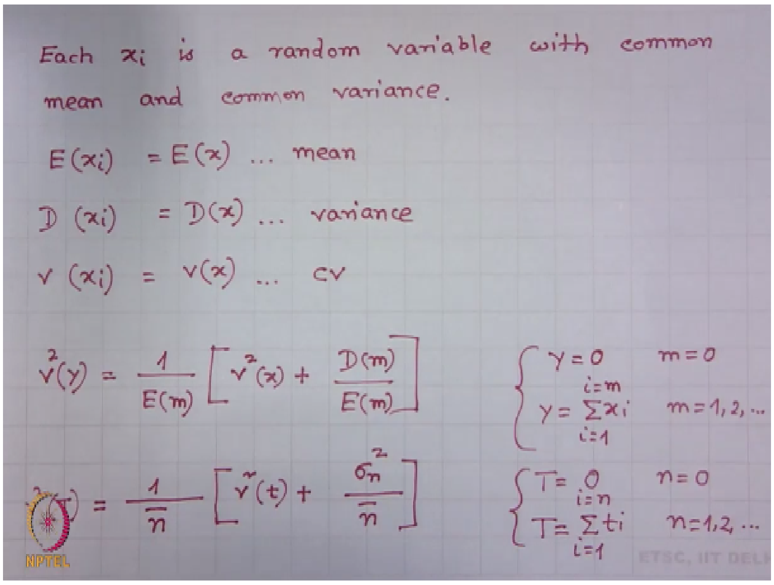

Then what we think is that each xi is a random variable with common mean and common variance. If we consider this, then mean expectation of xi becomes common. Similarly, variance of xi becomes common. Similarly, coefficient of variation of xi becomes common. So this is your mean, this is your variance and this is your CV coefficient of variation, right. Then we can write variance of Y, what is Y? Y is 0 when m = 0, y is summation xi.

When m is 1 2 and this, it such is the definition of random variable then its variance, variance of Y can be written as follows. Expectation of m, variance of x. No square of cv of, square of variance, CV of x + variance of m/expectation of M. This derivation you will see in a standard handbook of mathematical statistics. So if this is how we define Y, then the square of CV of Y is 1/expectation of m * square of CV of x random variable + variance of m/ expectation of m.

Now we compare this with our expression of fineness 0 when n = 0 and capital T = ti when i is 1 to n, n = 1, 2 and… right, then variance square of CV of this is = 1/expectation of n. Expectation of n is mean of n. So n bar * square of CV of fiber fineness + variance of n/expectation of n right. We will now work with this expression.

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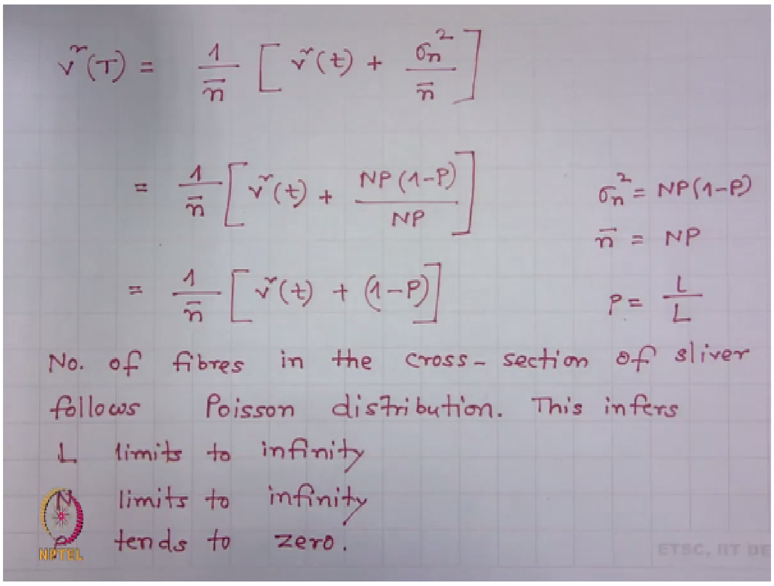

So the square of CV of sliver fineness is = 1/n bar v square t + sigma n square/n bar okay. We substitute variance and mean from binomial distribution. What was variance? this was the variance and what was the mean? N*P. So if we substitute what we get is. So if we assume that the number of fibers in the cross section of sliver follows binomial distribution then the square of CV of sliver fineness can be expressed by this form right okay.

So in order to find out square of CV of sliver fineness you need to know mean number of fibers present in the cross-section of sliver. You need to know the square of CV of fiber fineness and most importantly you need to know the probability P. What is this probability, this probability is small l/capital L, which is often difficult to find it out? So we introduce now an alternate assumption.

What is that? that is number of fibers in the cross section of sliver follows Poisson distribution. Number of fibers in the cross section of sliver follows Poisson distribution. This infers L limits to infinity. So N, total number of fibers also limits to infinity, when L limits to infinity the probability tends to 0. So under the assumption of Poisson distribution the probability becomes 0. Then we can write in this expression if you substitute P = 0, then we can write.

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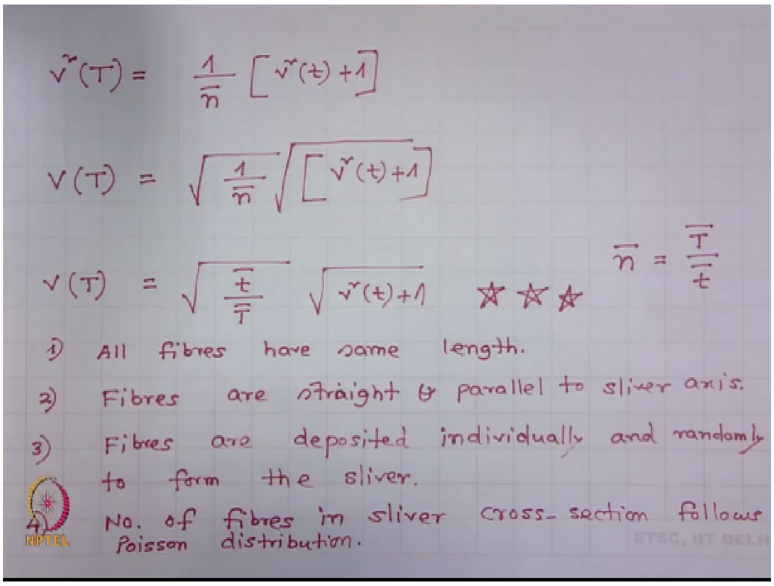

V square T = 1/n bar, v square t + 1, then VT is the CV of sliver fineness, root over 1/n bar V square t + 1, and what is n bar? All fibers are parallel, they are straight, so n bar is capital T bar/small t bar, means sliver fineness/mean fiber fineness. If we substitute here, then we obtain small t bar/capital T bar v square t + 1. So this is the most important equation till now. So this expression is developed based on 4 assumptions.

First assumption, all fibers have same length. Second assumption, fibers are straight and parallel to slavery axis. Third fibers are deposited individually and of course randomly then only this statistical distribution comes into picture. To form the sliver, you remember these 3 were the basic assumptions of Martindale’s model and the fourth assumption number of fibers in sliver cross-section follows Poisson distribution.

So this expression is true under these 4 assumptions. Now often in textile, books, research articles this VT is known as limit irregularity and it is expressed as follows.

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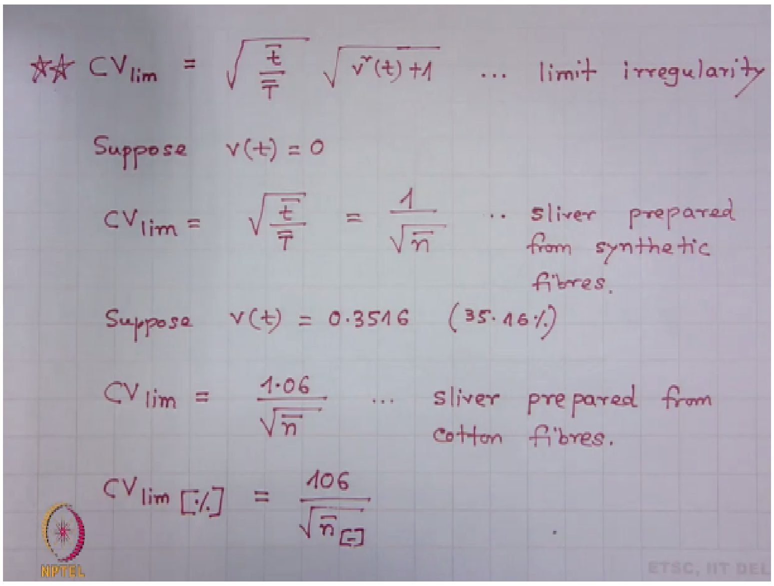

CV limit is equal to divided by this * V square t + 1. So CV limit stands for limit irregularity. So this is the expression for limit irregularity right. Suppose V t = 0, what does that mean. Fibers are uniform, they are fineness at different positions along a fiber are same, so there is no irregularity on fiber fineness. So the CV of fiber fineness is 0, if we substitute V t = 0 here then, what you will find, CV limit will be this.

This expression also you will see in many books. So in this case one more assumption is considered that is VT = 0. So often it is said that this expression is true for sliver prepared from synthetic fiber and suppose vt = 0.3516, so that is basically 35.16 %. So if you substitute this value here then you will find CV limit = 1.06/ this often said this is the limit irregularity of sliver prepared from cotton fibers.

By the way if we express in terms of percentage then this is dimensionless. So these expressions are quite well-known in traditional textile literature and this is how these expressions are obtained. So if we summarize Martindale's model. Martindale's model is based on 4 important assumptions, the basic assumptions are sliver is prepared from fibers, all fibers have same length.

All fibers are straight and they are parallel to sliver axis. The fibers are deposited individually and randomly to form the sliver and the special assumption is the number of fibers in the cross section of this sliver follows Poisson distribution. Under this 4 assumptions it is possible to derive this expression for limit irregularity, it is a quite general expression, if you know the CV of fiber fineness and you know the mean number of fibers present in the cross section of the sliver you will be able to calculate limit irregularity for that sliver.

Two special cases we considered. If we consider there is no irregularity of fiber fineness, then this limit irregularity becomes this. Often it is said that this expression is true for the sliver prepared from synthetic fibers. If we consider cotton fiber fineness CV = 35.16% then you will obtain CV limit = 106 root over n bar. Often said these expression is valid for the sliver prepared from cotton fibers. So this was Martindale's model. Now we would like to discuss a very interesting problem.

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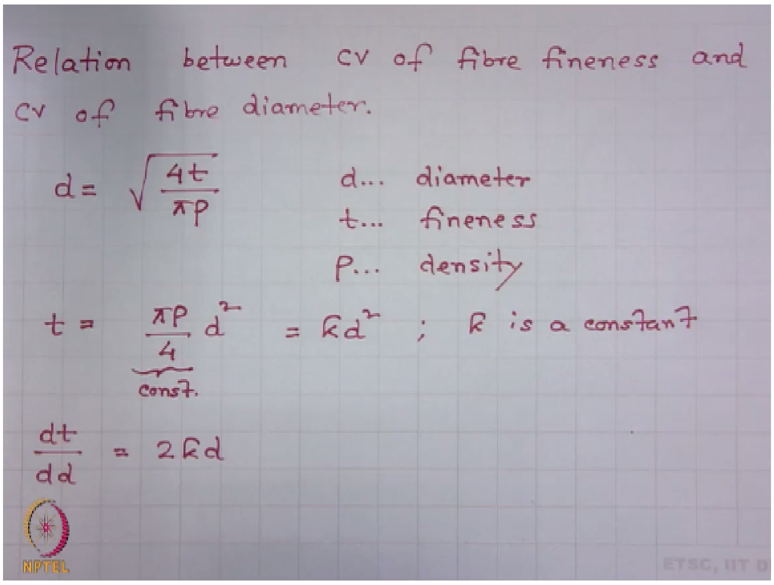

The problem is the relation between CV of fiber fineness and CV of fiber diameter. This problem comes from typically wool fiber. This size of wool fiber is often expressed by their diameter in the unit of micrometer. If we know the CV of fiber diameter is it possible to know CV of fiber fineness so that we can use Martindale's equation to find out the mass irregularity of the sliver prepared from such fibers.

So this is the origin of this problem. What is the relation between CV of fiber fineness and CV of fiber diameter? Now if we go back to our module 1, where we discussed about basic fiber characteristics, in that module we derived a relation of fiber diameter. Where d stands for fiber diameter, t stands for fiber fineness, rho stands for fiber density right. Then we can write fiber fineness = pi times rho/4 * d square.

For a given fiber rho is constant and obviously pi/4 there are also constants. So this becomes a constant. Suppose we write this constant by k, then we write kd square where k is a constant right. Then we differentiate t with respect of d then we obtain 2 kd, okay. Now we need to find out an expression for fiber fineness around mean value d bar.

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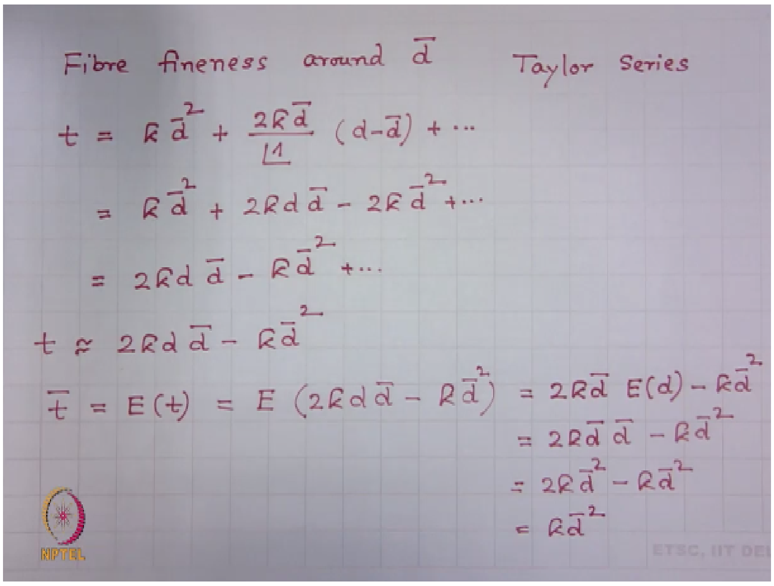

That means fiber fineness around mean value d bar. This can be obtained using Taylor series. What does this Taylor series say? This t = k times d bar square + first derivative at d bar/factorial 1 * d-d bar + other terms. So what do we obtain K times this + 2 kd d bar - 2 kd bar squared, that is = 2 kd d bar – kd bar square right. Approximately we can write taking the first 2 terms of Taylor series what will be the mean and variance of t.

Mean of t = expectation of t, that is = expectation of 2k d d bar – k d bar square, k is the constant and d bar is also constant. So we can write 2 k d bar * expectation of d – kd bar, then we can write 2 k d bar expectation of d is d bar - k d bar square. So 2k d bar squared - k d bar square = k d bar square. So this is the expression for mean fiber fineness and what will be the expression for variance?

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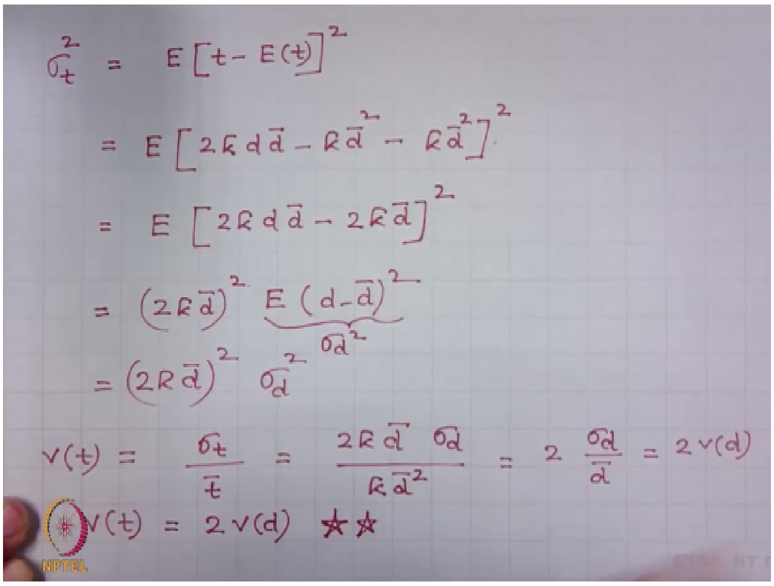

Variance is sigma t square right. So what is this sigma t square? Sigma t square is expectation of t - mean of t. So this is the definition of variance. We often write, so this we substitute, what is your t? t is your 2k d d bar – k d bar square – mean of t. What was mean of t, k d bar square. So what we obtained expectation of 2k d d bar - 2k d bar square. So here 2k d bar is constant. So 2k d bar * expectation of d - d bar square.

So 2k d bar square expectation of d in d bar square that is sigma d square okay. Then coefficient of variation of t is sigma t by t bar. What is sigma t? Sigma t is 2 k d bar * sigma d by what was t bar? t bar was k d bar square. So 2 sigma d/d bar. What is sigma d/d bar? Standard deviation of d/mean of d show 2 vd. This is cv of d, so vt = 2 * vd, this is another important relation. So if we substitute these 2 Martindale's expression.

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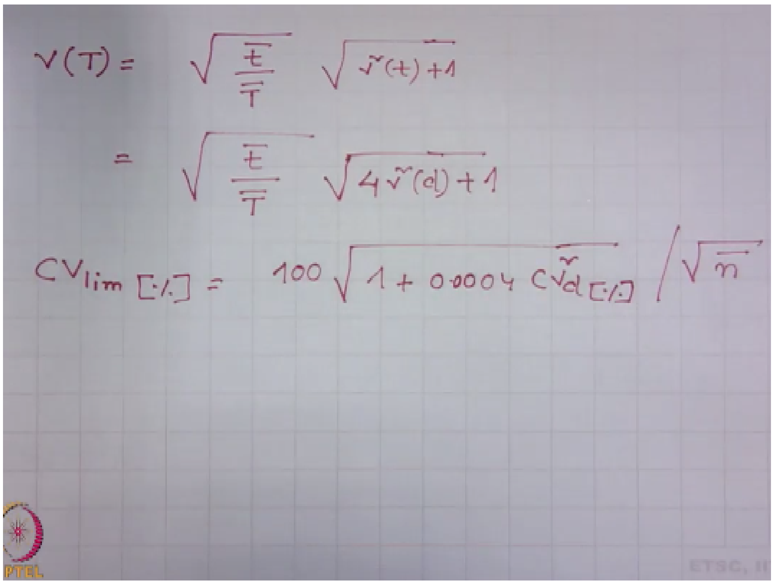

vT t bar/T bar v square t + 1, so t bar/capital T bar, v square t is 4, v square d + 1 right. In terms of unit CV limit = 100 * 1 + 0.0004 CV square d in terms of % / root over n bar right. This expression also you will see in many standard textbooks. We will stop here now in the next discussion we will talk about very interesting fact when the fibers are inclined then what will be the irregularity and also the doubling. Thank you for your attention.