Theory of Yarn Structure

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Lecture - 10
Helical Model of Fibres in Yarns (contd.,)

Welcome to you all to these MOOC's online video course theory of yarn structure. In the last class, we started with module 4 helical model of fibres in yarns. As you know, helical model is a very popular and (()) (00:38) models in the theory of yarn structure. This model basically explains the number of fibres present in the cross-section of yarn. It also explains the phenomenon of yarn retraction.

And finally it talks about limit of twisting. After insertion of certain amount of twist, if you still go on increasing your twist then this twist do not go inside the structure. So it gives a very different kind of structure, so there is some certain twist called limit of twisting. So this model also explains that. Now in the last class, we talked about how to determine number of fibres in yarn cross-section based on helical model.

Today, we will continue with helical model and we will study yarn retraction. What is yarn retraction? Yarn retraction refers to shortening of yarn length due to twist. You might have observed that if you twist a parallel bundle of fibres, then after insertion of twist the length of the resulting yarn decreases. So this phenomenon is called yarn retraction. Today, we will learn the principles, what governs yarn retraction and the associated relationship on yarn retraction. So let us start yarn retraction. Yarn retraction is schematically shown in this figure.

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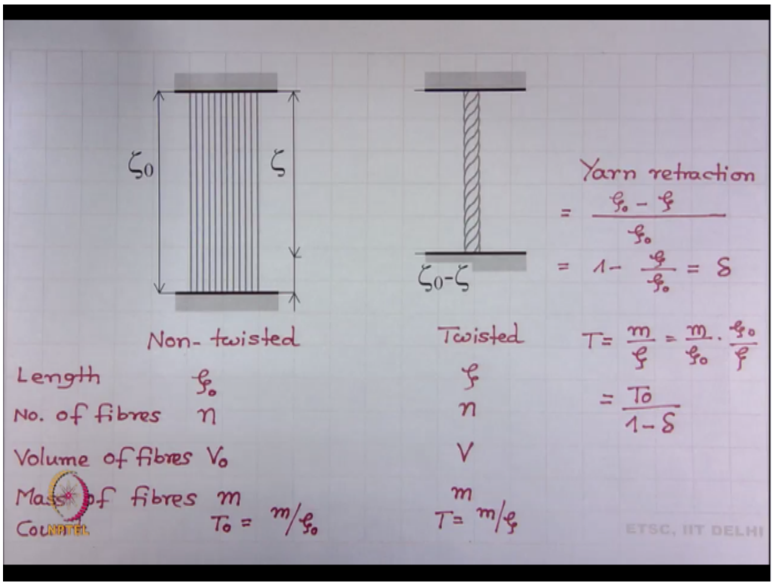

This is a parallel fibre bundle or non-twisted fibre bundle. So let us write here non-twisted bundle and this is the result after twisting. So this is twisted bundle. Now these two bundles, non-twisted as well as twisted they have certain characteristics. Let us establish those characteristics first. The first characteristic is length. What is the length of this bundle is zeta 0. What is the length of this twisted bundle?

This is the length is zeta right. So what is yarn retraction? Yarn retraction is defined by change in length/original length. So what is the change in length, zeta 0-zeta and what is the original length, zeta 0. So if we simplify, we obtain this expression. Let us use a symbol delta to denote yarn retraction right. Then, we come back to our characteristics of the bundles. Second characteristics is number of fibres.

Suppose there are n number of fibres here in the non-twisted bundle. After twist also there will be n number of fibres because fibres cannot fly out of yarn. So n number of fibres right. Then, volume of fibre, suppose initially the volume was V0 and after twisting fibre volume in yarn may change, may not change we will see what will happen V right. Then, mass of fibres, suppose initially fibre mass in the non-twisted bundle was m.

This will remain same in twisted bundle. So this will remain same okay. Then count, what is the count of this non-twisted one? Mass per unit length and this count will change because length is changing right. What is the relation between T0 and T? T is mass/length, let us write this as, so what is this, this is T0 and what is this, this is 1-delta, is not it? So T is=T0/1-delta. So this is the relationship between counts of non-twisted bundle and twisted bundle.

Twisted bundle is yarn okay. We will continue with this further in order to establish a few more characteristics.

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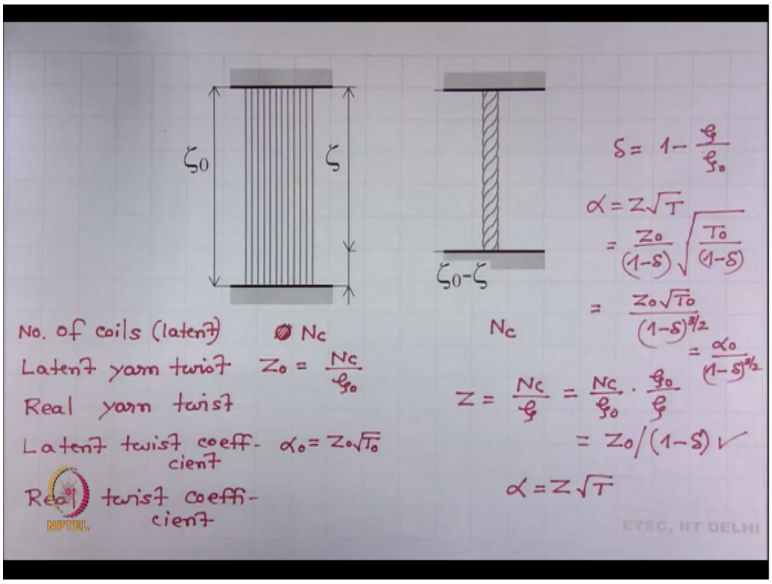

Number of coils, number of coils initially here is 0 and here it is suppose Nc right. Now actually let us imagine that this number of coils was also present here but in a latent manner right. So what we think that these number of coils are present also here, so number of coils let us write here as Nc. You can write here imaginatively these number of coils was latent. They were expressed later on right.

So then latent yarn twist, yarn twist is Z0, number of coils per unit, number of latent coils per unit length okay and here it is real yarn twist. Z is Nc/this, now we can write it as Nc/this manner. What is this? This is your Z0 and what is this? You remember yarn retraction was 1-final length/original length. So 1-delta, so this is the relationship between two twists. Now this is imaginative because it is parallel fibre bundle, it should not have any twist.

However, we imagined that this was latent twist. This twist will be expressed after in the yarn. So that is why we use this word latent yarn twist and here we used real yarn twist okay. Then, similarly if twist is there twist coefficient will be there. Latent twist coefficient, what is twist coefficient? Alpha 0=twist*initial count and here real yarn twist coefficient in alpha is equal to Z root T okay.

What is the relation between latent twist coefficient and real twist coefficient? What is the relation between alpha 0 and alpha? Let us establish that. So alpha is Z root T. Let us substitute. What is Z? Z is Z0 1-delta here and what is your T? T was expressed earlier, T was you remember T0/1-this but this is square root, so Z0 T0/1-delta to the power 3/2. So Z0 T0 is equal to alpha 0/1-alpha to the power 3/2 right. So you remember 3 relations that we will use later on.

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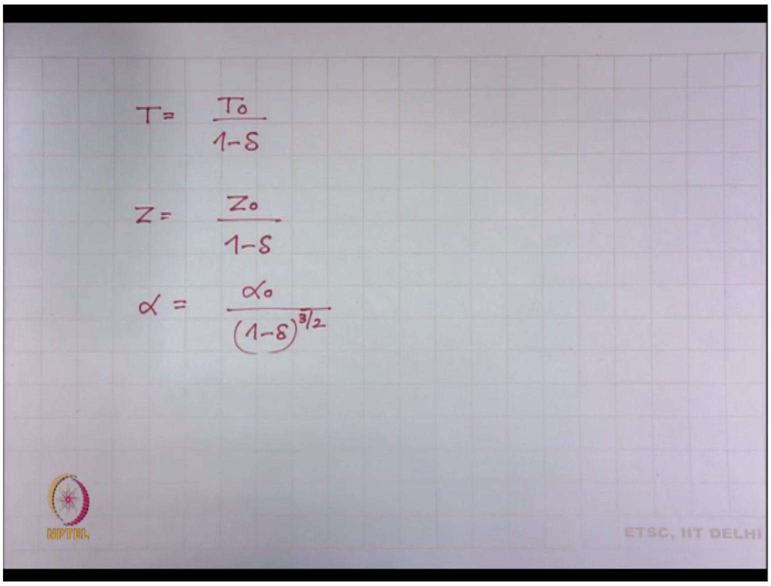

First was T=T0/1-retraction, second was Z=Z0/1-retraction, third was alpha=alpha 0/1-retraction to the power 3/2. So these 3 relations we have derived and you will use them in this module subsequently right. So we have established all characteristics in a non-twisted bundle as well as in a twisted bundle right. So now what we will do, now we will have one important assumption.

What is this important assumption? The important assumption is that the relation between V and V0. You remember what was the relation between V and V0?

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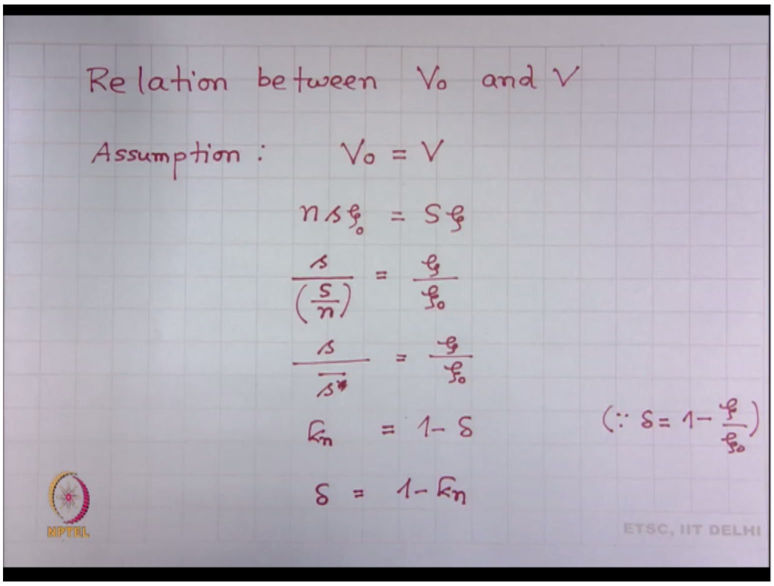

Relation between V0 and V. What is V0? V0 is the volume of fibres in a non-twisted bundle and what is V? V is the volume of fibres in a twisted bundle. How is the relation? Let us assume they are equal; we will see later on the consequence of this assumption. So our assumption is V0 is=V right. So what is now V0? V0 is the volume of all fibres. How many fibres in and what is the volume of one fibre? What is volume? Cross-sectional area*length.

They are parallel fibre bundle, so what is cross-sectional area? Small s and what is length? Zeta 0. So this is V0 right and what is V? V is the volume of fibres in the twisted yarn. They are not parallel right. So if they are not parallel, then that is the total substance cross-sectional area capital S*length. What is length? Zeta right. Then, we can write that s/S/n is= zeta/zeta 0, is not it? And what is capital S/n?

Total substance cross-sectional area of the yarn/number of fibres that is equal to mean sectional area of fibre. You remember in module 2, we discussed about that. So this is equal to this right and what is this expression s/s star bar? This is coefficient kn. All we discussed in module 2, module 3 and what is this? This is 1-retraction. Why? You know the definition of retraction, change in length/original length.

So this is this. So a very important relation we obtained here, coefficient kn is=1-retraction right or we can write retraction is=1-kn right. So retraction is 1-kn alright.

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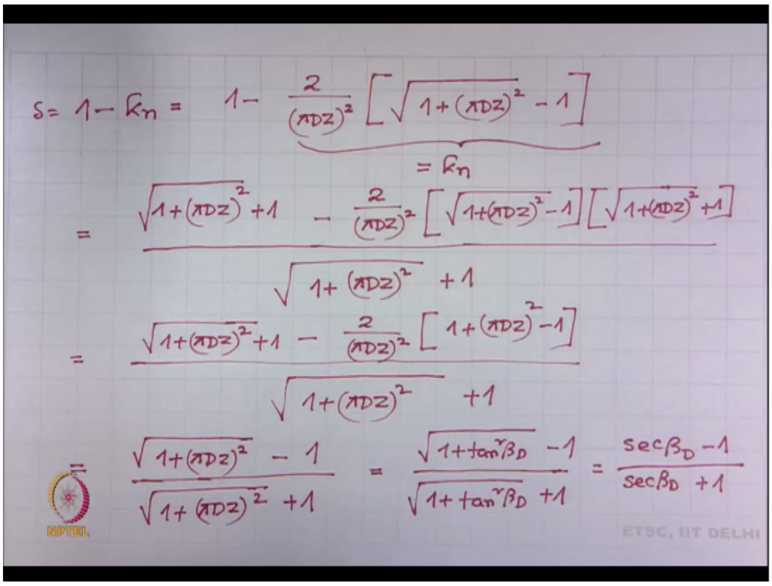

Now kn was derived by this form right, sorry 1-kn lambda, so this kn we derived in the last class okay. Now we need to work on this expression, so that we find a measurable expression for delta yarn retraction. How we do it? Let us see. 1+pi D z square+1 right-2/pi D z square*1+pi D z square-1*1+pi D z square+1/1+pi D z square+1. So what we did basically? We basically multiplied and divided by this expression.

Here we cancel so on will come minus here this will cancel, so this expression will come okay right. Let us see the consequence of this. This will remain as it is square+1-2/pi D z square. What will be this? This will be 1+pi D z square-1, is not it? So this 1 1 will cancel, pi D z pi D z square will cancel, this -2 will be there, +1 is here. So what we will obtain? 1+pi D z square-1/1+pi D z square+1, nice form is not it? Right.

Let us make it more nice. How we can do? What is pi D z? Pi D z is tangent of beta D, so if we continue 1+tan square beta D-1, here it is 1+tan square beta D+1. What is 1+tan square beta D? Sec square beta D, square root of sec square beta D is sec beta D. So write sec beta D-1/sec beta D+1 and sec beta D is 1/cos beta D.

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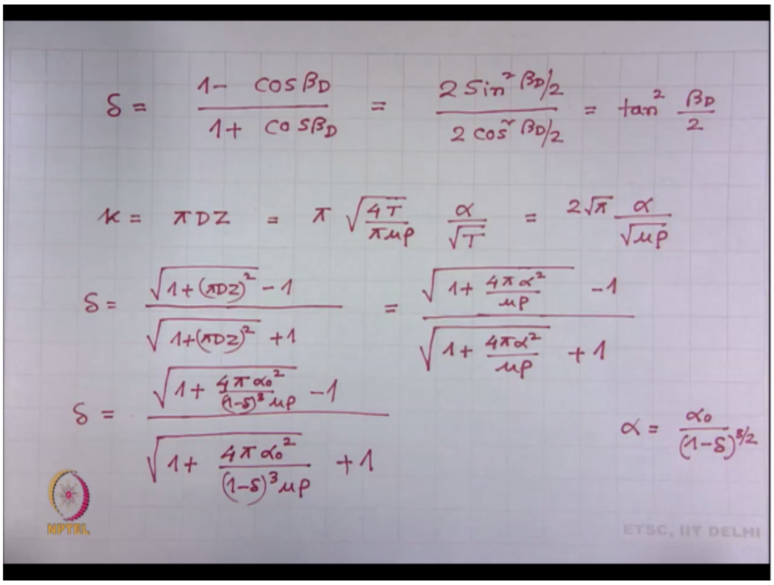

So what we obtain is delta is=1-cos beta D/1+cos beta D right. We can further work on it. What is 1-cos beta D? 2 sin square beta D/2. What is 1+cos beta D? 2 cos square beta D/2, so tan square beta D/2, very nice expression, tan square beta D/2. So this is the expression for yarn retraction. Now what we learned? We learned that yarn retraction is related to the coefficient kn which is further related to twist angle of surface fibres.

How is the relation? Yarn retraction is tangent square half of twist angle of surface fibre. So basically this is how twist dictates retraction because of twist length shortens. So this is yarn retraction. So once the twist is inserted, finally it is the angle of surface to each fibre that finally determines yarn retraction. This is what we understand okay. Now as I told you helical model explains 3 important things.

One, it helps us to determine number of fibres present in yarn cross-section. Second, it explains the phenomenon yarn retraction. Third, very important from practical point of view, it gives us an idea of limit of twisting. Have you ever tried experimentally to find out whether limit of twisting exists or not? You can do it easily. Go to a textile testing laboratory, there is yarn twister, mount on yarn and try to increase the twist.

Have a magnifying glass and look at the structural change in the yarn. After insertion of certain more twists, you will see the fibre coils are not going inside the yarn rather they basically bulge, so yarn structure bulges. So there is a limit of yarn twisting. Beyond that the structure does not absorb the twist. So it basically goes outside that is why you will see certain destruction of yarn structure, certain bulging of yarn structure.

So that is limit of twisting. That means practically it exists. Can it be explained scientifically? Yes, it is possible. Today, it is possible to explain scientifically. By using helical model, we can explain this phenomenon scientifically. So that is our third objective under this module to know about limit of twisting. So we will now proceed towards that. So we will continue with that in order to find out a limit of twisting but we will take a different strategy.

What is our strategy? We know that twist intensity kappa is=pi times D times z right. Now what is D? D we know 4 times T/pi mu rho. We already learned in module 2 and what is z? z is alpha/root T, is not it? So finally what we obtain is that we obtained this 2 root pi alpha/root over mu*rho right. So this is another expression for kappa. Now we will substitute this expression into the expression of yarn retraction.

So you remember we already derived this expression 1+pi D z squared-1/1+pi D z squared+1. So this is our starting expression in order to derive a condition for limit of twisting. We substitute kappa from here to here pi D z. So what we obtain, 1+4 pi alpha squared/mu*rho-1/1+pi D z 4 pi alpha squared/mu*rho+1 right. We obtained this expression. Now this alpha twist intensity is also related to yarn retraction.

How? You remember, alpha is=alpha 0/yarn retraction, so that means in this expression in one side retraction is present; also in the other side yarn retraction is hidden. So we need to take it out and put it in all in one side, then we will be able to solve for retraction, is not it? We have to basically do that. So let us do that. Retraction=1+4 pi sorry it was 3/2, 4 pi squared 1-Q mu rho-1.

You do it step by step, you will make it correct 1, is not it? Do it slowly step by step? okay sure. Now what is our goal? Our goal is very clear. We need to find an expression for delta. Now this delta is here also, here also. So we need to put all these delta in one side, so that is our aim and we will do it slowly. So then our next step we can write in this manner.

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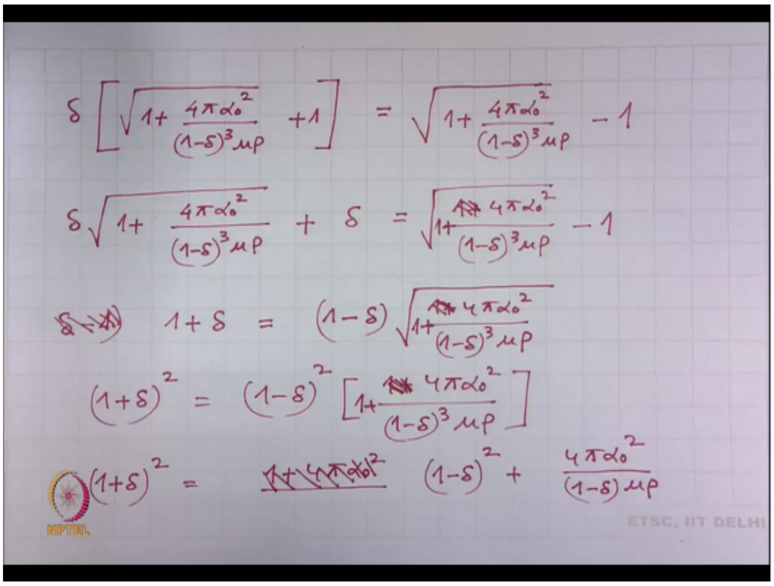

Delta*1+4 pi is=-1 right okay, so then we can write 1+4 pi this/1-this p+this 1+4 pi this. It is a quite long expression, you do it slowly but steadily step by step, you will get the answer okay. Now what do we do? This-1 does not good to write in this way, it is called so 1+delta is=1-delta this right. So what we did, we basically put this 1 in this side and we take this to this side.

Now we have to get rid of this square root. So we have to make square both sides. So let us do that, is not it? Right, so now what you see is that here it is 2, here it is 3, 1 can be canceled. We did a mistake; here is a mistake we did. This is 1+, this is also 1+, this is also 1+ is not it? So we have to write it in a different way. So basically 1-this square+4 pi this square 1-delta mu*rho correct.

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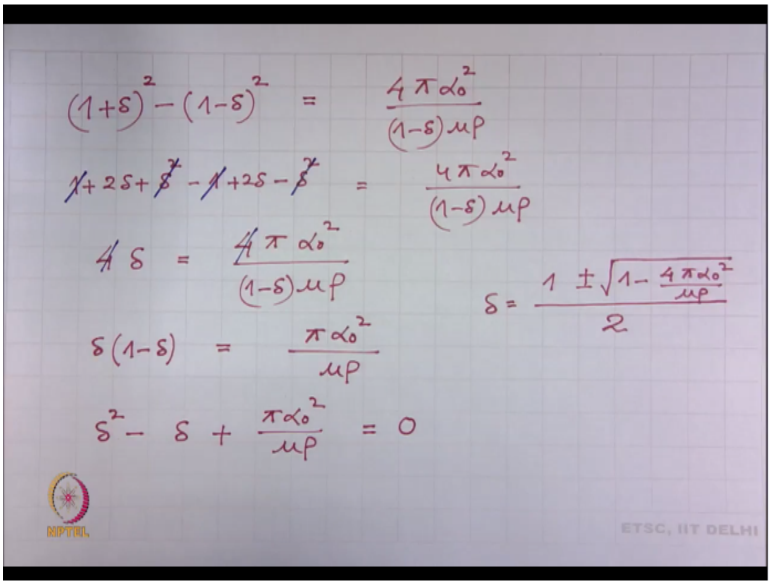

Then 1+this square-1-this square i=4 pi alpha 0 square/1-mu*rho right. So what is this then? 1+2 delta+delta square-1+2 delta-delta squared 4 pi squared 1-delta*mu*this. So what we see here? Here these terms are cancelling out, so what we obtain is 4 delta is=4 pi squared/1-delta*mu*p. This 4 4 will cancel out and we can write now all one side, pi alpha 0 squared/mu*rho okay.

Then, we can write delta squared-this+this=0, so we finally obtained a quadratic expression. So what is the root of this quadratic equation? Root of this quadratic equation is 2, so this is the root of this quadratic expression right. So step-by-step we derived this expression. What does this expression say? This expression is your yarn retraction. Delta is=1+-root over 1- 4 pi alpha 0 square/packing density mu*rho and whole divided by 2 okay.

Now let us see, this discriminant cannot be negative right. So it must be >=0. So let us now write that. So 1-4 pi alpha 0 square/mu*rho must be >=0 because this cannot be negative.

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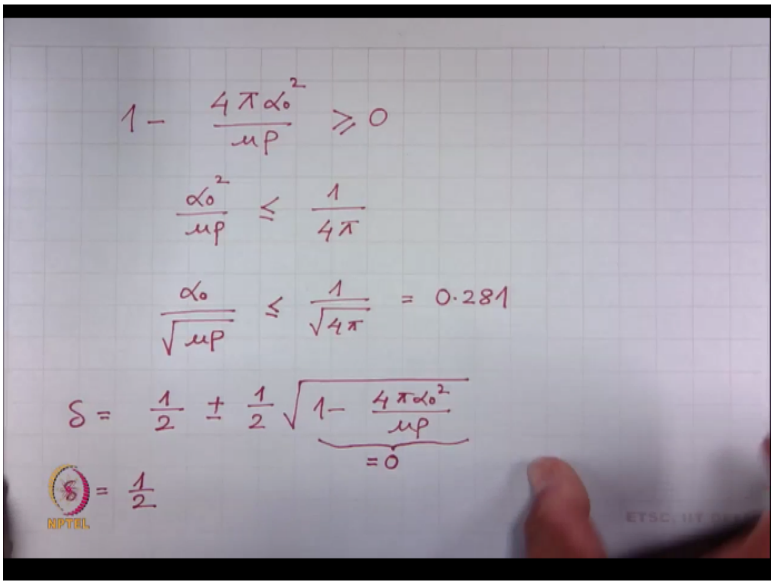

In that case, 1-4 pi square this must be >=0 right. So this must be <=1/4 pi. If we take the root, must be <=1/root 4 pi. What is this value? This value is=0.281 right. Now if we now substitute 0 then what we will get? We will get let this quantity is=0, it can be 0 or >0. Let us say the minimum value, let us work on the minimum value because we have to find out one limit of twisting right.

So this is equal to 0 minimum value. Then, what you will get? 1/2, so this is the limit of retraction okay. Now this is the limit of retraction.

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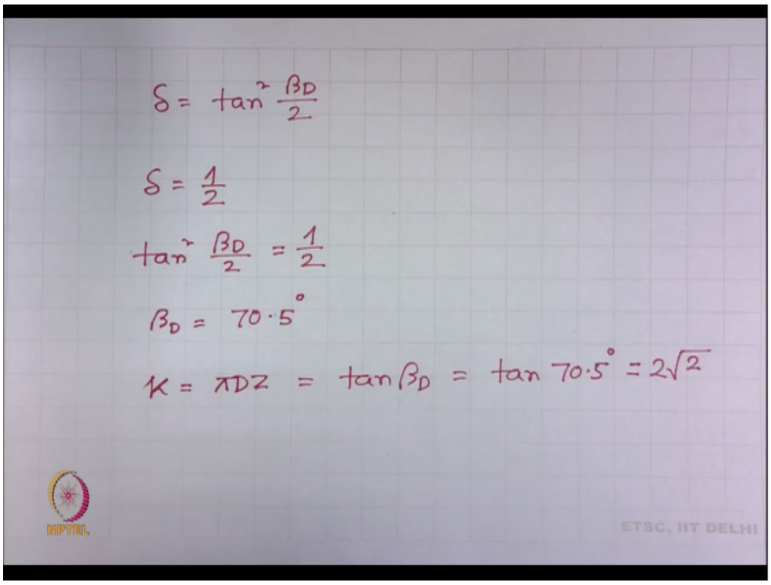

And we have already found out that tan square beta D/2 and if the limit of retraction is 1/2, so tan square beta D/2 is=1/2. Then, what will be beta D? Beta D will be 70.5 degree. That means the twist angle of surface fibre there is a limit and that limit is 70.5 right and then what is your twist intensity? Twist intensity is pi D z that is equal to tangent of beta D. Now if you substitute tangent of this what you will find out? 2 root 2.

So there is a limit of twist intensity also okay. So what we found is that there is a limit of retraction that is 1/2. Accordingly, there is a limit of twist angle of surface fibre that is 70.5 and also then there is a limit of twist intensity that is 2 root 2. So theoretically also limit of twisting exists. As I told you, you can determine it practically. It was done in a laboratory. Let me show you the image, very nice interesting image.

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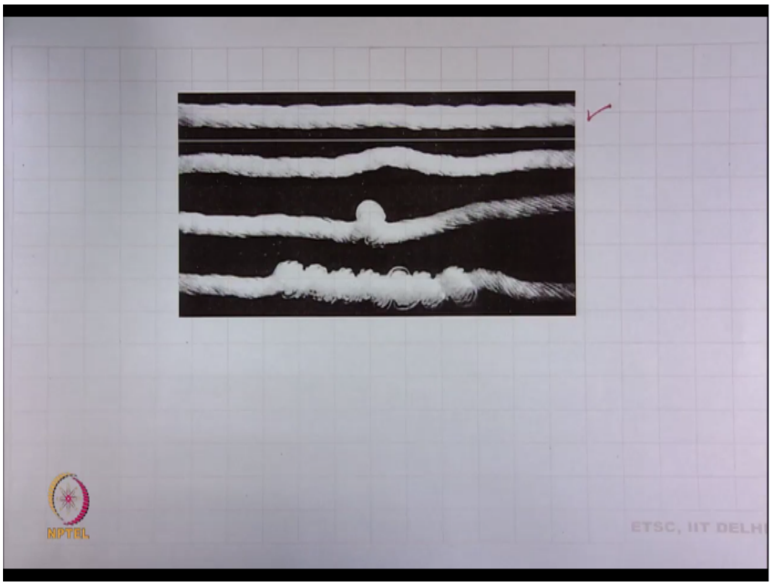

So this is the actual yarn, actual twisted yarn. What we do? We put in the twist tester and we try to increase the twist of the yarn. After a few rotations, what you will see is that you will see this kind of image. So the axis of the yarn will not be straight first. It will happen gradually. First it will be straight almost straight. Then, the axis deviates from linearity significantly. After a few insertions, you will suddenly see that the structure collapses.

So the fibres, the coils are not absorbed by the yarn rather they are deposited onto the surface. If you still go on increasing twist, then you will see total destruction of the yarn structure. So it is also possible to determine limit of twisting experimentally and by using helical model also we have learnt how to explain it scientifically. Now as we did every theoretical work must be compared with the experimental results. In this case also the theoretical results were compared with the experiment and this is the comparison between theory and experiment.

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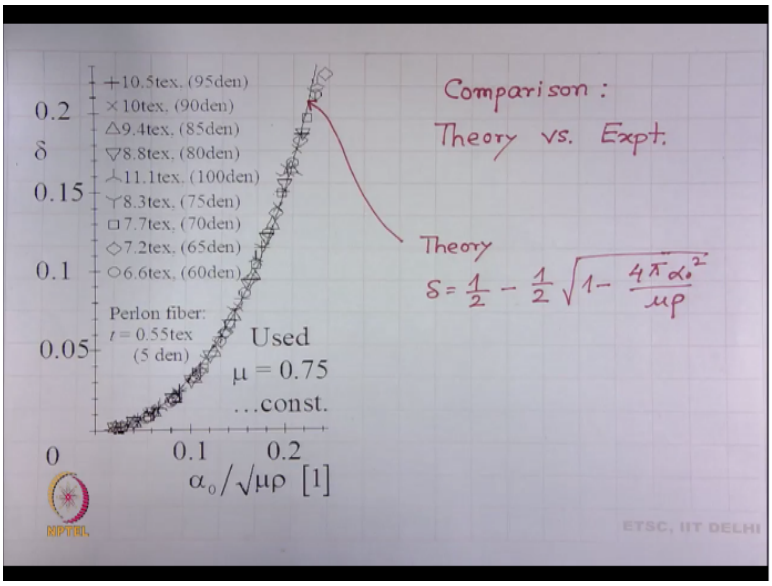

Theory versus experiment, a lot of yarns were studied and along the x axis alpha 0/square root of mu*rho is plotted, along the y axis retraction is plotted and all these points are basically the experimental results for different yarns and this line you see, there is a continuous thin line going on, that is basically coming from theory. What is this line? You have just now derived root over 1-4 pi alpha square/mu*rho right.

So this line is the theoretical line and all other experimental results. What we see is that throughout this region, this theory explains the experimental results quite well, is not it? So this basically completes the lecture part. Now we will solve a few numerical problems so as to understand the whole theory in a little better manner. Let us start with the first numerical problem.

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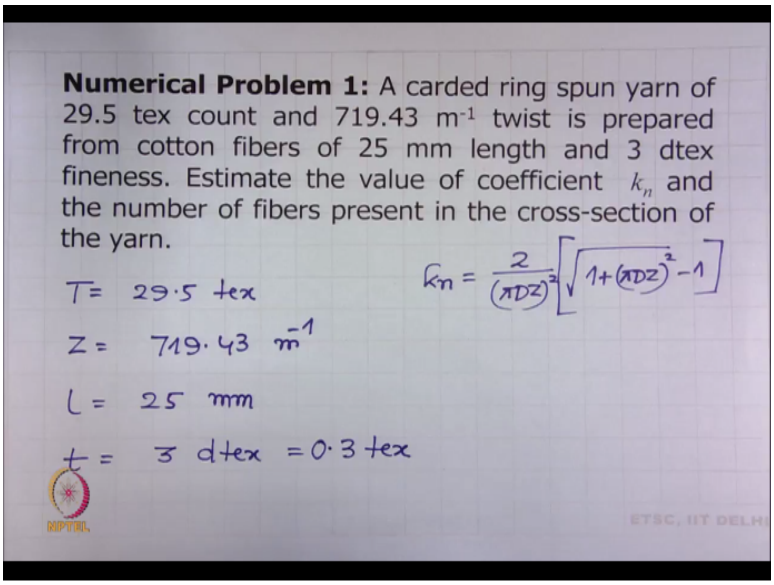

A carded ring spun yarn of 29.5 tex and 719.43 twist is prepared from cotton fibres of 25 millimeter length and 3 decitex fineness. Estimate the value of coefficient kn and the number of fibres present in the cross-section of the yarn. This you have to solve. How will you solve this problem? Now what is coefficient kn? Coefficient kn we have learned using helical model. The coefficient kn is 2/pi D z square square root 1+pi D z square-1.

This is your coefficient kn. So what you have to determine? In order to find out coefficient kn, you have to determine D and you have to determine z. Z is given 719.43, so simply you have to determine D yarn diameter. How to determine yarn diameter?

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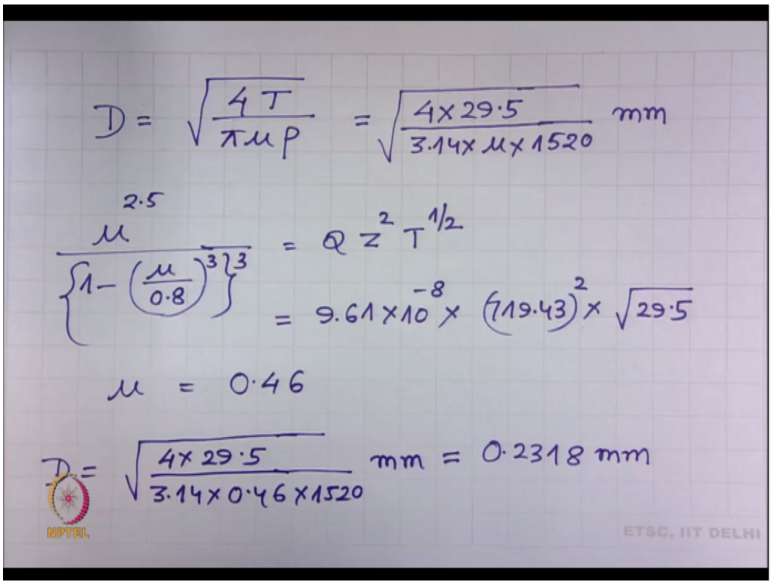

Yarn diameter is 4T/pi mu rho. That is your yarn diameter, T is given. What is T? 4*T is your 29.5 tex/3.14, mu is not given and it is a cotton fibre this millimeter. So mu is not given, how will you find out mu? Mu you have to come back to your module 3 in order to find out mu. You remember this formula in module 1, module 3. Now Q carded ring spun yarn 9.61*10 to the power -8.

We have already solved this problem there. What is z? Z is your 719.43 and what is your T? T is your 29.5 tex. If you solve this, you will find out mu is=0.46 right. Then, you substitute mu here 0.46, so you will find out diameter is equal to millimeter. This will lead to 0.2318 millimeter. So you obtained diameter, you know twist, you will obtain twist intensity.

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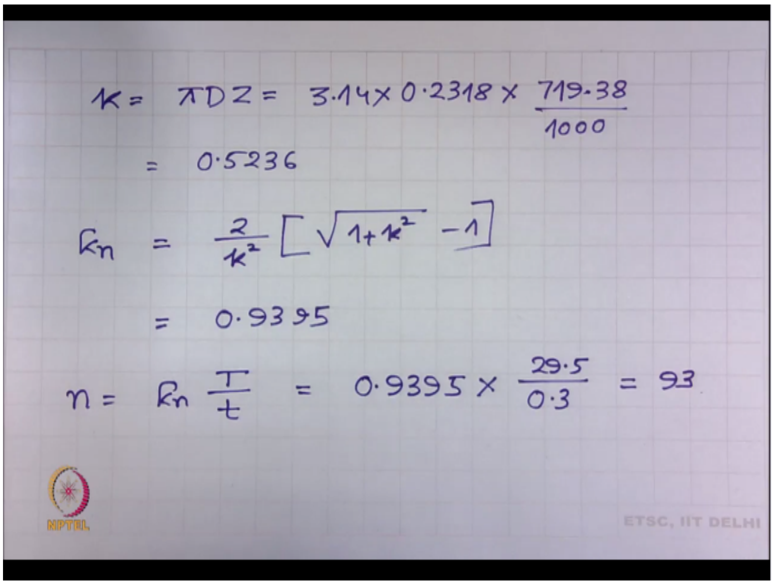

So kappa is pi D z 3.14*diameter 2318 millimeter*z, z is your 719.38 this is in one meter so this is in millimeter. So you will obtain kappa right. What will be this value? 0.5236. Then, you will go back and you will find out coefficient kn is=2/kappa square 1+kappa squared-1. So if you substitute kappa from here, you will find out the value 0.9395 right. So in this way you obtained coefficient kn.

The second part of the problem is number of fibres present in the cross-section of the yarn. Now if you know kn, number is kn times capital T/small t, so what is your kn? 0.9395. What is your capital T? Capital T is 29.5 and what is your small t? Small t is your 0.3. So if you solve how much fibre you will obtain? You will obtain in this case it will be approximately equal to 93 clear.

So this is how you will solve problem number 1. So we are left with two more problems. We will discuss those two problems in the next class. Thank you for your attention.