Advanced NMR Techniques in Solution and Solid-State

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Module-43
Product Operators for Two J Coupled Spins
Lecture-43

Welcome all of you, now we have been discussing quite a bit about product operators since last 2 or 3 classes. In the last class we discussed a lot about rotations and rotation of particular operator about a particular axis and what is the type of solution we are going to get? The general solution I said is the cosine of the old operator plus sine of the new operator. With the three diagrams which was given to show about rotation of each of the operators at about X axis, Y axis and Z axis we could get a solution very quickly without going to explicit calculation of the density operator rho of t which is the e to the power -iH of t rho of 0 into e to the power of iH of t. We do not have to go through all this rigorous maths and use trigonometric identities, everything is clearly available. We worked out for the one pulse sequence; wherein we applied 90 degree X-pulse and started collecting the signal; and we saw that by product operator analysis applying 90 pulse for the Z magnetization which is in thermal equilibrium the magnetization was brought to -Y axis. And then with the time t, when we saw the free precession during the time delay, during acquisition time, we found it generates cosine and sine components. That means the spin vectord start fanning out in the XY plane and exactly when alpha = 90 degree it comes to -Y axis. At t = 0 immediately after the pulse, the magnetization is -IY that is on -Y axis and then with time it creates MX and MY components; that is cosine and sine components.

And then we wanted to extend this further for spin echo analysis, spin echo is nothing but 90 pulse tau, 180 pulse tau. So, with 90 pulse tau what happens? After 90 pulse and free precession we already understood. Next, the effect is we have to see what happens if you apply 180 pulse, how we are going to refocus the offsets, we are not worried about couplings here as we are taking uncoupled spins; in which case we do not have to worry about coupling at all. If there is a refocusing it has to be offset or chemical shift. So, now we will continue further from that with 180 pulse.

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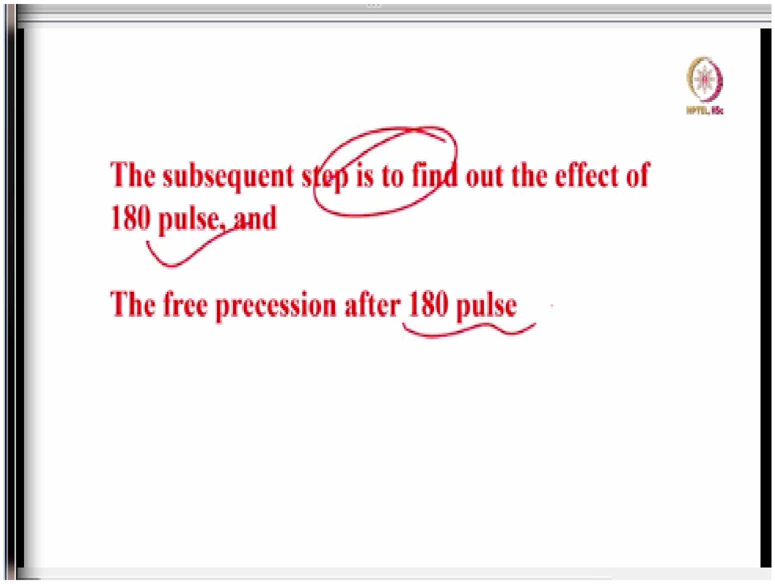

Now we wanted to find out in the second step effect of 180 pulse, and free precession after 180 pulse.

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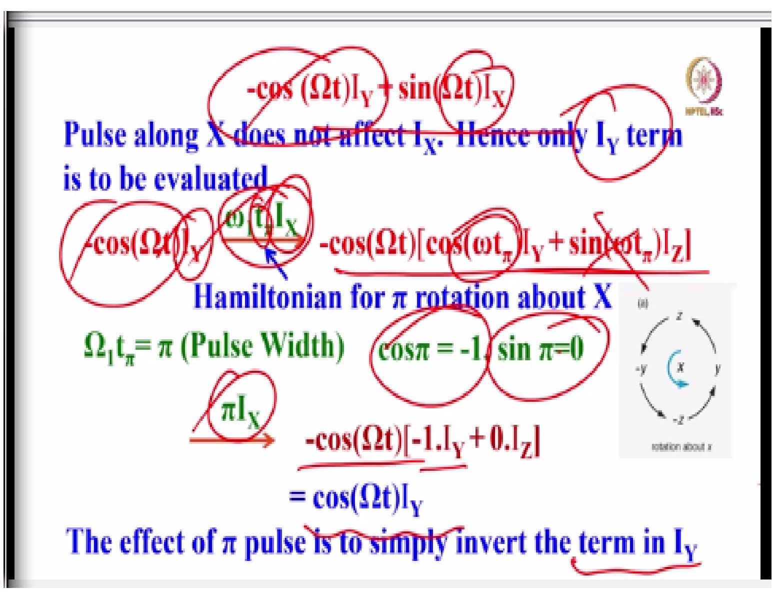

So, this is the solution we have got up to time t after the 90 pulse and first tau during free precession. Pulse along X does not affect X, so we do not have to worry about IX, hence we have to consider only IY term, I told you IX pulse about X, IZ pulse about Z, all those have no effect at all, Z pulse along Z, X pulse along X, Y pulse along Y, has no effect. So, now we are considering only -cosine omega of t and how it evolves with the 180 pulse. Of course, we have to see the figure, go back to the drawing, you have to see what is, now is the rotation of IY about IX.

So, IY term is there, you are rotating about X axis because you are applying a 180 pulse along X axis. So, this is the term you are going to get, if you see the figure -cosine omega t is there and IY is rotated and we are going to get cosine omega tau pi into IY + sine omega tau pi into IZ. And this is the Hamiltonian for pi rotation, 180 pulse about X axis. Same omega 1 tp; but now tp is 180 degree, so it is omega 1 pi. So, use this diagram omega 1 tpi = pi; pulse width, cosine of pi = -1 sine of pi = 0, substitute it there. Then in arrow rotation we can say the application of pi pulse along Y axis gives you -cosine omega of t, this is -1 and this is sine pi 0, this term will go and we are going to get only -1 into IY, -1 into IY. This will finally minus into minus, it will turn out to be cosine of omega t into IY. This is what we are going to get. This is the application of 180 pulse after the first delay, ok. What does it do? The pi pulse has simply inverted the IY. From -IY it became +IY; that is all.

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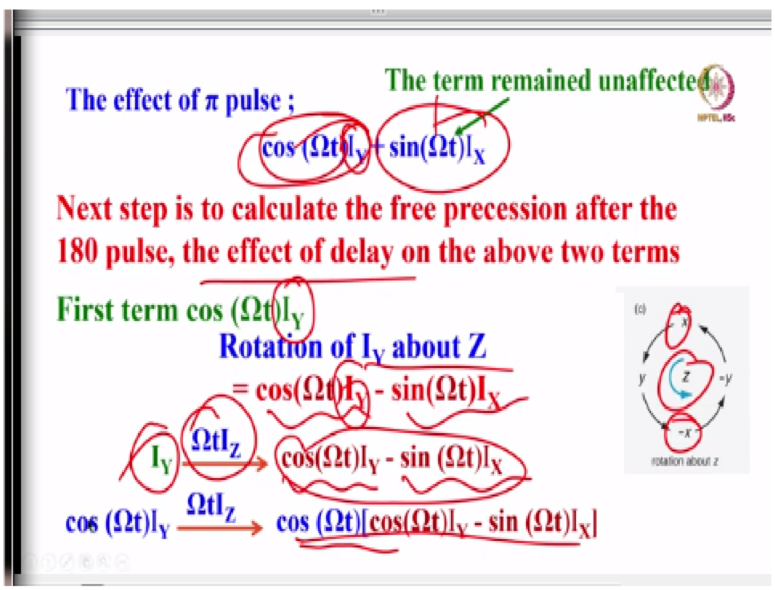

Now the effect of pi pulse, we have seen this term remained unaffected, we have to now consider that. Next step is to calculate the free precession after the 180 pulse; this is what we got with the 180 pulse. Now this term which remain unaffected we consider because you cannot ignore that. Now the total precession during free precession, ie. during second time we have to consider the free precession for both the terms you have to take.

First term will take cosine of omega t into IY, again that means rotation of IY. IY is rotated about Z axis rotation of IY about Z axis you write cosine omega t into IY - sine omega t into IX by using this formula. Cosine of the original operator IY and sine omega t of IX. IX when it is rotated about Z axis which goes to –X, so it will become minus of IX.

So, this is what we are going to get, IY when it is rotated during free precession about omega t IZ about Z axis, omega t is the offset, t is the time; you are going to get cos omega of t into IY - sine omega t into IX; this is the first term. All we did is rotate to IY about Z axis. So, this is what cosine omega t into IY, now I will take cosine omega t, multiplication still there, we cannot ignore that.

Now you only saw rotation of IY, now it turns out to be, this is the solution of rotation of IY with cosine of omega t you are going to get cosine omega t into IY - sine omega t into I X this is what the solution you are going to get. So, this is what for the first term.

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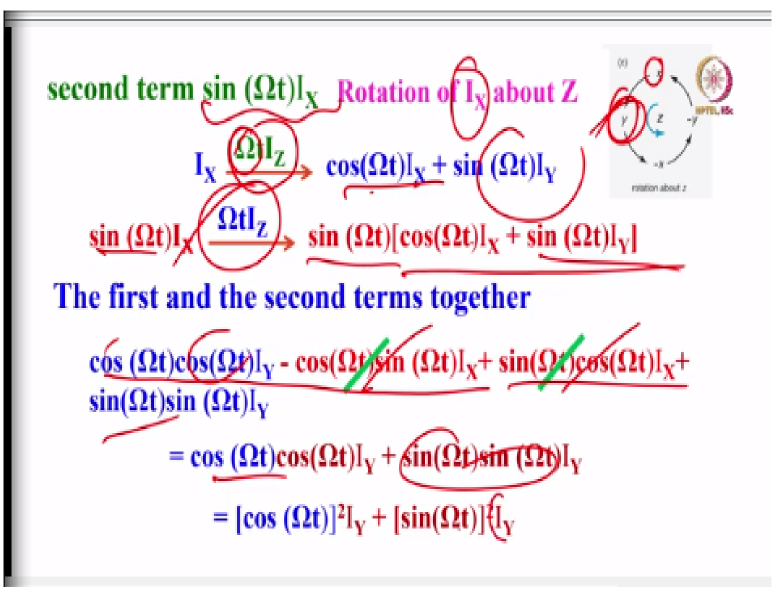

Now let us see second term, second term is sine omega t into IX, what is happening here? You are rotating IX about Z axis, now the rotation is about Z axis during free precession. IX is rotated about Z axis, this is drawing you have to use, this figure. Now you are rotating IX about Z it should go to IY; that is a solution. So, IX when you rotate about IZ with omega into t is the offset. See, now cosine of omega of t into old operator plus sine omega t to new operator into IY, this is the solution for it. So, now sine omega t into IX if you take that product was also there sine omega t which you cannot ignore. Now during free precession this turns out to be sine omega t into cosine of omega t into IX + sine of omega t into IY, this is what you are going to get.

Now we have to bring both these terms together. We have evaluated individually each of these terms, now combine both of them you have to see what is the effect of that. Now bring both the terms together this is the term which you obtained from the first one, this is the term you have obtained in the second one. Sine omega t into cos omega t + sine omega t into sine omega t into IY, this is what the term you are going to get.

But interestingly you can see this term and this term will cancel out, plus and minus. So, what you are going to left with is, cosine omega t into cosine omega t into IY, sine omega t into sine omega t into IY, so what is this? Cosine omega t into cosine omega t is cosine square omega t, this is sine square omega t.

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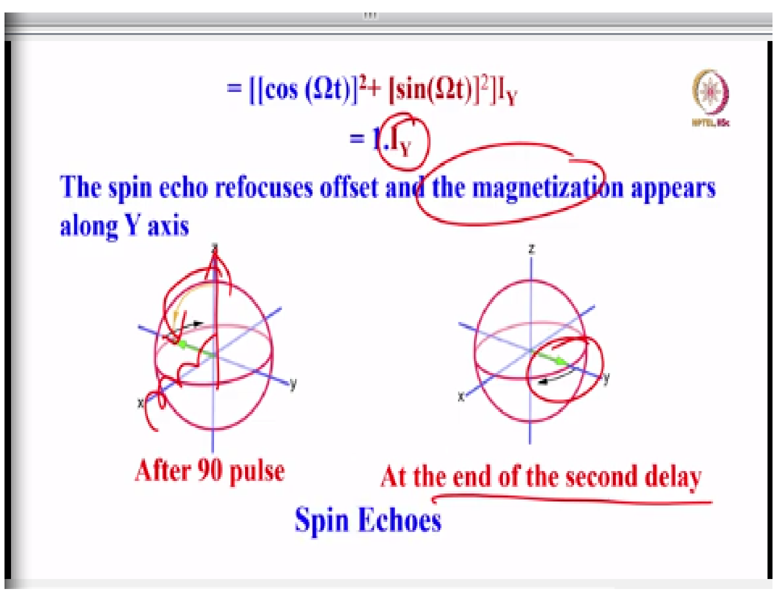

Now take out IY as a common factor, what are you going to get? cosine omega square + sine omega square, what is cos square theta + sine square theta, it is 1. So, this term turns out to be 1 and solution is going to be 1 into IY, that is IY, so what happened now? See, what happened is when you applied 180 pulse and allow to evolve for equal amount of time another time period t and in a workout during free precession what happened? The magnetization remained as IY, what happened? The spin echo refocuses the offset and the magnetization appears along Y axis, the chemical shift term will go, it is not there, offset term is not there, it is removed, only the magnetization which is along -Y axis you brought to Y axis. This is called refocusing of the offset or chemical shift, please remember this. Now we are considering the homonuclear case, now in the spin echo sequence that is 90 tau, 180 tau sequence, it refocuses the offsets or chemical shifts and make sure that the magnetization which is along -Y will goes to +Y axis, that is the spin echo sequence.

Diagrammatically you can understand like this, look at it what is happening? The magnetization which is along Z axis is brought to -Y axis by applying an RF pulse along X axis, that is what we did. And then at the end of the second delay this is what happens, the magnetization has come back to +Y axis. tThis is what is called in an echo, how it works out? You can see vectorially like this, there is a vectorial picture.

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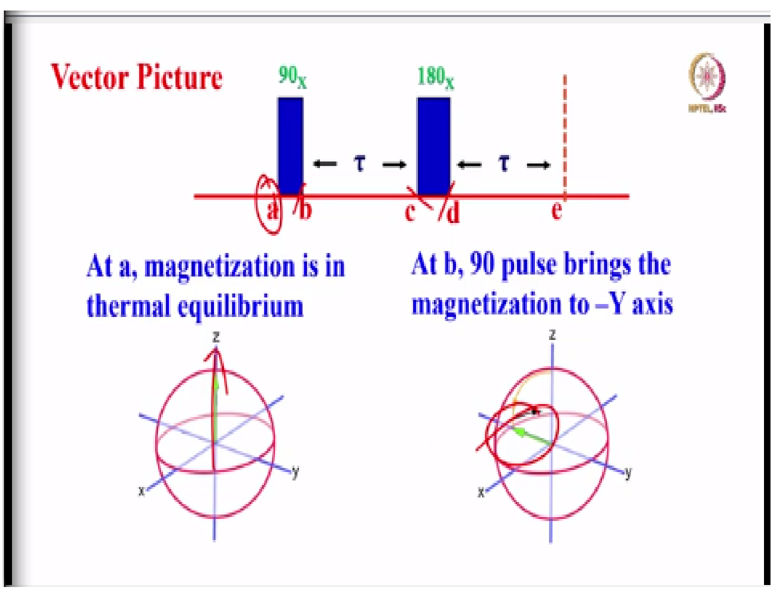

We have worked out by product operators for different sequences. Vectorially also we can understand to get a picture. But we cannot do this for all pulse sequences, especially for the complicated sequence easily; it is a very complex thing. But just for this I will show you, we start with different time periods a, b, c, d etcetera, I call these a, b, c, d, I call this as e.

What is the state of magnetization at a? It is in thermal equilibrium along Z axis, at a the situation is the magnetization is along Z axis in thermal equilibrium. At b, what is happening? You are applying a 90 degree pulse; you brought the magnetization to -Y axis; that is the state of magnetization at b.

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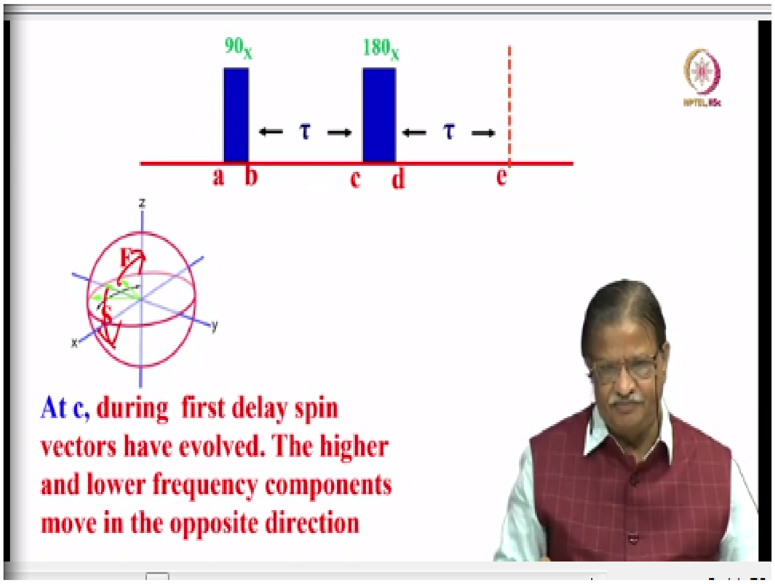

At c, what is happening? During the time period there is a dephasing, we said you know magnetization starts fanning out; it creates cosine and sine components; oscillating components will be produced as a function of time, as time evolves. So, now magnetization vectors start moving on either side. First this one which move this side, other which moves other side, we call them as fast moving and slow moving components; one is at higher frequency other is at lower frequency; they start moving in the opposite directions. This we call it as fast moving and slow moving components.

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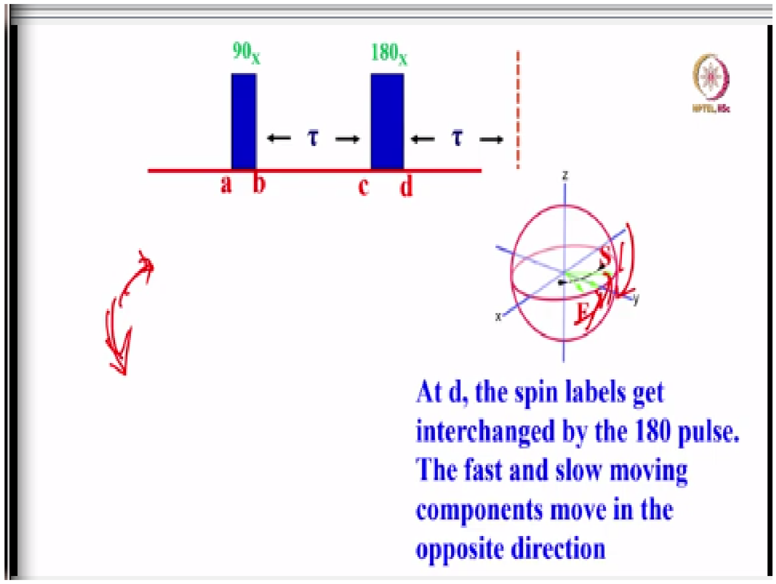

Now after a certain time, now we are going to apply 180 pulse. With the 180 pulse what you are going to do? You simply invert the magnetization from +pi to –pi, the entire vectors which were here it is like turning, turning a pancake you completely turn it on other side, from -Y axis to +Y axis. Then what is happening is the fast moving and slow moving components interchange the directions of motion, they start moving in the opposite direction now. Fast moving earlier it was going like this, slow was moving like this, now fast starts moving like this, slow starts moving in the opposite direction. Then what is going to happen?

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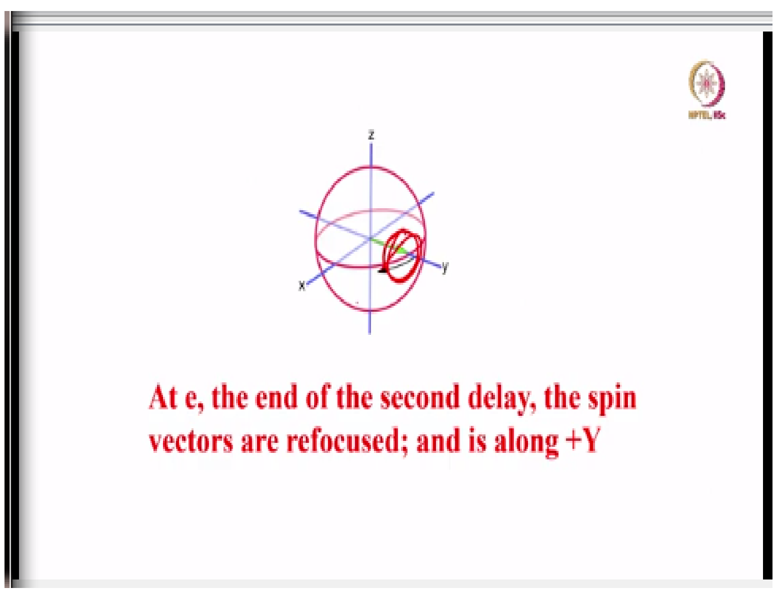

After the exactly same amount of time they will come back, refocus along Y axis. So, the vectors get refocused under +Y. Started with -Y for a given time they start dephasing and created cosine and sine components; and apply 180 pulse, invert the magnetization from -Y axis to +Y axis. Fast and slow moving components start moving the opposite direction and after exactly same amount of delay, they refocus along Y axis.

This way, this is the diagrammatically how it works can be seen like this animation of a spin echo; it is like this; you see like this. Now magnetization is along Z axis 90 pulse brought like this and then rotated by 180 pulse and they can go back and assemble, fantastic, this is what is called spin echo sequence.

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Now we got some hang of how to analyze the pulse sequence using product operators. But remember till now we have taken a single spin or more spins without any J coupling; there is no interaction between the spins. You are only concentrating on evolution the magnetization under free precession or under offset. Now consider the case of a coupled spin system, take example of two couple spin system. Again weakly coupled I and X spin or we call it 1 and 2, does not matter, this is a weakly coupled case.

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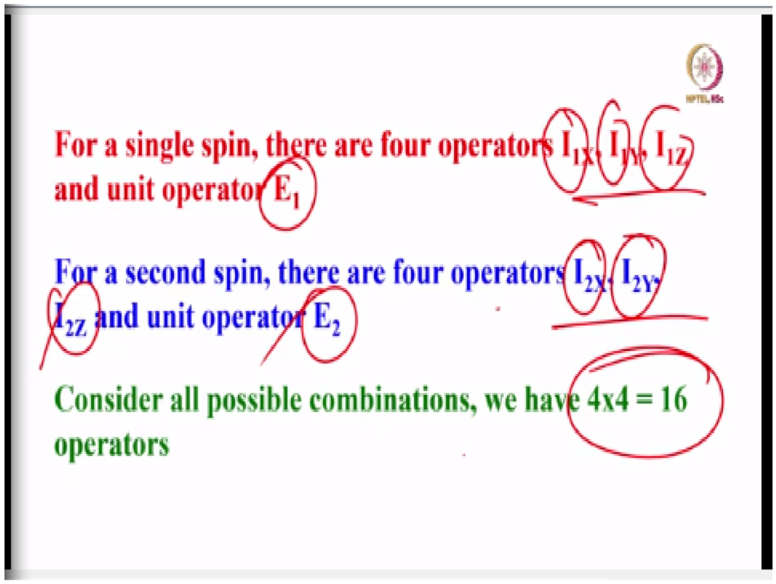

Now for a single spin, there are 4 operators. Of course, you know I 1X, I 1Y and I 1Z, there is also called the unity operator E 1. Of course practically it does nothing, it only helps in mathematical operation. For a second spin, there are 4 operators, again I 2X, I 2Y and I 2Z and a unit operator E 2. So, for a second spin, there are 4 operators like this, now when 2 spins are interacting, we consider all possible combinations. We have 4 operator for spin 1, 4 for spin 2, 4 into 4; we have 16 operators. There are 16 operators.

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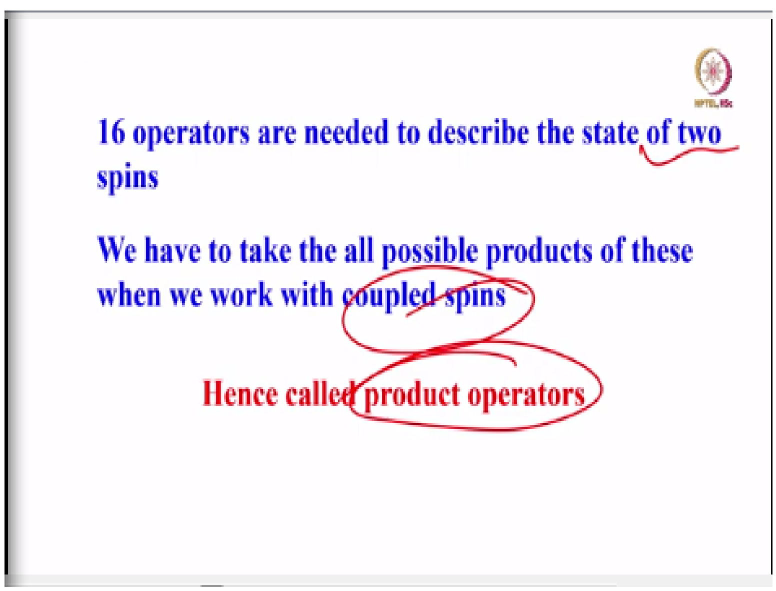

The 16 operators are needed to describe the state of 2 spins completely. So, we have to take all the possible products of these when we work with coupled spins. All 16 operators we have to consider. That is the reason since we get product of all these things, these are called product operators, why product operator name comes? Because when you consider multiple spins, for example 2 spins here, each spin has 4 operators we have to take the combination of all the 4 with all remaining for the other spin, so we will have 16 operators; that is why they are called product operators.

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Now what are the operators we require for 2 coupled spins?

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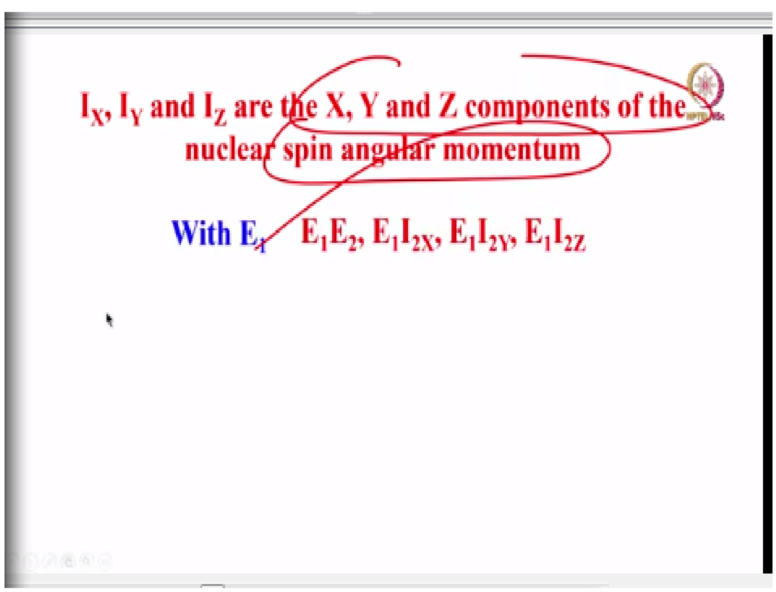

One thing is I X, I Y and I Z are the X, Y and Z components of the nuclear spin angular momentum, that we know I X, I Y and I Z. And with E 1, considering E 1 also, E 1 is a unity operator.

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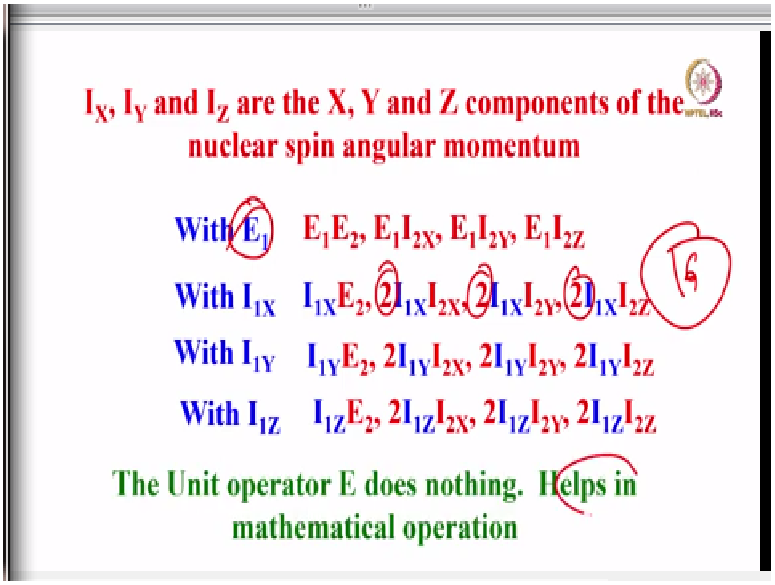

As I told you it is E 1 is only to simplify the mathematical operation. Ynity operator is there for easing the maths. With E 1 now what are the 4 possible product operators E 1 into E 2 for the second spin, E 1 into E 2, E 1 into E 2X, E 2Y and E 2Z. Because now I am considering for the second spin E 2 I 2X, I 2Y, I 2Z are the 3 operators angular momentum operators, when it combines with E 1 we get 4 possibilities E 1 into E 2, E 1 into I 2X, E 1 into I 2Y, E 1 into I 2Z.

With I 1X of the first spin, first X component of the angular moment of the first spin, again 4 combinations I 1X E 2 etcetera like this, I 1X, 2 times I 1X, I 2X. Now the question here why 2 comes, it is only for normalization, do not worry, that is a number. Similarly, with I 1Y we have 4 operators, I 1Z we have 4 operators, so 4, 4, 4, 4 there are 16 operators we have generated together for 2 spins. The unit operator E nothing is only helps in mathematical operation.

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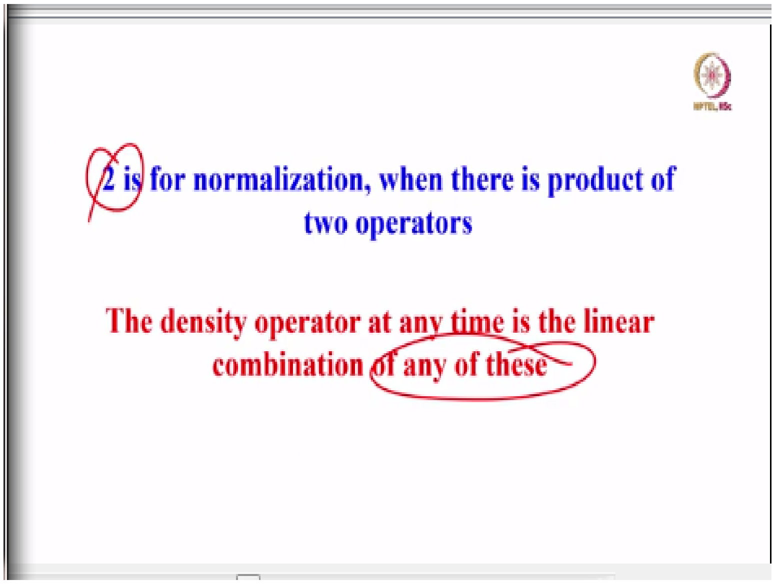

What is the 2? I showed you for normalization when there is product of 2 operators 2 comes, we use it for normalization. So, now if I have to consider the density operator at any time when the 2 spins are interacting including J coupling. It is a linear combination of any of these, all the 16 operators; it is a linear combination of any of these together.

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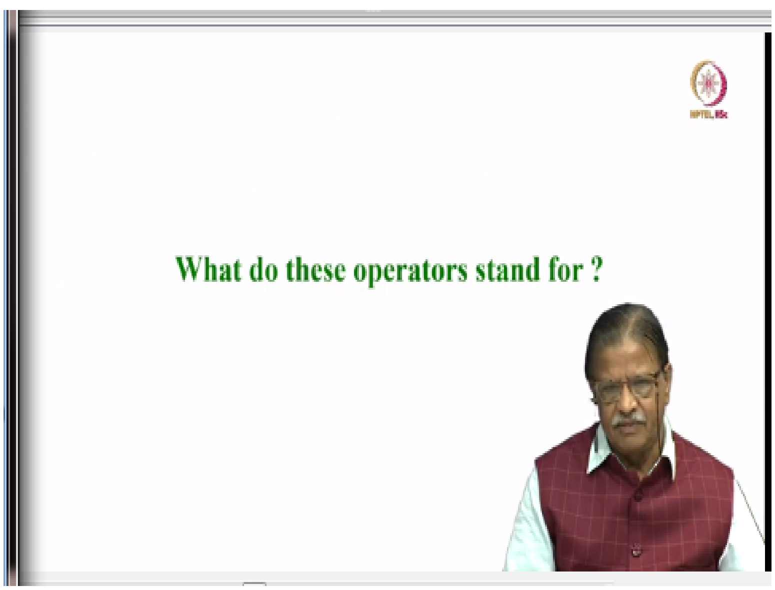

What do these operators stand for, how do you understand these operators?

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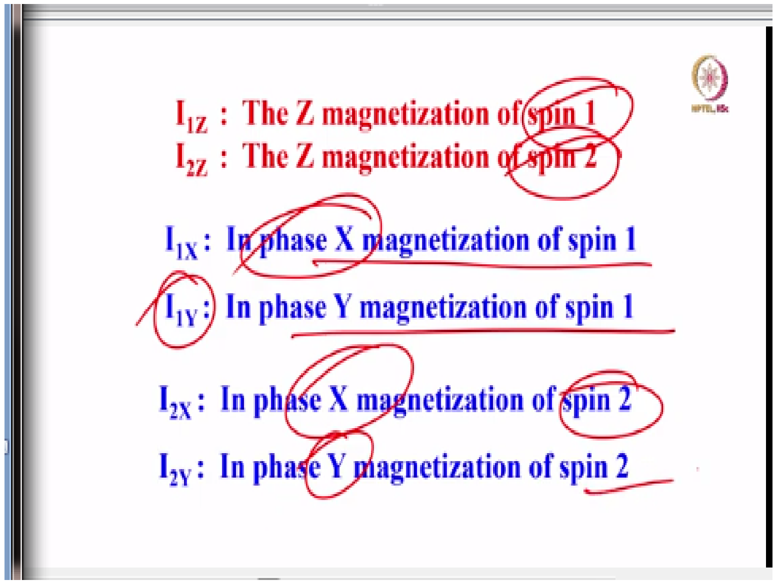

See I 1Z is a Z magnetization of spin 1, I 2Z is called Z magnetization of spin 2, I 1X is called in phase X magnetization, please remember I 1X is the in phase that is the word I am stressing; an in phase X magnetization of spin 1. Similarly, I 1Y is in phase Y magnetization of spin 1, I 2X is in phase X magnetization of spin 2 and I 2Y is in phase Y magnetization of spin 2.

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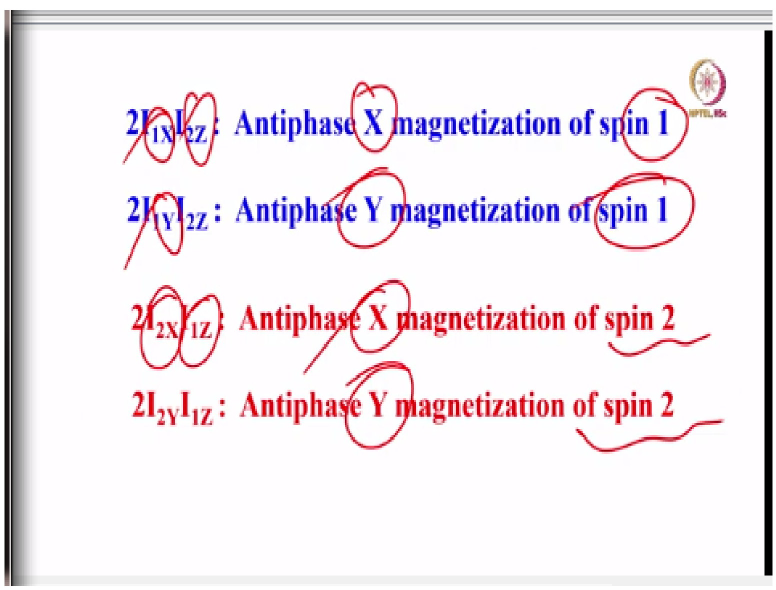

2I 1X I 2Z, for this product if you consider it is an antiphase X magnetization of spin 1, you look at this spin 1. For spin 2 it is along Z axis here, spin 1 is along X axis, this is called antiphase X magnetization. Same thing along Y axis it is called antiphase Y magnetization of spin 1, you understand. Similarly if I consider I 2X I 1Z for the second spin also we have antiphase X magnetization of spin 2 and antiphase Y magnetization of spin 2, this is what each of these operator mean.

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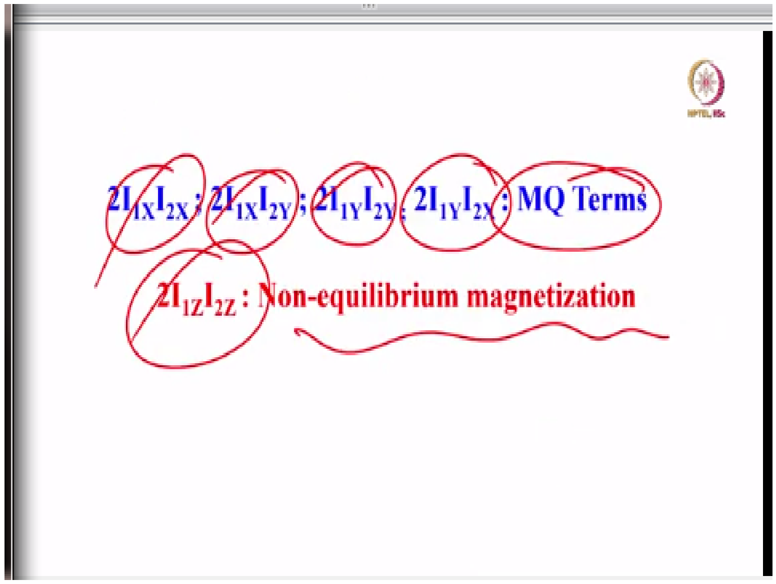

Now there are also combination of other operators like this I X into 2X, I 1X into I 2Y, I 1Y into I 2Y, I 1Y into I 2X. Here both the X and Y operators of both spins are present in the XY plane; these products are called multiple quantum terms. Since there are 2 spin operators, these are called as double quantum terms. And the product 2I 1Z and 2I 2Z is called a non-equilibrium magnetization along Z axis, it is a non-equilibrium magnetization, that is all, you do not have to worry about it. That will not generate any observable magnetization, this what it is.

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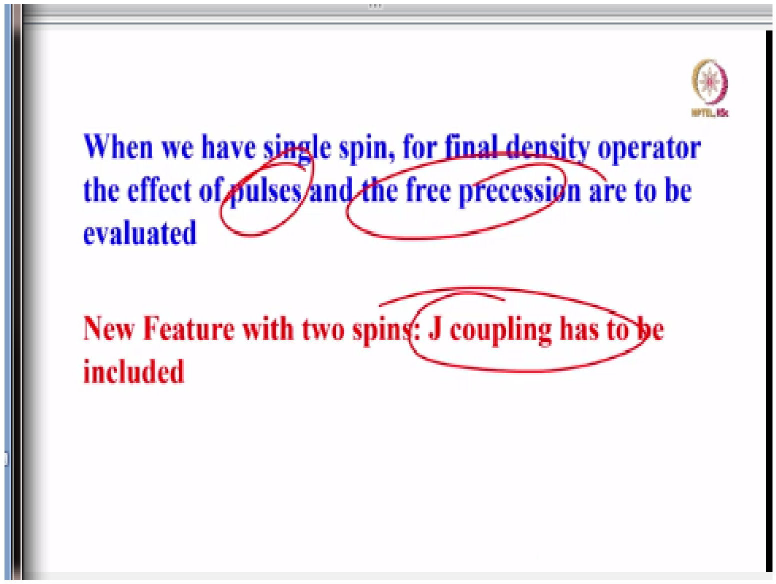

And when we have a single spin, for final density operator we have to consider the effect of pulses and free precession. But when we have 2 spins the new feature is, we need to consider J coupling also, that has to be included.

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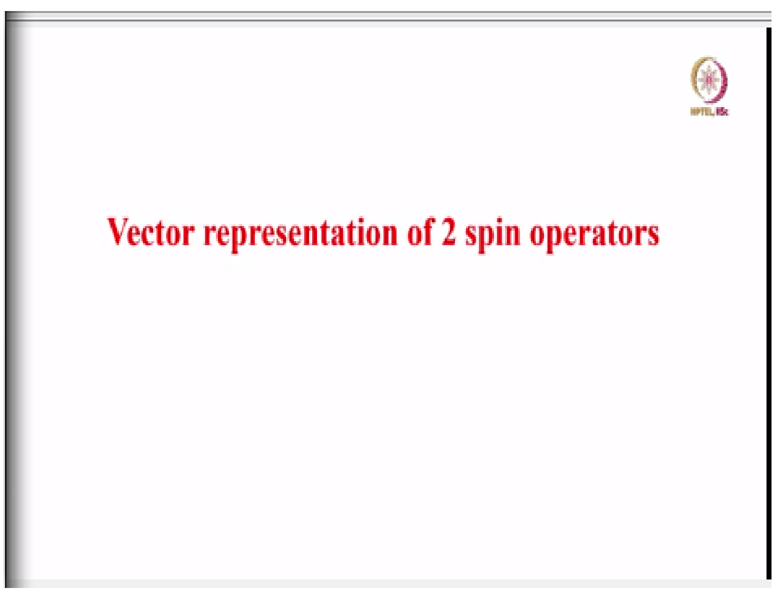

So, now vector representation of all these 2 spin operators can be made, this is what we said; I 1Z, I 2Z what are the I 1X, I 2X etcetera. Graphically also we can see how are they. So, we can represent vectorially these 2 spin operators.

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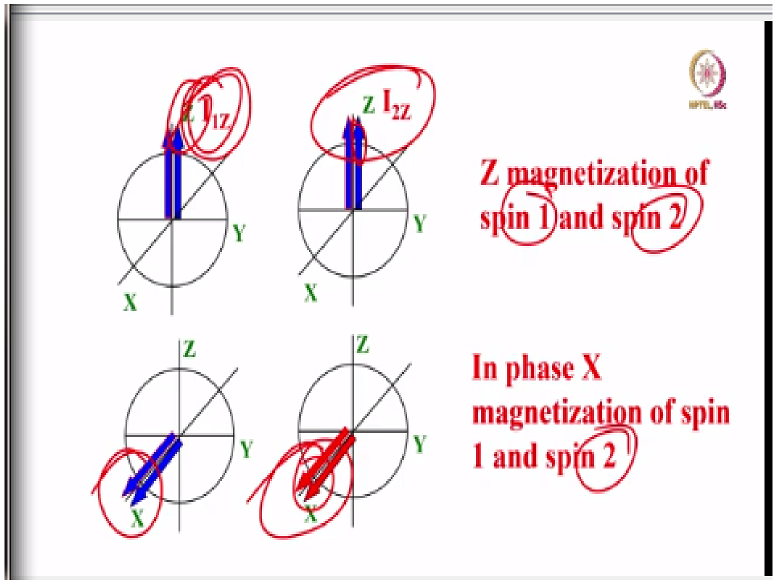

For example if I write like this, this is I 1Z, the 2 J split multiplets are along Z axis, it is I 1Z, the Z magnetization of spin 1; This is called Z magnetization of spin 2 both along Z axis. Here in phase X magnetization of spin 1, let us say, both along X; this is in phase X magnetization of spin 2. I have given you different colours. Of course this should have been red, I have given red colour for spin two. Of course does not matter the colour code is only to make you understand clearly, this is for spin 2.

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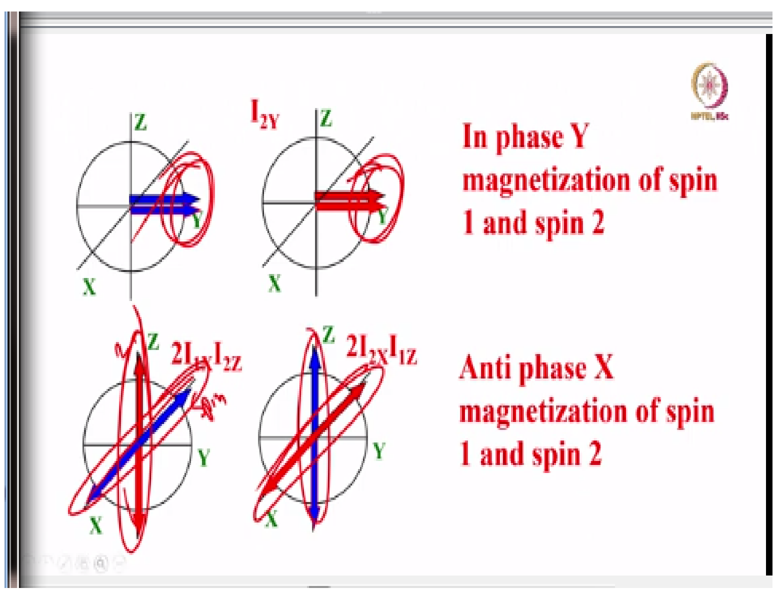

Now this is I 2Y, this is I 1Y and this is I 2Y the in phase Y magnetization of spin 1, this is in phase Y magnetization of spin 2. And this is important; X magnetization and Z magnetization here 2I 1X and I 2Z, this is for spin 2, this is for spin 1. This is anti phase X magnetization of spin 1, remember spin 1 has X magnetization in the antiphase and this is Z magnetization of spin 2. Similarly for spin 2, X magnetization is antiphase like this and for spin 1, Z magnetization is antiphase along Z axis.

Remember, one thing you should clearly understand these antiphase terms are very, very important, especially for transferring the spin polarization in experiments involving polarization transfer. Please understand, 2I 1X I 2Z means it is antiphase X magnetization, see X components are antiphase along X axis and for spin 2 it is along Z axis. Spin 1 along Z axis, spin 2 along Z axis both are antiphase. For the spin 2, X components are anti phase and for spin 1, Z components is antiphase like this.

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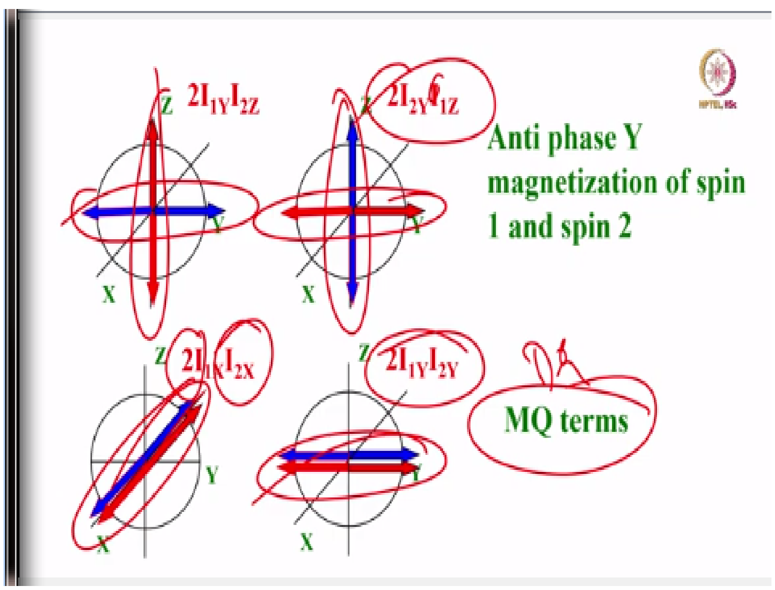

Similarly anti phase Y magnetization of spin 1 and 2. Now spin 1, both the vectors are anti phase along Y, The spin two is anti phase along Z. Similarly I 2Y and I 1Z it is for second spin, Y magnetization is anti phase, for spin 1 Z magnetization is anti phase. And here 2I 1X and 2I 2X both X components of both spin 1 and spin 2 are anti phase.

Please understand, X components of both spin 1 and spin 2 are anti phase, that is 2I 1X I 2X. Similarly 2I 1Y I 2Y for spin 1 and spin 2, both the Y components are anti phase like this. These are called multiple quantum terms or in our case it is a DQ term, double quantum term, because both these spins are either along Y axis or X axis anti phase with each other.

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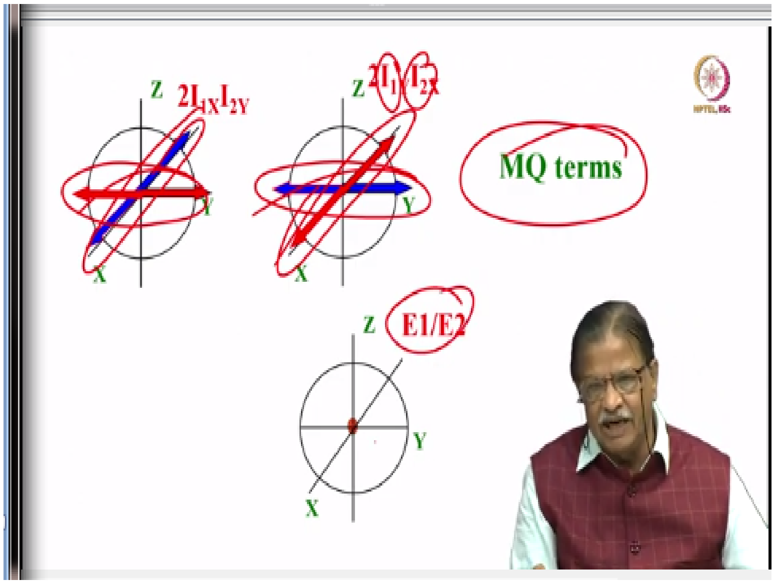

So, now continuing further I can write 2I 1X and I 2Y, that means spin 1 both X vectors are opposite, anti phase. whereas spin 2 they are in Y axis anti phase. Similarly 2I 1Y I 2X spin 1 is anti phase along Y axis, spin 2 is anti phase along X axis, all these are called MQ terms. E 1 and E 2 is nothing but just a vector representation for mathematical operation, this is diagrammatically represented like this.

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So, the free precession Hamiltonian for 2 couple spins we have to consider now.

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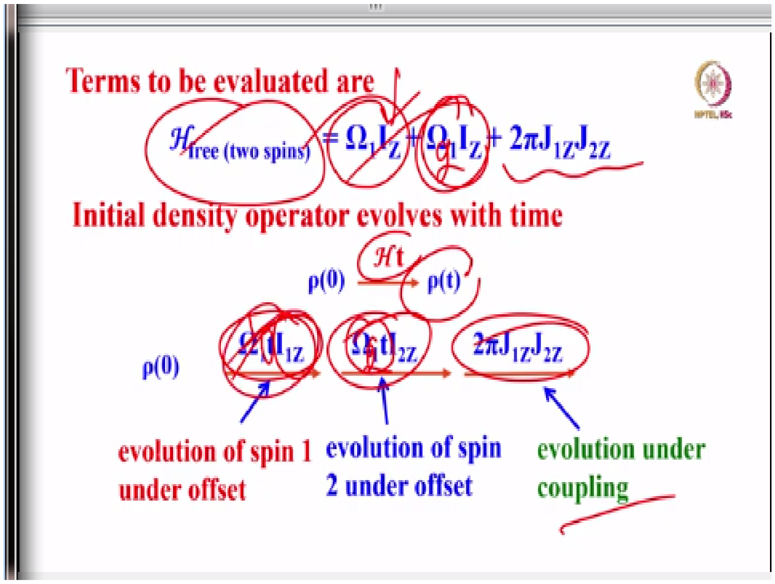

Terms to be evaluated are free precession of 2 spins. Now when I consider first free precession of spin 1 that is rotation about Z axis omega 1 IZ. And then this should be omega 2 IZ, omega 2 IZ that is our spin 2 and then J coupling 2 pi J 1Z J 2Z. These are the terms which you need to evaluate. So, initial density operator now evolves with time, initial density operator rho 0 when it evolves with time finally what we have to evaluate is rho of t.

Now rho of 0 first evolution of spin 1 under offset omega 1 into t I 1Z, omega 1 is the offset with time t, how it is evolving we know. And this is for the spin 1, that is first one evolution of spin 1 under offset. Now evolution of spin 2 under offset is again should be omega 2 t into I 2Z. This evolution has spin 2 under offset; it need not be same for both omega 1 and omega 2, they could be different. And also third term is evolution of J coupling, J I1Z J2Z, so this is the evolution of the coupling term, this is the evolution of offset under spin 2, evolution of offset under spin 1.

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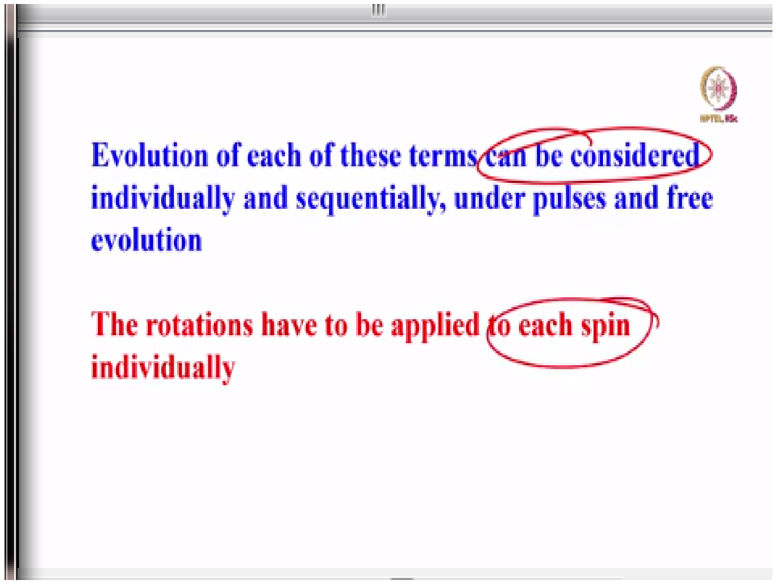

As I said evolution of each of these terms can be considered individually and sequentially under pulses and free precession, both. Applying pulse individually you can take evolution of spin 1 and spin 2. There is no correlation between 2. When you are considering spin 1, spin 2 can be ignored, when you are considering spin 2, spin 1 you do not have to worry. Similarly when evaluating evolution a chemical shift you do not have to worry about J coupling. When you are evaluating J coupling evolution you do not have to worry about chemical shifts. So, all these rotations have to applied to each spin individually.

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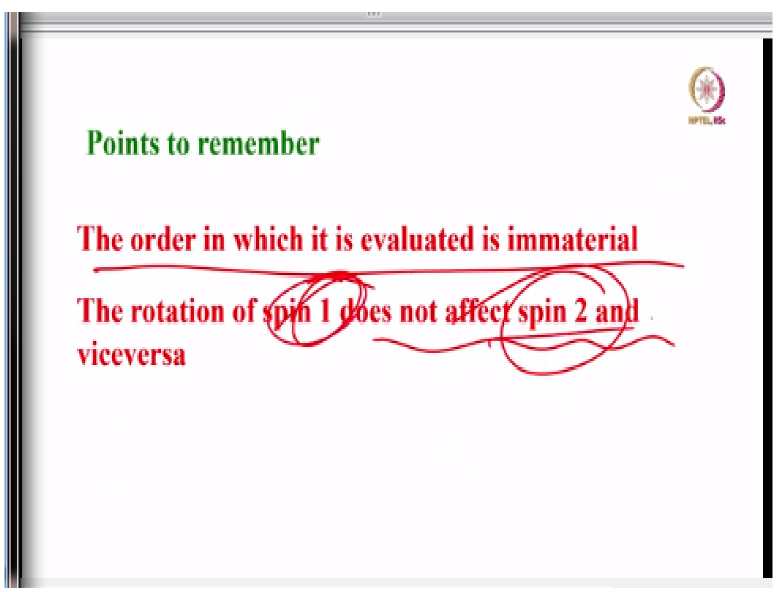

And the order in which it is evaluated is immaterial that is not very important. And when you consider the rotation as spin 1, it does not affect the rotation of spin 2. Similarly when you are considering rotation of spin 2 does not affect spin 1. So, both are important points in order to evaluate whether chemical shift first or J coupling first, free precession, all those things does not matter. Individually spin 1, spin 2, chemical shift, and J coupling anything you can do in any order. And another important thing rotation of spin 1 does not affect the rotation of spin 2 and vice versa.

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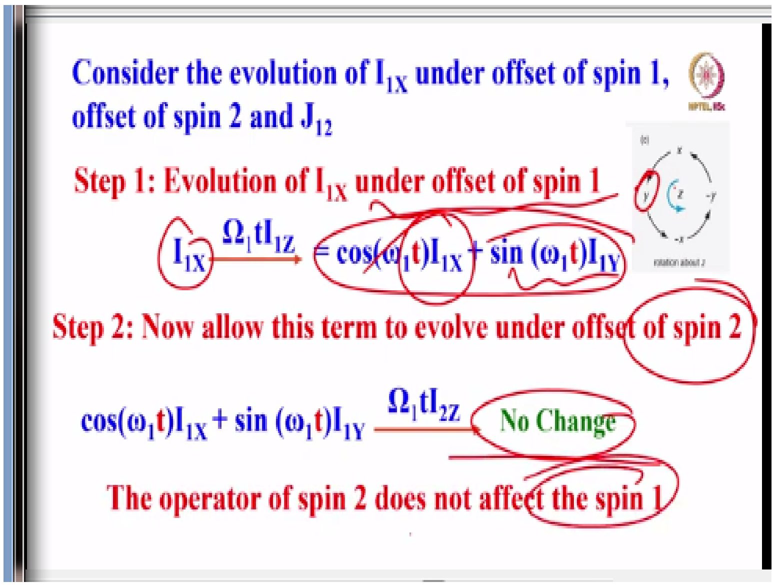

Now let us consider the evolution of I 1X. what is I 1X? Under offset of spin 1 and offset of spin 2, and J 12. What is I 1X? It is in phase X magnetization of spin 1, I told you, in phase X magnetization, diagrammatically I showed you. I 1X is for first spin, spin 1 in phase X magnetization. Now let us see how does it evolve under offset. So, now what is that evolving rotation is about Z axis, evolution of I 1X rotation about Z axis. So, I 1X when you rotate about Z axis, it goes to Y.

So, cosine of the old operator it becomes cosine omega 1 t I 1X + sine omega 1 t I 1Y, this is I 1X I am considering. Now allow this term to evolve under the offset of spin 2, now these 2 both have to be evaluated under offset of spin 2. Now what is going to happen? Absolutely no change you will see, the reason is as I told you, when you are considering rotation of spin 1, it has no effect on spin 2, and vice versa. So, now you are considering rotation of spin 1 and it is effect and spin 2 is not there at all, it is ignored. So, that is why no change is there, you can ignore, it does not affect spin 1.

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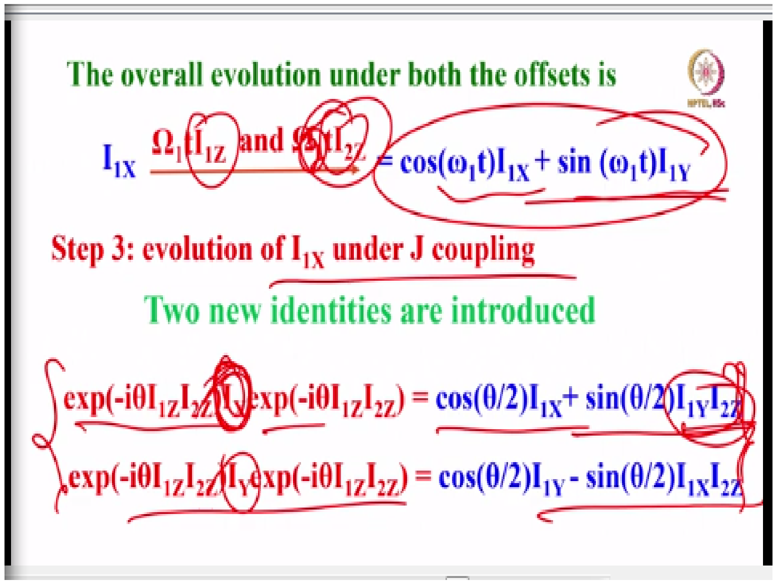

Now overall evolution under both the offsets is given as omega 1 t I 1Z and omega 2 t I 2Z should be equal to this, t I 2Z. Now it should be like this sine omega 1 tI 1X + sine omega 1 t I 1Y because this has no effect. Step 3: evolution of I 1X under J coupling we have to see, there are now 2 new identities introduced in phase term, we have considered evolution of I 1 X under omega 1 t I 1Z omega 2 t I 2Z that we have considered. Now evolution of the same term under J coupling, so that means these 2 terms should be considered under J coupling evolution.

So, in this case the 2 new identities are introduced like it was introduced for the single spin case. Remember, 3 drawings were given finally to summarize this thing. Exactly these 2 terms, new identities are introduced, this is cosine of -I theta into I 1Z I 2Z into I X is into this one is given as cosine of theta by 2 into I 1X + sine theta by 2 into I 1Y I 2Z. Similarly, the rotation about Y axis this is what it is, so this is for the rotation of I X and this is rotation of I Y, these are the things we have to consider.

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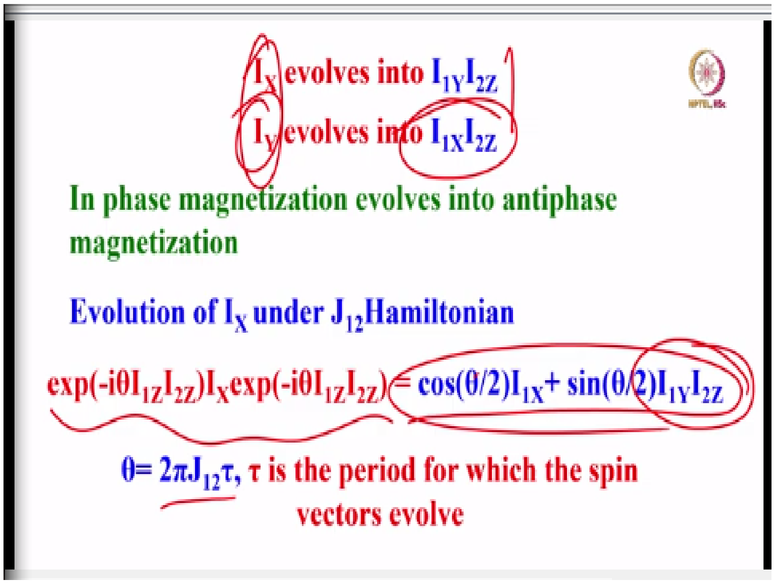

Now I X what do you understand from this? When I X is evolving under J coupling, remember it evolves into this one. I X when it evolves the in phase term evolves into I 1Y I 2Z, what is I 1Y I 2Z? I told you it is an anti phase term. I X evolves into I 1Y I 2Z, similarly I Y evolves into I 1X I 2Z; both are anti phase terms. What it means under J coupling? In phase terms evolves into anti phase terms.

So, in phase magnetization became anti phase magnetization because of J coupling evolution. So, under J 12 Hamiltonian if you write how the evolution of I X takes place, this is the formula I X is being rotated under J 12 in which case this is the identity we have to use. Now I X component which is in phase turned out to be antiphase. Now exactly at theta = 2 pi J period for which spin vector tau is the period for which spin vector evolves, we can work it out, what happens if you put this value in this theta and calculate cosine and sine terms.

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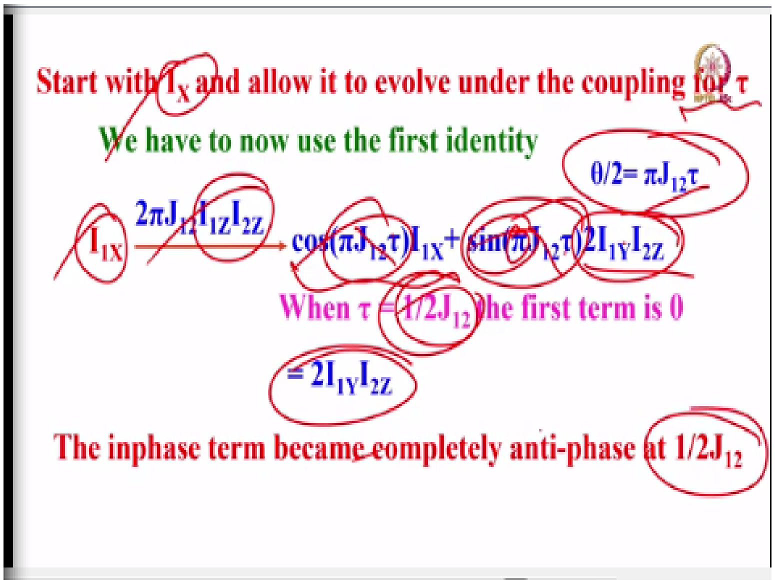

Now we will start with IX and allow it to evolve under coupling for tau. I started with IX and see IX means it is the first one you have to consider, first identity you have to consider because that is the rotation of IX. So, now we have first term, we have to consider I am worried about IX which is rotated under J coupling. So, now with that if I consider, go to the next one, here.

Consider an I X, allow it to evolve under coupling for a period tau, we are using the first identity, this is a first identity. And now theta = pi J into tau J 12 is between 1 and 2 and that is J, theta by 2 = pi J into tau. So, now I am evolving I 1X in phase magnetization of spin 1, in phase X magnetization of spin 1 under J coupling, you get cosine of I 1 X of this term into sine of this J term into anti phase term, this is what you are going to generate.

Now put tau = 1 over 2J here, what happens 1 over 2J? This gets cancelled out J, it will become pi by 2, what is cosine of pi by 2? 0, so we are going to left with only again here tau = 1 by 2J if you put this J will cancel out pi by 2 you get, sine pi by 2 = 1. So, you are going to left with only this term. Exactly when tau, the delay for which you allow this coupling to evolve is 1 over 2J then in phase term became anti phase term, very important point. The in phase term becomes completely anti phase at exactly tau = 1 over 2J.

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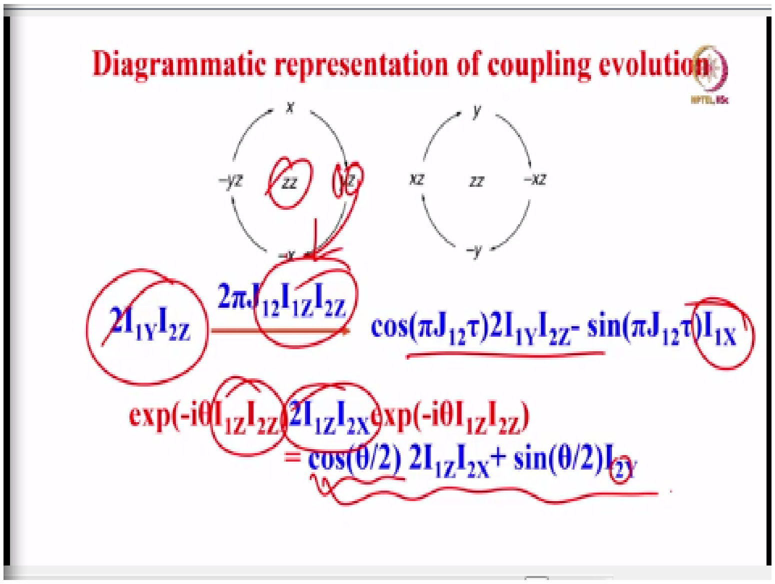

And this is diagrammatically you can represent the coupling evolution, it is like this. For example I 1Y I 2Z if it is evolving like this I 1Y I 2Z if you consider, now Y and Z if you consider, Y is the first spin, Z is the second spin. Now I am considering Y and Z which is rotating like this, in this direction. Now when it evolves under J coupling this is going to be -I X, you put it -I 1X and now this is a anti phase term.

So, exponential of this term if you put it, if you work it out using diagram it turns out to be now what I am doing? This 2I 1Z I 2X is evolving under I 1Z and I 2Z, the solution is cosine of theta by 2, 2 of I 1Z I 2X + sine theta by 2 into I 2Y, very simple.

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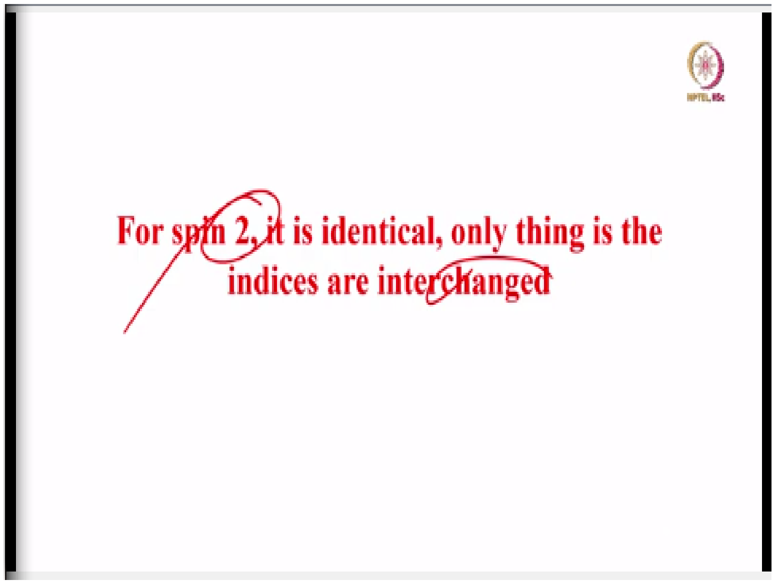

So, what we worked out for this spin 1? This is for spin 1 we worked out. We took this spin 1 evolution under offset and then under J coupling. Now what about spin 2? I said both can be individually taken, individually you can calculate, rotation of spin 1 does not affect spin 2, and vice versa I said. Now if what happens if I calculate for spin 2? Identical term you get, only thing is indices which you saw here get interchanged; 1 become 2, 2 become 1, that is all. Here 2 become 1, that is what is going to happen. So, indices get interchange when you work out for spin 2.

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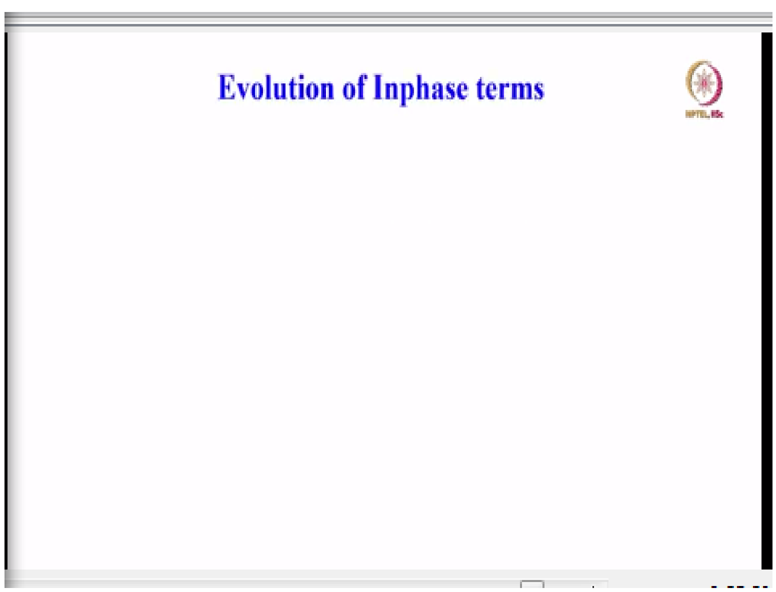

So, now we can work out how does in phase term evolves after this. So, now since the time is getting over what I am going to do is I will stop here. We will come back and continue the evolution of in phase term and then work out how product operators can be applied for heteronuclear case in the case of 2 spins when the J coupling is present. So, right now we have understood quite a bit about product operator, especially when J coupling is taken into account.

We got product operators, diagrammatically represented, we understood I 1X I 2X, I 1Y I 2Y all those things are called in phase magnetization of X and Y for spin 1 and 2. Similarly we understood what is the anti phase magnetization I 1X I 2Z, I 1Y I 2Z, I 2X I 1Z, I 2Y I 1Z all those things; they are anti phase term. Similarly I 1X I 2X, I 1Y I 2Y I 2X I 1Y, all those terms are called multiple quantum terms.

So, altogether there are 16 possible combinations we saw, we could see their product operators when there are 2 spins coupled. With the 2 spins, evolution we have to consider under J coupling also. So, now under J coupling all the 16 operators will be there and how it evolves we understood by taking a simple example of two spins which are coupled; how they evolve under the offset and J coupling.

And then we understood when you consider the evolution of the J coupling the in phase term turned out to be anti phase term, very interesting. When we took I 1X term evolution of I 1X term, the in phase term of spin 1 and during J coupling we found it turns out to be anti phase term. So, same thing happens to I 2Y term also, if you consider for spin 2 also. So, identical things you are going to see.

So, now we are going to understand evolution of in phase terms, so far we understood how in phase term evolved everything, how it become anti phase terms? We will continue further and understand few more concepts that are involved in this. So, I will stop here, I will come back and continue in the next class, thank you.