Advanced NMR Techniques in Solution and Solid-State

Prof. N. Suryaprakash
Professor and Chairman (Retd)
NMR Research Centre
Indian Institute of Science – Bengaluru

Module -13
Fourier Theorems
Lecture - 13

Welcome all of you. In the last class, we discussed about the complex form of the Fourier series. And then we found out how to obtain the Fourier coefficients in the complex form, which are defined as C0, Cn and C-n, these are the coefficients Cn, C-n and C0. We also knew how to get these values for C0 for example, simply you have to integrate the function from -pi to +pi f of x dx and we obtain C0.

And then for Cn and C-n, we have to multiply by a certain exponential function, and then in one case positive exponential in the other case negative exponential, and then we can get coefficient Cn and C-n that was well known. And we also understood how to solve the square wave using the complex form and we also obtained Fourier series, I am sorry Fourier coefficients. Then, we came to a function called square function which is non periodic, only a single square function and it does not being non periodic does not have a Fourier series. Then what we wanted to see is, we just periodized this function at regular intervals 2, 4, 16 like that, and calculated the Fourier coefficients and then plotted the Fourier coefficients and saw as the period increases, the frequencies become closer and closer. And then, by doing the simple mathematical operation, we have showed that this turns out to be this rectangular function, turns out to be a sort of sin x / x function, it is called a sinc function, which goes like that.

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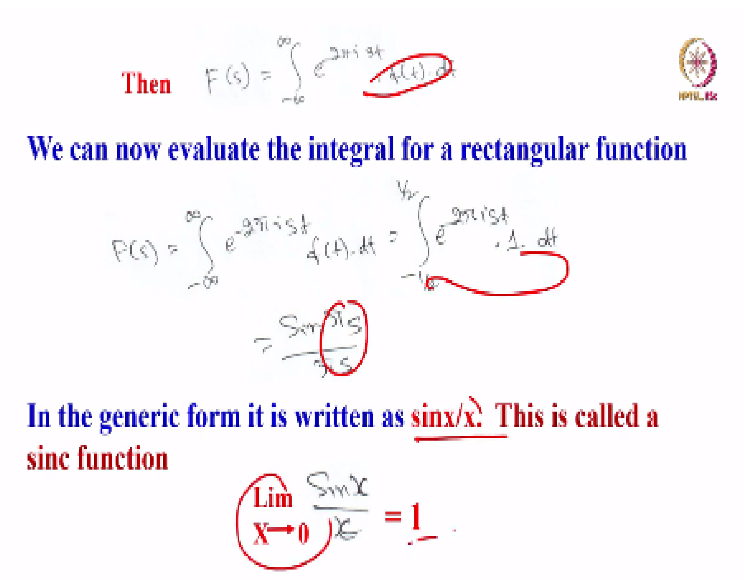

For example, this F(s), this is a function which is F(s). These F(s), for example and then we can evaluate this integral and then when you evaluated this integral we saw and it turns out to be sin of pi s over pi s and the generic form it was called as sinx /x; as the limit tends to x = 0 it is 1 and this function is called sinc function.

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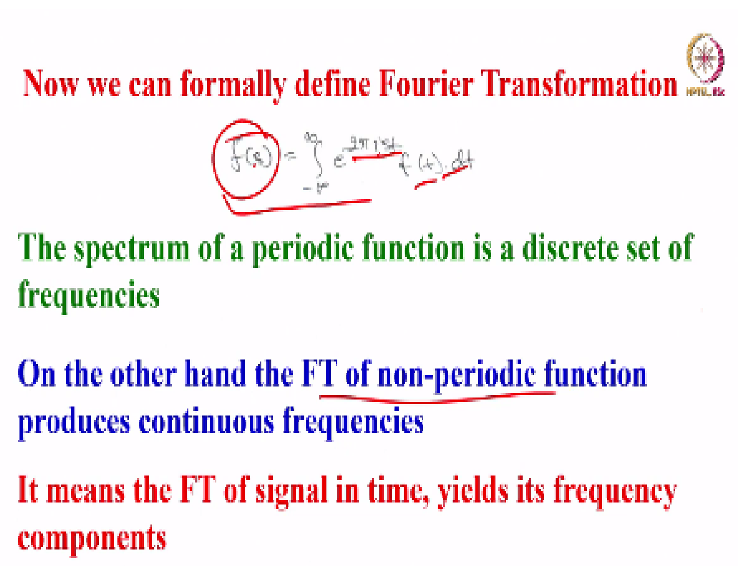

So, we formally defined the Fourier series transformation like this. A time domain function can can be expressed as a frequency like this. This is a time domain function and then you do the integration from minus infinity to plus infinity, e to the power - 2 pi ist f(t) dt, you are going to get f(s). This is a spectrum of a periodic function which is a discrete set of frequencies. And I take a Fourier transformation of non periodic function, it produces the continuous frequencies for us.

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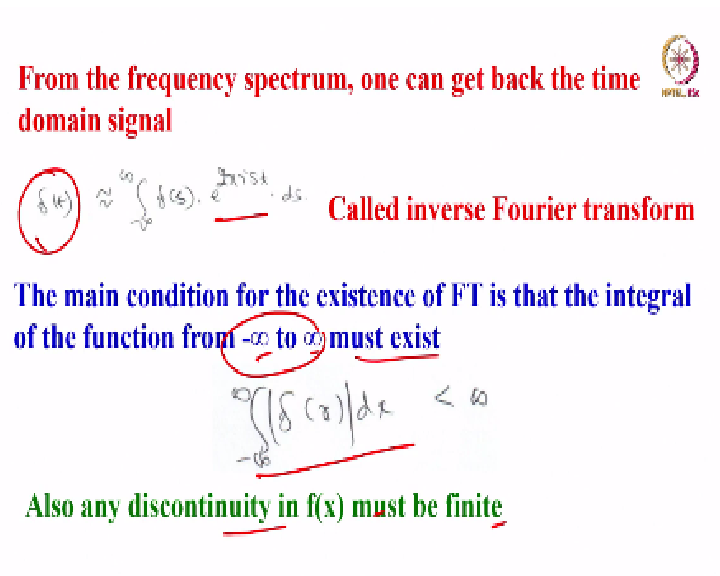

It means what I was trying to tell you was, you can take the time domain a signal, do the Fourier transformation and you get the frequency components present in that. That is what Fourier transformation does. That is what I wanted to show you and I gave you in NMR that is what we are going to do. In NMR spectroscopy, we apply a radio frequency pulse and collect the signal as a function of time; will collect time domain signal called free induction decay. We do the Fourier transformation of it and you get the frequency spectrum.

And now, you may ask me a question I have a frequency spectrum; can I get the time domain part of it? Of course, it is possible, we can do that also, if I have a frequency like this, I can get back the time domain function, this is called inverse Fourier transformation. What it means is, please remember time and frequency are called Fourier pairs, if I have a time domain function, I can do the Fourier transformation and go to frequency domain; if I have a frequency function, I can do the Fourier transformation and go back to time function.

What it, how it behaves as a function of time? The only condition is if I have to do the Fourier transformation of a particular function, it is an integral function what we need to get there must be an integral value for this function from -∞ to +∞; integral value must exist. If it does not exist, it is not possible, at the same time and this is the integral value and then if there is any discontinuity in f of x. Thus it must be finite. These are the 2 things which we need to understand. Remember this thing. So, if any integral value for the function from minus infinity to plus infinity must exist for the condition for the Fourier transformation.

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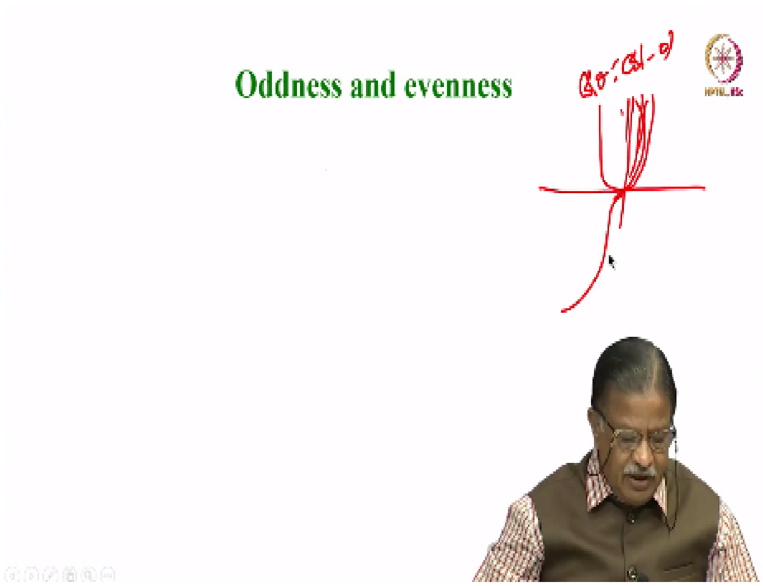

Now, I will introduce a term called oddness and evenness of the function; which is basically all of you know, what is the odd function and even function. If I take the cosine of theta, it is equal to cosine of minus theta; and we know that even function is one which is symmetric with respect to the origin, odd function I am sorry with respect to the ordinate; I am sorry odd function is one which is symmetric with respect to the origin, it is very well known.

For example, if I take a function like this, it is symmetric with respect to origin, it is an odd function. Whereas, if I take a parabola like this, it is an even function, it is all well known. So, now, similarly, I can introduce oddness and evenness for this also here. So, what we are going to do with this thing?

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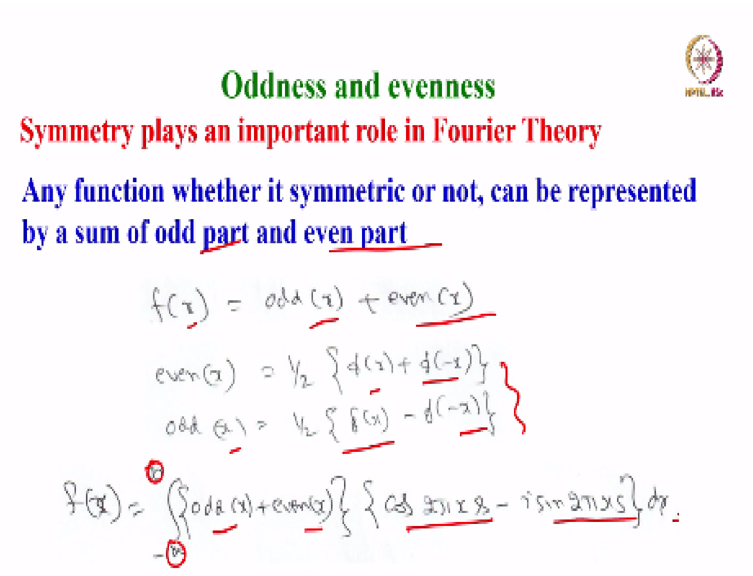

This is because symmetry plays an important role in Fourier theory, any function whether it is symmetric or not, it may not be symmetric, but we can represent them by the sum of odd part and even part; any function you take, does not matter it need not be symmetric. We took the example of rectangular function we solved it; it was a symmetric function from the origin we took. The origin was taken to be the centre of the rectangular function, it need not be symmetric it can be anything, but, we can represent them as a sum of odd part and even part, both we can do.

So, fx can be written as sum of odd x + even x both can be done. So, even x you can write like this, even of x = half of fx + f of –x whereas, odd of x = half of f of x minus of f of -x. This is a simple way to express an even function; the odd function which you all know in basic arithmetic I do not need to explain this very often. Now, I can express this fx as a function which I can integrate from minus infinity to plus infinity which was discussed, I can call it as odd of x + even of x into cosine of 2 pi xs - i sin pi xs into dx I can write like this.

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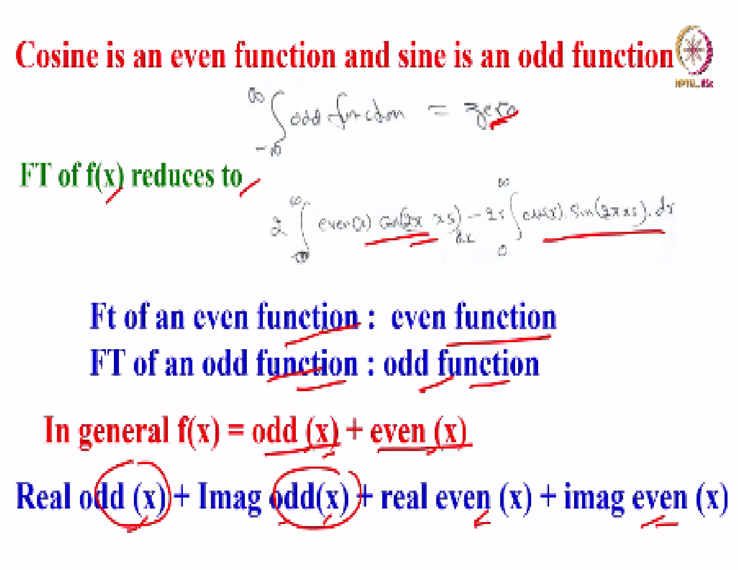

So, cosine is an even function and sin is an odd function. Now, what will happen if I take odd function integrate from minus infinity to plus infinity? you all know in basic integration this value is 0, if I take the Fourier transformation of fx, which reduces to 2 times 0 to infinity even of x into cos of 2 pi xs minus of this function 2i, 0 to infinity odd of x into sin 2 pi xs dx; because odd function integral minus infinity to plus infinity is 0. So, what you understand from this is the Fourier transformation of an even function is an even function. The Fourier transformation of odd function is an odd function. This is important condition you should know that; this we use it very often when you understand the NMR spectrum, we discuss real part, imaginary part even function, odd function, everything. So, please remember Fourier transformation is invariably used in many of our NMR spectroscopy techniques and Fourier transformation of an even function is an even function, for Fourier transformation of odd function is an odd function.

So, in general if I have a function fx, I can write it as odd of x and plus even of x; can write like this. So, when I do the Fourier transformation you get both real and imaginary. You can get and this function I can write it as odd x, real odd, imaginary odd, real even and imaginary even; both I can write like this, because when I do the Fourier transformation, I get both real part and imaginary part. So, if I write the general function f of x as odd x + even x. If I try to do the Fourier transformation, I can write it as real odd x, imaginary odd x, real even x, imaginary even

x. Of course, we can also express in a simple way like this. Now, I will introduce some important Fourier transformation theorems. These are all very, very important to understand.

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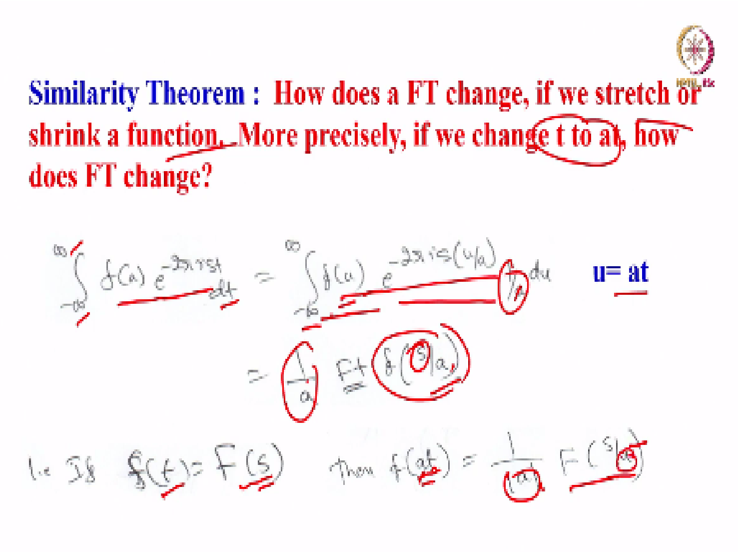

One is the Similarity Theorem. How does a Fourier transformation change if we stretch or shrink a function? What it means is I divide this function by some number or multiply by another number to expand it, let us see more precisely what will happen to the function t. If I change t to at, a could be some number it could be multiplied, or it can be divided; then how does the Fourier transformation change?

It is a question which you have to answer; understand my question clearly, I have a function I want to stretch or shrink a function. That means, I will multiply by the number or decrease by some number. Let us see how the Fourier transformation change, simple thing Fourier transformation is minus infinity to plus infinity, f of a into e to the power of - 2 pi st dt, this can be written as minus infinity to plus infinity, f of u into this number. What I have done here is I have assumed u = at, I have substituted u as at and then rearranged this function; that is all I have done, nothing extra here. There is nothing new I have done here. And this can be written as the integral form is 1 over a; I will take it out, 1 over a. This is nothing but the Fourier transformation of s/a, u/a, I can call it as a frequency; this is a time domain function, I can do the frequency I write it as s/a; very simple.

So, this was a time domain function, I put it as s/a, because I already told you Fourier transformation of time domain gives you a frequency spectrum. So, I represented s as the signal frequency, so, this is the FT form of it; this one; this is FT of it. So, I will have taken out 1/a outside integral; this entire integral now turns out to be Fourier transformation of this one. So, that means, the Fourier transformation of the time domain function is F(s).

The Fourier transformation of the function multiplied by a = 1 over modulus of a function of s / a with frequency; this is the Fourier transformation. So, you take the function f(t); do the Fourier transformation you get f(s), it is the frequency. Now, multiply t, this function by a; call it as f(at); then do the Fourier transformation, you are going to get 1 over modulus of a in to Fourier transformation of s/a, so that is what it is going to happen. This is what is the similarity theorem, whether you stretch it or compress it, how it behaves after the Fourier transformation, you can understand from this, if I multiply by this function, the frequency it is just dividing like this.

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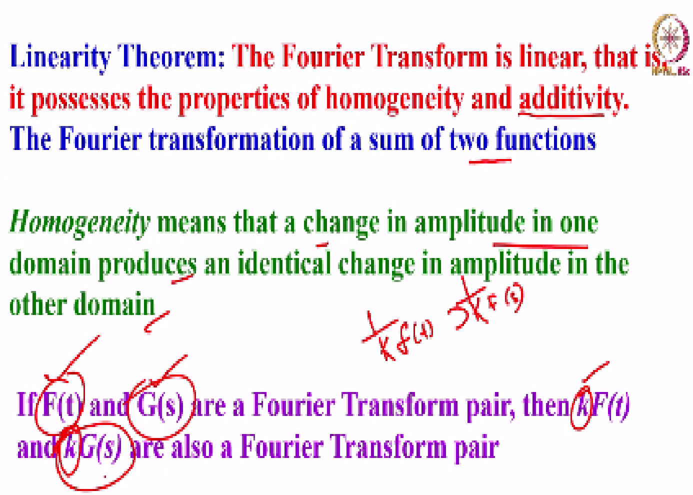

We have another theorem called Linearity Theorem. The Fourier transformation is the linear function, it is linear. That means, it possesses the properties of homogeneity and also additivity; both are present. The Fourier transformation of sum of any 2 functions if you take, what will happen? Homogeneity means that a change in the amplitude in one domain produces an identical change of amplitude in the other domain. That means, in the time domain, I change the amplitude of this function, I do the Fourier transformation. In the frequency domain also, it remains same. For example, take your function, f of t, I do the Fourier transformation of it, I will get there f of s. Now, what I am going to do? I am going to change the amplitude of this function by some number, and then do the Fourier transformation; you are going to get this. So, what we are going to do is change the amplitude of this; in the frequency domain also the amplitude correspondingly gets changed.

This is what you must remember, in the homogeneity theorem. Now, if the 2 functions F of t and Gs, F of t is the time domain function; G of s is the frequency domain function, we call them Fourier transform pairs, I do the Fourier transformation of this, I am going to get this one, these are Fourier pairs. Now, I multiply this by a function k and do the Fourier transformation, what you are going to get? the frequency function is also multiplied by k. This also forms a Fourier pair.

For example, I told you here, k of Ft you will take; it become k of Fs, If you take 1 over k of Ft it will become 1 over k of Fs. So, if I take 2 functions, F of t and G of s, which are Fourier pairs, now I alter the function, the first function, I am going to stretch it or compress it by multiplying with some other number, some constant k let us say, or some function, then the Fourier transformation of that in the frequency domain is also multiple by the same number. This is what is the linearity theorem, which I wanted to tell you, I hope it is clear.

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Additivity of the Fourier transform means that addition in one domain corresponds to addition in the other domain

$$F(f+g)(s) = Ff(s) + Fg(s)$$

Shift Theorem

A shift of a variable or a delay in time has a simple effect on FT. The frequency does not change

However, there will be a change in the phase

Now, additivity of the Fourier transformation means, addition in one time domain correspond to addition in the other domain. It is a very simple theorem here, if I take 2 functions, let us say function of f and function of g; these are 2 functions. I take the sum of these 2 functions, which I call as function of f + g, 2 functions I do the Fourier transformation of this and I am going to get in the frequency domain Fourier transformation of this function f. And Fourier transformation of the function g, I am sorry they cannot be s, it has to be t Fourier transformation of this. If I take this function f + g as a function of time; and do the Fourier transformation individually, you are going to get the frequency components. This is exactly what we are going to discuss; as we go ahead further in the NMR spectrum, you are going to take a time domain signal, there are n number of frequencies present in that. You are going to collect an interferogram and then each of this is an exponential function. So, many frequencies are present; they are overlapped. Now, you do collectively the Fourier transformation of all of them; and you are going to get the frequency components of each of them present there. So, this is nothing but the additivity theorem of Fourier transformation, which is telling you in time domain, I have added all the frequencies. In the frequency domain I get the same number of frequencies added up. That is if the different frequencies are there, after Fourier transformation, we get that many number of frequencies. So, this is additivity theorem

There is also what is called a shift theorem. A shift of a variable or a delay in a time has a simple effect on the Fourier transformation, I take a function and then I shift it by a certain value.

Remember, when we were trying to understand the Fourier series, I told you what happens if I shifted this wave square function by pi/2, we also got to know, everything shifted by pi/2, the sin part became cosine, because it shifted by pi/2. If you remember, what we discussed in the last class, I shifted the variable or a delay in time, it has simple effect on the Fourier transformation. Only thing is the frequency does not change. But however, there is a change in the phase; you saw that, you know the sin function became cosine, because it changed by pi/2. There was a change in the phase but the function remained same, the amplitude did not change, only frequency changed, I am sorry phase change; frequency did not change only phase change. So, this is a shift theorem in Fourier transformation.

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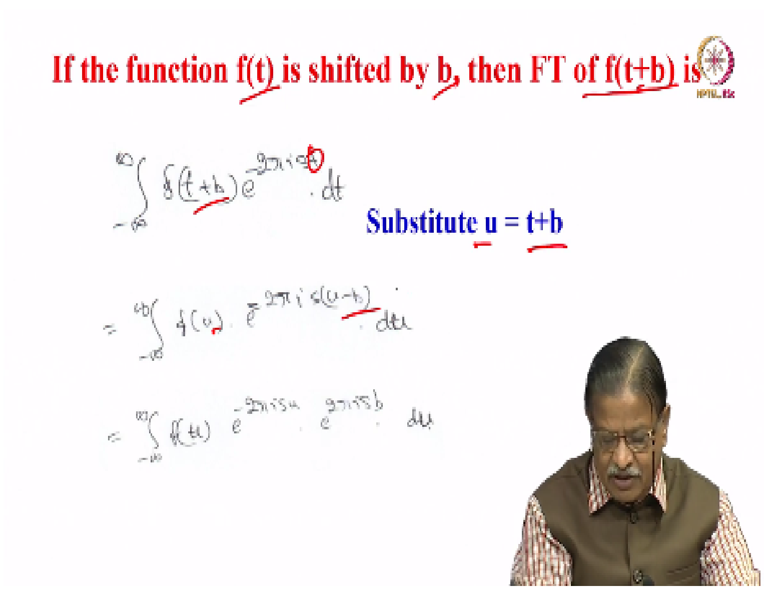

So, if the function of ft is shifted by b, then Fourier transformation f + b, if I am sorry, t + b if you work it out, you will substitute u = t + b here, then it becomes f of u and t you can write as u - b here, and then do the integration and separate out this function here.

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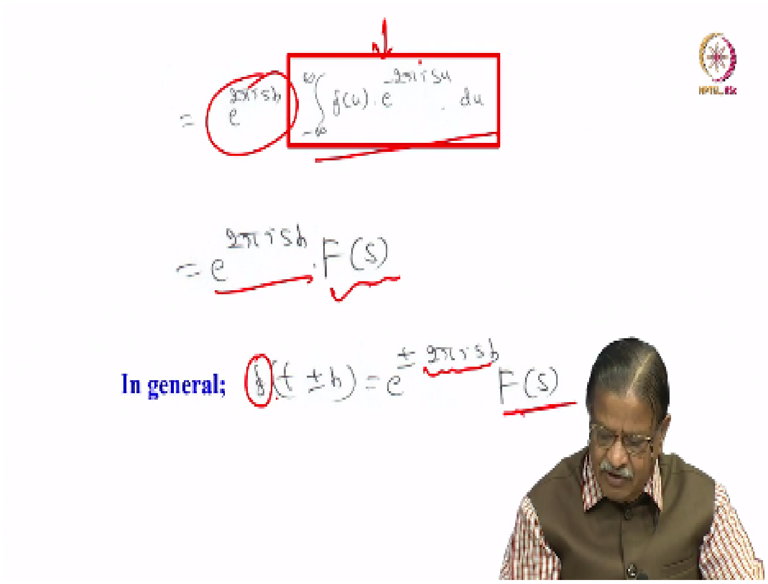

And then what you are going to get is; you are going to get this, you can take out e to the power of 2 pi isb and this is what you are going to get. This is again in Fourier transformation of this u. So, this is the Fourier transformation of the function u; I call it as F of s, a frequency. So, what did we do? Here you see that when substituted this function by t multiply by some function, and then you get this one; e to the power of 2 pi isb into Fourier transformation of the function or this function, which is frequency.

So, in general, I can say, if I have a function f, which is t plus or minus b, I can write it as exponential to the power of plus or minus b, e to the power of 2 pi isb into Fourier transformation of this frequency spectrum. So, first you take the time domain and add or subtract another thing for this function, the Fourier transformation will be there, when it is going to be multiplied by the exponential function, either it is plus or minus depending upon whether you are adding or subtracting from this function. That is the simple theorem what I said about shift theorem in the Fourier transformation; please remember that.

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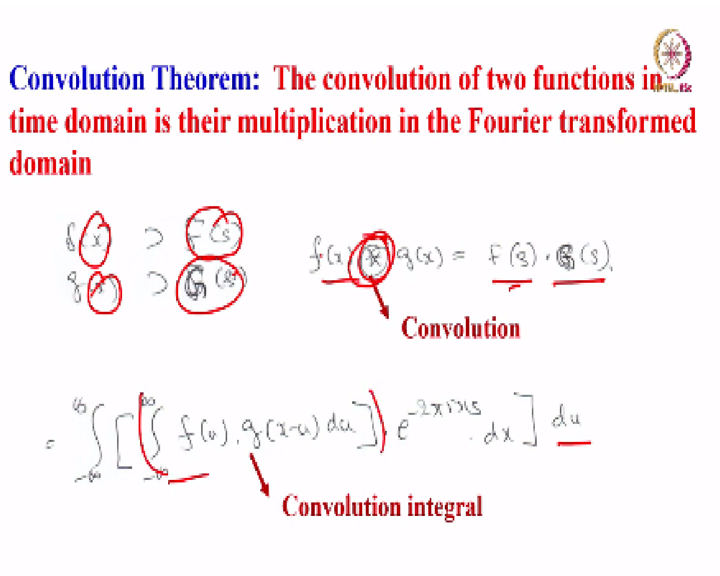

Now one another important theorem is there in Fourier transformation, which is called convolution theorem, it is a very important theorem. The convolution of 2 functions in time domain is nothing but the multiplication in the Fourier domain. This is the convolution theorem. it tells you the multiplication of 2 functions in time domain, I am sorry, convolution of a function in time domain is the multiplication in the frequency domain. So, what does it mean? what do you understand?

If I have a function f of x, when you do the Fourier transformation, you are going to get frequency F of s. Similarly, I have a function g of x; do the Fourier transformation of it, you are going to get G of s, that is the frequency part. Now, convolution means I will do this, this is a symbol for convolution. I will do f of x; convolute with g of x. What are you going to get? F of S into G of s. See, I convoluted these 2 functions. But I got multiplication of these 2 frequencies in the frequency domain; before Fourier transformation I convoluted these 2 functions.

And after the Fourier transformation, these 2 functions come as multiplied frequencies. This is a convolution. So, it is very simple; you can write like this, I will take the integral of f of u g of x - u du, and this is one function. Simple integral, it is different from the previous part, I have taken out and this is called as a convolution integral.

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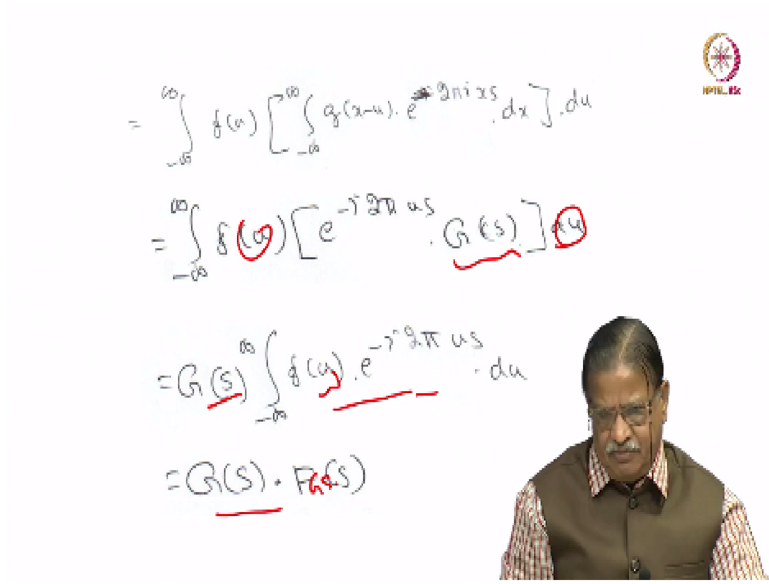

So, if I write like this, expand this function; and this convolution integral is the frequency part; I write as G of s. And this, if I take it out; Gs here, because I am integrating over u, and Gs has no role, I take it out. And I write it as G of s in to minus infinity to plus infinity for this function. This is the Fourier transformation of the function u. So, this is going to give you frequency. So, G of s multiplied by F of s. So, you convoluted the function here; this is a convolution function, and then do the simple integration. And afterwards, you take out; this is Fourier transformation of the one function and then this is the Fourier transform of other function, these 2 appear as multiplied here. So, the convolution of 2 functions in the time domain got multiplied, they got multiplied in the frequency domain. So, the convolution theorem tells you, when the 2 functions are convoluted in the time domain, in the frequency domain they get multiplied.

This is the important theorem in Fourier transformation called convolution theorem. You must remember this, because we keep on using these very often; without your knowledge when you take the NMR spectrum, all these theorems are used. That is why I wanted to discuss these theorems for you. So that you get the gist of what is going on here.

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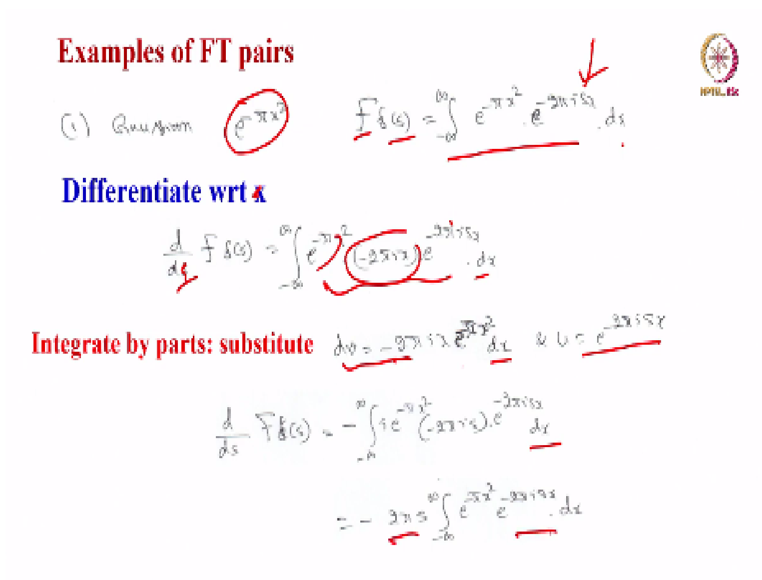

Now, with this knowledge of Fourier transformation, we can work out Fourier transformation on one or 2 simple functions. See, for example, I told you, I am going to take the free induction decay, an exponentially decaying signal; which is a time domain signal in NMR spectroscopy. I do the Fourier transformation, I get the sharp peaks, which are Lorentzian I call it, so it clearly tells me the Fourier transformation of exponential function gives me Lorentzian.

We can work out and see that. Take an exponential function, do the Fourier transformation you get a Lorentzian. See, take 1 or 2 examples of that for the Fourier pairs here. Now, we will start with a simple function, a Gaussian function. A Gaussian function is simply represented as e to the power of - pi x square, it is simple Gaussian. Now do the Fourier transformation of this; in the frequency domain it can be written like e to the power of - pi x square, e to the power of - 2 pi isx into dx. This is basic Fourier transformation integral. We want to do the Fourier transformation, so you need this integral. Now, I am going to differentiate this function with respect to x; I will do differentiation with respect to this one, not with respect to this. When I do that, see now I will do this one, this integral also I have to now do the differentiation, I will do this one, I will integrate by parts. What I will do for integration by parts; substitute dv as this function, e to the power of - 2pi ix, e to the power of - pi x square dx; this I call it as dv. And u as e to the power of -i pi sx; this say call it as u, this one. So, this one, this one into dx I call it as dv, this is called as u now. I am going to integrate by parts. So, I am doing differentiating this function from minus infinity to plus infinity, I am going to do it, I will integrate this function by parts, you know how to do the integration by parts, you have to simply substitute these things and do the simple integration.

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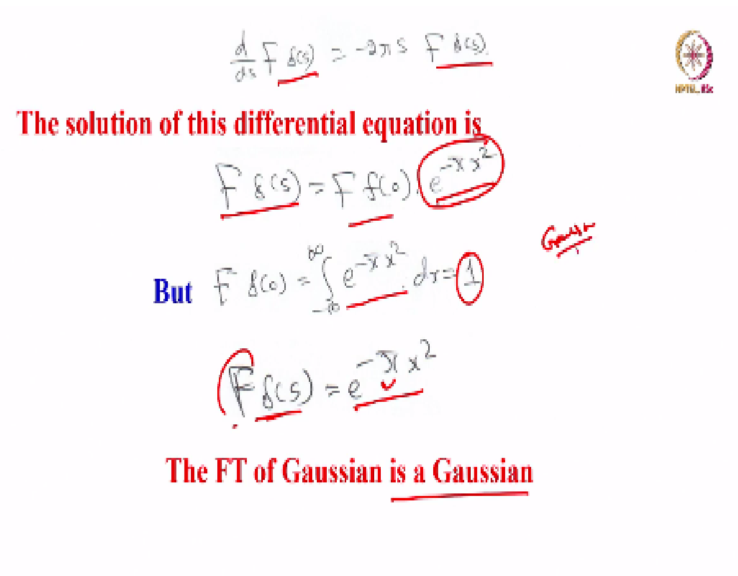

It turns out to be d/ds of function of f of s = - 2 pi s into f of s. So the solution of this differential equation is a very well known equation, this is nothing but e to the power - pi x square. So, I took the function f of some function, did the Fourier transformation which is nothing but this function; e to the power of - pi x square, it is a Gaussian function. I do the Fourier transformation, what I am going to get? the frequencies f of s. But it is e to the power of - pi x square is coming.

Now, we know what is F of f0, which is nothing but e to the power of - pi x square dx. This is the original function which I chose; which is nothing but 1. So, that means F of gs is nothing but e to the power of - pi x square. What did you understand from this? We started with the time domain function e to the power of - pi x square, which is a Gaussian. You do the Fourier transformation, in the frequency domain again its Fourier transformation is f of s, you again you get the same function; same equation you got; what does it tell you?

The Fourier Transformation of a Gaussian is a Gaussian, you understand; see the Fourier transformation of Gaussian, if you go and work out these integrals, I did not go into every step for you, step by step you can work out if you want; it is simple integration by parts you have to do; it is a elementary integration which you all know in PUC also, and you work out you will see that Fourier transformation of the Gaussian is a Gaussian.

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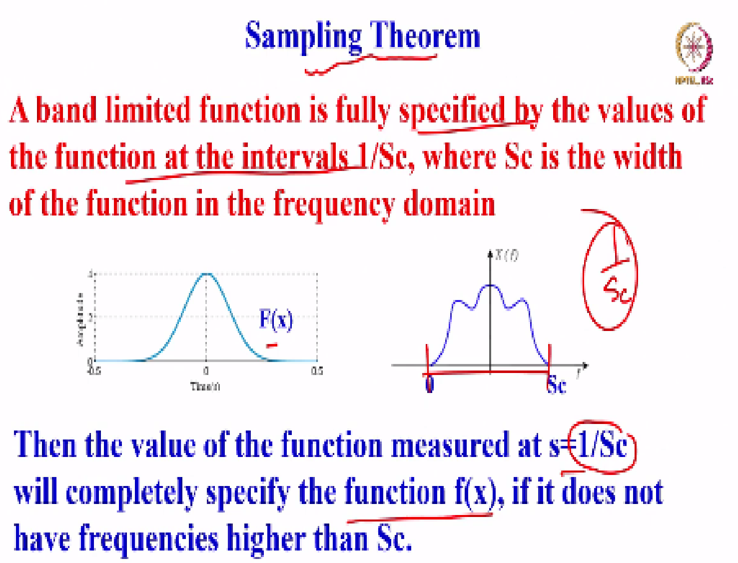

Like that you can work out several pairs take a comb function, do the integration will get a DC function. Take exponential, you get a Lorentzian like this. All these functions you can try to work out, and get a practice for what happens to these functions, when you do the Fourier transformation in the time domain; what do you get in the frequency domain? that is one thing.

There is another interesting theorem, which we generally adapt in the Fourier transformation by acquiring the data, especially in NMR when acquiring the data we come up with this theorem, this is called sampling theorem. What the sampling theorem tells is, if I have a band limited function, let us say there is a band limited function, it is not function which is extended from minus infinity to plus infinity. I said it is band limited; if I take this band limited function, the values of this function can be completely specified in the interval 1 over Sc, what is Sc? Sc is the width of the function in the frequency domain, this is a function in the time domain f of x or f of t whatever we call; and when I do the Fourier transformation, this is the frequency spectrum; this is the band 0 to Sc, I have taken.

If I takes 0 to Sc, 1 over Sc if I take this band limited function can completely specified by, if I digitize this data, in the intervals of 1 over Sc or in other words, simply remember a band limited function can be completely specified. You can understand what it is, understand the values of the function, if you know its value at 1 over Sc, at the regular intervals, where is Sc is the width of the frequency domain, not the time domain, do not get confused; the band limited function is time domain that can be completely specified.

If I know what is a Sc? Sc is the bandwidth of the frequency spectrum. So, if I know that, then at regular intervals of this function if I can completely define this function. So, band limited function is fully specified at the intervals of 1 over Sc. Remember this is a very important theorem in Fourier transformation. So, then the value of the function at 1 over Sc will completely specify this function; what happens if it is not going to be sampled properly, like this. We have lot of problems, I will tell you; when we discuss what is called a folding theorem or Nyquist theorem, when I go further in the Fourier transformation; I am going to explain that later.

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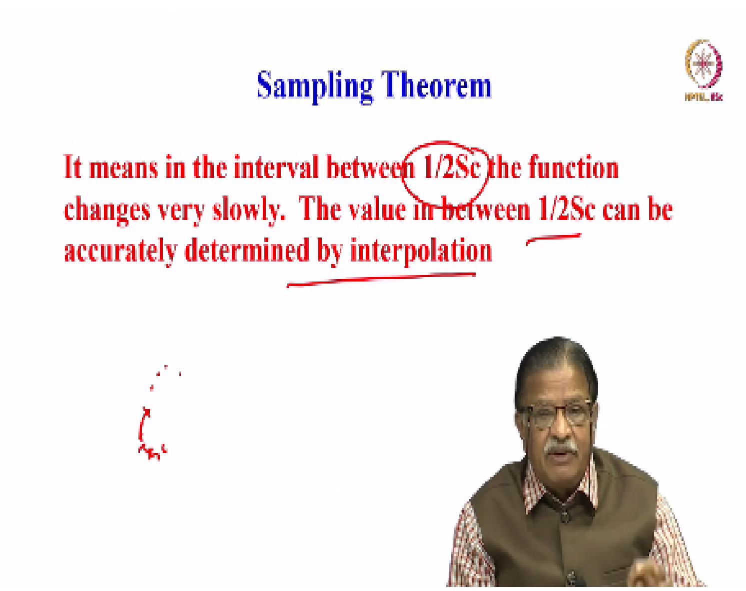

So, with this sampling theorem, you simply understand it like this, it means in the interval of 1 over twice of Sc, the function changes very, very slowly, the value in between 1 over 2Sc can be accurately determined by the interpolation. If I have a function which I know the values are different intervals, this is 1 over Sc let us say, between these 2, it can be a smooth function, I can define the function between these 2 intervals by interpolation. So, the interval 1 over 2Sc of the function between these 2 changes very, very slowly. That is what the sampling theorem tells you.

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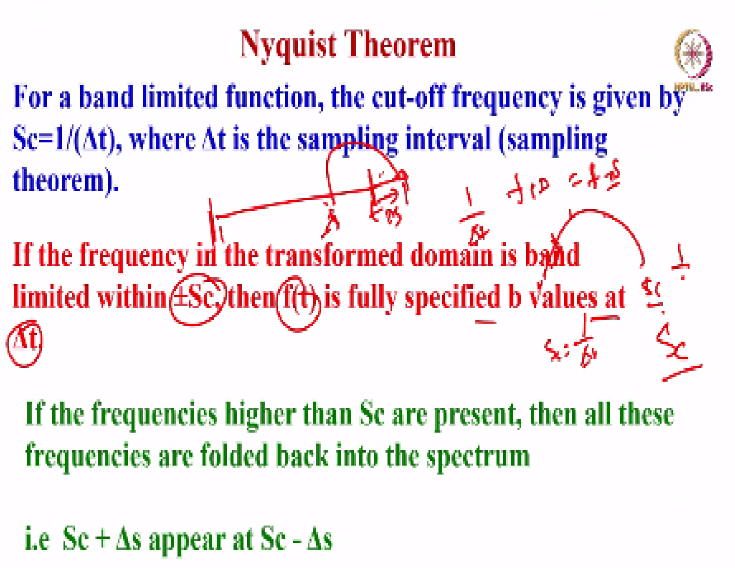

So, simply remember, mathematically I did not go into the details thinking that would take enormous amount of time, I cannot cover everything in this short course. But remember what the sampling theorem says, if I have a band limited function in the time domain it can be completely specified by 1 over Sc where Sc is the bandwidth in the frequency domain. And of course, in 1 over 2Sc if I consider between these 2 points, then function changes very, very smoothly. And they can be, obtained by interpolation. That is what you should know.

Now I will use another theorem called Nyquist theorem. If I have a band limited function, there is a cutoff frequency for it. I told you it is 0 to Sc if I take, let us say Sc is the cutoff frequency which is 1 over delta t, delta t is the sampling interval, let us say, between these 2. I keep on sampling it at different intervals, and this is delta t. So, then what will happen? The cutoff frequency is always 1 over delta t for the band limited function. So, the frequency in the transformed domain is band limited within Sc; then f(t) is fully specified in the intervals of delta t. Please understand this theorem, very easy, it has a lot of meaning in it. The Fourier transform signal if I consider, if it is a band limited within this function, within this interval 1 over Sc, then this time domain function can be fully specified value at the intervals of delta t. So, that means, if

I have the frequencies, in a simple logic I will explain; let us say, I take a range here 0 to let us say, some 0 to t is the range for me. And I have a frequency beyond this; it is extending beyond this, and this is my limit I have chosen, this my band limited function. But when you are choosing it for Fourier transformation only this region, and you will not sample it properly, and there is a frequency here, then what is going to happen is, the frequency which is extra; delta + s here, we will come back here; that is f + delta appear as f – delta.

If you fold back the frequencies which are higher than the band limited function, that limit which is defined as Sc, if there are frequencies more than that, they will fold back into the spectrum. So, that means, Sc is the limit it is a cutoff limit, cutoff frequency, then any frequency above that Sc + delta S will appear as Sc - delta S. This is the Nyquist theorem in Fourier transformation.

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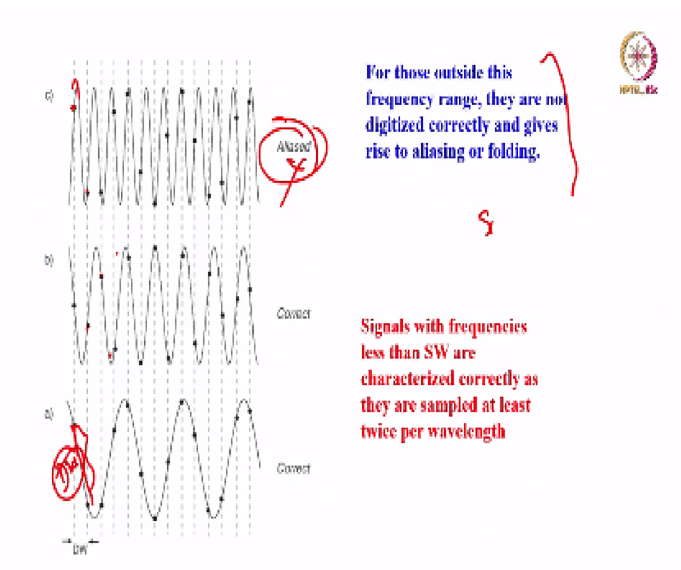

For those things, which are outside this frequency range; for example, like this, this is not fully specified, you see, it is the oscillating like this; specified only once here. So, it is not completely specified, whereas this is correct, fully specified, this is even more correct in one cycle you are specifying it 4 times; whereas, in this case what will happen? the bandwidth it is not fully specified within the band interval, within the cutoff frequency of Sc, you are not completely specifying it. So, what will happen this frequency which is beyond of Sc, Sc + delta s will come back as Sc - delta s here. So, that is why this is not correct. This is called Aliased; whereas, here you see digitized properly in every cycle; here it is even more, much better here. And this interval is called DW, is called the dwell time; this is the specified interval; within this interval I said the function very smoothly; and you can obtain this value by interpolation; you do not need to go through millions of points. You can just take 2 or 3 points within this cycle, it is enough; at least minimum required is 2, more than 2 is better and then if it is not full completely specified within this interval, then you are not going to clearly identify the frequencies, the frequencies beyond certain limit f + delta will come back as f – delta. This is what the Nyquist theorem says.

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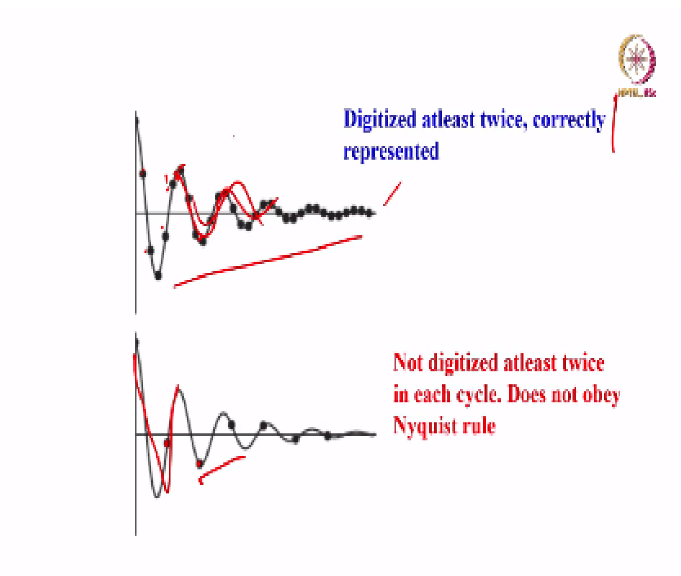

And this is the free induction decay which is collected in the NMR spectrum. You can see there are number of data points which are digitized; at least twice it is digitized you see in the cycle, this is a cycle which is undergoing periodicity. This is the periodic function, you can see here. Of course, the amplitude is getting changed; exponential it is decaying, it is an exponential function. But you see here, and it is specified very clearly here, digitized at least twice, so, it is correctly represented. Whereas look at this one, this function, here digitize once here, only once here, once here. In this cycle, it is not completely specified. So, this do not obey Nyquist rule, whereas this obey Nyquist rule.

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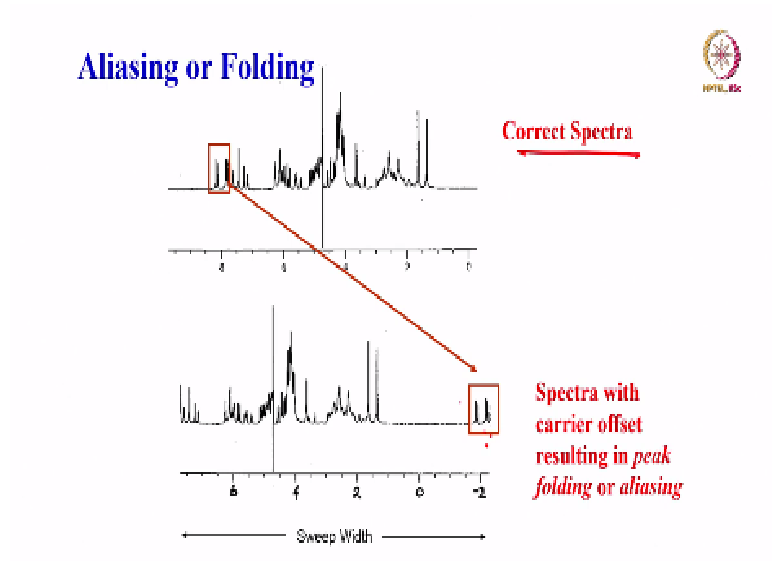

So, this is the reason what happens, if the Nyquist rule is followed, you get a correct spectrum See, otherwise, the spectrum if it Nyquist rule is not followed, what happened, see here in this case, it will fold back like this, the one which was here, will come back here, this is called folding, the frequency which is beyond the band limit, will come back within the limit, with f + delta as f - delta.

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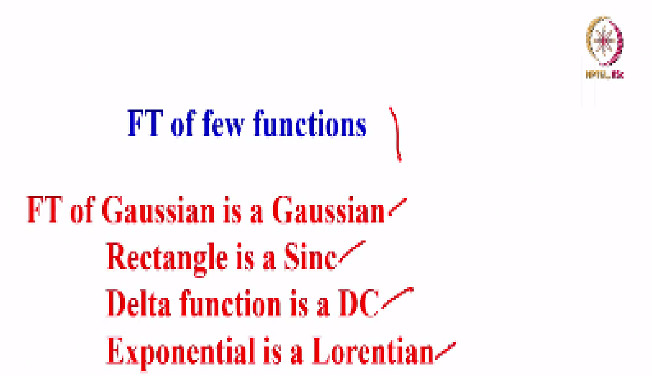

So, with this, please remember, these are some of the things which we very often utilize. The Fourier transformation of a Gaussian is a Gaussian, few functions we regularly utilize. Fourier transformation of rectangle is a sinc function. Fourier transformation of a delta function is a DC it is a comb function, I told you. FT of Exponential is a Lorentzian. These are some of the important function which you remember, so what I am going to do is that time is getting up, I am going to stop it here.

We will come back and continue this in another class. I will try to finish Fourier transformation as there is no point in stretching too much. But what I wanted to tell you is in the Fourier transformation, today first we understood what is Fourier transformation. And afterwards, we introduced and took an example of a rectangular function, and found out its Fourier transformation in the sinc function, then got the Fourier transformation of the Gaussian and showed this as a Gaussian.

And we understood lots of theorems related to Fourier transformation, which we use in the day to day our experimental methods when we are trying to do this shift, viz., theorem, linearity theorem, convolution theorem, Nyquist theorem; all these things we understood. Of course, Fourier transformation is linear function, there are 2, 3 things which are present, simultaneously, if you add it up in the time domain, the frequency that will number of components will be present.

And similarly, if you convolute 2 functions in the time domain, do the Fourier transformation, in the frequency domain they get multiplied. And of course, in the folding, I told you that the band limited function can be completely specified within the band range, if you know the frequency band Sc; within 1 over Sc we can completely define the time domain function. So, that is what is called the sampling theorem, I explained the sampling theorem.

And also I explained to you what is the Nyquist theorem, what happens for the frequency which is present beyond the limit f + delta, which appears f - minus delta; this is called folding or also called aliasing in Fourier transformation. So, with this I am going to stop now, we will come back and continue for some time and then I will complete this Fourier transformation. Then we move on to something else. Thank you.