Advanced NMR Techniques in Solution and Solid-State
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Module-12
Complex Form of Fourier Series
Lecture - 12

Welcome all of you. Last week, we were discussing about Fourier transformation, Fourier series I introduced. As you know, Fourier transformation is one of the basic requirements in NMR spectroscopy, it is one of the important mathematical tool that we use; as we have been discussing right from the beginning we have apply radio frequency pulse, collect the free induction decay as a function of time, the time and frequency are related to each other by a mathematical operation called Fourier transformation.

So, it is good to know something about Fourier transformation, what is Fourier series, what is the difference between Fourier series and Fourier transformation and how we can utilize Fourier transformation in some important applications in NMR spectroscopy. So, in the last class, I started introducing the Fourier series and then discussed about periodic function, we can express this periodic function as a series called Fourier series, where we have summation a0 + summation as a function of an into cosine of x + b n into sin of x, all those things we wrote. And then we also understood how to calculate these Fourier coefficients that we understood. And we took an example of a periodic function, which is square wave and tried to arrive at what are the Fourier coefficients for this thing. And, in fact, this is what we did last time.

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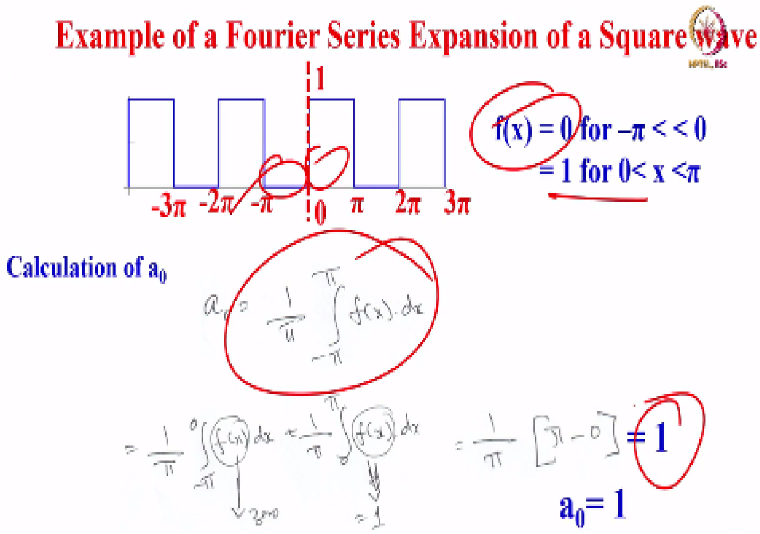

See, we evaluated for this square wave we took and this, what we define the function f of x, which is equal to 0 in this region and it is 1 in this region. And of course, I calculated a0, a0 is simply the integral of this function. And I showed it, I do not want to go again, to be equal to 1. So, a0 is 1 for this function.

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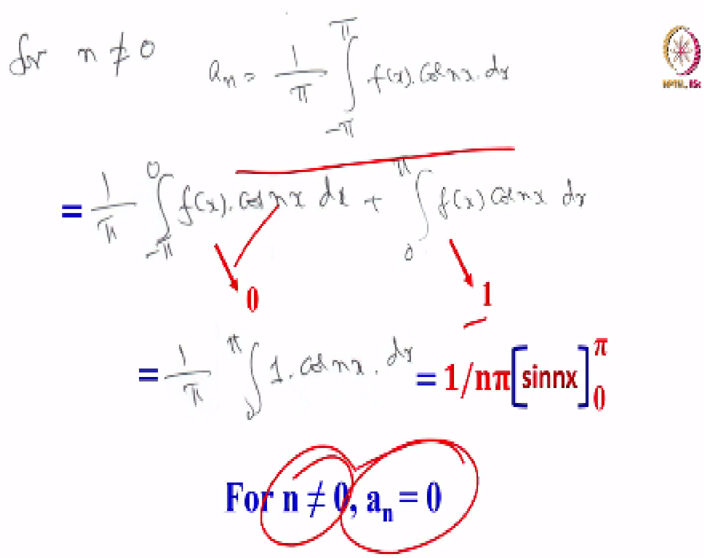

And then, for n not equal to 0, we have to evaluate this integral. And again, separating into 2 parts - pi to 0 and 0 to pi, we know how the function is behaving in different regions. And of course, this will go to 0. And we also worked out, this turns out to be 1 and we calculated what is the an, for n is not equal to 0, we showed that n equal to 0, I do not want to go through the steps again, because it was discussed last time.

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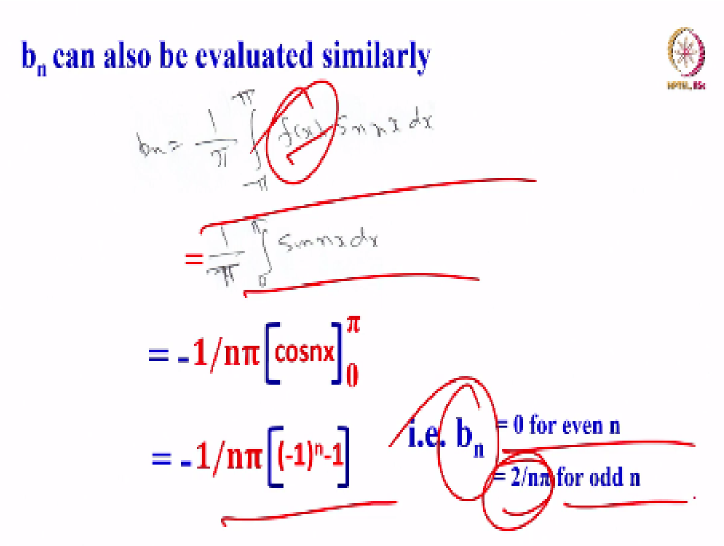

Similarly, we also evaluated bn, bn is again integral of the sin function; and then again, what will happen you resolve into -pi to 0 for the function that part will go to 0; and the other integral survives. 0 to pi sin nx dx will survive; and then substitute the limits and finally, what we are going to get is this one; this I said the value of bn is 0 for even n. And it is 2 / n pi for odd values of n. This is what we worked out last time.

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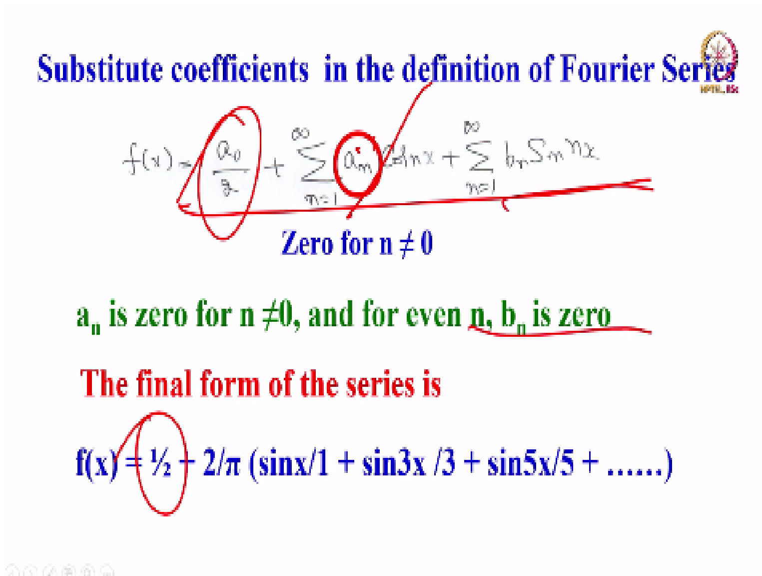

But now we substituted this coefficient in the basic equation, our expression for the Fourier series; in that we know what is already a0. a0 is 1. So, we got it and this is equal to 0 for n not equal to 0. So that means for any value other than n not equal to 0, that means a0 survives, a1 to an all the values will become 0, this term will not survive, it will go to 0. And we have to deal with only bn sin nx. And again, if you want to see this bn; for even values of n, we also showed bn is 0.

So, if you substitute all these Fourier coefficients, what we calculated, the final form of the series is like this, because a0 = 1 and a0/2 is half and then this is 0. And the same part, even part is zero; only odd part will survive. The odd part we are writing like this, this is the Fourier series for the square wave which we considered.

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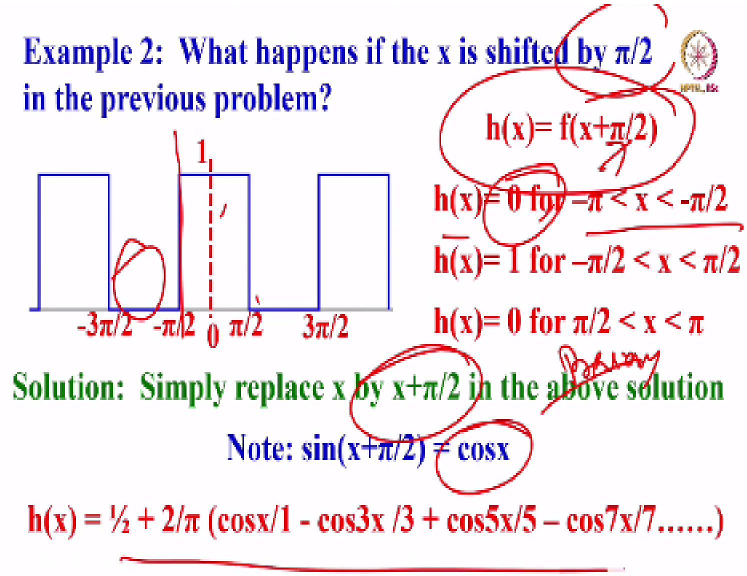

Of course, what happens if I shift, let us say x in the function by pi/2, in the previous problem. Now you write this thing, earlier it was here, I took this as 0. And I knew this was 0, and this was 1; but now I shifted by pi/2. So, I call this function h(x) which is equals to f(x) + pi /2; shifted by pi/2. So that means you can define values for this h(x) equal to 0 for - pi less than x less than -pi / 2. In this region and cos series equal to 1 for minus pi/2 to plus pi/2, and equals 0 again, for plus pi/2 to pi.

So, these values can be defined for this function, all we have to do is in the previous equation, previous problem is solved, we worked out the Fourier coefficients final form, simply replace x + pi/2, that is all you have to do with the previous solution, not in the above solution in the previous solution. So, if you do that, then what you have to do, please remember the trigonometrical operation, sin of x + pi / 2 = cosine of x.

So, whatever you wrote there previous, it is written like this, is the sin series, you are going to write it as a cosine series you see here, it was a sin series here, but we are going to write as a cosine series. That is what I am going to write here. So, this is an example for this.

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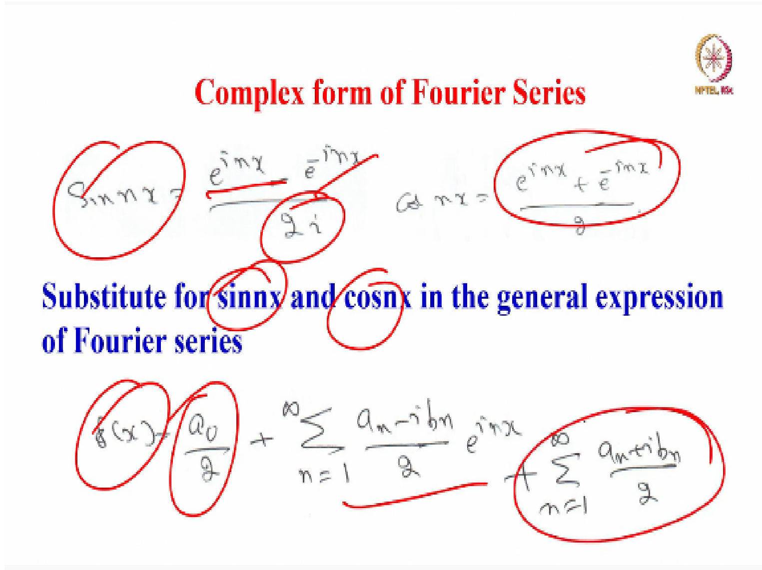

Next, I will go into another form of Fourier series called complex form of the Fourier series. You should know what is the complex form of the Fourier series also, because when we do the Fourier transformation, we get both real and imaginary part like the complex number. So, we should know something about the complex form of the Fourier series before getting into Fourier transformation.

So, what we are trying to do is of course, you know the basic equation, what if we have a sin of n x, it can be written as exponential e to the power inx minus of e to the power inx over 2i. This is well known trigonometrical formula. Similarly, cos of nx also well known it is e to the power of inx + e to the power of -inx / 2. These two sin nx and cos nx are well known formula, it is available in all basic trigonometry books. Now, what we do is we have to substitute for sin nx and cos nx, in the general expression, what we wrote for the Fourier series.

We wrote Fourier series, remember, fx = a0 plus of summation over cosine of nx and sin of nx, all those things, an, bn and everything we wrote. For that basic Fourier series expression, we have to substitute these things. So, if you substitute these, fx turns out to be a0/2, and then of course, do a simple mathematical jugglery after substitution, then it turns out to be (an – ibn)/2 because we are talking about sin and cosine expression in terms of (e to the power of inx + e to the power of -inx) / 2 and other things like this. And then this is another part you are going to write; this for the cosine part and this is for the sin part.

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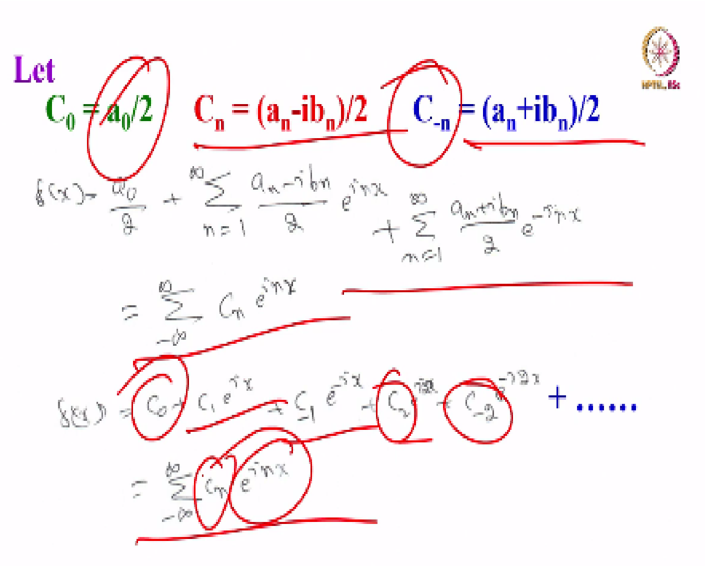

Now what I am going to do is, for solving this equation, I am going to let C0 equal to a0/2, just I am defining; and then Cn I am going to defined as (an – ibn) / 2, to ease of my operation, ease of my calculation. I am defining like this similar C-n, I am going to define it as (an + ibn)/2. So, now substitute this in this basic expression for the Fourier series. If you do that, then it turns out to be minus infinity to plus infinity Cn e to the power of inx; a very simple mathematical jugglery you can do by substitution these terms.

Then what you are going to get is, you are going to get C0 + C1 e to the power of ix C-1 e to the power of – ix; alternately n = +1, n can be +1 and –1, +2 and -2. So, you can write down C as C1, C2, C–1, C-2 etc., expressed in terms of exponential form like this. This is simple jugglery of mathematics, I have not done anything new; it is all well known simple arithmetic. Then it turns out to be summation of minus infinity to plus infinity Cn of e to the power of inx, this is what finally we are going to get for fx.

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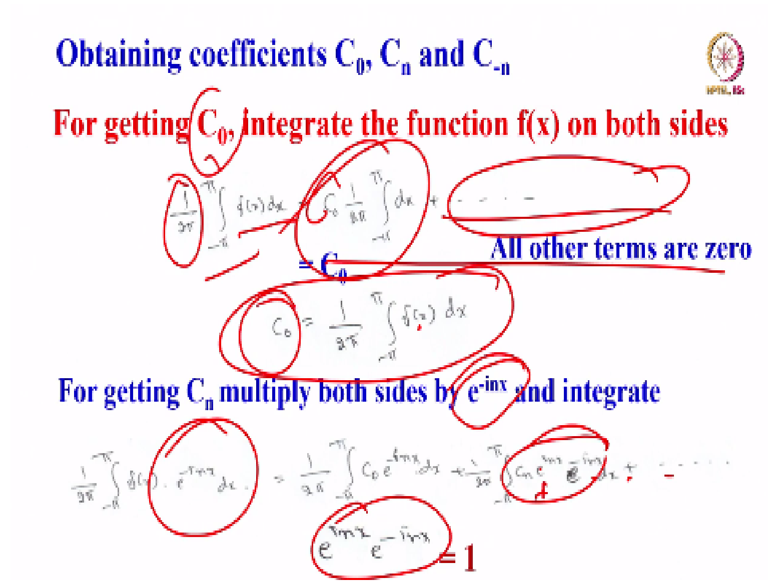

Now, the question is, you know, this is fx, but we need to work out Cn; that means, we have to work out C0, Cn and C-n also. All these 3 coefficients we need to work out; how do we go ahead and do it? For getting C0 all we have to do is integrate this function of fx on both sides, we have written fx now, complex form of it, what I had to do is take 1/2pi integral from -pi to +pi fx dx; integrate, then this turns out to C0 into 1/2pi integral of -pi to +pi dx, plus all that terms should come into picture.

This is only a previous expression written here; you have already worked out you; see from this. So, now, this what I did; the remaining terms what happened to them you can work out; and see all the terms here are 0, only this term will survive, this is called C0. So, that way C0 survives all other term goes to 0; and integrate the function fx on both sides from -pi to +pi. That is integral limits. So, C0, I can write as C0 = 1 over 2 pi integral of fx dx. This is the expression for C0.

So, if I know our fx, if I integrate that function over –pi to +pi, I can calculate C0; I can find out coefficient C0. How do you get Cn now? For getting Cn, what we have to do is, instead of just integrating the function on both sides from -pi to +pi, multiply by e to the power of – inx and integrate on both sides. That is what we have to do to get Cn. So, if you do that simply multiplying the function by e to the power of -inx and dx.

And then if you do that, of course, we can resolve this into different terms like this, 1 over 2 pi -pi to +pi, C0 Cn and C-n of course, it can take all the values from -n to +n etcetera. But here we are considering plus of n; Cn, so now we know that basic operation, it is exponential function, e to the power of inx into e to the power of -inx equal to 1. That is well known. Of course, let us use this here, it turns out to be e to the power of inx + e to the power of -inx; as you know, that will become 1.

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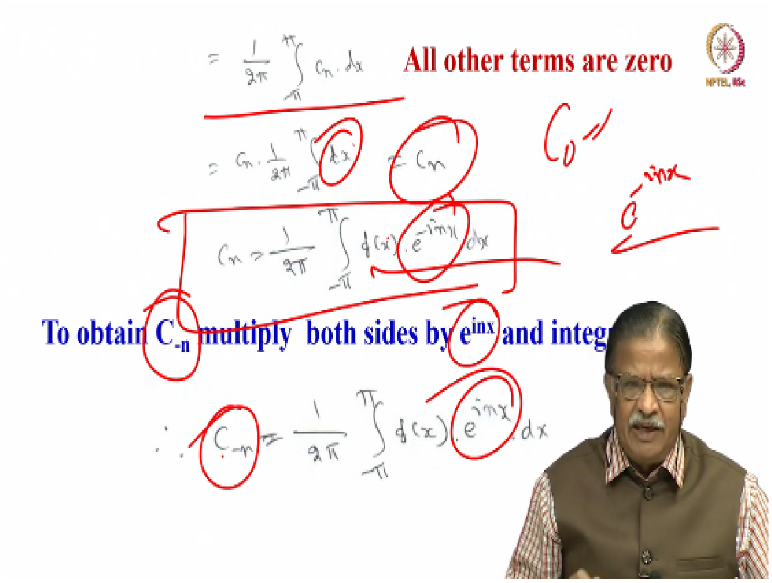

So, using this operation, you will show that this turns out to be equal to 1 over 2 pi into - integral of -pi to +pi Cn dx. All other terms are again 0; it turns out to be 0. So, it is a simple basic elementary integral calculus I did; and showed that Cn is equal to this function; is equal to Cn into 1 over 2 pi into integral of -pi to +pi dx. So, this is our Cn. So now, what I have to do is, I substituted Cn; I got the Cn. So, essentially I write Cn as like C0 I wrote, which is integral of – pi to + pi f(x) dx; I wrote 1 over 2 pi.

Similarly now, integral of Cn if I have to obtain; 1 over 2 pi integral of -pi to +pi f(x) dx inx, if I integrate, I will get Cn this what I want to show you. Now, I need to get C-n also; remember, I also have to obtain the coefficients for -n values. In this case, you have to do different; previously, we multiplied that function by e to the power of -inx to get the Cn. Now, to get C-n multiplied by e to the power of inx and integrate; simple jugglery we are doing nothing else, now substitute e to the power of inx, to update C-n, like we did here.

So, Cn if we want to get I do this; C -n if I want I will do this one. I know, I told you already what to do if I want to get C0. So, all the 3 types C0, Cn and C-n; all the coefficients we can obtain by simply integrating the function from -pi to +pi with this fx dx. One case there is no multiplication is C0, but in other cases, you have to multiply by corresponding exponential function either e to the power of -inx or e to the power of inx, and then substitute in the integral.

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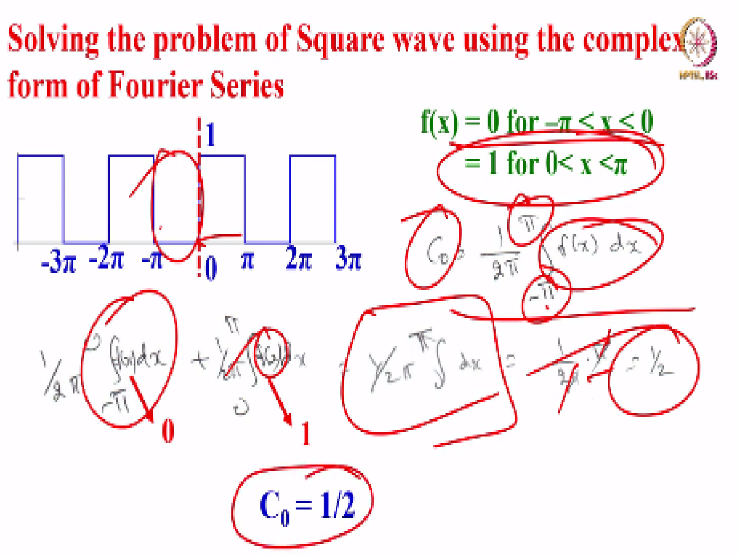

So, this is very, easy so, this is what we have to do to obtain the coefficients C0; Cn and C-n. This is a way to express the complex form of Fourier series. Now, we will do one thing, we can solve the square wave problem which is solved using the Fourier series in a conventional way. Now, can we solve the same problem using complex form of Fourier series that means, instead of a0; an and bn we need to work out C0; Cn and C–n.

And we I already told you we have discussed how to work on the coefficients that is to this exercise. Of course, earlier we have already defined this function square wave and this is the value for fx equal to 0 for -pi to 0 and then equal to 1 in this range, these function values are valid in different ranges are known; value of these how the function behaves in different ranges.

Now, C0 as I told you integral 1 over 2 pi just integrate the function from - pi to 2 pi fx dx without multiplication by exponential that is what we I told you, so, do that one.

Now, again this function we can resolve into 2 terms -pi to 0 and integral of fx dx + 1 over 2 pi integral 0 to 2 pi fx dx. Same – pi to pi integration I have divided it 2 steps; 2 parts - pi to 0 and 0 to 2pi; 2 integrals that is all. Of course, this part is already known fx = 0 in this range, so now we have to deal with only this one; here fx = 1 so what does it mean? This turns out to be 1 over 2 pi 2 integral of 0 to pi dx, this is nothing but the x. So, it is nothing but 1 over 2 pi.

If we substitute the limita; it turns out to be pi. So, you cancel this pi, what you are going to get is half. So that means for this function for C0, we got half; very easily we worked out using the complex form for this one. So, C0 we know. What are the other things we have to do? We have to work out now Cn and C– n.

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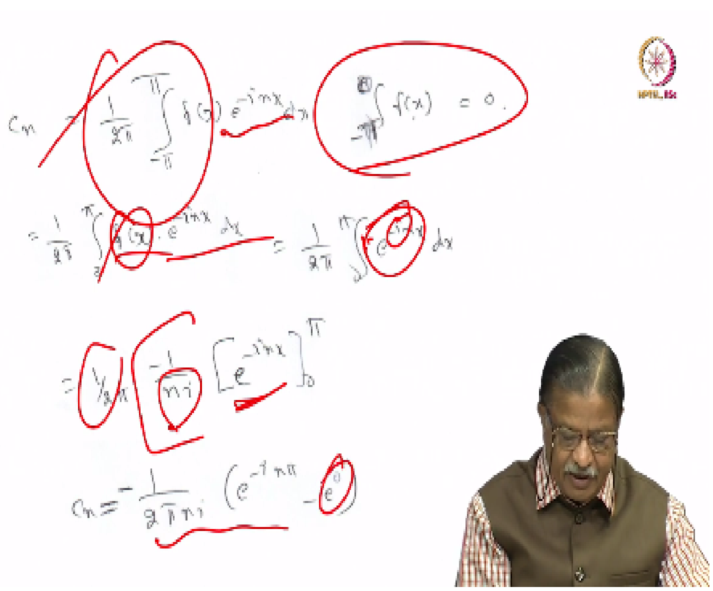

So, Cn I already told you take it from -pi to +pi, e to the power of -inx. Again, you know, in this range integral from -pi to 0 for this function fx = 0, you do not have to consider that; consider only from 0 to pi, fx e to the power of -inx dx. And this one, if you write like that, of course, this value is 1 in this region. So, it turns out to be just 1; this one, I did not write one but it is all simple integral calculations, integral calculus, I guess you all know this.

So, this turns out to be 1 over 2 pi into integral 0 to pi; e to the power inx dx, which is nothing but 1 over 2 pi into -1 over pi into e to the power of –inx. This is what I did, the integration of this function. Now, this limit is from 0 to pi. Substitute this limit -ni you know that if this comes in the denominator -1 over ni into e to the power of -inx dx, this is the simply integral form for the exponential function. So, substitute the limits, when I substitute the limit for n which is equal to 0 to pi, you see, that this turns out to be 1 over 2 pi ni into e to the power of –in pi into -e to the power of 0; this is 1 again.

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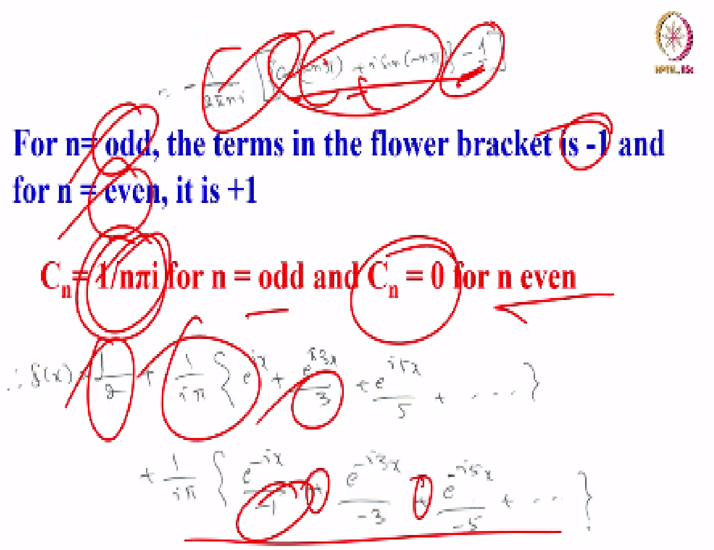

So, when you do that, this part we did some jugglery, this we can resolve into sin and cosine components. We know that trigonometric e to the power of pi x, we can express in terms of sin and cosines. So, this we resolve and expressed as cos of n pi + i sin - n pi, this is the e to the power of –i n pi; this basic trigonometry you should know that cos n pi + i sin - n pi. This is the way we write simple exponential form, expanded as cosines and sines; of course -1 we already got it.

So, now, what will happen if you look at this function for n is equal to odd number? the terms in the flower bracket here goes to 0, I am sorry goes to -1 for n is equal to odd. Calculate this term for putting n equal to 1, 3 and 5, it is simple trigonometrical function you put the n equal to 1, 3 and 5 all odd terms, then you find out the bracket turns out to be -1. So, what will happen? then for n equal to even if you consider it is going to be +1.

Now, when it is +1 and -1 what is going to happen you look at it for Cn because simply it is 1 over n into pi into i for n equal to odd; Cn is equal to 0 from this equation I am sorry, this is 1 over n pi i for n = odd; and see it is 0 for n equal to even. So, from this you can find out when n is equal to even is +1, +1 and -1 here cancel, this term becomes 0. That is what happens. So, for n is equal to even number Cn = 0; For n is equal odd number Cn turn out to 1 over i into n into pi. These terms we know.

So, we have worked out Cn and we know C0 and simply substitute for this fx and express this one, this is when you substitute for this function in exponential form. We already wrote the exponential form of the Fourier series, simply substitute these coefficients and write down C0 we got as 1/2 and for the odd values of n, we got this one. And simply for n = +n and -n substitute these, n = -n, -1, 2, 3. It is going to be -1, -2, -5, so this all turns out to be negative, that is all it is going to happen.

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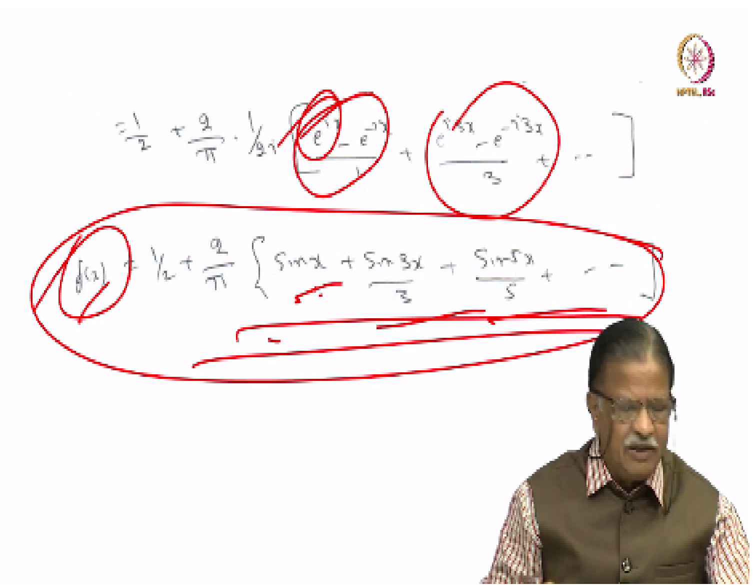

So now, you can simplify it in a simple way, when you have exponential function like e to the power ix - e to the power -ix you can combine them like this; e to the power ix - e to the power -ix / 1. Similarly, this term like this, very simply, you can combine. All these terms like this,

because I have +n and -n both the values are there, as you can see here, for +n e to the power of i pix, for -n e to the power of -i pix. Similarly, for all odd terms, you have plus or minus combine them like this.
And then make a simple jugglery if you combine them, these turns out to be remember in the very first time I showed you, how do you express sin term, sin and cosine in terms of exponential function. This is nothing but sin x; sin 3x / 3 sin 5x / 5 etcetera. It is a series like this. So, the fx if you only expresse for the square wave in the form of the complex number ,complex series it can be written like this.

So, this is simple complex formula for square wave, the expression of this square wave using the complex formula of the Fourier series, then a function fx turns out to be this. So, you people should simply know what we have been doing; we are solving this using a simple trigonometry, integrating the function from -pi to +pi; in one case we do not multiply, in other case multiply by e to the power of - i pi ni, in other case you multiply by e to the power of + i pi x; like that very easily you substitute this thing and make a simple integral calculus.

Afterwards, when you do the integration all you have to do is to combine the terms based on the terms like e to the power of ix + e to the power of -ix; all those things. Then you can express in terms of sines and cosines. In this case for even number of cosine part was 0. So, we got only sin part. So, this is all I wanted to tell you something about Fourier series, this is the Fourier series we were trying to do only for the periodic functions we can express as Fourier series.

Suppose, the function is not periodic what can we do? So, we will discuss something about what the new term and new mathematical operation called Fourier transformation, this is something which I am going to discuss now. This is of course, is a function, which is non periodic also, we can express this Fourier series; somehow we can do the mathematical operation and get a continuous function. And this we call it as a Fourier transformation; a mathematical operation which we are going to discuss now, the Fourier transformation.

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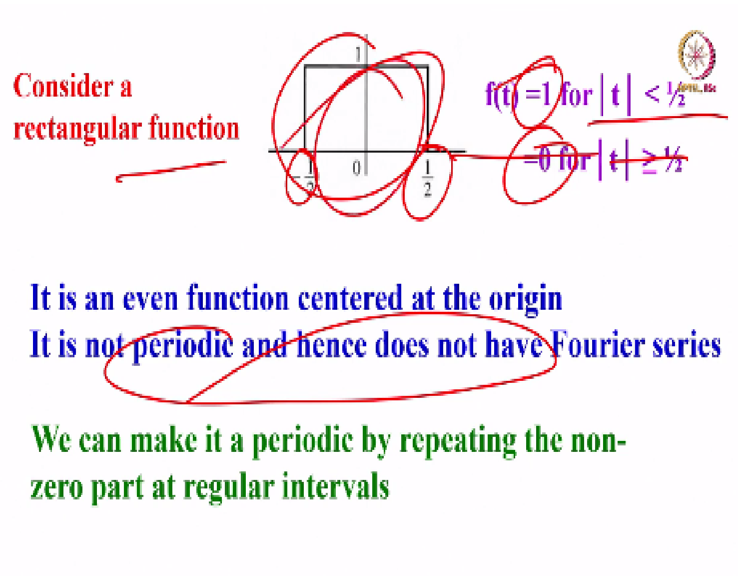

What we are going to do? again consider a simple rectangular function, this a rectangular function; please note it is not periodic, it is just a pulse. I have written rectangular function it is from minus half to plus half is the limit, and this is 0 to 1. I have this is function I have written. So, this function value can be written as f(t) = 1, for modulus of t less than half equal to 0; for modulus of t greater than half this is zero; a simple thing what we have written for the function, it is a square function I have written; it is not periodic as you can see it in the single square pulse.

Now, it is an even function centered at the origin. Remember it is an even function like a cosine function; all even functions are symmetric with respect to the y axis. Now, it is not periodic and hence, does not have a Fourier series. But if I have to solve it, what do I do? So, what I am going to do is we can make it periodic artificially. What I do is just for the calculation purpose, for understanding Fourier transformation, I am going to do some simple jugglery.

What I am going to do is I will periodized this type of function by repeating it at regular intervals. What I mean is, I take this function repeat it after some time here this side, this side, this side, you can keep on repeating considering this as the base function, it is possible to do that.

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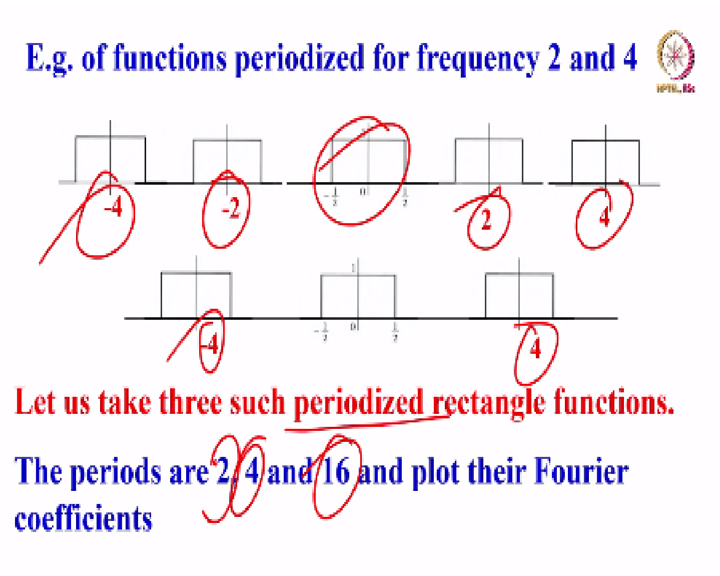

Like for example, I am periodizing the function at regular frequency intervals of 2 and 4 for example, here this is a base function and this is periodized at period 2, it is repeated and period 4 it is repeated similarly, -2 and -4 it is repeated it is not periodic, I deliberately made it periodic, I periodized just to make my mathematical operation simpler or to understand some concept. Same thing if I want to do periodicity 4 like this, instead of 2 I have written it +4 and -4 then again +8 and -8; it goes like that.

So, this intentionally artificially I made it periodized, this function at different intervals at different frequencies. Let us do one thing, let us take 3 rectangular functions which I have periodized at regular intervals, like the periods could be 2, 4, 16. Why I took these numbers any number is fine, I have already shown you for 2 and 4 some another number big number. In fact, I wanted to go to 32 or 64 to show as the interval becomes you know bigger and bigger, the frequency becomes closer and closer and then it goes, more towards the continuous function.

That is where I want to take a bigger number; but anyway 2, 4 and 16 I have chosen. These are the periods I have chosen to deliberately periodized these type functions at 3 intervals.

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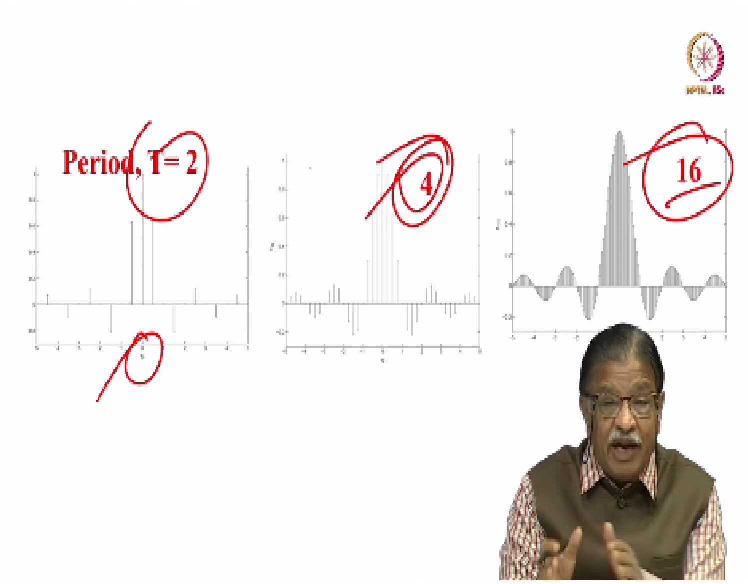

Now what I will do? I will calculate the Fourier coefficients for this; you know how to calculate the Fourier coefficients. Already we worked out, A and B and C and all those things in a complex form C0, Cn and Cn-1, we know how to calculate; it is simple integral calculus, we did that integral calculation. So, we can do that artificially periodized for period 2, 4 and 6; calculate all the Fourier coefficients.

And then plot it, now plot the Fourier coefficients like this, for period T, for different values of n, n equals 0, 1, 2, 3, 4; calculate the values and then start entering here; and then plot like the stick plot. For period 4 it is like this, for period 16 it is like this. Already you can feel it what is going on, as the period keeps on increasing you know, here, the period was only 2, the frequencies are well separated, they were dispersed more and more, as increase the period 2 became 4, it became closer and closer; see. I go to 16, it became even closer. Now I take a large number; instead of 16, 64 or 32, 64 or 128, what will happen? This type of period function become more or less continuous, you can treat it as a continuous function, so that is what happens.

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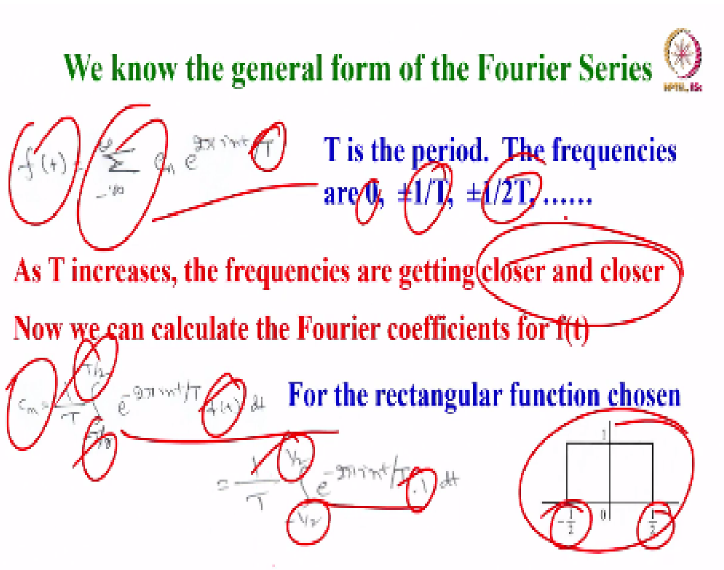

So, now we know the general form of the Fourier series; and then what we are going to do is, we will see in this case, we put T is a period ,I already explained to you, and it is in the denominator, the frequency is 1 over T that you know, period. If you know the period, I can calculate the frequency which is 1 over T. So now, for the frequencies are 0, 1 over T, 1 over 2T, 1 over 4T it keeps going, you can calculate for this T. As T increases what is happening, we saw in the previous graph I showed you as T becoming bigger and bigger, larger and larger, the frequencies become closer and closer. Again I will show you, you can see here, here frequency became very close, here, well separated here more dispersed. So that is the advantage. So, when T becomes larger and larger, the frequency becomes closer and closer. That is a concept you understood.

Now, what we can do is we can calculate this function t, f(t) Fourier coefficients for this. This is the Fourier series; let us calculate the Fourier coefficients. But remember, I started this as a non periodic function, I intentionally periodized, just to understand what it is, I wrote in this form. And now I want to calculate the Fourier series, once I make it periodic, you know, there exists a Fourier series for that. So, once there is a Fourier series, I can calculate the Fourier coefficients that is what we will do. So, I calculate Cn when expressed in the form of exponential form; a complex form of the Fourier series. So, we can take this from minus T/2 to plus T/2 and integrate this. Of course, the function was known already and for this is the general expression for different T periods you have chosen. Now, for the rectangular function what is chosen, the values are going from minus half to plus half. So, accordingly, we change the limits of integration minus half to plus half and this is the function and f(t) in this region is 1.

So, in simple term 1 over T integral of minus half to plus half e to the power of -2 pi int over T this is what I would taken, I would put it here.

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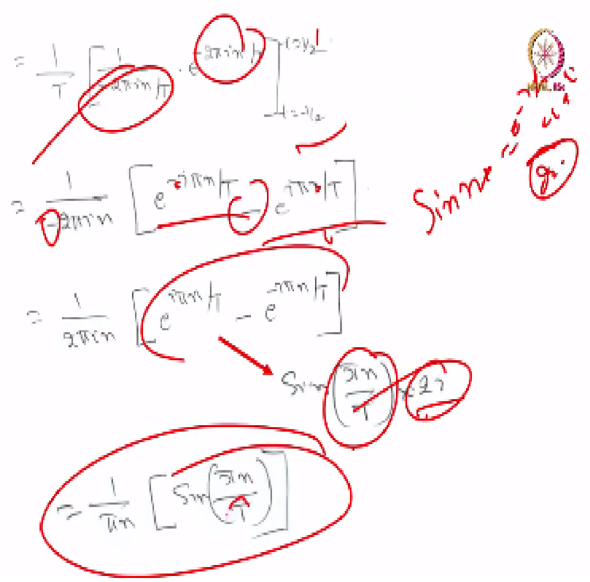

So, now, express this as 1 over T of course. We will integrate this one; this term comes in the denominator and the limit is known actually, integration limits minus half and plus half substituted; then this turns out to be this one. When you substitute this one, then what will happen? again further simplified; little bit do the simple mathematical operation. I have not done anything more here. So, this minus sign I removed it by bringing here this bringing here, that is all I did nothing else. Now, this can be written as sin pi n /T.

That is very well known, this can be written as sin pi n / T into 2i, a trigonometrical simple operation which we have discussed already, otherwise if I write sin nx I told e to the power of -iT + e to the power of +iT; 2i comes you know at the bottom. So, that one if you want to express this, then this can be written as sin pi n / T into 2i. So, in simple form, it turns out to be 1 over pi n sin of pi n over T. This is a simple expression. If you work out this integral and when you write down the equations of this Fourier series, this turns out to be like this.

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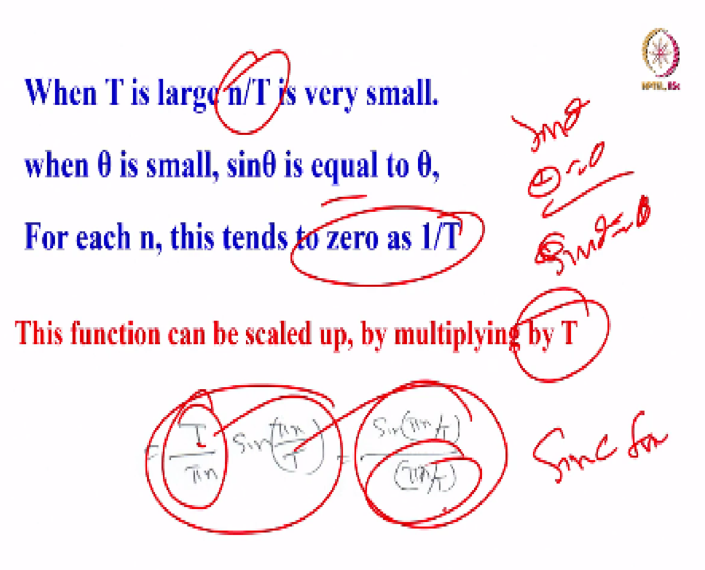

Now when T is large, here, go back here, when T becomes larger and larger n / T becomes very very small here, because it is T is in the denominator. Now, when this become very very small, in trigonometry we know that ,let us say sin theta is there, if you have sin theta when theta is equal to very very small, so, sin theta can we simply written as equal to theta, that you know in trigonometry, basic trigonometry. So, now, for each one this tends towards 0. As a function of T as T keeps on increasing, this tends towards 0.

So, this is what tends more towards 0. And this function what I am going to do which I got it here, I will scale it up, I multiply by T; does not matter simply scale it out just for the sake of understanding. So, this is scaled up function; all I did for the Fourier series which I have obtained, multiply this by T, that is all. And this I can write like this sin pi n / T over pi n / T what I did is I brought it to the denominator, why because I know there is a special name for this function. You must be knowing from your elementary trigonometry, this is called sinc function. So, sin c by c function.

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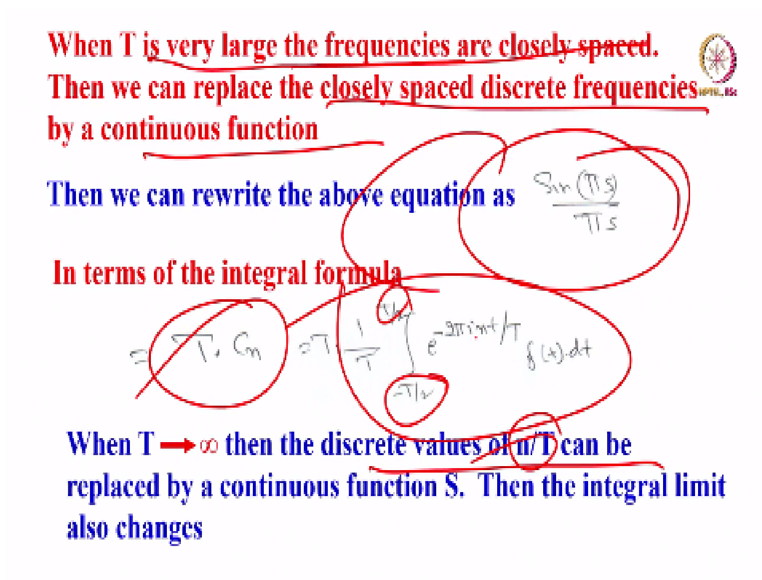

So, that is why I wanted to do that mathematical jugglery. So, when T is very, very large, the frequencies are closely spaced then we can replace the closely spaced discrete frequencies with continuous function; that is what I wanted to tell you. See now, we took as a single square function, it is not periodic, just to understand we periodized at different intervals 2, 4 and 16. We understood, as T increases, i.e., as the period increases, what will happen? this frequency becomes closer and closer.

So, what I understood finally, it turns out to be a sinc function and T is very, very large as I told you 1 over T comes close to 0; sin theta can be written as just theta or when T is very very small, then we wrote sin of nx / T is whatever the function we wrote here sin of pi n / T or pi n / T and that is called a sinc function. You understand from that, when T is very, very large, the frequencies are very closely spaced that means we can replace this discrete frequency by continuous function; you understood.

I started with a function which is only just a square wave; it was not a Fourier series, it was not periodic there was no Fourier series; just to understand how we can do; artificially periodized and then obtain this thing. And then we can write the above equation as a simple term sin pi s over pi s; in the integral formula you can write it as T into C of n. what is T? Cn is already obtained like this. I multiply just by this T and then as T tends towards 0, the discrete values of n/T can be replaced by a continuous function.

So, what I am trying to say is, there is a discrete value it is changing only from minus T/2 to plus T/2 that the integral values you have to take limits of the integral from minus T/2 to plus T/2. But now, as T tends towards infinity tending towards infinity, the discrete values of n/T I can replaced by continuous function, I call it as S, some f(x) also, then the integral also changes because the continuous function is minus T/2, I can go from minus infinity to plus infinity also.

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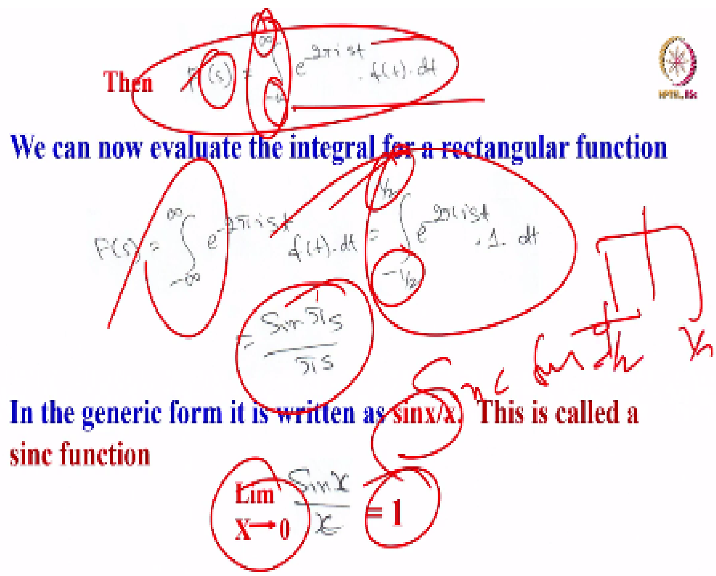

So, accordingly I will change it, I will call this function as in continuous function S, you should have f(t) call it as f(S), because it is continuous. I called as some other function and now change the integral from minus infinity to plus infinity; the same thing remains. Now, we can evaluate this integral for a rectangular function very easily, we know that rectangular function, again minus infinity to plus infinity if you take; this you already worked out minus half to plus half before; the rectangle function what you have chosen.

And this turns out to be sin pi S over pi S, this is called a sinc function, this is also called a sin x / x function. So, for a rectangular function which we took it and when we knew the function from minus half to plus half, we worked out and then periodized and then calculated the Fourier coefficients by artificially periodizing and then after simple mathematical jugglery we understood that as the period keeps on increasing the discrete function we can replace by a continuous function.

So, a limit of integral also can be changed from minus infinity to plus infinity; it is the general form for a function which is continuous, but in the special example of the rectangle function which was taken it is from minus half to plus half so, then it turns out to be sin pi S / pi S or in the simple form it is called sin x / x function; it is called a sinc function. And as the limit of this function as x tends to 0 it become 1. This is what the rectangular function, we saw, it is 1 from minus half to plus half we took.

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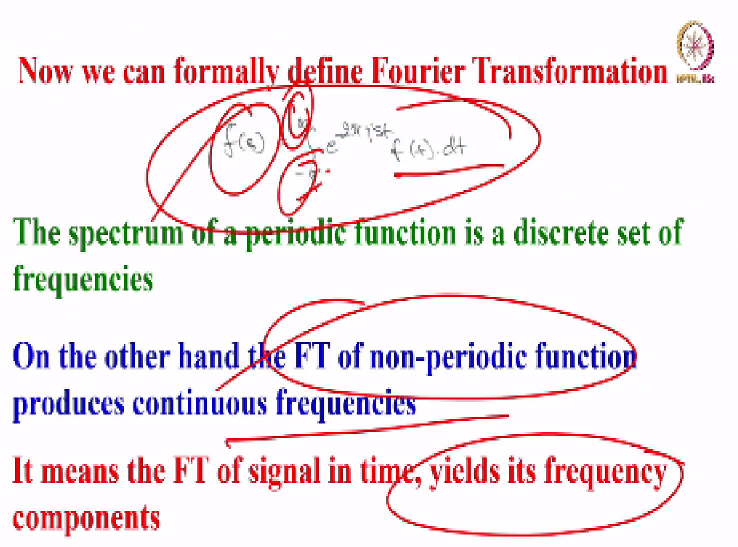

So, we can now formally define what is called a Fourier transformation, the Fourier transformation is given by a function like this; f of x is equal to minus infinity to plus infinity e to the power of -2 pi ist f of t dt. So, it is a discrete function; we integrate with the entire range from minus infinity to plus infinity, it becomes a continuous function. So, discrete function can be expressed as a continuous function by doing Fourier transformation, the spectrum of a periodic function is a discrete set of frequencies.

On the other hand, the Fourier transformation of a non periodic function produces a continuous frequencies; you understood. So, this means the Fourier transformation of signal in real time gives you its frequency components. This is the conclusion which I want to tell you; see the time is running out. What I am going to do is I will stop here. Today, what we tried to do is we continued from yesterday and understood what Fourier series and express the complex form the Fourier series understood what are C0, C1 and C–n.

How to calculate them by simple integration for C0 of the function f of -pi to +pi, for Cn and C-n, you have to multiply by e to the power of -inx, otherwise e to the power of i pi x and e to the power of +i pi x; that simple thing we did, and then worked out coefficients Cn and C-n and then workout the Fourier series and solve that for a square function. And now what happened is, we took a simple square function which is not periodic and then periodized for different values and worked out the Fourier series and plotted. And found out as period increases, it becomes continuous. So, what we tried to understand is formally we can define the Fourier transformation of this discrete function. And the spectrum of this periodic function is discrete set of frequencies. But if you take the Fourier transformation, non periodic function, it produces the continuous frequencies. So, what we are trying to say is the Fourier transformation of a signal in time gives the frequency components present in that, this is exactly what I wanted to tell you.

So, this is what you come across in NMR. You collect the signal in time domain, do the Fourier transformation, you are going to get frequency domain. So, I will stop it here. We will come back and continue later from this point. Thank you.