Control System Design.

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Lecture -9
Introduction to Feedback Control (Part 1/2)

Hello in this clip I am going to introduce the fundamentals of a feedback control to you. If you want to start with the assumption that we do not know too much about feedback control and ask ourselves what do you mean by feedback control or more generally control. One would probably conclude that to control something is to get it to do your bidding. And that precisely also what the objective of feedback control is. You want a system to do your bidding. And in this course as I outlined earlier, we are focused with single input, single output systems.

So, in the context of single input, single output systems, what we mean when we say that we wanted to do our bidding is that if you provide a certain input waveform, let us call it reference r of t.

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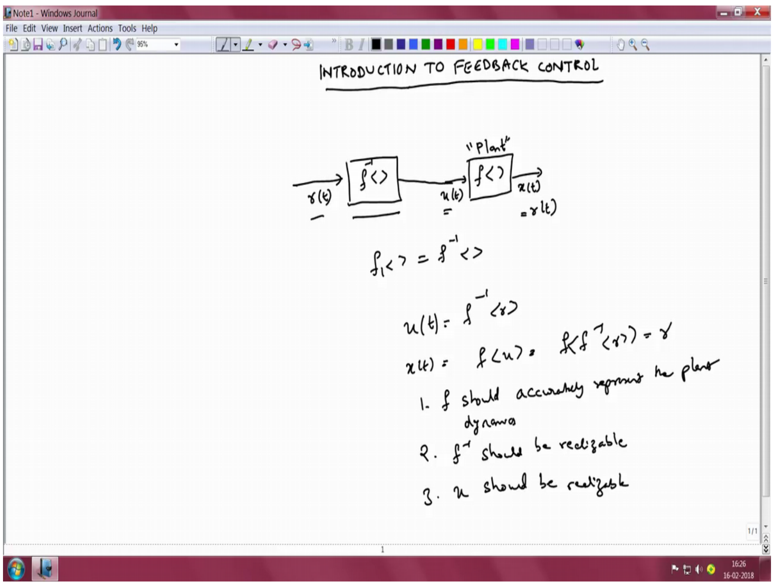

Then we want the output of the system. So, r of t is some reference wave form that we want to feed to our system, this could be any physical system that is of interest was it could be a motor; it could be a car, it could be an aeroplane, it could be a chemical plant, r of t is something that you want it to track.

So, the output of the system which we shall call x of t should ideally be equal to r of t, if we have design our control system very well. So, this is the ideal, this is the goal of all control engineers to get the output of the system to track our reference. Now what is our system?

As I said it could be any dynamic system. So, depending on the particular discipline that we are talking about, the physical appearance of the dynamic system might be different. But fundamentally from a mathematical perspective each a single input single output system. So, there is a single input u that you can provide to the system.

And in response to u of t, you get an output x of t. And the relationship between u and x is characterised by a certain input output mapping which we shall represent by means of this function f. So, this function f could possibly include derivatives of output with respect to time and so on and so forth. So, differential operators and double differential operators and others can also be part of the structure of f.

So, the reference I want to track is r of t, the input I am providing to my plant is u of t. And it is response is x of t and ideally I would like x of t to be equal to r of t. How do I do that? Of course, this will not happen on it is own. I need to introduce some mathematical function here, which manipulates the reference in such a way that when we feed the output of this block to my plant. Then that becomes u of t, then r of t will become equal to x of t. So, we should therefore, have some mathematical function mapping f 1 between r and u which modifies or distorts my reference r of t such that when it is pass through my plant I get it back at the other end.

So, what should f 1 of or what should f 1 look like? A moment start would review that essentially f 1 should be the inverse of f. So, if I want to replace f 1 with the inverse of f, then you will notice that when I u of then you will notice that u of t is equal to f inverse of r. And therefore, x of t will be equal to f of u which will be equal to f of f inverse of r that is therefore, equal to r.

So, in a sense all the controls problems can be reduced to this one simple mathematical goal of obtaining the inverse of the system that we are trying to control. So, the system that we are trying to control is called the plant. And the goal of all control engineers is to get the plant output to be equal to some desired reference r of t.

And that can be accomplished by insulting an electronic system that inverts the model of the plant and cascading that with suitable actuators to get x of t to be equal to r of t.

So, there it is very simple objective, that guides the work of all control engineers both control engineers who use feedback control as well as those who do not use it and that is to invert the plant. As simple as this objective looks there is a catch, it is not always possible to invert the plant and indeed we will see that it is almost never possible to perfectly invert the plant.

But apart from that, there are a few other restrictions as well for one thing we need to have f to be an accurate mathematical model of the plant, f should accurately represent the plant dynamics. I make this statement to point out that ultimately when we write down an equation to determine the input output relationship for a physical system. That equation is written on the basis of some assumptions we make simplifications we ignore terms that are not very significant and so on and so forth.

Now, it is assumed that we have been careful in doing our simplifications and ignoring some terms. In that the final output of the mathematical model is almost equal to the actual output of the physical system. So, that is one prerequisite for us to be able to invert the dynamics of the plant reasonably well and to get x of t to be equal to r of t.

The second condition is that f inverse should be realizable and this we will discover is a very big condition. So, as simple as the problem of control appears, these conditions of f being able to accurately represent the plant dynamics and f inverse being realizable become major road blocks, especially in the context of feedback control. And the third condition is that you should also be realizable, the control effort u just equal to f inverse of r should be realizable.

So, to give a practical example, if the input u is so large that your actuators cannot generate that kind of that magnitude of control effort, then even though it is possible to compute mathematically f inverse of r, physically it cannot be possible to realise f inverse of r. So, you should be physically, you should be realizable f inverse should be realizable and f should accurately represent the plan dynamics. These are the basic prerequisites for us to be able to get x of t to be equal to r of t which is the goal of all of control engineering. However, as I discussed this requirement that we simply obtain the model of the plant and invert it is deceptively simple.

Let us illustrate this by means of an example. Let us consider for instance a first order system and let us take the example oven of an oven and assume that it is governed by first order dynamics. And therefore, the relationship between the input apply to the oven which could be in terms of voltage or current or fuel or whatever to the temperature raise is governed by a first order differential equation.

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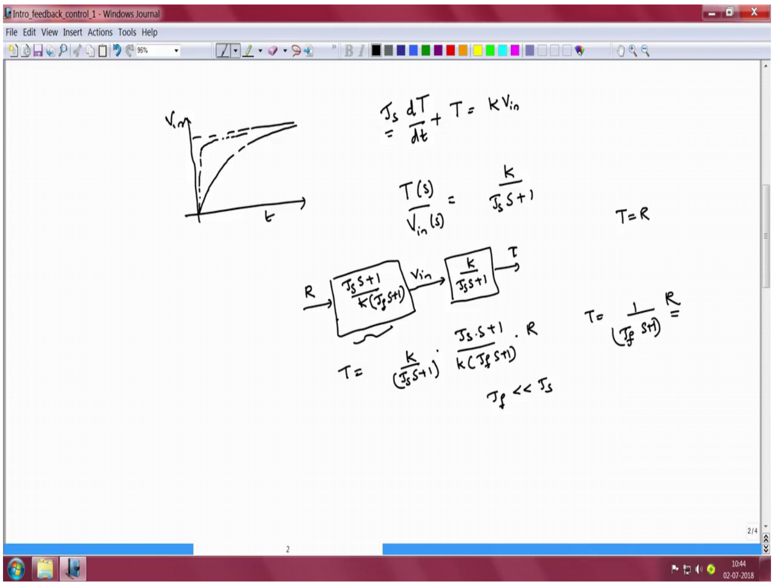

Let us say the differential equation has the following form tau s d capital T by d t plus capital T is equal to K times v in where v in represents the input voltage. And tau s here represents the time constant of the oven.

So, if you provide any step voltage input, then it takes some time for the response of the system to reach its steady state value and that time is characterized by the time constant tau s. So, the first order response of the system to a step input would look something like this. So, if this is a step input v in and this is time and the response of this oven might look something like that. And we say that the response does not really track the reference and hence, we need to invert the dynamic model that we have here for us to be able to track this reference perfectly.

Let us attempt to do this in the Laplace domain by first obtaining the transverse function for the system. So, it is evident from this time domain representation that the transverse function of this system would be something like this namely, that the Laplace transform of temperature T divided by the Laplace transform of the input voltage V in is going be equal to K by tau s times S plus 1.

So, this is the input output relationship for this particular plant. So, I shall write it out here, the input is v in the output is the temperature T. And the transverse function is of the form K by tau s S plus 1. The subscript s for the time constant tau is intended to denote the fact that this is a slow system; most thermal systems such as ovens and coven are slow. Therefore, if I am going to provide a step input to this oven it takes quite some time for the temperature to build-up and reach its steady state value.

Hence, we have denote it the time constant tau with a subscript and called it tau s. Now, if we want this oven to track a certain reference R, so let us draw this reference R here. And we want the temperature T to be equal to R, what do we need to do? Ideally what we need to do is to insult a transverse function here that inverts the plant. In other words, you should have a transverse function of a kind tau s times S plus 1 divided by K.

However, we note that there is a problem in this transverse function namely, that this is a non causal transverse function. The degree of the denominator polynomial is 0, because there is no term in the denominator of this open loop controller transfer function and the degree of the numerator polynomial is 1. And hence, it is not even a proper transverse function let alone ping a strictly proper transverse function.

So, therefore, we see that even for the most simple case of a first order plant, it is not possible for us to perfectly invert its dynamics. What can we do? Instead is this the end of the road for us actually not, we can do a little bit to improve matters and invert the plants dynamics approximately if not exactly.

In order to do this, let me choose the denominator polynomial of this same controller which at this present moment cannot be physically realized to be of the tau f times S plus 1. So, let that be the denominator of this controller.

Now, the numerator degree is 1 the denominator degree is 1 and hence the controller transverse function is a proper transverse function and in principle we can realise this transverse function. So, if we now look at the relationship between the input and output, we note that we would have the relationship to be given by T is equal to K by tau s times

S plus 1 times tau s times S plus 1 divided by K times tau f times s plus 1 times the reference r.

So, what does that tell us? That tells us that tells us that T is equal to 1 by tau f times S plus 1 times R. Now, if we choose the time constant tau f to be much smaller than tau s, then the time constant associated with the temperature raised for the certain input command R of S is going to be much lesser than the time constant associated with the temperature raised for the original oven without any control.

So, if we want to plot the step response of this new case where we have this controller cascaded with the original oven, the step response would look something like this. So, the time constant tau f being much smaller than the time constant tau s, the step response raises much faster and settles down close to its steady state value much faster than for the original oven. Now if we compare the step input and the response, we note that the response is much closer to the input in it is appearance than the original response was.

And hence, we can conclude that we have succeeded in approximately inverting the dynamics of the plant. So, we see therefore, that although at the end of the day all control problems can be boiled down to finding out inverses to the dynamics of the systems that we are trying to control. Even for the simplest case of a first order plant, we cannot think of a planted is simpler than this a planted is more simple and this is a simple proportional plant and that is a trivial case.

So, beyond that the next simple plant is a first order plant and even for the first order plant it is not possible for us to come up with an exact inverse for the dynamics of the plant. We can only come with an approximate inverse and make the response of the overall system relating the output to the input to track the reference much better than what the original plant could, but it is not possible to get it to track the reference exactly.

Now, these problems only get exacerbated when we are dealing with plants of higher order. So, second order, third order and so on. And even worse when we are dealing with plants, that have nonlinearities or that have parameters that are varying with time.

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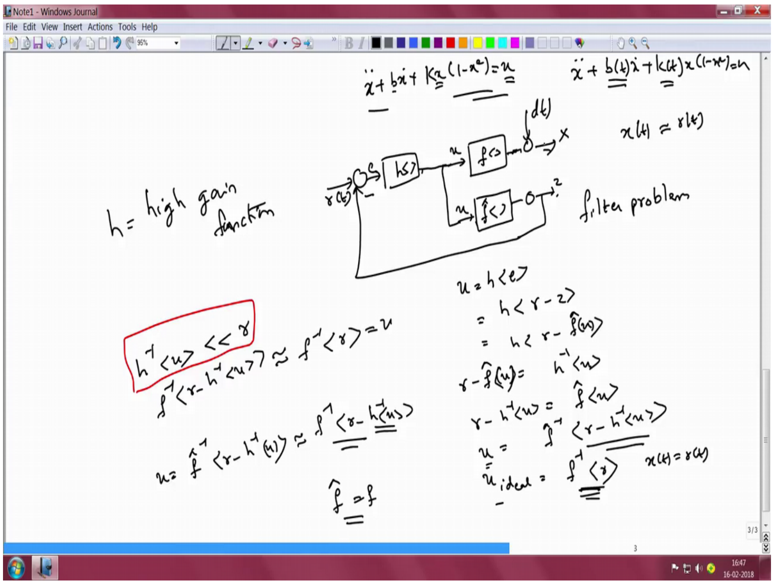

So, for instance, if you want to consider another system whose mathematical model that relates the input and output look something like this, x double dot plus b x dot plus k x times 1 minus x square is equal to u where u is the input and x is the output. Let us assume in this case that we know the coefficient b and k very well. Then we know everything about the differential equation. So, we know the input output relationship very well, but how does one invert this relationship, but is not clear.

So, even though we know, f that relates x and u very well. We do not know how to compute f inverse that relates r and u and therefore, we are still stuck with this problem. So, in the first example, we could obtain the inverse, but that inverse could not be perfect. In this particular example we are able to model the system very well. So, we can realise f very well, but we are not able to realise f inverse. So, it in this context, that we shall explore a slightly different and interesting technique to invert the model of the plant.

So, let us say, we have our plant f which represents the mapping between the input u and output x. Our goal is to somehow get it to track a reference r of t, somehow we have to get x of t to be equal to r of t that is our objective. Now we shall approach this problem in a slightly different way, we shall first make a copy of the plant. So, when I say a copy of the plant, we are coming up with the mathematical model whose input output relationships we are compute, we are computing using an electronic circuit or a computer or some such method.

So, I shall call the input output relationship of the copy as f cap. Ideally we want f cap to be equal to f, but if you do not understand the dynamics very well of the actual system. Then there will be small differences and to underscore the fact this is likely to be a difference between the output of the actual plant and output of its mathematical model I have called is f cap. And the output of this model is z. So, both of them are given the same input u and ideally both the outputs should be identical.

Now, in order to get x to be equal to r, what I shall do is compare the output of my copy with the reference r. So, I shall compare them in other words take the difference between them, I shall do z of t minus r of t. And in the ideal case, we want an error e between the two to be zero, because, we want output of the plant to be equal to r of t. But then that is not going to happen naturally, we have to distort this error in some manner using some function h.

And what our quest now would be to investigate the general properties of h that will allow the output of that namely, u to invert the model of the plant. So, what kind of functions h do we need to choose to get x of t to be equal to r of t. If you look at this block diagram, it is reminiscent of a feedback block diagram, but this is not a feedback system yet. Because we are implementing all of this inside a computer and we are therefore, not depending on external sensors and other such inputs for us to modify the input to the plant. And therefore, this is still what we call as a filter problem.

We are coming up with a suitable electronic circuit or a computer based implementation that allows us to invert the dynamics of the plant. And the prototype solution that we are discussing now has it is particular structure.

Now, from this block diagram, you see that u is equal to h of e. So, h is the mapping, that relates the error e to the output of that block namely u. And e is the difference between r and z, and z we know is f cap of u. From this equation we see that r minus f cap of u is equal to h inverse of u which intern means, but r minus h inverse of u is equal to f cap of u or equivalently u is equal to f cap inverse of r minus h inverse of u.

Now as control engineers, what do we desire u to be we want u ideally, to be equal to f inverse of r. That is for we wanted to be, if you choose this as the ideal u, then our x of t will be equal to r of t. Of course, we cannot do this ideal inversion; we are hoping that this particular prototype solution will at least help us get an approximate inversion. And we are investigating what properties it is needs to have for this approximate inversion to happen.

So, if you compare, what we have got as an expression for u with what we desire as a expression for u. You notice that the actual u will approach the ideal u under two conditions. Firstly, we should have f cap to be equal to f. So, in other words our copy should be a very good replica of the mathematical model of the plant itself that is one prerequisite.

So, that I can replace f cap inverse here by the term f inverse and the second thing is that if you look at the expression u equal to f cap inverse of r minus h inverse of u. This would be approximately equal to f inverse of r minus h inverse of u when this particular condition is valid. And if you compare this expression with what we desire, We see that we are very close to what we desire. With the exceptional we have this extra term as one of the independent variables for the function f inverse.

Now, therefore, if h inverse of u is a very small quantity, in other words if h inverse of u is much less than r. Then what you see is that f inverse of r minus h inverse of you would approximately be equal to f inverse of r itself. And if this is so, we know that this guy is going to be equal to u and so approximately therefore, we end up we succeed in inverting the plant.

So, the necessary condition for this prototype block diagram to help us in approximately inverting the plant is to get this to happen, h inverse of u should be much smaller than r. What this indicates about a general nature of the function h or the mapping h is that the input output relationship for the function h inverse should be such that the output of the function should be much smaller than the input.

Equivalently what this indicates about the function h itself is that the function h should be a high gain function. In other words if h inverse is a function which whose output is going to be a very attenuated version of it is input. Then the mapping h should be one where the output of h should be highly amplified version of the input to that function.

So, in other words h should be a high gain function; what is very interesting about this prototype solution where we have h as the high gain function is that, h need not in any way be related to the plant f which was trying to invert. So, you can choose any high gain function and in principle we able to invert the plant dynamics. However, there is a small catch. So, we have a certain dynamics for this copy here and this dynamics can potentially be unstable and the concern of instability is exacerbated when you are having a high gain function in the forward path.

So, assuming that the dynamics of this copy model that we have in our computer is stable, then our problem of obtaining an approximate inverse to f is done. So, what is interesting about this is that it allows you to obtain the approximate inverse even for plants such as this for which you cannot directly compute f inverse. But the application of this is even more general.

So, suppose we had the differential equation to be of the kind x double dot plus b of t times x dot plus k of t time x times 1 minus x square is equal to u. Even for such a situation we can employee, this particular prototype solution to obtain the inverse and get the plant to follow the reference r of t.

But there are accessions when these coefficients b and k are not known very well. And it is in that scenario that this prototype solution, this prototype approach takes now. And that is because; in order for this entire strategy to work, we need to have an accurate copy of the plant dynamics.

Now, when our plant model changes in time and in a manner that we cannot easily predict then, we do not have a good enough of copy at all times t for the plant model. This is one circumstance under which this breaks stop. There is one other circumstance namely, when we have uncertainty associated with the environment in which the plant operates. Mathematically what that means, is that my plant is afflicted by a disturbance d of t. And if I can measure d of t I can incorporate that measurement also in my copy and eliminate it is effect.

So, this can be x, this can be z, I can eliminate it is effect using the same technique. However, if I cannot measure d of t; so, if this cannot be measured then, this particular approach will not allow me to compensate for it is effect. So, it is in this context that as good as this prototype solution is we cannot invert the plant the way you would like it to be inverted.

So, the solution to this problem is to replace the copy of the plant by the actual plant itself and the copy of the disturbance by the actual disturbance itself. And for that to happen we have to invest in the sensor. So, that instead of measuring z or instead of looking at the variable z, we directly look at the variable x itself.

So, if you invest in the sensor and measure x and implement the same prototype solution, then you will be able to invert plants, whose parameters might change with time in a manner that we do not understand and which might be afflicted by disturbances that we cannot measure. And this approach to inverting the plant is called feedback control. And this we shall look at in greater detail in the next clip.