Control System Design
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Lecture - 44
Introduction to nonminimum phase systems

Hello, in this clip we will talk about a particular class of physical systems which impose special constraints on our ability to design controllers for them. So, these systems come under the umbrella of what are known as nonminimum phase system and the reason that a, are called nonminimum phase systems is something that we shall get to in a few minutes time. So, if you look back at a all the control techniques that we have discussed. We have noticed you might have noticed that the plant generally takes a backseat. So, we assume a generic plant and, we do not assume that the plant imposes any special restriction on our abilities has control engineers.

That is no so, in the case of nonminimum phase system. It turns out that nonminimum phase plants are particularly difficult to control and they impose fundamental limitations on our abilities as control engineers. So, for instance, it is sometimes possible that if you have a nonminimum phase plant it may not be possible to achieve a desired level of, error in rejection of disturbances or a desired level of accuracy in tracking references or a desired level of robustness, when it comes to plant parameter variations.

So, in contrast if, you take a minimum phase plant you can accomplish all of these things, but often if, the requirements for disturbance rejection and, robustness and so on are very stringent. We would I have to invest in fairly high bandwidth and expensive controllers, but theoretically for a minimum phase plant which are the kinds of plants that we have been looking at so, far there are no fundamental limits as to what we can accomplish has control engineers. In contrast for as I said in nonminimum phase plants there are fundamental limits and there are cases when it is actually impossible for us to reject disturbances or achieve robustness to plant parameter variations or track certain references.

So, there are 2, kinds of plants which come under the umbrella of nonminimum phase plants one are plants with time delay and second are plants with right half plane zeros or in other words if, I have a plant P that is given by P 1 of s times e to the power minus sT then, such a plant is afflicted by time delay. So, this term e power minus sT represents, Laplace transform of the delay operator and hence the input output relationship for this plant has a delay incorporated into it.

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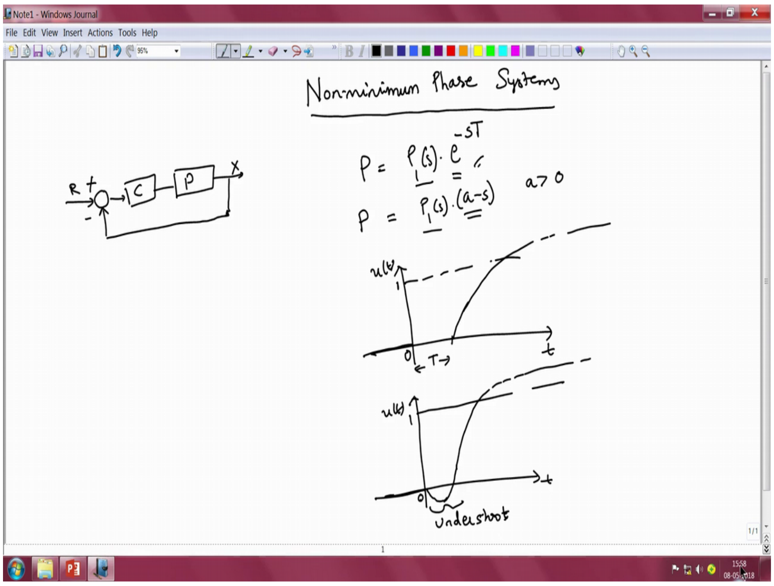

So, this is one class of plants that come under the umbrella of nonminimum phase plants; the second are plants with, right half plane zeros. So, for a plan to the right of plane zeros the transfer function would look like P is equal to P 1 of s times a minus s, where, the 0 namely s is equal to a is greater than 0. So, if P 1 in both cases is a transfer function all of these polls and 0's are on the left half of the complex plane. Then a plant with of the kind P is equal to P 1 e power minus sT is a plant with a time delay and a plant of the time P is equal to P 1 times a minus s is a plant with one, right half plane 0. So, such plants are called nonminimum phase plants.

We shall first take a look at plants with time delay and then subsequently, take a look at, plants with nonminimum phase zeros and see what special problems both of these cause for us control engineers. Before we start discussing plants with time delay let us first see, how we can tell whether term of this kind either time delay or, right half plane zeros exist in our plant or not from our identification exercises, from our model identification exercises.

So, for instance if a plant has a time delay and if I provide a step input to the plant. So, as function of time I have the input u of t which is a step it was 0 from for t less than 0 and became 1 for t greater than or equal to 0 if I have a time delay capital T as indicated here the plants response will start t units after application of the input. So, there is a duration of t units during which time the plant would not respond and it responds only after 2 units and the response is going to be characterized by the step response of the plant P 1 of s. So, the plant P 1 of s let us say, has a step response that look something like this then the plant with time delay would respond in a manner that is shown in this graph here it responds T seconds after the application of the input. Similarly, if a plant were to have a right half plane 0 then, one can show that if one to apply a, step input.

So, let us say we apply the same step input u going from 0 to 1 at time T is equal to 0. A plant with a right half plane 0 of the kind that we have shown here we will respond by first moving in the opposite direction in this particular manner and then coming back up and then finally, moving in the right direction and reaching it is steady state value this phenomenon is called undershoot. So, a plant with a right half plain 0 can be easily dissent can be easily identified by looking at it is step response, for such a plant when you apply in a step input the plants response first changes in the opposite direction as the sign of the input that has been provided and subsequently reverses it is direction and moves in the same direction as that of the, as that of the apply input.

Likewise if, you have a plant with the time delay the plant does not respond at all for some duration of time and subsequently you see it is response. Thus, this time domain responses, of plants can be use to tell if these plants have been afflicted by these particular melodies; namely time delay and right half plane 0. I call them melodies because these particular terms right half planes 0 and the time delay impose fundamental restrictions on our abilities as control engineers to control plants, but include them. So, with this preamble let us first look at how, time delay can affect the performance of a plant.

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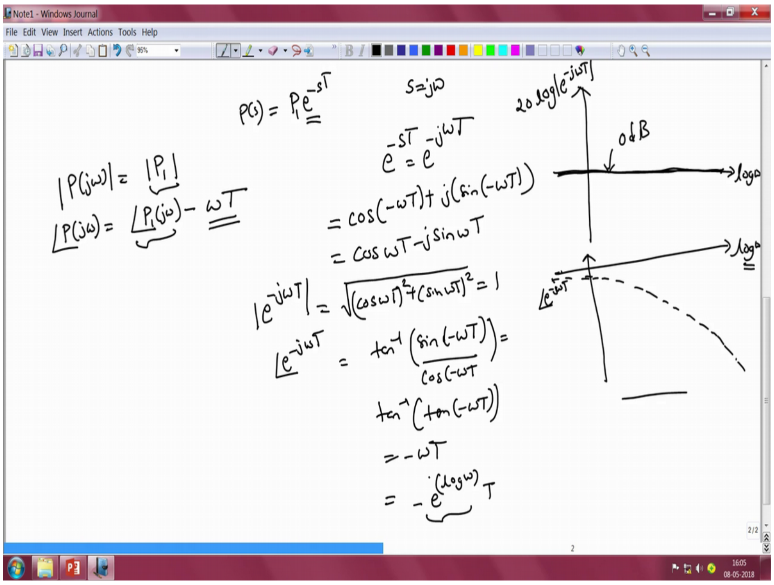

So, let us return to the transfer function of the plant with a time delay which is going to be equal to P 1 e power minus sT and this is going to be the plant transfer function P of s. Now, let us focus on the term e power minus sT and sketch the Bode plot of just this term alone because, the Bode plot of the plant P is going to be equal to the Bode plot of the plant P 1 plus the Bode plot of the term e power minus sT. And if, P 1 is a transfer function all of whose poles and zeros are on the left of the complex plane the Bode plot of P 1 is something that we must be very familiar and comfortable with having discussed such plants over the entire of this course. So, let us focus on what is new as for as we are concerned, namely the term e power minus sT.

So, in order to draw it is Bode plot up course we have to first set s is equal to j omega, in which case we would have e power minus sT to be equal to e power minus j omega T. Now, we know that e power minus j omega T is essentially given by cos of minus omega T, plus j times sign of minus omega T, or in other words it is given by cost of omega T minus j sign omega T.

So, what is the magnitude of the term e power minus j omega T? The magnitude of that is the square root of the squares of real part plus the imaginary part namely cos of omega T, the whole square plus sign of omega T the whole square that equal to 1. Therefore, this term e power minus j omega T does not modify the magnitude characteristics of the plant P 1 at any frequency, but all frequency is the gain of this term e power j minus omega T is going to be equal to 1. What about the phase of the, term e power minus j omega T. So, to determine the angle of the term e power minus j omega T we should take the tan inverse of, the imaginary part by the real part that is given by tan inverse of sin of minus, omega T divided by cos of minus omega T or in other words is going to, be equal to tan inverse of, tan of minus omega T, and this is going to be equal to minus omega T.

Hence, this term contributes a phase lag that is proportional to the frequency omega. So, if we want to draw the magnitude plot of the term e power minus sT. So, the x axis is log omega, the y axis is 20, log magnitude of e power minus j omega T. We get a flat curve that is coincident with the x axis of the plot because it is going to be 0 dB the gain is going to be equal to 0 dB or equivalently again it going to be equal to 1 at all frequencies omega. As for as the phase plot is concerned, x axis again log omega and the y axis will be the angle of e 2 the power minus j omega T.

We note that the angle of e power minus j omega T is minus omega T and this can be written as minus of e to the power log omega times, T. I have done this manipulation because the x axis of our Bode plot is log of omega what this means therefore, is that the Bode plot of the phase plot of time delay is going to be an exponentially increasing function multiplied with minus T, and if we look something like this. So, this is going to be the phase plot of e to the power minus sT and the magnitude plot is going to be equal to 0 db. So, therefore, if we have the plant P of s which has a time delay, in the magnitude of t of j omega is always going to be equal to the magnitude of the minimum phase transfer function P 1. However, the angle of P of j omega, is going to be equal to the angle of P 1 of j omega, the minimum phase part of the, expression for P of s minus the angle of e power minus sT or minus e power j omega T and which we discover is equal to minus omega times T.

So, this term adds only phase lag and does nothing to the magnitude characteristics. And this is precisely why this term is called a nonminimum phase term; it is because if you look at the plant P the magnitude characteristics of the plant P has a certain phase characteristics associated with it in the absence of the time delay and that is given by the angle of P 1 of j omega. Now, if t were a minimum phase transfer function then we can use Bode's gain phase relationship to extract the phase namely angle of P 1 of j omega given the magnitude namely magnitude of P 1 and j omega at all frequency omega; however, because we have a time delay as part of the terms of function for P of j omega.

We note that the phase lag associated with the magnitude characteristics of P 1 is larger than the phase lag of a minimum phase a transfer function P 1 of j omega and that phase log that excess phase lag is given by omega T. So, that extra phase lag that is associated with the magnitude characteristics of a transfer function of has that a time delay unit is what earns it the name of a non minimum phase system. So, let us now, see what, this kind of a Bode plot this kind of a magnitude and phase characteristics combination can do to the stability of our closed loop system.

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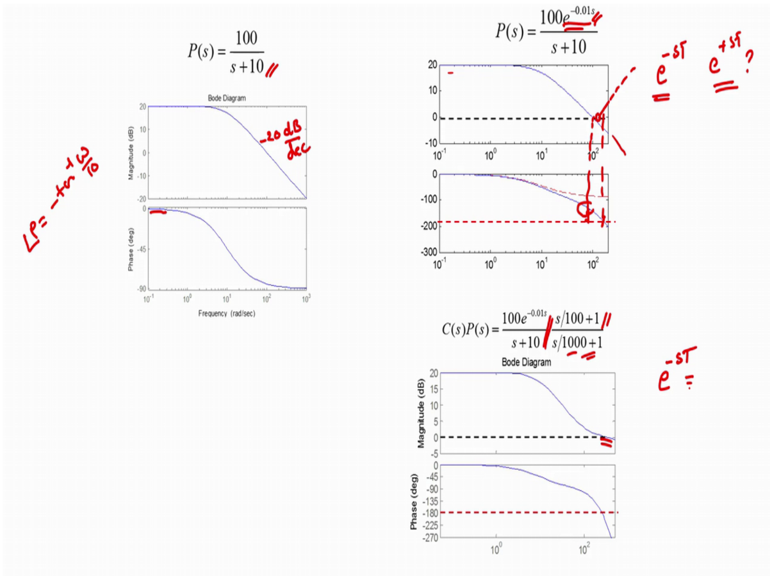

So, what I have taken here is a first order plant P of s is equal to 100 by s plus 10 and it is evident from inspection that the dc gain of this plant is 10 units and it is corner frequency is at 10 radians per second. So, the Bode plot of, this such a first order plan, is as shown on the left hand side it has a dc gain of 20 decibels and at the corner frequency of 10 radiant per second there is a role of and the magnitude role of is at a rate of minus 20 decibels per decade.

And as far as phase is concerned it is obvious from inspection that the phase of P is going to be given by minus of tan inverse omega by 10, and if this were to be ported as function of frequency you note that at very low frequencies of phase is 0 and as omega tends to infinity the, phase tends to minus 90 degrees. I have pointed out in past that a plant of this kind a first order plant is a dream come true for us control engineers simply because such a plant does not have any phase crossover at all.

Since it does not have any phase cross over there is no issue of instability induced by simple controllers such as a proportional controller. So, it is very easy to work with a plant that has a first order dynamics governing it is input output relationship and is therefore, a blessing and a boon for us as control engineer. If, we are having to, if we have to control first order plants. Now, suppose the same first order plant were cascaded with a small time delay. So, this time to a could arise as result of, the process which has first order dynamics, but there is some delay in making measurements after the actuation has been done.

So, to give an example of how such a time delay could arise one can think of the example of a factory, where steel sheets are being fabricated. So, you can control the thickness of a steel sheet only when the sheet is red hot, but you can measure the thickness of the, steel sheet only after it has cool down; because, such hot sheets are likely to damage your thickness sensor therefore, they has to be some distance that the steel sheet has to travel, during which time it can cool and subsequently you can measure it is thickness. So, there is extra time delay, that is involved between the action of controlling the thickness of the steel sheet and making it is measurement, and that time delay is what could contribute to a term of the kind e power minus sT that is cascaded with the dynamics of the plant itself.

So, in this particular example I have assumed that there is a very small delay of just 10 milliseconds. So, that our time delay term e power minus sT is of the form e power minus 0.01 s. Now, if you were to draw the Bode plot of this new plant which is now afflicted by this time delay. We first note that there will be no difference between the magnitude plot of this plant and the magnitude plot of a plant which has only the first order dynamics namely 100 by s plus 10 because the time delay term does not contribute to the magnitude characteristics; it has a gain of 0 dB at all frequencies.

So, when we add that to the magnitude characteristics of the plant we just get the magnitude characteristics of a plant itself. So, the dc gain will be 20 decibels and at the corner of frequency of 10 gradients per second we will start a role of minus 20 decimals per decade for all future frequencies. What about the phase characteristics? The phase characteristics of the plant with the delay will be equal to the phase characteristics of the plant itself minus omega times capital T; where, capital T is a time delay in this particular example that I have considered I have chosen capital T to be 0.01 seconds or in other words 10 milliseconds and as a consequence of having chosen a plant with time delay what you notice is that the phase characteristics is gradually decreasing as function of frequency and there is one particular frequency at which the phase crosses over or in other words the phase lag becomes greater than 180 degrees.

So, what you see therefore, as the effect of time delay is that initially you had a plant which did not have any phase crossover and therefore, stability of a plant in cascade with a proportional controller was not even a concern for us as control engineers. But, as a consequence of introducing a small amount of time delay of just 10 milliseconds we have got a plant which had no phase crossover to now poses a phase crossover and because you know have a phase crossover we need to worry about the gain margin and whether the gain margin is adequate or not in the interest of stability of the close loop system. So, what was earlier a problem that did not have stability as a concern at all has now become a problem where stability has to be worried about as a result of the phase lag contributed by the term e power minus sT.

Now, you might wonder whether it is possible to make this phase lag go away for instance can we multiply e power minus sT with a term of the kind e power plus sT is this possible a moments thought will revile that this is not, practically possible because the term e power plus sT is a non causal transfer function; if, you are saying that you are able to multiply the existing plant with the term e power plus sT. So, essentially, you are essentially stating that you are able to read into the future you are able to predict the response of a system even before an input has been apply which is not a practical proposition.

So, e power plus st is a non causal transfer function and hence we cannot implement it and hence we have to live with the phase lag characteristics at a introduced by the term e power minus sT. Now, in order to see why it is difficult to deal with a plant with a phase characteristic and magnitude characteristics such as the one that is shown here what, I have done is try to implement a controller for a plant of this type. Now, suppose at, gain crossover frequency you have a certain phase margin, but you want a improve the phase margin then, in the previous lectures, we have noticed that adding a 0 slightly to the right off the gain crossover frequency almost as to add a phase lead of plus 45 degrees near the gain crossover frequency and hence improve the phase margin.

Now, when we did this in the case of a minimum phase plant we already had a certain role of for the minimum phase plant beyond the gain crossover frequency and that role of tended to become lesser as a result of having added a 0, slightly to the right off the gain crossover frequency. Now, in this particular case we already have just a first order plant. Now, if I want to add a 0 to a first order plant I will have a 20 dB per decade raise for frequency is greater than the frequency at which 0 has been added. So, that for frequency is beyond the 0 the 20 dB per decade raise will cancel the minus 23 dB per decade role of and the gain will flatten out and what is means it that, I cannot add any further 0's to improve the phase characteristics. Because, any further 0 will cause the gain characteristics once again increase and not role of as we intended it to do and that is precisely what has happened when I try to multiply this particular transfer function namely the transfer function of the plant with a lead compensator I noted the gain crossover frequency to be somewhere close to 100 radians per second.

So, I added a 0 at 100 radians per second and in the interest of causality I added a pole very far away at around 1000 radians per second and what we notice is that unlike in the minimum phase case where the magnitude characteristics resulted in a certain phase characteristic in a nonminimum phase case for the same magnitude characteristic we have a much larger phase lag characteristic. So, when we add this, lead compensator we note that the magnitude character slope is tending to 0 and it is preventing us for adding any further 0's in order to improve the phase characteristic beyond what has been accomplished here. So, we note that even the tricks that your able to employees so powerfully in the case of control of minimum phase plants find difficulty, when we try to apply them in the case of nonminimum phase plants.

So, all of these examples have been taken to underscore the fact that nonminimum phase terms such as a time delay, cause curious problems to us as control engineers because the modify only the phase characteristics and not the magnitude characteristic and the modification is also not in favor of improving control performance; the add excess phase lag. And therefore, they cause the phase to either cross over, crossover earlier if, there was already a phase crossover frequency for the plant and therefore, reducibility margins of the overall system.

Now, suppose one wants to analyze how the, term e power minus sT affect stability of our close loop system using tools such as the root locus where, needs to be able to locate the open loop poles and zeros of this term; however, since we cannot represent the term e power minus sT as the ratio of two polynomials. We can only come up with approximate rational transfer functions or ratios of two polynomial which will result in a magnitude and phase response that is close to what is produced by the term e power minus sT. So, let us now look at, how we can come up with transfer functions which are the ratios of two polynomials or which have poles and zeros in them the approximate the magnitude and phase characteristics of the term e power minus sT.

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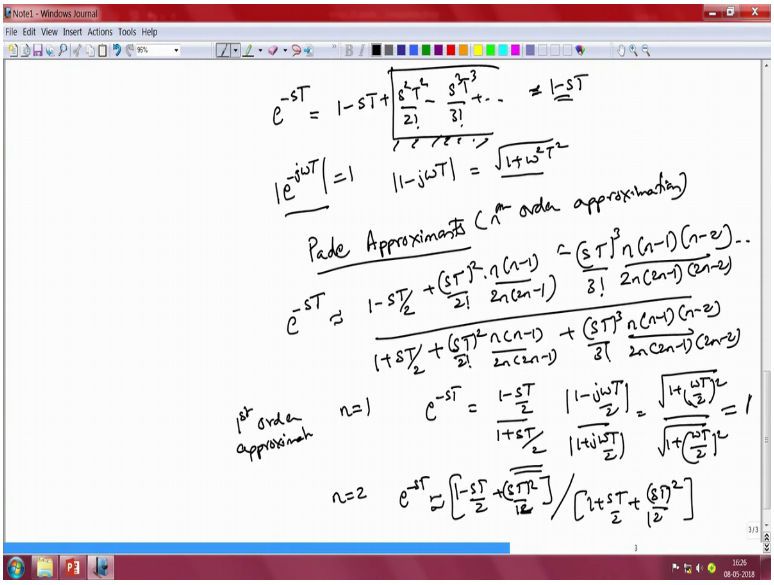

So, if one wants to approximate the term e power minus sT or in particular if one is interested in coming up in, approximation in the interest of drawing a Bode plot or the root locus what might be tempted to simply obtain a Taylor series expansion for e power minus sT as a polynomial in s and write it out as 1 minus sT plus s square P square by 2 factorial, minus s cube t cube by 3 factorial, and so on and so forth. So, one might wish to do this and then in order to approximate e power minus sT one might choose to ignore all the higher order terms and simply approximate it as 1 minus sT.

Now, is this a correct approximation; you will notice that is not a very good approximation because the magnitude of e power minus sT when s is equal to j omega, which is given by, magnitude e power minus 0 omega 2 is going to be equal to 1 for all frequencies omega. So, on the other hand if, you look at what we have on the right hand side 1 minus sT. The magnitude of 1 minus sT for s is equal to j omega is given magnitude 1 minus j omega T, and that is going to be equal to square root of 1 plus omega square T square.

So, unlike the magnitude characteristics of the function that we are supposed to be approximating namely, e power minus j omega T which has a constant value of one at all frequencies; the approximation that we have attempted to make namely 1 minus sT has a magnitude characteristics that increases with frequency and hence this is not a good approximation for the term e power minus sT.

So, a good approximation is one which whose magnitude characteristics are going to be identical to the magnitude characteristics of the time delay, but, of the time delay transfer function; but, the phase characteristics approximate the phase characteristics of the time delay only up to a certain frequency. Such approximations were provided by Pade, and they go by a name of Pade Approximates. So, I shall first right out the general, transfer function that, Pade came up with in order to approximate the time delay transfer function e power minus sT.

So, e power minus sT is approximately equal to 1 minus sT by 2 plus sT by whole square by 2 factorial times n times n minus 1, divided by 2 n times 2 n minus 1, minus sT the whole cube divided by 3 factorial, times n times n minus 1 times n minus 2, divided by 2 n times 2 n minus 1 times 2 n minus 2, and so on and so forth, divided by, 1 plus sT by 2, plus sT by 2, sT the whole square by 2 factorial times n times n minus 1 by 2 n times 2 n minus 1 plus sT the whole cube by 3 factorial.

So, whatever had a negative sign in the numerator will have a positive sign in the denominator and whatever had a positive sign in the, numerator we will continue to process a positive sign in the denominator as well. So, you would have the third term to be sT the whole cube 3 factorial times n times n minus 1, times n minus 2 by 2 n times 2 n minus 1, times 2 n minus 2, and so on and so forth. So, this is the general n th order approximation, for the time delay.

So, for obtaining the 1 st order approximation we set n is equal to 1. So, this is the 1 st order, approximation; and if, you do that we notice that the terms that contain sT the whole square by 2 sT the whole cube by 3 factorial so on will get we will vanish because n minus 1 will become equal to 0 in all those terms and we would be left with the first order approximation of e power minus sT, to be equal to 1 minus sT by 2, divided by 1 plus, sT by 2. And in this case if you were to evaluate the magnitude of the approximation namely magnitude of 1 minus j omega, T by 2 divided by magnitude of 1 plus j omega, T by 2 we will notice, is equal to square root of 1 plus omega T by 2 the whole square, divided by square root of 1 plus omega T, by 2 the whole square that is going to be equal to 1.

So, unlike our first, amateurish attempt at approximating e power minus sT the more accurate approximation namely 1 minus sT by 2 by 1 plus sT by 2, gives the correct magnitude characteristic for the transfer function for all frequencies; because the term one minus sT by 2 by 1 plus sT by 2 as a magnitude equal to 1 at all frequencies, just like the term e power minus j omega T. Now, the 2 nd degree approximation is obtained by settling n is equal to 2 and that is given by e power minus sT, is approximately equal to 1 minus sT by 2, plus sT by sT the whole square by, 12 whole divided by 1 plus, sT by 2 plus sT by 2 the whole, sT by whole square divided by 12.

So, this is a 2 nd degree approximation, we can have higher degree approximation higher order approximations as well; and these each of these approximations are valid over different frequency ranges. So, the 1 st order of approximation is valid over a certain frequency range it is determine by a corner frequency of the pole and 0 that for using for approximating the, ah, transfer function e power minus sT. And the 2 nd order approximation is valid over a wider frequency range the 3 rd order approximation is valid over an even wider frequency range.

So, the larger is the value of n that we picked the larger will be the frequency range over which the approximation would be valid. So, all of this work very well provided the time delay is rather small in which case we will have some small problem associated with instability, but it will not be fatal. It will not prevent us from stabilizing the closed loop system and achieving to a reasonable extent our performance specifications; however, when the time delay is very large it can literally cripple the performance of a closed loop control system. So, let us illustrate this by means of a numerical example.

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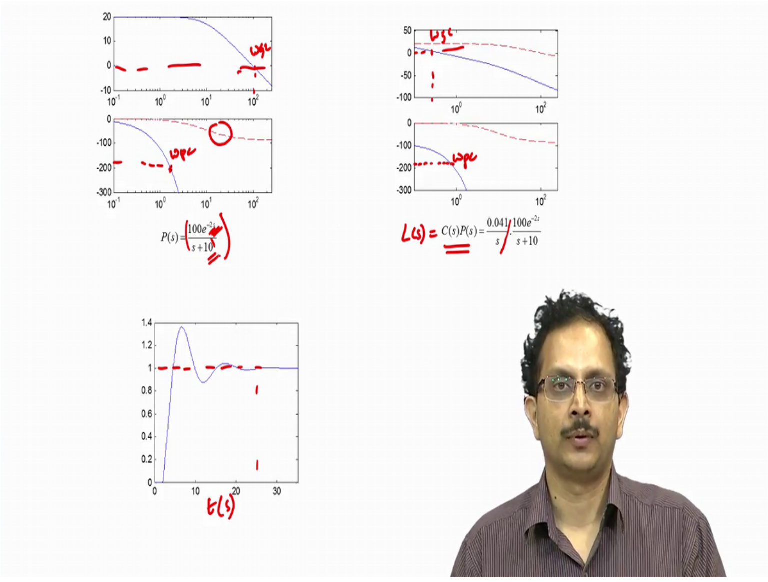

In this numerical example, I have consider the same plant namely, P of s is equal to 100 by s plus 10, the minimum phase plant is exactly same as what we discussed, a few minutes back. But, this time instead of assuming that it is the time delay of just 10 milliseconds; I have assume that it is a time delay of about 2 seconds. So, that the term associated with the time delay transfer function would be given by e to the power minus 2 s.

Now, if we have this as our actual plant, we note that the magnitude characteristics of this plant will be identical to the magnitude characteristics of the minimum phase component of the plant transfer function namely 100 by s plus 10 that is not going to change. The phase characteristics of the minimum phase part is shown here it changes only from 0 to minus 90 degrees exactly as we discussed before. But, the phase characteristics of the overall plant which includes that of the time delay is going to result in much larger phase lag when compared to the phase characteristics of just the minimum phase part alone. In fact, because the time delay is rather large namely 2 seconds the phase lag is exceedingly large and very quickly it reduces to fairly large negative values and it crosses over at a frequency that is much less than the gain crossover frequency.

So, the gain crossover frequency here is somewhere near 100 radians per second whereas, the phase crossover frequency occurs much before 100 radians per second somewhere near somewhere between 1 and 2 radians per second. And the phase lag continues to increase as we increase frequency and it becomes very large unmanageably large at higher frequencies.

So, what is the way forward? We discuss that we cannot really deal with extremely large phase lags by using the tricks that we have adopted when we were dealing with, minimum phase transfer functions; especially, in a case like this where the phase lag near 100 radians per second may be so large that adding 0 for a controller is not even a practical proposition; we have to all together give up hopes of being able to improve the phase characteristics of the overall system. So, the only way forward for us is to reduce the gain characteristics to such a value that the gain crossover frequency omega gc which is now much greater than the phase crossover frequency omega pc; now, becomes less than the phase crossover frequency unless the gain crossover frequency is less than the phase crossover frequency we do not have a stable closed loop system on our hands.

So, our first and foremost goal as control engineers now has to be to attenuate the gain characteristic to such a point that the phase crossover frequency becomes greater than the gain crossover frequency. Now, we choose a simple proportional controller to attenuate the gain then we end of attenuate in the gain over the entire frequency range, including at very low frequencies where there is a possibility for us to get some respectable performance in terms of tracking of references or rejection of disturbances. So, instead of using a proportional controller I have chosen to go with an integral controller and chosen the gain of the integral controller to be an extremely small value. So, in this case I have chosen the transfer function of the controller to be 0.041 divided by s and the gain of the integral controllers 0.041 and that small gain has been intentionally chosen. So, that the phase crossover frequency of the overall system, is going to be larger than the gain crossover frequency.

So, the phase crossover frequency happens somewhere around, 1 radian per second and as a consequence of choosing the controller to have a gain of 0.041, the gain crossover is happening at a frequency that is a little bit less than 1 radian per second. So, our omega gc is now going to be less than omega pc and we will end up having a stable control system on our hands. But what is the price that we have paid in order to stabilize our closed loop system, it is evident from the bandwidth that we have accomplished. The bandwidth of a plant itself the gain crossover frequency of the plant itself was around 100 radians per second, but as a consequence of having a time delay cascaded with it namely e power minus 2 s. We had to attenuate the gain of the overall open loop system to by such a large magnitude that we now have a gain crossover frequency that is more than 2 orders of magnitude smaller than the gain crossover frequency of the plant itself. So, were closed loop bandwidth therefore, is going to be over 100 times less than the bandwidth a plant itself.

So, if we have this as our open loop system then, if we plot the step response of the closed loop system with C of s times P of s being the loop gain namely, L of s of our open loop system. Then pc the step response to look something like this, of course, it is stable and of course, it is dc gain is also unity as a consequence of having chosen, an integrator as our controller. But look at the times gain in which the, closed loop system settles down to it is final value. If you look at the x axis which is the time plotted in seconds you note that the time it takes to settle down to it final value is a little bit more than 20 seconds.

Now, to decide whether this number is large or small we need to only pay attention to the pole of the plant which is the one that normally determines the time scale of response of the plant itself. If you look at the pole of a plant you notice that the plant has a pole at, s is equal to minus 10 which implies that the settling time of the plant itself is simply one-tenth of a second or in other words 100 milliseconds. So, the plant is so fast that it can settle down to it is final value in 100 milliseconds.

But, because we have a time delay associated with the plant because as a 2 second time delay, when we tries to obtain when we try to close the loop and obtain a feedback control system we notice that the settling time of the feedback control system becomes exceedingly large, it becomes something close to 25 seconds. And this implies therefore, that such a feedback control system cannot be employed to track references of frequencies that are greater than the gain crossover frequency which is namely around 1 radian per second also. And, hence and also reject disturbances, whose frequency content is greater than the gain crossover frequency of our closed loop system.

So, we see that a charge time delay, you can have a fatal effect on our closed loop system and what we shall see a little while later is also that these, numerical examples which have revealed to us certain problems associated with plants control of plants that have time delays. Actually, result in fundamental limitations on our abilities to control engineers; although, the fundamental limitations are not evident in the examples we have taken. The examples reveal that there are problems associated with their control. We shall see a little while later that there are fundamental limitations on the achievable gain crossover frequency, the achievable phase margin and so on and so forth if, you are given a nonminimum phase plant.

Thank you.