Control System Design
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Module - 09
Lecture - 43
Ideal Bode Characteristic (Part 2/2)

Hello. In this clip, we will continue our discussion on the Ideal Bode Characteristic, which attempts to estimate the minimum price that we need to pay as control engineers for the benefits that we would want to reap by employing feedback control. So, if we have been specified that a certain performance requirement, again of M units is required up to a certain frequency omega 1. In the previous clip, we saw what the minimum gain crossover frequency that we can achieve given these specifications is. If we know what kind of phase margin expectations, we have for our closed loop control system.

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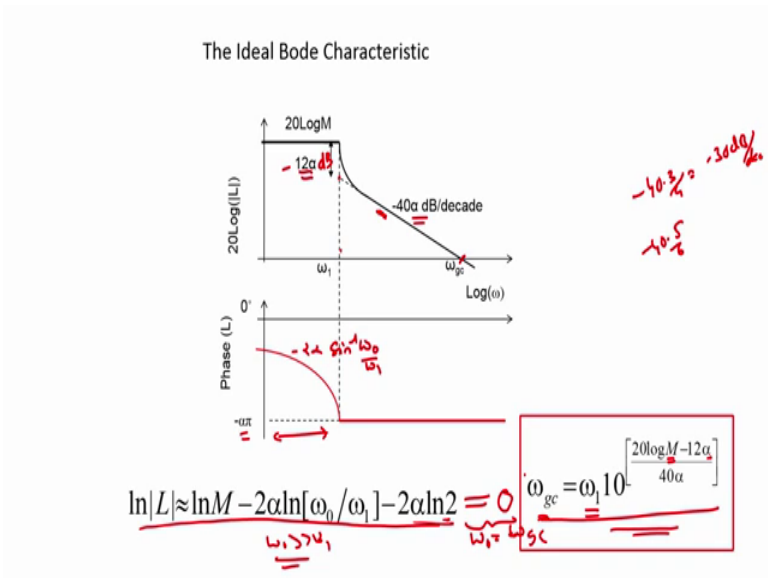

So, I have shown in this slide, the magnitude of L and the phase of L as function of frequency. So, as we see up to omega 1, our specification was that the magnitude of the loop gain had to be M units. And beyond the frequency omega 1, the phase had to be equal to minus alpha pi, where alpha was a number that was less than 1 and this ensured our loop gain has a adequate phase margin. So, we wanted to decrease the magnitude characteristic as quickly as possible, while ensuring at the same time that our phase lag requirement of minus alpha pi is met in the frequency range greater than omega 1.

And we discovered that the magnitude characteristic looks something like this as shown in the schematic here. And for frequencies that are much greater than omega 1, the magnitude characteristic rolls off at minus 40 alpha dB per decade, where minus alpha pi represents the maximum permissible phase lag for our loop gain. So, the minimum gain crossover frequency is the one at which this particular magnitude characteristic crosses the 0 dB line and we derived that to be given by this particular expression at the right bottom corner here.

But, if you look at this expression, we note that this term alpha is in general, not really an integer value. So, if you want a phase margin of 30 degrees, we saw in our example in the previous clip that alpha would be 5 by 6. So, the role off would be minus 40 times 5 by 6 decibels per decade, which is a very strange number. And how does one practically accomplish this kind of an unnatural roll off, it is by choosing strategically located poles and zeros for the controller that injure that the final characteristic would roll off at this unnatural slope of minus 40 times 5 by 6 decibels per decade.

Likewise, when the phase margin requirement was 45 degrees, we saw that we got alpha to be 3 by 4. In other words, we would have the roll off to be minus 40 alpha or in other words minus 40 times 3 by 4, which is essentially minus 30 dB per decade. Now, once again a roll off of minus 30 dB per decade cannot be realized by using a simple distribution of poles and zeros for the controller. One needs a fairly complicated distribution and strategically located positions for the poles and zeros of the controller, in order to get the roll off to be this unnatural value of minus 30 decibels per decade.

So, therefore, the question that we need to answer is fine, we need this unnatural roll off in order to get our gain crossover to happen as early as possible. We wanted a gain crossover to happen as early as possible, because we wanted to minimize the closed loop bandwidth and thereby minimize the effect of measurement noise and also reduce the cost of the control system.

But, the next question that we need to answer is how long do we continue to have this kind of an unnatural roll off of minus 30 dB per decade or minus 40 times 5 by 6 whatever the number might be. So, these kinds of roll offs require as to strategically locate poles and zeroes and choose many poles and zeros for our controller in order to realize such roll offs.

So, the natural question is up to what frequency do we need to continue to have this particular kind of roll off. And beyond what frequency, can we allow the final characteristics of the open loop system to set in. So, by final characteristics, I mean the characteristics of the loop gain at frequencies omega that is much greater than all the corner frequencies of the different poles and zeros of the loop gain. So, what is the smallest frequency omega 2 at which the final characteristics of the loop gain can be allowed to set in. This is the question that we will try to answer in this clip.

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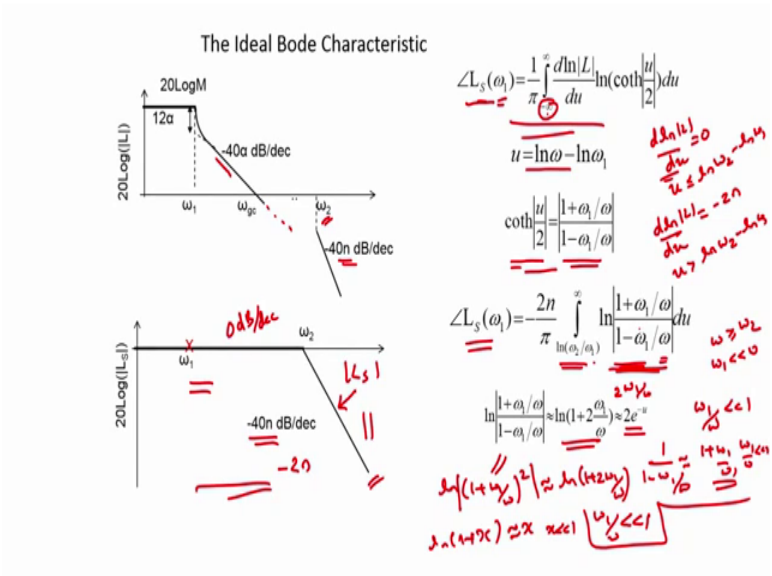

So, I have shown the same problem here. We have our magnitude to be a constant equal to 20 log M decibels up to frequency omega 1. And in the interest of reducing the gain characteristics as quickly as possible, we discover that the characteristics should look as shown in the schematic here. So, this characteristic will continue down at minus 40 alpha decibels per decade. But, as we discussed minus 40 alpha decibels per decade is an unnatural roll off, because they does not correspond to the final roll off of any particular transfer function, may it be an integrator or a first order filter or any such terms of function.

Hence, one needs to have several poles and 0's to realize a slope of approximately minus 40 alpha decibels per decade. So, there needs to be a certain frequency omega 2 at which we stop continuing to have this kind of a roll off and allow for the final roll off of the loop gain to set in. So, as I said the final role of refers to the roll off of the loop gain at frequencies that are much greater than the corner frequencies of the loop gains poles and 0's.

And the question is what is that frequency omega 2 at which the final roll off can be allowed to set in. So, in order to understand, what is the frequency at which we can allow for the final role of (Refer Time: 06:33) set in, we need to first understand the consequences of allowing the final roll off to set in. So, since the final roll up has a slope of let us say minus 40 n decibels per decade, where 2 n represents the relative degree of the loop gain in the limit that omega tends to infinity.

Then we note that this particular roll off of minus 40 n decibels per decade will introduce a certain phase lag. And this phase lag will affect the phase characteristics over the entire frequency range. And in particular in the frequency range between omega 1 and omega gc, where stability of the closed loop system is a major concern. Hence, the consequence of introducing the final roll off, no matter at what frequency omega 2 we introduced it. Is that it might potentially affect the phase characteristics the phase lag of the loop gain at lesser frequencies in particular frequencies between omega 1 and omega gc, where stability is a real concern for us.

So, what we shall do first is therefore, take a look at this term separately. So, we shall assume that there is a magnitude characteristic, whose slope is equal to 0 dB per decade up to frequency omega 2. And beyond frequency omega 2, it rolls off at minus 40 n decibels per decade. And we shall ask ourselves what exactly is it that this kind of a characteristic would do to the phase characteristics of our open loop system at less of frequencies, namely in the frequency range between omega 1 and omega gc, where stability is a concern for us.

To answer this question, we need to go back to the relationship between the magnitude and the phase. So, we need to derive the relationship between the magnitude and the phase for this particular magnitude characteristic here and in particular in the frequency range around omega 1. What we need to note is that when we are doing the derivation, we note that omega 1 is much less than omega 2. And hence, that will permit us to make a few approximations, when we are doing the derivation.

So, the starting point as I said is the famous Bode's gain phase relationship, which we have alluded to in the past. So, the phase of L at any frequency in particular frequency omega 1 is given by 1 by pi times integral minus infinity to infinity d by du of lawn of magnitude of L times lawn of cotangent type of (Refer Time: 09:12) magnitude of u by 2 d u, where the variable u is given by lawn of omega minus lawn of omega 1. So, this is the starting point. We have to use this expression to determine what this final characteristic of the loop gain would do to the phase characteristics of the open loop system in the vicinity of the gain crossover frequency and the frequency omega 1.

Now, in order to simplify this integral, we need to note a couple of things. First we note that cotangent hyperbolic u by 2 is actually given by this particular expression here that is because u is given by lawn of omega minus lawn of omega 1. And based on the definition of the function cotangent hyperbolic of x, we can write out this function in terms of the variable u by 2 and simplify it. And show that cotangent hyperbolic of magnitude of u by 2 is simply equal to magnitude of 1 plus omega 1 by omega by 1 minus omega 1 by omega, where omega is the variable of integration in this integral on the right hand side. So, this is the first realization.

Next, we note that although the limits of this integral are minus infinity to infinity, we note that up to frequencies omega less than or equal to omega 2. The slope of the magnitude characteristic for this particular characteristic here is 0 dB. So, in other words, dA by du is going to be equal to 0. For frequencies omega less than or equal to omega 2 or in other words for u less than or equal to lawn of omega 2 minus lawn of omega 1. So, for frequencies for u up to this particular value, we would have dA by du to be equal to 0.

The second simplification arises from the specific form of the magnitude characteristics. We note that the magnitude characteristic has a slope of 0 decibels per decade for up to frequency omega 2 and the slope of minus 40 and decibels per decade beyond frequency omega 2. So, in other words, d of lawn of magnitude of L by du will be equal to 0 for u less than or equal to lawn of omega 2 minus lawn of omega 1. So, this is one simplifying step that we can undertake given the special magnitude characteristics, whose phase characteristics we are trying to derive.

Furthermore for frequencies omega greater than omega 2, we have a constant slope in the bode plot of minus 40 n decibels per decade. Now, when we are talking of the slope as being minus 40 in decibels per decade, we know that the x axis is log to the base 10, log of frequency to the base 10 and the y axis is 20 times log of magnitude of L to the base 10.

However, if you were to represent the same slope of minus 40 in decibels per decade on another logarithmic scale, well x axis is log to the base e of frequency and not log to the base 10 of frequency. And the y axis is simply log to the base e of magnitude of L and not 20 times log to the base 10 of magnitude of L. We can show that a slope of minus 40 n dB per decade in the new graph will essentially amount to a slope of minus 2 n units. So, what that means is that d of lawn of magnitude of l by du is going to be equal to minus 2 n for u greater than lawn of omega 2 minus lawn of omega 1.

So, we have the first fact that cotangent hyperbolic of magnitude of u by 2 is given by magnitude of 1 plus omega 1 by omega by 1 minus omega 1 by omega. And second we have that given the special magnitude characteristics; we have the slope of the magnitude characteristic to be 0 up to frequency omega 2 and to be equal to minus 2 n for frequencies beyond omega 2. So, the same integral that we have at the top, essentially gets simplified a little bit, because the lower limit no will no longer be minus infinity. Because, from minus infinity to lawn of omega 2 by omega 1, we have the slope of the magnitude characteristic being equal to 0, so that integral vanishes.

So, what we would be left with is that the angle of L s, where L s represents the magnitude characteristics that I have shown here. So, the phase of L s at the frequency omega 1 is going to be equal to minus 2 n by pi integral lawn of omega 2 by omega 1 up to infinity of essentially lawn of cotangent hyperbolic magnitude of u by 2, which is given once again by simply lawn of magnitude of 1 plus omega 1 by omega by 1 minus omega 1 by omega du.

Now, we note that the frequencies omega in this integral will always be greater than or equal to omega 2, because the lower limit for the integration is the frequency omega 2 or the variable u would be lawn of omega 2 minus lawn of omega 1 or equivalently lawn of omega 2 by omega 1. So, since the frequency omega is much greater than omega 2, we note that the frequency omega 1 is also going to be much less than the frequency omega that is because omega 2 is to the right of the frequency omega 1. And the variable of integration here omega is always greater than or equal to omega. And hence, omega is always significantly larger than omega 1 and that will allow us to make a further simplification via in approximation.

Now, we note that lawn of magnitude of 1 plus omega 1 by omega by 1 minus omega 1 by omega is approximately equal to lawn of 1 plus 2 times omega 1 by omega. In the limit that omega 1 by omega is much less than 1 that is because 1 by 1 minus omega 1 by omega will approximately be equal to 1 plus omega 1 by omega, when omega 1 by omega is much less than 1. This you get from Taylor series expansion.

And hence, we would have this to be lawn of 1 plus omega 1 by omega the square. So, this term will approximately be equal to lawn of magnitude of 1 plus omega 1 by omega the square and which in turn will be approximately equal to lawn of 1 plus 2 times omega 1 by omega, once again from Taylor series expansion. In the limit that omega 1 by omega is much less than 1 and that is precisely what has been written here.

And we note that once again lawn of 1 plus x is going to be approximately equal to x, where x is less than less than 1. And by exploiting this fact, we can conclude that lawn of 1 plus 2 times omega 1 by omega is going to be approximately equal to simply 2 times omega 1 by omega. And noting that u is equal to lawn of omega minus lawn of omega 1, we would conclude that 2 times omega 1 by omega is essentially equal to 2 times e to the power minus u. So, this is how we are able to reduce the expression that is being integrated to the term 2 times e to the power minus u.

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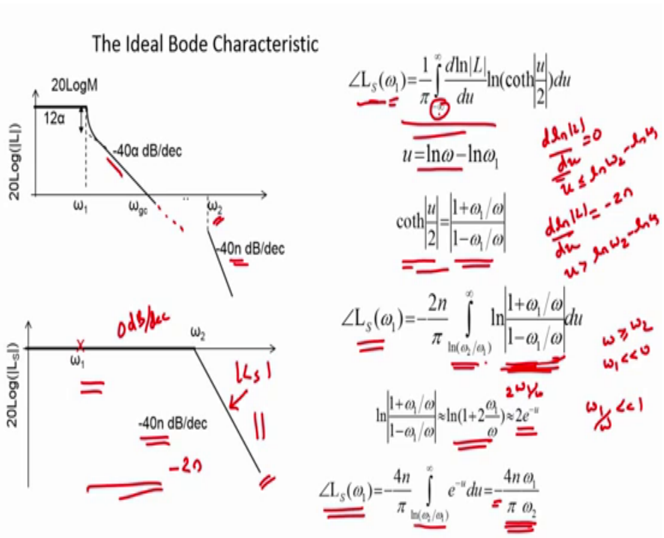

So, having performed the simplification, we can proceed with the integration. We would have the angle of L of s at omega 1 to be equal to minus 4 n by pi times e to the power minus u du with the limits of u being lawn of omega 2 by omega 1 to infinity. And if one computes its integral, then one can show that the angle at the frequency omega 1 is given by minus 4 n by pi times omega 1 by omega 2.

So, now this has one particular important message for us as control engineers. What it tells us is that the phase contribution at all frequencies because of this final characteristic like we have chosen to introduce at the frequency omega 2 is always negative, we always have this particular negative sign. And what is multiplying, the negative sign is always a positive number. So, this final characteristic will always therefore add a phase lag at all frequencies. And in particular at frequencies between omega 1 and omega gc, where stability is a concern for us.

So, this final characteristic is therefore guaranteed to spoil our phase margin specification that we had set for ourselves in the previous derivation. So, how do we address this issue. To address this issue what we need to notice that no matter at what particular value of omega 2 we introduce this characteristic, our phase margin requirement is going to be affected.

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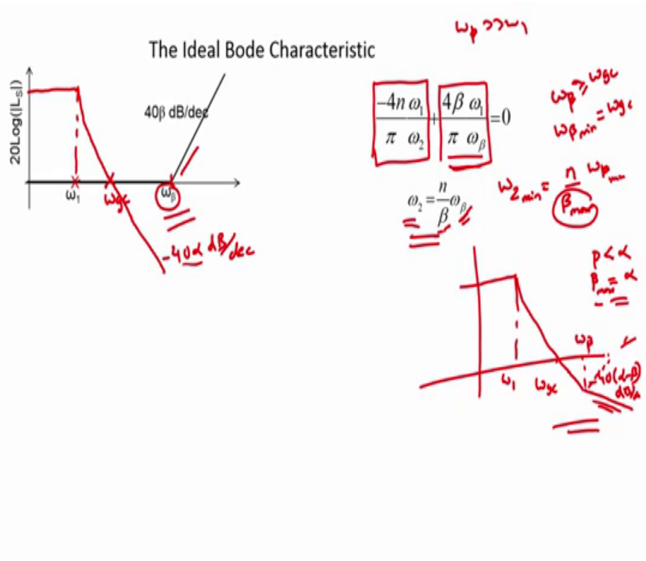

And hence, to compensate for it, we need to add another positive characteristic of the kind that is shown in this schematic here, which contributes a positive phase at the frequencies between omega 1 and omega gc, I shall mark out omega gc also on the same graph. So, we should add a positive characteristic to cancel out the negative phase that is added by the final characteristic at the frequency omega 2.

Now, let us say the corner frequency of positive characteristic is omega beta. Then we can from the by using the same derivation that we undertook in the previous slide, we can show that in the limit omega beta is much greater than omega 1. The phase lead provided by the characteristic of the kind that has been shown in this slide on the left is going to be given by 4 beta by pi times omega 1 by omega beta.

So, because the final characteristics resulted in a phase lag at all frequencies and in particular at frequencies between omega 1 and omega gc. We had to choose to add a positive characteristic or choose a controller that provides this kind of a positive characteristic, such that the phase lead provided by this characteristic cancels the phase lag provided by the final characteristics of our loop gain. And together, they will ensure that the phase margin specification remains unaffected between the frequencies omega 1 and omega gc, so that is why we have introduced this positive characteristic.

And as a result of this introduction, this positive characteristic will add a phase lead given by 4 beta by pi times omega 1 by omega beta, which we obtained with in a very similar manner as what when we obtain the phase characteristics, for the example that we just considered some time back. And the phase characteristics that we considered some time back, we saw which is the final characteristics of the loop gain results in a phase lag and that is given by 4 n by pi times omega 1 by omega 2.

And the positive characteristic here has been to cancel the negative phase added by the final characteristic. So, we would have minus 4 n by pi times omega 1 by omega 2 plus 4 beta by pi times omega 1 by omega beta, the 2 should add up to 0, so that at the frequency omega 1, the net phase lag is simply going to be given by minus alpha pi and not by a larger value. So, if you simplify this expression, you would get that the frequency omega 2 at which we can introduce the final characteristic is dependent on the frequency omega beta at which we introduce this positive trend to the controllers characteristic. So, omega 2 is therefore, going to be given by n by beta times omega beta.

And hence, omega 2 minimum is going to be given by n by beta maximum times omega beta minimum. So, since we want the final characteristics to set in as early as possible, so that we can stop controlling this unnatural roll off of the loop gain beyond the frequency omega gc. We want to have beta to be as large as possible and we want omega beta to be as small as possible.

Now, the question is what is the maximum possible value for beta and what is the minimum possible value for this frequency omega beta at which we can introduce this positive slope? Now, to answer this question, we need to understand what this kind of positive characteristic does to the already existing characteristic of our loop gain. To refresh your memory, the already existing characteristic has a constant gain of 20 log M decibels up to a frequency omega 1. And below this frequency omega 1, it rolls off and crosses over at the frequency omega gc and continues rolling off. And this rolling off is going to happen at minus 40 alpha decibels per decade.

Now, if you notice here, when we add this particular positive characteristic to the already existing characteristic that gives us the smallest possible value for the gain crossover frequency. The effective characteristic would look something like this. So, we would have the magnitude characteristic being a constant up to frequency omega 1 and then after that we would have this roll off up to frequency omega gc.

And at frequency omega beta the roll off will be lesser, because we would have minus 40 alpha dB per decade roll off and that gets added on to plus 40 beta decibels per decade rise in the magnitude characteristic. So, the roll off will beyond this frequency omega beta, it will be minus 40 times alpha minus beta decibels per decade. Now, this particular bode plot on the right tells us everything that we need to know to determine what beta max can be and what omega beta minimum can.

We not hear that for the characteristic after omega beta to continue to roll off, we need to have beta to always be less than alpha. And hence, the maximum permissible value of beta is going to be equal to alpha, it cannot strictly speaking be exactly equal to alpha, but it can be made arbitrarily close to alpha. So, beta max is going to be alpha. If beta assumes values greater than alpha, you will not have a role off at all beyond the frequency omega beta, but instead the magnitude characteristic will start to rise and will crossover again at a new frequency omega gc prime and which is not acceptable to us.

Hence, in order for the magnitude characteristic to continue to roll off, we need to have alpha to be always greater than beta and at most beta can be equal to alpha. So, the maximum permissible value for beta max is going to be equal to alpha. Likewise, when we have to determine what the minimum possible value for omega beta is. We note that if omega beta is less than omega gc, then we would have the part of the magnitude characteristic between omega beta and omega gc rolling off at a much lesser slope, namely at minus 40 times alpha minus beta decibels per decade and that would cause the gain crossover to happen at a much higher frequent now, then what is happening now.

So, therefore, in order to make sure that the gain crosses over at the smallest frequency possible, we need to make sure also that omega beta is always greater than or equal to omega gc. And hence, the minimum value for omega beta, which I shall call as omega beta minimum is equal to omega gc. So, the smallest frequency omega 2 at which we can introduce the final characteristics is dependent on the smallest possible value for omega beta and the largest possible value for beta. And our argument here shows that the largest allowable value for beta is equal to alpha and the smallest allowable value for omega beta is equal to omega g.

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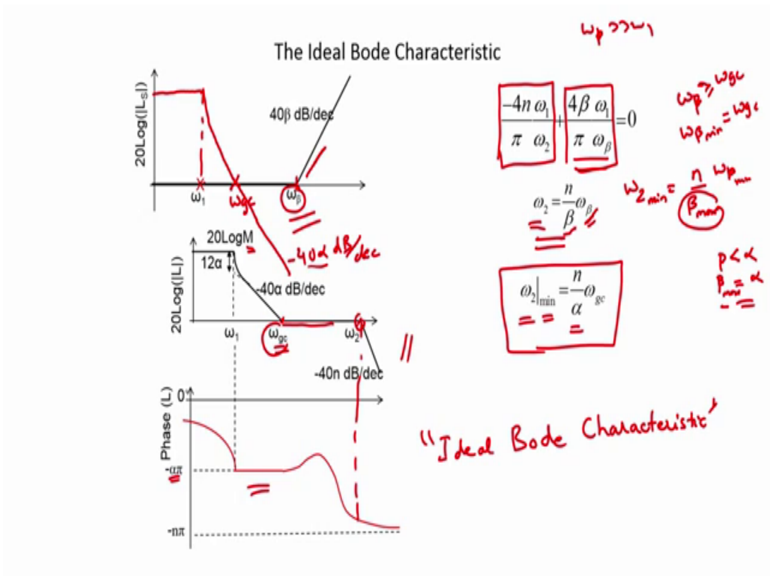

So, from using these two facts, we can come to the conclusion that the minimum frequency omega 2 at which we can allow the final characteristics to set in is given by n by alpha times omega gc. So, it is up to this frequency that we have to control the loop shape. Although the gain has crossed over at the frequency omega gc, we have to continue to maintain the loop shape and get it to roll off at minus 40 alpha decibels per decade up to a frequency omega 2 in the interest of stability. In an interest of making that our open loop system has a specified amount of phase margin. And the minimum frequency omega to up to which we have to maintain the loop shape is given by this particular expression.

So, the final bode a plot, which includes the final characteristics as well as the increasing characteristic that we have introduced here to cancel the effect of the final characteristics on the face in the frequency range between omega 1 and omega gc is shown here. So, up to omega gc, it rolls off at minus 40 alpha decibels per decade. And this is the minimum gain crossover frequency possible for a specified performance and for a specified phase margin.

Now, between omega gc and omega, we would have the positive slope of plus 40 beta decibels per decade getting added on to the magnitude characteristic, which is rolling off at minus 40 alpha decibels per decade. And in the limited beta is approximately equal to alpha, the net slope will be 0. And hence, we would have this to be a flat curve between the frequencies omega gc and omega 2. And at omega 2, the final characteristics would set in. And the minimum frequency that means the final characteristics can set in is given to be n by alpha times omega gc. The associated phase characteristics are shown in the second plot at the bottom.

Now, this particular magnitude and phase characteristic together is called the ideal bode characteristic that is because it is specifies the minimum possible gain crossover frequency or in other words the minimum possible closed loop bandwidth for a specified performance, namely M units. And for a specified phase margin, which has been indirectly specified here by indicating the phase lag to be minus alpha pi beyond the frequency omega 1. So, we determine the minimum possible gain crossover frequency and also the minimum frequency omega 2 up to which we have to maintain the loop shape in order to make sure that our system is stable by the specified amount.

So, the stability once again is specified by the maximum permissible phase lag, which is minus alpha (Refer Time: 29:38). And to maintain this phase lag at minus alpha pi between the frequencies omega 1 and omega gc, we need to include this positive characteristic as well. And the two together result in this flat response between omega gc and omega 2. And omega 2 which is given by this particular expression here specifies the minimum frequency up to which we have to maintain the loop shape in the interest of stability.

Now, there is one last consideration that we have not taken into account yet and that is the gain margin of this open loop system. If you notice the magnitude characteristics that I have drawn here y7ou will be very quick to realize that the gain margin associated with this characteristic is 0 0 dB that is because any small increase in gain of the overall system will quickly change the gain crossover frequency from this particular location, namely omega gc to somewhere near omega 2. Because, between omega gc and omega 2, we have a flat magnitude characteristic. And at the frequency omega 2, you can see that the phase lag is rather large, which means that if the gain were to increase just a little bit, our closed loop system would become unstable. So, we have no gain margin whatsoever for the ideal bode characteristic as it has been drawn here.

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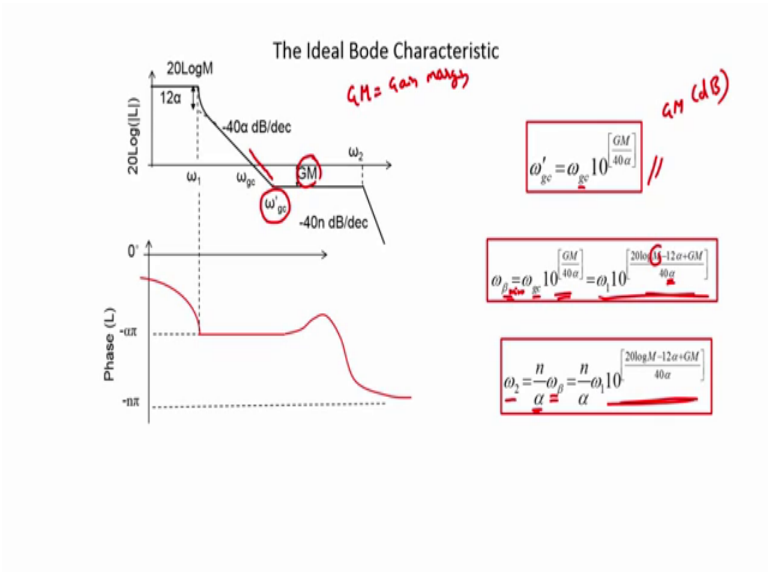

So, to remedy this small problem with the ideal body characteristic as it has been drawn here. What we would do is to continue the magnitude characteristic at minus 40 alpha dB per decade up to a frequency omega prime gc. And this omega prime gc is a frequency at which the loop gain falls to minus GM decibels, where GM stands for the Gain Margin.

Hence, even if there is uncertainty in the gain of our open loop system and the gain were to increase, we would still have the new plant with a different and a higher crossover at the minimum frequency possible for that particular gain. So, we continue the magnitude characteristic up to a frequency omega prime gc and the frequency omega prime gc is related to omega gc by the equation that is given here. Omega prime gc is equal to omega gc times 10 to the power GM by 40 alpha and that simply obtained by noting that the slope of this curve between the frequencies omega gc and omega prime gc is given by minus 40 alpha decibels per decade. And here we note that GM has been represented in decibels.

So, we have to continue this magnitude characteristic up to the frequency omega prime gc, in order to make sure that our open loop system has a specified gain margin GM. The positive characteristic which we earlier introduced at a frequency omega can now be introduced only at frequencies that are greater than or equal to omega prime gc. So, earlier when we had no gain margin specification, the minimum possible value of omega beta was equal to omega gc. But now, when we have this gain margin specification, the minimum possible value for omega beta will be omega prime gc.

Hence, omega beta is going to be given by omega prime gc, which is equal to omega gc times 10 to the power GM by 40 alpha. Noting that omega gc can be expressed in terms of omega 1 and the performance M that has been expected of us along with a stability specification alpha. We note that omega beta is related to omega 1, M and alpha according to the expression on the.

Now, since we allow the positive characteristic to set in at this particular frequency omega beta minimum, we would have that the minimum frequency at which the final characteristic can be introduced, which has a slope of minus 40 n decibels per decade is going to be given by omega 2 equal to n by alpha times omega beta, exactly as we had it in the previous slides. But, this time omega beta is no longer going to be equal to omega gc but since it is going to be equal to omega gc prime, which is given by the first expression here.

So, if you plug in that expression in the equation that relates omega 2 to omega beta, we would get the final expression on the right hand side, it is equal to n by alpha times omega 1 times 10 to the power 20 log M minus 12 alpha plus GM by 40 alpha. So, this is the smallest frequency at which we can follow the final characteristics to set in or equivalently this is the frequency up to which we as control engineers have to maintain the shape of the loop for the loop gain L. In order to understand the implications of these particular equations, let us take a numerical example.

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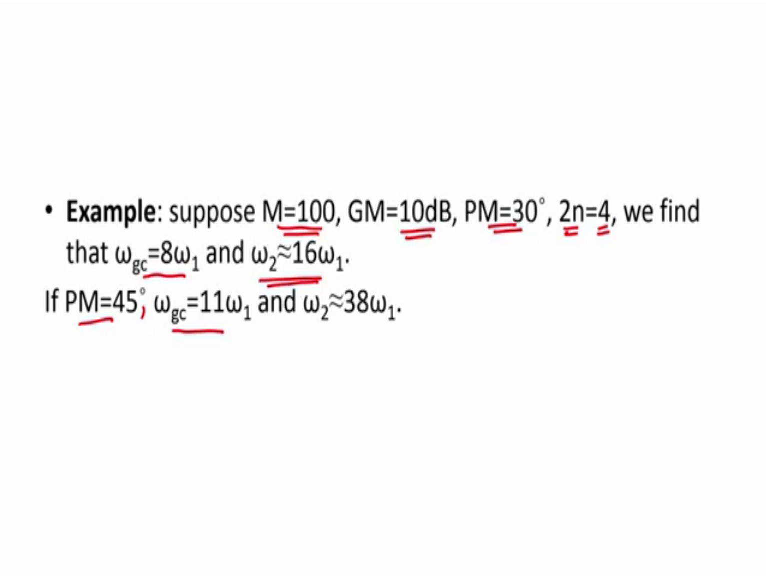

So, in the next slide here, we have considered the case one, we expect a magnitude M of 100 up to some frequency omega 1. So, in other words, 40 decibels up to some frequency omega 1 and we expect a gain margin of 10 decibels and a phase margin of 30 degrees. So, let us say these are our expectations. And the complexity of the loop gain as characterise the relative its relative degree 2 n, let us say is equal to 4.

Then we find that the minimum frequency at which the gain can crossover or in other words the minimum possible bandwidth for the close loop system is going to be given by omega gc is equal to 8 times omega 1. And the minimum frequency at which we can allow the final characteristics to set in or in other words the minimum frequency up to which we as control engineers have to maintain the loop shape in the interest of stability is given by omega 2 is equal to 16 times omega 1.

Now, these numbers only become larger, if our phase margin specification is higher. So, if our phase margin is 45 degrees instead of 30 degrees, then we note that our alpha will be a smaller number. Alpha will be equal to 3 by 4 as we discussed in our previous clip. And with that smaller alpha, we would have a larger value for omega gc. So, the minimum bandwidth of our closed loop system or equivalently the minimum gain crossover frequency of the open loop system is going to be given by omega gc is equal to

11 times omega 1. And omega 2, which is the frequency up to which we have to maintain the loop shape is going to be given by 38 times omega 1.

So, from this numerical example what is revealed is that even in the ideal case, if we are expecting a certain performance such as a gain of 100 up to a frequency omega 1, what it implies is that our minimum gain crossover frequency is at least 10 times larger than the frequency omega 1 up to which we are expecting this performance. And the frequency up to which we have to maintain the loop shape is even larger, it is 16 times omega 1 in the first case and 38 times omega 1 in the second case, so nearly 100 times more than the frequency omega 1.

And this frequency range is entirely the range, where we are paying the price for expecting the benefits of feedback up to a frequency omega 1. So, if you are expecting performance up to a frequency omega 1, be prepared to pay the price for it, in other words be prepared to have a closed loop system, whose bandwidth has to be at least 10 times larger than the frequency omega 1 and whose and an open loop system, whose loop shape has to be maintained up to a frequency, which is anywhere between 20 to 40 times larger than the frequency omega 1.

And all of this is revealed from our analysis of the ideal bode characteristic. And as the name itself reveals, the ideal bode characteristic is an ideal characteristic, so it is not something that is easily realized in practice. If one more to go with simple controllers such as proportional controllers or PID controllers or lead like compensators or such other simple controllers, then these numbers will be far worse. The minimum frequency up to which we need to maintain the loop shape and the minimum bandwidth that we can achieve for our closed loop system will be much bigger than the numbers that have been indicated here.

So, the ideal body characteristic allows us to quickly calculate the minimum bandwidth that we need to alert for our closed loop system. And the minimum frequency at we will have to maintain the loop shape for the open loop system, in order to reap the benefits of feedback between 0 and omega 1.

Thank you.