Control System Design

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Module – 09
Lecture – 42
Ideal Bode Characteristic (Part 1/2)

Hello, the topic of today's clip is the Ideal Bode Characteristic and in this clip we shall ask ourselves an important problem that is of concern to all of us control engineers and that is what is the minimum price that we need to pay to reach the benefits of feedback. Now, what do you mean by benefits of feedback volume of certain amount of disturbance rejection you want you might want to a certain amount of robustness to variation in plant parameters you might want to track certain reference to desire level of accuracy.

So, these are all the benefit that we seek from our feedback control system and associated with this benefits we need to maintain the loop gain of our system high in the frequency ranges where this benefits are sought and what is the price that one that one needs to pay in order to realize this benefits. One has to invest in a controller which has a sufficiently high bandwidth and one has to make sure that the close to bandwidth is sufficiently high in order for us to track the references, reject disturbances, achieve robustness and so on and so forth.

So, the frequencies up to which we have to either control the shape of the loop or in other words maintain the structure of the feedback controller and the close loop bandwidth of our close loop system could count as the price of we might the paying in the for reaping the benefits that I just talked about. So, in this lecture we would try to addresses the important question as to what the minimum price we need to pay is in order to reap the benefits of feedback. So, let us try to peck this problems little bit more concrete.

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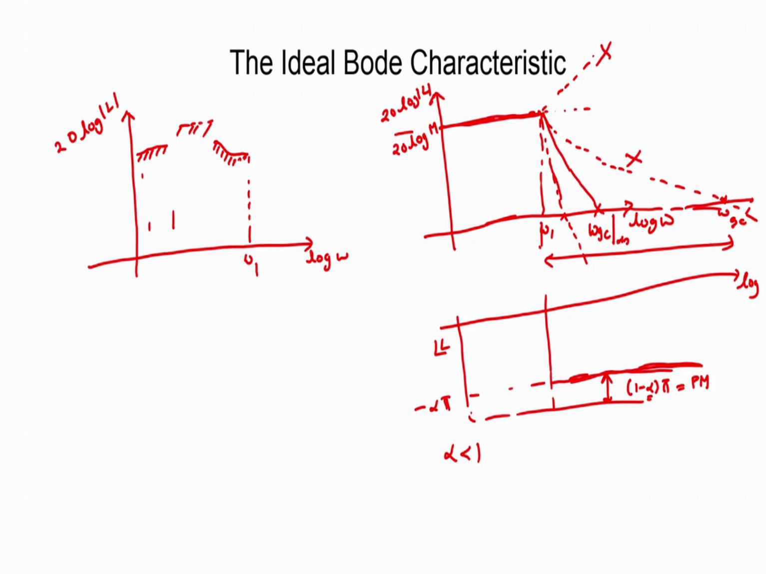

So, what I shall draw here is a Bode plot where the x axis is also omega in y axis is log of, magnitude of L 20 log magnitude of L. Now, in the interest of the performance I might require 20 log magnitude of L to be above I certain magnitude in one particular frequency range above a different magnitude in a different frequency range and may have to have a certain functional dependence, the magnitude need to have certain functional dependence of frequency in a third frequency range and so on and so forth.

So, this may all be part of our specification in order to either track references or reject disturbances or you know achieve robustness to plant parameter variation. Now, there will be one frequency which I shall call omega 1 at which we stop expecting any performance from our close loop system. The question is how should the loop gain appear after the frequency omega 1 so that it is most economical in terms of the price that one needs to pay to reap the benefits in the frequencies below omega 1 where we are expecting the performance specification I just talked about.

So, how should the loop shape look that is the question we are trying to answer. Now, stated as such this problem is mathematically intractable. So, you may have certain functional dependence of magnitude of loop gain as function of frequency in some frequency range, in another frequency range you might have a constant magnitude function of frequency, in a third frequency they you might have again constant magnitude, but of a different value.

So, to solve this problem of determining the shape of loop gain beyond frequency of omega 1 when you have all such complicated and competing specification of frequencies below omega 1 makes this problem difficult to tackle. So, before we solve this problem of obtaining the most economical loop gain let us first simplify the specification have been given to us and solve it for a very simple kind of specification and that itself is going to give us some important insights into the price that one needs to pay need to pay to rip the benefits of feedback.

So, rather than specify that our gain requirements or frequency dependent and they may be different and different frequencies let us assume that our gain requirement is some constant value M up to the frequency omega 1. So, if I were to plot the Bode plot of my system then I want the gain of the open loop system to be equal to 20 log M for frequencies below omega 1, I want it to be a constant.

So, unlike the kind of dependence that the loop gain would have frequencies in the conventional case when we actually have to tackle a real life problem in order to make the analysis of the optimal loop gain mathematically tractable we shall take a step back and simplify the problem of how the loop gain is supposed to appear for frequency is below omega 1. We shall assume it is supposed to have a magnitude again of value M in order to satisfy the various performance specifications made be disturbance rejection or reference tracking or robustness to plant parameters. Whatever it may be we shall assume that having a loop gain M will allow us to meet those specifications either exactly meet them or do better than what was expected of us.

So, we have a constant magnitude characteristic for our loop gain below frequency omega 1 and the magnitude is M and the question now that we are asking is what happened to the loop gain for frequencies beyond omega 1, how should it look like? Well, if you think a little bit there are a few possibilities which we can easily exhaust one possibility is of course, that we allow it to increase for frequency is beyond omega 1 and clearly does naught make any sense because we are stating explicitly that there is no performance requirement beyond frequency omega 1, so why would one want to needlessly increase the loop gain of frequency is below beyond omega 1. There is also the price associated with controllers gain and bandwidth that needs to be paid in order to keep this loop gain is high and increased it for frequency is beyond omega 1.

So, clearly increasing the loop gain at for frequency is beyond omega 1 makes no practical sense hence this option is altogether ruled out from our consideration. So, we have to therefore, either keep it constant which once again does not make sense because we are not expecting performance beyond frequency omega 1 and therefore, we can allow it to drop. Now, there are two ways in which loop gain can drop one is you can drop gradually and finally, crossover at some fairly high frequency and this may the gain cross over the frequency and continue to the drop over continue to decay beyond the frequency. This is one manner in which loop gain can reduce with frequency.

Now, there is one more way in which it can reduce and that is a very drastic and dramatic reduction with frequency. So, it can we can try to reduce the loop gain as quickly as possible to 0 for frequencies beyond omega 1. Now, which of these to do we pick or should be go for some intermediate rate of decreases of loop gain with frequency. To answer this question we first shall take the case when we were allowing for fairly slow decrease in loop gain. If the rate of decreases is very low and then what happens is that our gain cross over frequencies will be quite large.

So, it will be significantly to the right in this particular graph here and what that essential means is that all the time from frequency omega 1 2 frequency omega gc over this entire frequency rage we have to keep the loop gain high for no particular reason; we are not expected to reject any disturbances or attract any reference for frequency omega 1.

So, needlessly we had to expand in a controller that gives this fairly large closed to band width. You need to remember the close to band width is closely related to gain cross over frequency of open loop system hence, having large gain cross over the frequency is anonymous having a large close loop bandwidth. A close loop width much larger than the frequency omega 1 up to which we are expecting performances and that does naught make any sense. There is one more reason why you do not want the gain cross over the frequency to be very large, if the gain cross over frequency is large as we just discussed the close loop band width of our system will also be exceeding the large. If our close loop band width is large then our close loop system will end up letting him lot more measurement noise than what is unavoidable.

So, in order to achieve the benefits of feedback up to frequency omega 1, we will have to settle for feedback control and we will therefore, have to let in some measurement noise know if we needlessly reduce the loop gain at a very slow rate then we are having a large closed loop bandwidth and over this entire closed loop bandwidth up to frequency omega gc we will be letting in measurement noise. And this is want to be disastrous especially if the measurements noise has some high frequency component which will in which case I will they will be letting in significantly larger amplitudes than the case when we have a low gain crossover frequency and low closed loop bandwidth.

So, clearly this alternative of reducing the loop gain at every low rate is not a desirable one for two reasons; one is it will result in the controller having to have a fairly large bandwidth so that the closed loop system will also have a fairly large bandwidth and secondly, the fairly large bandwidth the closed loop system will make the closed loop system more sensitive to measurement noise. The output will be affected by measurement noise especially when the noise is either white noise or it has high frequency content. So, therefore, this alternative is ruled out.

How about the lost alternative that we discussed, namely where we reduced the loop gain very rapidly? In some sense this is what we desire because we are not anyway expecting performance beyond frequency omega 1. So, the sooner we reduce the loop gain and get it to cross over the better it is. The lesser will be our close to bandwidth and therefore, the lesser will be the effect of noise on our close loop system, but can this rate of reduction of loop gain as function of frequency the arbitrarily steep, can we make this exceedingly steep?

If you think a little bit about this and go back to the previous clip where we discussed the relationship between the magnitude characteristic and a phase characteristic of a Bode plot, you might recall, but the phase characteristic of the Bode plot is related to the rate of decrease of magnitude characteristic in the Bode plot or another words the slope of the magnitude characteristic approximately gives the phase characteristics.

Now, this can tell us what is the likely price that we might pay, if we reduce the loop gain too quickly beyond the frequency omega 1. Now, if the loop gain reduction is very fast or another word is a very steep curve then it is slop will be a very large negative number and associated with this very large reduction in slope as function of frequency you will have a very large phase lag and the phase lag by potentially big greater than minus pi radiant in which case would have phase crossing over at frequency that is less than the frequency at which gain cross over or in other words it would result in the close loop system by being unstable.

So, if the reduction is very steep we have this problem of excessive phase lag associated with steep reduction and that could make the system unstable if the reduction is very shallow then we end of the large closed loop system with very large bandwidth and that results in more expensive controller and more sensitivity to measurement noise. Therefore, the ideal case is to reduce it as fast as possible so, as quickly as possible, but without jeopardizing the stability of the close loop system. In other words if we assume that our phase namely angle of L should not drop below a certain value, I shall call that value as minus alpha pi where of course, alpha is lee than 1, if our phase is always greater than minus alpha pi where alpha is less than 1 then we are making sure that our magnitude characteristic will at more points result in instability of the close loop system.

So, therefore, we should reduce the magnitude characteristic at the highest rate that is possible to ensure that the phase lag does not become smaller than minus alpha pi, where alpha is a number that is less than one that prevents the close loop system from becoming unstable. So, the first question that we will be looking at is to determine that minimum gain crossover frequency omega gc minimum that we can have for a specified value of the frequency omega 1 and for a specified value of the magnitude M and for a specified amount of stability that we might want to have. Remember, that if minus alpha pi is the designated phase lag then the phase margin is going to be given by 1 minus alpha times pi.

So, if I have been specified a certain phase margin or equality specified a certain alpha what is the smallest frequency at which I can get the loop gain into crossover given the frequency omega 1 up to which I am expecting performance and the magnitude M of the performance that I am expecting up to frequency omega 1. This is the mathematical problem that we shall first end over to solve and this will gives us crucial insides on the minimum price that one needs to pay in order to maintain the loop gain at M units up to frequency omega 1. The price in a sense is the frequency range between omega 1 and omega gc up to which we have to keep the loop gain high, despite the fact that there is no performance expected in this frequency range.

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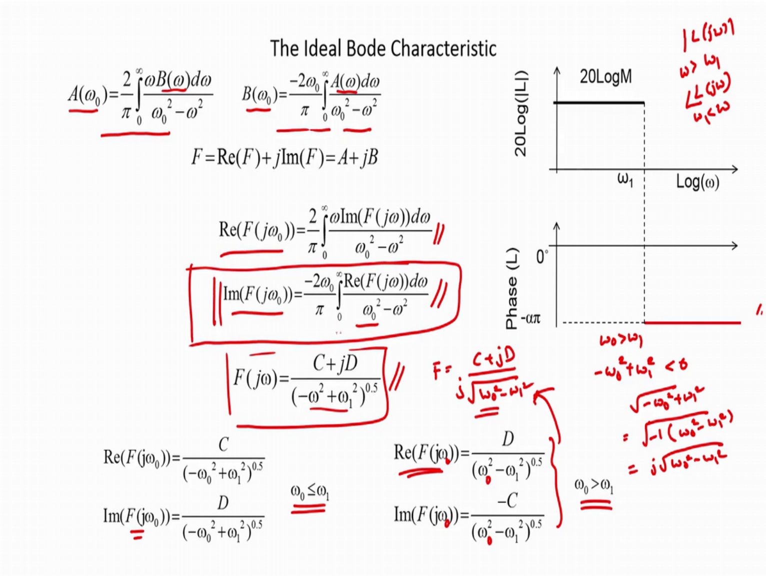

So, the mathematical problem that we are trying to solve has been shown graphically on the right, we expect that the magnitude characteristic could have a constant value of 20 log M up to some frequency omega 1 and this is the frequency up to which performances is expected and beyond frequency omega 1 in the interest of stability we want the phase log one to be either greater than or equal to minus alpha pi. Now, the larger is phase log the larger is the rate of reduction of magnitude characteristic.

So, in order to get the gain cross over as early as possible beyond the frequency omega 1, we insist that the loop gain should have a phase log of exactly what is permitted, exactly the largest negative value permitted which is essentially equal to minus alpha pi. So, the question is how does the magnitude characteristic look in the frequency is beyond omega 1 that ensure that the phase log is going to be equal to minus alpha pi at frequency is beyond omega 1 and how will the phase characteristics look at frequency is below omega 1. Given that the magnitude characteristic is a constant equal to 20 log M for frequencies below omega 1 this the question that we are to trying to answer.

Once again you will be depending on some of the mathematical equipment that we have developed and used over the last couple of clips in order to determine the answer to this question of what the minimum gain crossover frequency achievable is for a specified performance and stability requirement. To answer this question as I said we shall go back to the relationship between the real and imaginary parts of a complex function F of s which has certain special characteristics that we employed in our derivations in the previous clips.

So, for such functions we noted that the real part of the function which is A of omega naught is related to the imaginary part of it by the expression given here and the imaginary part which is given by B of omega naught gets determined completely if we specify A of omega over the entire frequency range. And that relationship is given by this particular equation and I want to remind you that A and B are essentially the real and imaginary parts of a complex function F for which we carried out the derivation and obtained this result.

Now, I can rewrite the two equations here in terms of real part of F and imaginary part of F for reasons of clarity that we will get to in a minute. So, instead of writing it in terms of A of omega and A of omega naught, B of omega and B of omega naught I shall write it in terms of directly the real part of F and the imaginary part of F and that rewritten equation looks this way. Real part of F of j omega naught is equal to 2 by pi times integral of omega times imaginary part F of j omega naught by omega naught square minus omega square. Imaginary part of F of j omega naught is equal to minus to omega naught by pi times real part of F of j omega d omega by omega naught square minus omega square.

So, I shall write it in this manner and subsequently we shall define F the function F to be of the form F is equal to C plus j times D divided by omega 1 square minus omega square under root or divided by square root of omega 1 square minus omega square. Now, I want to remind you before I get into further algebra and simplifications for the whole purpose of undertaking all of this algebra and integration and so on is to answer the question of how the magnitude characteristic of the loop gain. In other words how will magnitude of L of j omega look like for frequencies omega that are greater than omega 1 and how will the angle of L of j omega look like for frequencies omega less than the frequency omega 1.

Having been specified that the magnitude characteristic for frequency is below omega 1 is a flat line of magnitude 20 log M and the phase characteristic for frequencies is beyond omega 1 is again a flat line and to the phase lag of minus F of pi. Given these magnitude and phase specifications in those particular frequency ranges how do the magnitude and phase characteristics in a complementary ranges appear. That is the problem we are trying to solve and the mathematical equipment we were employing now is all geared to solve in this particular problem.

So, let us say we choose to apply the results of these two equations to a function F that looks something like this. F of j omega is equal to C plus jD by square root of omega 1 square minus omega square. Now, what are these functions C of j omega and D of j omega you we will get to that in a few minutes time, for the moment though let us just assume that there are two complex functions for which these results are valid and we are now trying to apply these results for this particular function.

Now, this function is quite peculiar because the real part of F of j omega naught is from inspection given by C by minus omega naught square plus omega square under root for frequencies omega naught less than or equal to omega 1. Likewise the imaginary part of F of j omega naught is given by D by omega 1 square minus omega naught square under root for this in the same frequency range.

However, what is interesting is if you look at frequencies omega naught that are greater than omega 1 in which case we would have minus omega naught square plus omega 1 square to be less than 0, that happens when omega naught is grater then omega 1. Then we would have that square root of minus omega naught square plus omega 1 square and this can be written as square root of minus 1 times omega naught square minus omega 1 square which in essence is equal to j times square root of omega naught square minus omega 1 square.

So, for frequencies omega naught that is greater than omega 1 we can right F as equal to C plus jD divided by j times square root of omega naught square minus omega 1 square. Now, if you simplify this we will note that. So, for frequencies omega naught greater then omega 1, the real part of F of j omega interestingly is going to be equal to D by omega naught square minus omega 1 square under root and imaginary part of F of j omega omega naught is going to be equal to minus of C by omega naught square minus omega 1 square under root. And, this is obtained directly by looking at the real and imaginary parts of the expression that has been given here.

So, we have chosen such a function F that in the frequency range up to omega 1 we have the real part of F to be given by C by omega 1 square minus omega naught square under root while as the frequencies greater then omega 1 we have real part of F to be given by D by omega naught square minus omega 1 square under root. And likewise also the imaginary parts the terms C and D get flipped between frequencies omega naught less then omega 1 and omega naught greater then omega 1.

The reason we are choosing this special kind of function is because we have been specified the magnitude characteristic of frequencies omega naught less than omega 1 and the phase characteristic for frequencies omega naught greater then omega 1. Suppose, we apply the second result here namely imaginary part of F of j omega naught is equal to minus 2 omega naught by pi times integral 0 to infinity real part of F of j omega D omega by omega naught square minus omega square. Suppose you apply this part to the special function F of j that we have picked then the right hand side of the expression within this integral would look something like this.

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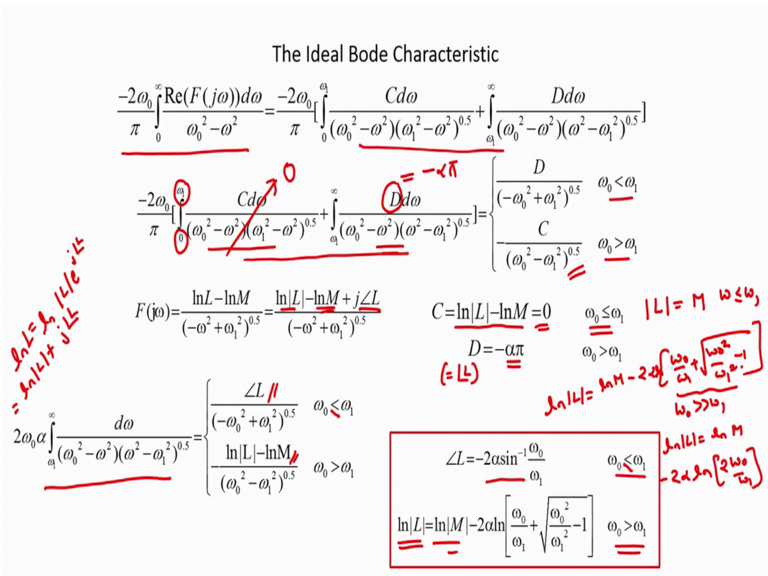

Minus 2 omega naught by pi integral 0 to infinity real part of F of j omega D omega by omega naught square minus omega square will look something like this because the real part of F of j omega assumes a certain functional form between the frequency 0 and omega 1 and a different functional form between the frequencies omega 1 and infinity as we just discussed in the previous slide. Indeed for frequencies up to omega 1 it is given by C by square root of omega 1 square minus omega square and for frequencies beyond omega 1 it is given by D by square root of omega square minus omega 1 square.

So, we have there for written out the right hand side of that integral as given here at this there for is going to be equal to the imaginary part of F of j omega naught and once again depending on whether omega naught is less then omega 1 or greater then omega 1 the imaginary part of F of j omega naught would be different. So, for omega naught less then omega 1 we would have the integral to be equal to D by square root of omega 1 square minus omega naught square and for omega naught greater then omega 1 we would have the integral to be equal to minus of C by square root of omega naught square minus omega 1 square.

Now, these are the mathematical preliminaries that we have to put in place in order to determine the magnitude characteristics of the loop gain for frequencies beyond omega 1 and for and determine the phase characteristics of frequency below omega 1. Our goal now as control engineer is to apply it to a carefully and cannily chosen functions C and D, so that we can extract the magnitude characteristic for frequencies beyond omega 1 and phase characteristics of frequencies below omega 1 and we shall choose the function F to be of the kind F is equal to of ln of L minus ln of M divided by square root of omega 1 square minus omega square.

Now, since we know that ln of L is going to be equal to ln of magnitude of L e power j angle of l we would have ln of L to be equal to ln of magnitude of L plus j times angle of L and precisely these two terms here and we also have additional term minus ln of M why these have this term will become evident in a few minutes time. So, from this expression we can see that the functions C of j omega is given by ln of L minus ln of M and interestingly we see that for frequencies omega naught less than or is equal to omega 1 this term we going to be equal to 0 because our problem specifies that the magnitude of L is equal to M for frequencies omega less than or equal to omega 1. So, this is the part of problem specification we would have C to be equal to 0 for frequencies omega naught or less or equal to omega 1.

Likewise for frequencies omega naught greater than omega 1 we would have the term D to be equal to minus alpha pi, where D is essentially equal to the angle of L that is going to equal to minus alpha pi of frequencies omega naught greater than omega 1 or more generally for frequencies omega greater than omega 1. So, we can employee these two realizations in simplifying the appearance of this particular integral. We note that the first integral here goes from 0 to omega 1 and it integrates the term C by square root of omega 1 square minus omega square times omega naught square minus omega square and denote that C is equal to 0 within the limits 0 to omega 1 and hence what we would naught is that this integral essentially goes to 0.

As far as the second integral is concerned we would have that the term D would be a constant for frequencies between omega 1 and infinity and this term D essentially would be equal to minus alpha pi in this frequency range because that is the phase lag that we have assumed for our loop gain between omega 1 and infinity in order to make sure that in our attempt to reduce our loop gain as quickly as possible. We are not going to be destabilizing the closed loop system.

So, if we substitute that, then the integral on the left hand side gets reduced to expression that has been shown here and on the right side for frequencies omega naught less than omega 1 we would have it to be equal to D by square root of omega 1 square minus omega naught square. And noting that for the particular F of j omega that we have to chosen the function D is essentially given by angle of L, it is going to be equal to angle of L by square root of omega 1 square minus omega naught square for frequencies is omega naught less than or equal to omega 1 and it is going to be equal to minus of ln of magnitude of L minus ln of M divided by square root of omega naught square minus omega 1 square for frequencies omega naught greater than omega 1. Because it is going to be equal to essentially minus of C by square root of omega naught square minus omega 1 square according to this expression and see our particular problem is essentially ln of magnitude of L minus ln of M and that is how you get this particular expression.

Now, we can solve this integral to directly obtained the angle of L for frequencies omega naught less than or is equal to omega 1 and the magnitude of the L for frequencies omega naught greater than omega 1. Now, the procedure to solve this particular integrally relatively straight forward and it is given in the reference that I have indicated in the notes and if one word compute this integral namely 2 omega naught alpha times omega 1 to infinity D omega by omega naught square minus omega square times square root of omega square minus omega 1 square. We would note that the angle of L upon solving the integral is going to be given by minus 2 alpha sin inverse to omega naught pi and omega 1, for frequencies omega naught less than or equal to omega 1 and ln of magnitude of L is going to given by ln of M minus 2 alpha ln of omega naught by omega 1 plus square root of omega naught square by omega 1 square minus 1 for frequencies omega naught greater than omega 1.

So, we have succeeding obtaining the angle of L as function of frequency, for frequencies omega naught less than or is equal to omega 1 and the magnitude of L as function of frequencies for frequencies beyond omega 1. So, if you going to graph the angle of L the magnitude of L in these respective frequencies ranges it will look something like this.

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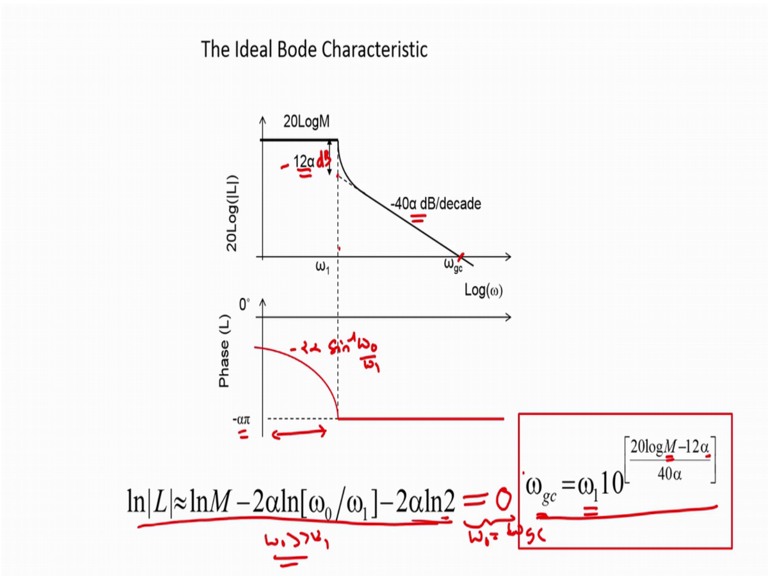

In the frequencies range beyond omega 1 we would have magnitude characteristic rolling of and the role of rate is going to given by minus 40 alpha D be per decade where alpha pi was the specified phase lag of our open loop system and phase response for frequencies below omega 1 is given by minus 2 alpha sin inverse omega naught by omega 1. Now, to check whether the phase response is right, we note that if you substitute omega naught equal to omega 1 we get that the phase lag at exactly the frequency omega 1 is going to be equal to minus alpha pi. And, therefore, there is going to be no discontinuity in our phase estimate for frequencies is beyond omega 1 and exactly at omega 1.

So, if we return to the magnitude characteristic we note that this magnitude characteristic causes over at a certain frequency and this frequency is a smallest frequency at which we can allow the magnitude characteristic to crossover. That is because if the magnitude characteristic were to cross over any frequency less than this frequency then the associated the phase log could be so large that the requirement that the phase lag be at most minus alpha pi would be violated.

So, how do we obtain the gain cross over the frequency omega gc? In order to obtain omega gc let us first make an approximation to the magnitude characteristic of frequencies beyond omega 1. If we written to the expression of ln L we have that ln of magnitude of L is equal to ln of M minus 2 alpha times omega naught by omega 1 plus square root of omega naught square by omega 1 square minus 1. Now, in the frequency range omega naught greater than greater than omega 1, we would have that omega naught by omega 1 will be much better than 1 and hence the term within the bracket here can be simplified us ln of magnitude of L is equal to ln of M minus 2 alpha times ln of 2 times omega naught by omega 1.

And, I have written the same approximate expression here. So, for frequencies omega naught which is much better than omega 1 we would have ln of magnitude of L to be approximately equal to ln of M minus 2 alpha ln of omega naught by omega 1 minus 2 alpha ln 2. And when we convert the x axis to the as log of frequency to the base 10 and the y axis as log of magnitude of L to the base 10 times 20 the term 2 alpha ln 2 essentially becomes equal to 12 alpha decibels. And, the role of is given by minus 2 alpha if the x axis and y axis for simply ln of magnitude of ln and ln of frequency, but if we were to inspect plot 20 log to the base 10 magnitude of L verses log to the 10 base of frequency the slope which earlier minus to alpha essentially becomes minus 40 alpha dB per decade.

So, the magnitude characteristic roles of approximately linearly for frequency is omega naught that is much greater than omega 1 and if were to extend this linear characteristic up to the frequency omega 1 we would notice that linear trend will intersect the vertical line at omega 1 at a point that is 12 alpha dB lower than the 20 log M. Hence in order to determine the gain crossover frequency we need to note that at the gain crossover frequency you would have 20 log of magnitude of L to be equal to be 0 or equivalently ln of magnitude of L will also be equal to 0. So, we just have to set this entire expression to be equal to 0 to obtain the frequency omega gc.

So, what omega naught equal to omega gc you would have this to be valid, namely this entire ln of magnitude of L will be equal to 0. Now, if we want to solve this equation we get the minimum gain crossover frequency that is given by this expression omega gc minimum is equal to omega 1 10 to the power 20 log M minus 12 alpha divided by 40 alpha. So, if we specified a certain phase margin requirement then we fix the value for alpha and if we specify the performance requirements in terms of the extant of disturbance rejection or the maximum permissible error in tracking we essentially specify the term M. And with these two pieces of information along with the information of the frequency omega 1 up to which the performance is expected we can compute the minimum gain crossover frequency that is possible for our close loop system for it to achieve the performance M up to frequency omega 1 and achieve a phase margin as characterized by the constant alpha.

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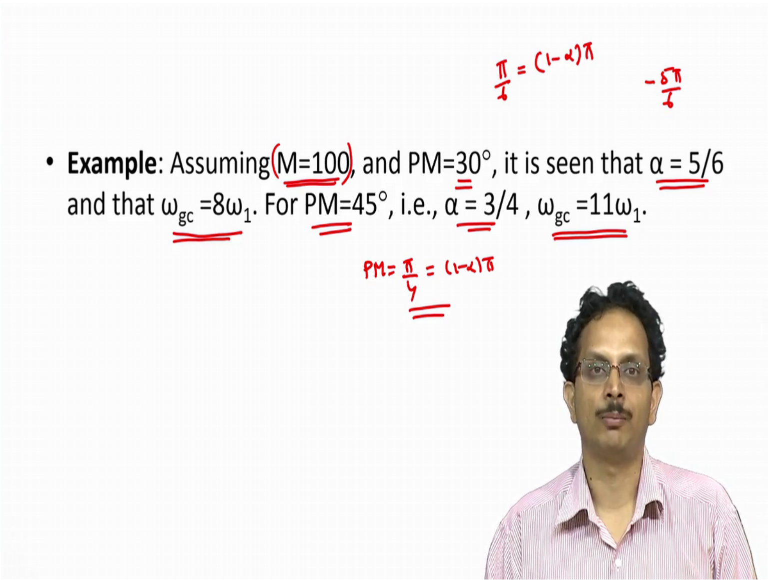

So, to drive home the significance of this fact let us take a numerical example. So, let us assume that as control engineers we are expected to achieve a gain of 100 which corresponds to 40 dB and up to some frequency omega 1 and have a phase margin of at least 30 degrees. Now, we note that if the phase margin is 30 degrees or in the other words 5 by 6 radian, then we would have that the maximum phase lag which is characterized by the term minus alpha pi can be obtained by solving the equations pi 6 is equal to 1 minus alpha times pi and if rearrange it we get that alpha is equal to 5 by 6 in other words the maximum log permitted in order for us to achieve a phase margin of 30 degrees will be minus 5 pi by 6.

So, for this particular permitted phase lag and for this particular magnitude requirement up to some frequency omega 1, we obtain the gain cross over the frequency the minimum gain cross over the frequency to be 8 times omega 1. So, almost nearly order of magnitude greater than omega 1. Now, suppose we want a slightly more conservative design and desire a phase margin of 45 degrees or in other words the phase margin is going to equal to pi by 4, then we can show that alpha can be calculated as 1 minus alpha times pi if t is equal to the phase margin or is equal to pi by 4 and that gives alpha to be equal to 3 by 4.

Now, we substitute this particular value for alpha along with the particular required value for the gain M in the equation that I showed in the previous slide you will obtain the gain crossover frequency to be equal to 11 times omega 1. So, what this indicates to us in general is that as a rule of thumb the price that we pay for achieving the benefits of feedback up to a frequency omega 1 is that we have to keep the loop gain high, in other words greater than 0 dB for a frequency range that is nearly 10 times larger than the frequency omega 1 up to which we are expecting performance and this is the best that we can do.

Any smaller value of gain cross over frequency and hence the closed loop bandwidth would result in a phase lag that is going to be lesser than minus alpha pi in the phase margin requirement is will be violated. So, as a rule of thumb therefore, the frequency range over which we pay price or in other words the closed loop bandwidth of our system of our control system is in general going to be about 10 times larger than the frequency range up to which were expecting performance from our close loop system.

Thank you.