Control System Design
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Lecture – 39
Tutorial on QFT Toolbox software (Part 3/3)

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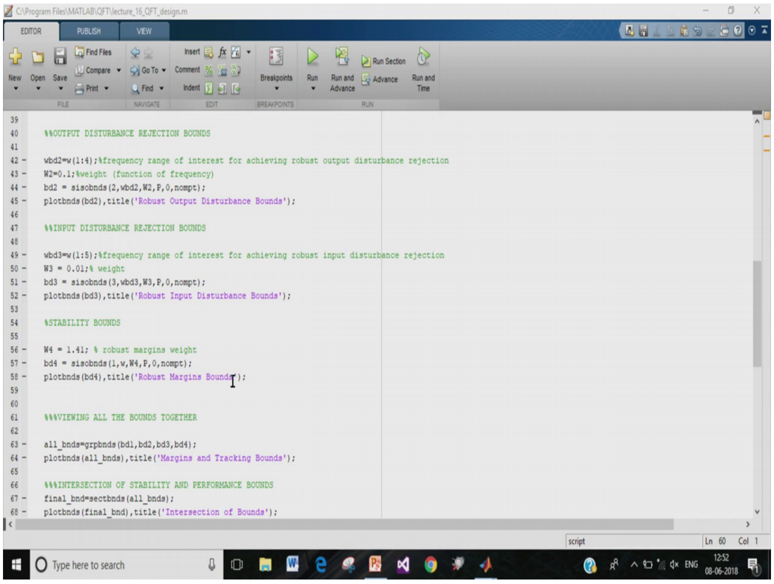

So, after plotting the bounds for the different performance specifications namely for robust tracking, output disturbance rejection and input disturbance rejection and also for stability you would wish to view all these bounds. So, it is for this purpose that the function grpbnds can be employed. So, we have defined the bounds for each of these different performance and stability specifications and the function grpbnds allows us to view all these bounds together.

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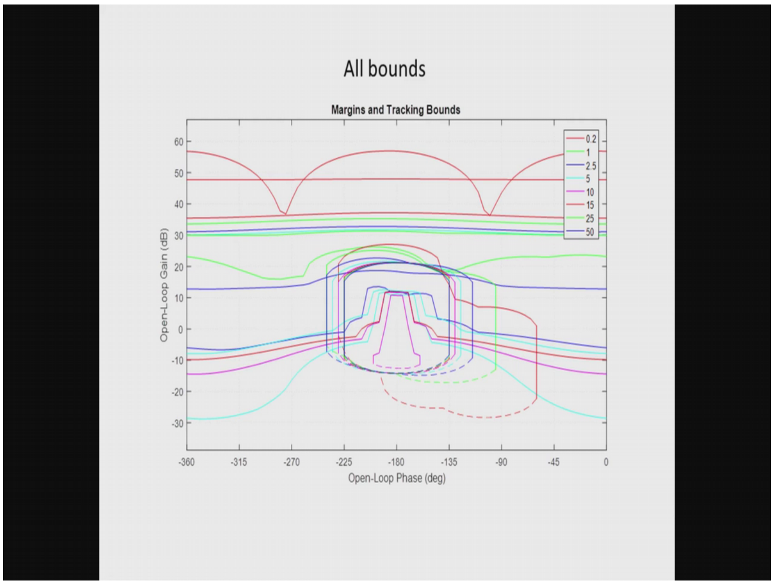

And all the bounds have been plotted together here by once again using the command plot bounds or plotbnds. So, after executing the function grpbnds the output is plotted and it shows the bounds on the loop gain owing to robust tracking, output disturbance rejection and input disturbance rejection over the entire frequency vector from 0.2 to 50 radians per second. So, in order for our loop gain to satisfy simultaneously all the different performance and stability specifications at any particular frequency, it should lie in a region that is going to be the intersection of the permissible regions in which it needs to lie in order for it to satisfy each of the individual performance specifications.

So, in order to obtain therefore, the intersection of this region and the boundary of this intersection, a new function is used that is that is called sectbnds and that is highlighted here. So, the function sectbnds helps us to obtain the intersection of the different bounds on the loop gain at each of the frequencies, where were interested in performance and stability.

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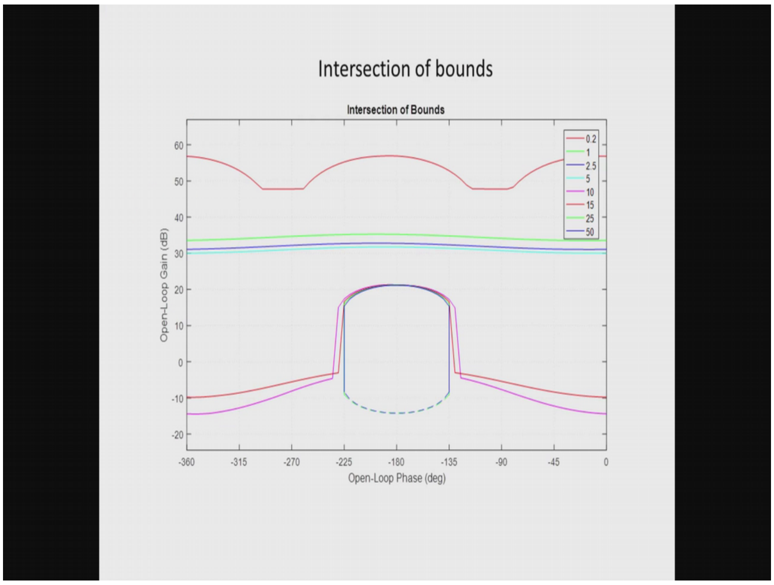

So, after executing the function sectbnds and then we plot the intersection of bounds it looks as shown in this slide here. So, this indicates that the nominal loop gain has to lie above the red curve at the top, which I am showing by means of the arrow here at 0.2 radian per second in order for it to satisfy all the 4 specifications simultaneously. Namely robust tracking, output disturbance rejection, input disturbance rejection and stability specification at 0.2 radians per second. Likewise, at 1 radian per second it should lie above this green curve at 2.5 radian per second should lie above the blue curve at 5 radian per second should lie above the cyan colored curve, at 10 radian per second should lie above the purple colored curve and so on and so forth.

Now, at 25 and 50 radians per second there were no performance specifications, we only had the stability specification. And, the corresponding stability bound has been indicated here and the loop gain has to lie outside this stability bound at 25 and 50 radians per second; for the control system to be stable to the specified extent at these frequencies. The next step after obtaining the intersection of bounds is to perform loop shaping in order to determine the structure of the controller that allows for the loop gain to lie within the permissible regions at each of the frequencies that have been shown here.

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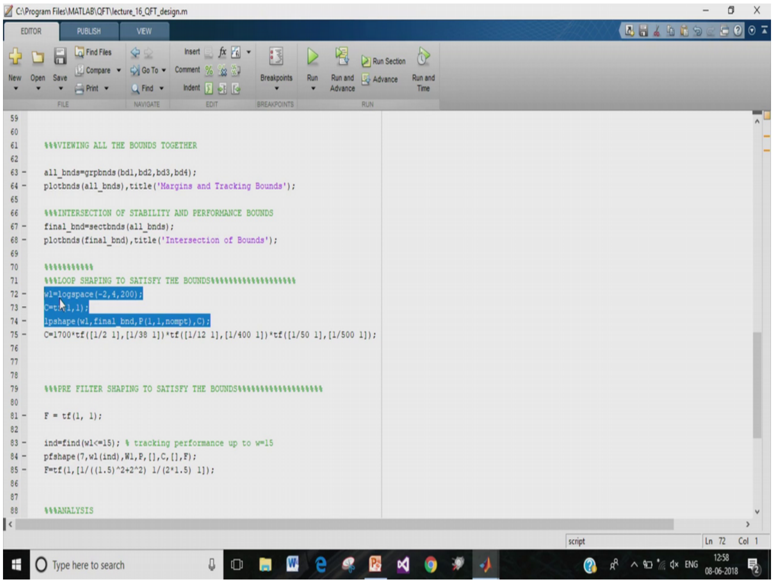

In order to perform loop shaping we have to use this command lpshape, which I am highlighting. Now, lpshape allows us to plot the Nichols plot and overlay upon the Nichols plot the different bounds that we have computed now and along with that also the loop shape of the nominal plant. So, the input arguments to this command are the frequency vector over which we want to plot the loop shape. And it so, happens that we have chosen that frequency vector to be frequencies starting from 10 power minus 2 radians per second to 10 power 4 radians per second.

And, we have chosen 200 points in between in order to plot the loop shape and the second input to this command is the intersection of all the bounds. And, the third input is the nominal plant and the fourth input is a controller. Now, at the start of the design our controller transfer function is just C is equal to 1 because, you have not yet obtained the controller that satisfies all the bounds. However, if we run this part of the code we will be led to a window where we can interactively perform control design.

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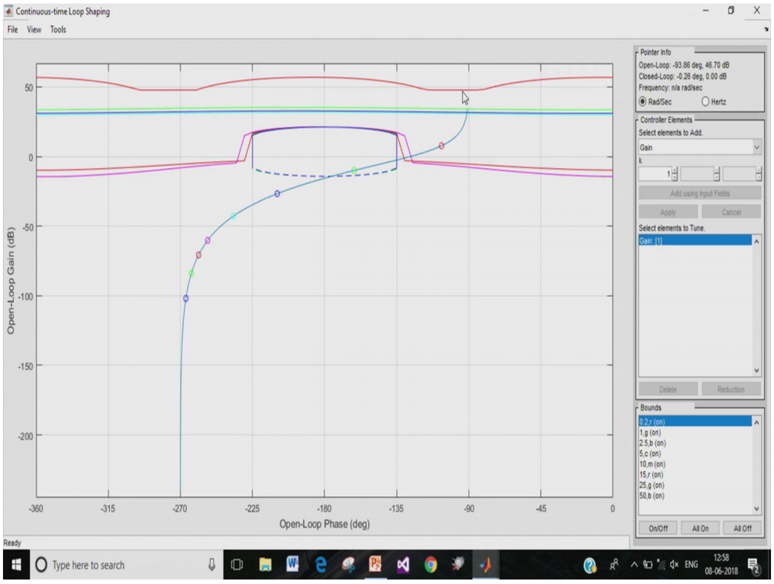

So, what you see in this window here is the Nichols plot of the open loop system. So, the Nichols plot of the open loop system is shown here and the colors here of the open circles indicate the loop gain at the different frequencies. And, these colors match the corresponding colors of the bounds at each of these frequencies. For example, at 0.2 radians per second the loop gain is here somewhere, as I am showing it by means of the arrow here. And, the bound on the loop gain at 0.2 radian per second is indicated above it here, likewise also at other frequencies.

So, when the controller C is equal to 1 we note that the open loop system is for all practical purposes violating all the bounds. So, the red circle which has to be above the red bound is actually below the red bound and therefore, the performance specification at 0.2 radian per second is violated. The green circle is below the green bound, the blue circle is below the blue bound, the cyan colored circle which is which corresponds to the loop gain at 5 radians per second is below the corresponding the cyan bound and so on and so forth. Therefore, with the controller C is equal to 1 we will not be able to achieve any of the performance specifications that we have set out for ourselves.

Furthermore, what you see is that the loop shape is entering this closed curve which represents the stability bounds at 25 and 50 radians per second. And therefore, even the extent of stability that we desire will not be achieved by using the controller C is equal to 1. However, what you have on the right hand side is this controller elements which can be added and we can interactively change the structure of the controller in order for us to achieve the desired performance specifications.

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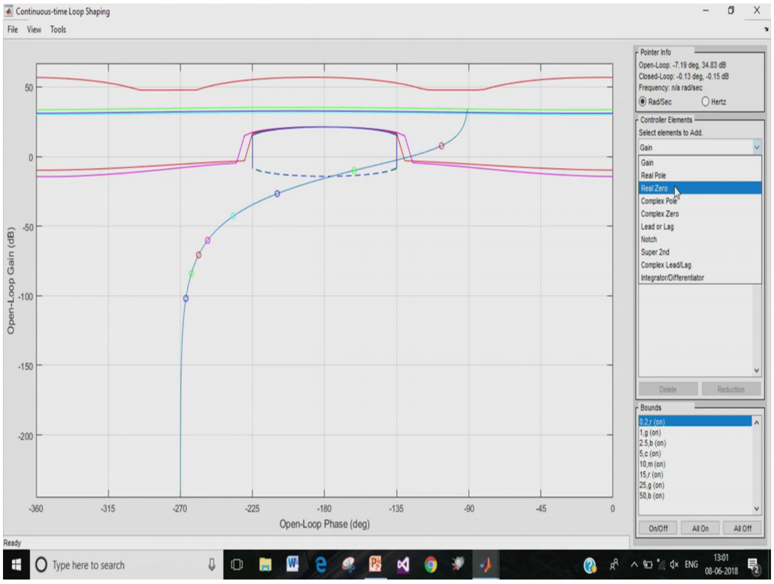

For instance, we can choose to add gain for the controller or we can choose to add a real pole for the controller, a real 0 for the controller, a complex pole, a complex zero, a lead, lag, a notch. So, all these different controller structure configurations are available for us to tune and by using these configurations we can appropriately modify the loop gain of the nominal open loop system and make sure that the loop gain satisfies all the performance bounds.

So, it will start with let us go with a simple gain element. So, we let us first try to make sure that the loop gain at the lowest frequency that we have a picked namely at 0.2 radians per second is within its specified bound. Namely, the top most curve that you see in the Nichols plot here.

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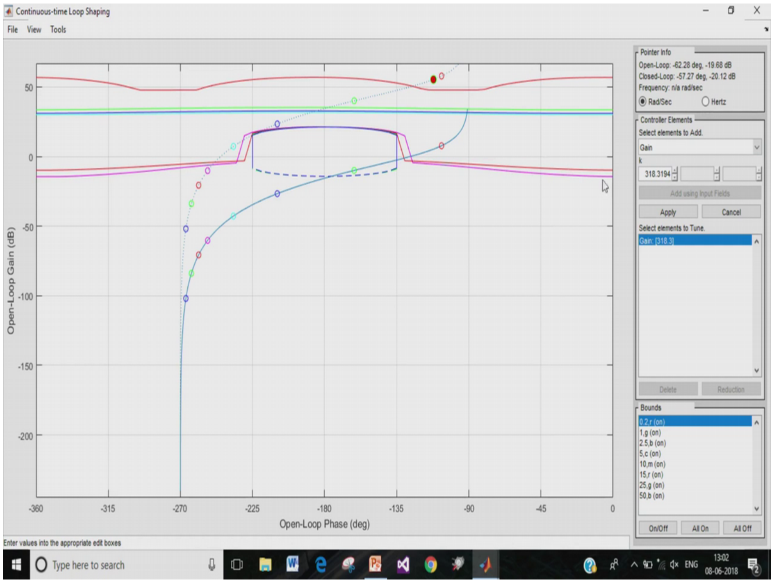

So, what I shall do here is I can take the entire loop gain and drag it up in this particular manner. And, the software will allow us to place this loop at whatever location that we want. And, if we do that then on the right hand side we see the kind of gain that we need to choose in order for our loop shape to be located as indicated by the dotted lines here. So, suppose we choose to go with this particular gain and hit apply.

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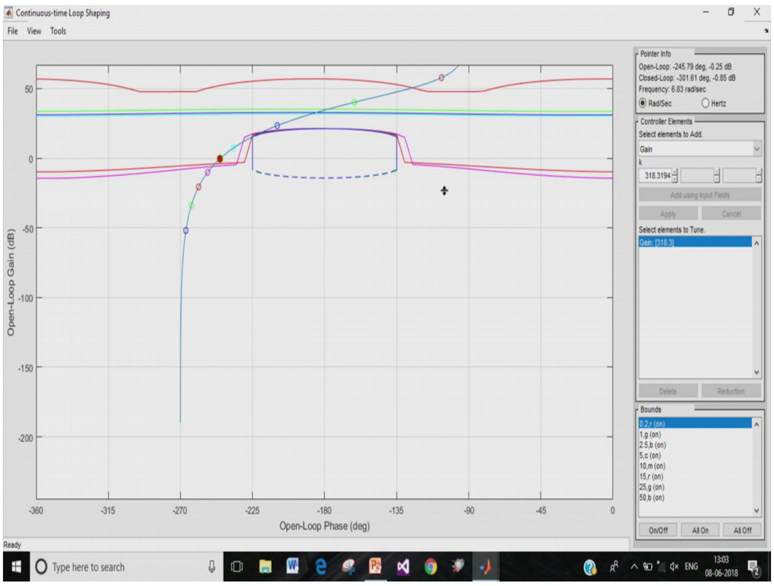

Then we have chosen a simple proportional controller with a gain of about 318 units and we note that with this proportional controller the red circle is above the red bound. So, we are able to satisfy the performance specification at 0.2 radians per second, the green circle is above the green bound. So, at 1 radian per second also we are able to satisfy the performance specification. The blue circle is actually below the blue bound and the overall loop is encircling the critical point namely 0 dB comma minus 180 degree in such a way that our closed loop system will be unstable.

So, what we would like to do therefore, is to drag this entire loop back towards the right in order to prevent this loop from encircling the critical point, namely the point 0 dB comma minus 180 degrees in the way that it has done now. Because, at the present moment if you compute the phase margin you will discover that the phase margin is negative or in other words the close loop system is going to be unstable. So, in order to drag the entire loop towards the right we can choose to add a 0 because, we know that and 0 adds a phase lead.

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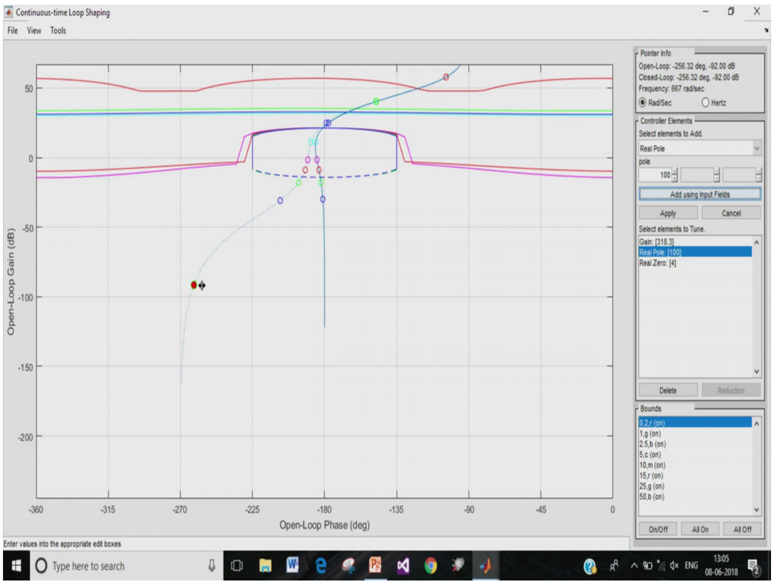

So, we can choose to pick a real 0 and suppose we choose to add this real 0 at some particular frequency 4 radians per second. So, if I were to enter 4 in the box that has been provided to me on the right top corner and hit add using input fields then the dotted curve here will show how the loop shape will change upon adding this 0. So, if I am with this change I can hit apply and when I hit apply the controller structure changes to 1 which has a gain element cascaded with a real 0 of course, we know that a controller with a single 0 is a non causal controller.

So, we have to also pick another pole. So, I shall now go and pick a real pole as an extra element in my controller and I shall choose to locate this pole fairly far away, I shall choose to locate it around 100 radians per second. So, I shall hit add using input fields to see how the loop shape changes, when I include a term s by 100 plus 1 as part of my controller structure and that is indicated by the dotted line dotted curve that you see here.

So, the solid blue curve here shows the loop shape with a single 0 and a gain term cascaded with the plant. The dotted curve here shows the loop shape with a 0 and a pole along with the gain cascaded with the loop shape. So, if I hit apply then the controller structure will now include both a real pole as well as a real 0.

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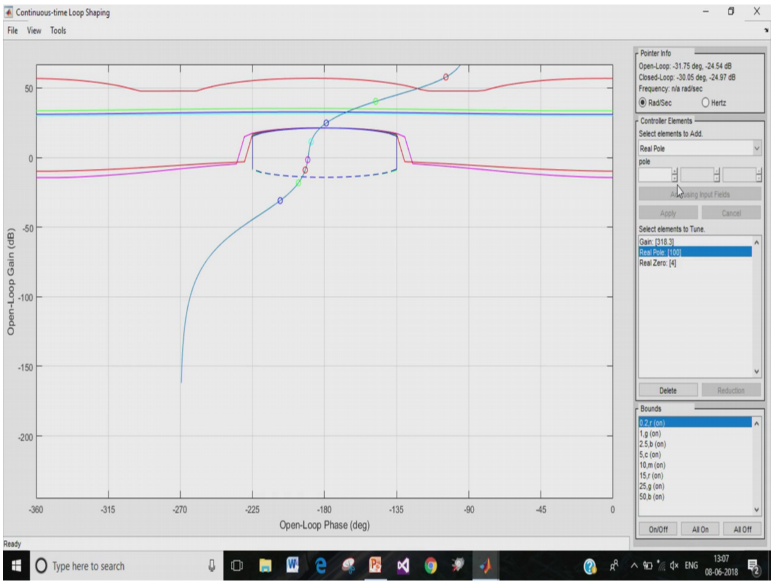

Now, we see that with this combination we are still not able to meet the stability specifications or the performance specifications at several of the frequencies. So, we are able to meet it only at one end point to radian per second. So, we have to add many more poles and 0’s for us to be able to meet them. So, I have undertaken this iterative design in order to pick the poles and 0’s of our controller. And, what is attractive about this tool is that it will allow us to interactively study how the loop shape changes when we change the structure of the controller or the parameters of the controller.

So, before we freeze the structure of the controller or its particular parameters we can see by means of the dotted curve, how the loop shape will change upon making the changes that we desire to make. And by iteratively making the changes in order to ensure that each of the performance and stability specifications are met, we can finalize the controller structure. I have already undertaken this design and the controller structure that meets all the different controller specifications has been shown here. So, it is given by C is equal to 1700 times s by 2 plus 1 divided by s by 38 plus 1 times s by 12 plus 1 divided by s by 400 plus 1 times s by 50 plus 1 divided by s by 500 plus 1.

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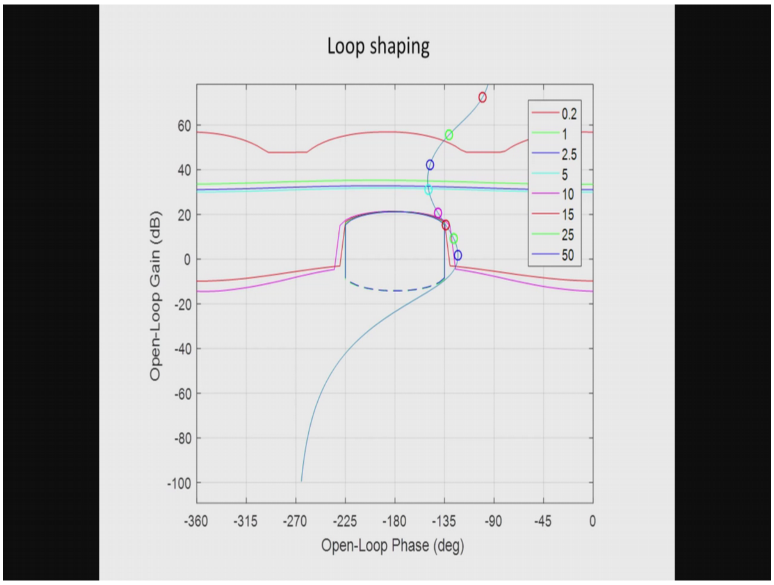

For this particular case the loop shape is shown here, you see that the red circle is well above the red bound. So, actually at 0.2 radian per second here ended up designing a control system significantly more conservative than what we would like it to be. But, it meets the performance specifications better than what we had intended it to do. And likewise, the green circle represents a loop gain at 1 radian per second and that is well above the green bound here.

The blue circle is above the blue bound the cyan circle is just on the cyan bound. So, at 5 radians per second the performance specifications are just exactly met. Likewise, you can also notice that the loop gain at other frequencies are well above the bounds at those specific frequencies. And, the loop shape just touches this closed curve which represents a stability bounds at 25 and 50 radians per second which implies that the stability specifications are just met as a consequence of this particular controller structure.

So, having performed loop shaping to determine the structure of the feedback controller, the next step is to perform shaping of the prefilter, in order to determine the nominal dominant dynamics of the overall closed loop system and that is performed using a function pfshape. So, the input argument to pfshape are indicated here. The first input argument is the index of the problem that we want to solve and since we are interested in restricting the variation of the transmission function between the specified limits the problem type is 7.

And, the second input is a frequency range over which you want to restrict the variation and we have specified in the line above, that the frequency range has to be less than or equal to 15 radians per second. W 1 is a specified weight for robust tracking and that is basically the combination of t upper and t lower and so on and so forth. P here represents the uncertain plant, C here represents the feedback control that we have already design and F is a prefilter. So, at the moment we have not yet designed a prefilter so, F is equal to 1. So, on another words F is tf of 1 comma 1.

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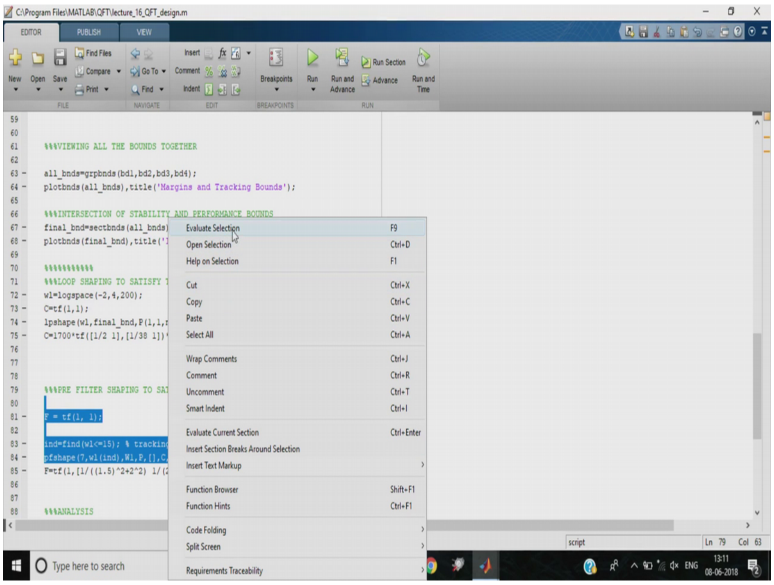

We can now run this part of the code and we will be led to a window, where we can perform prefilter shaping in a manner similar to the way we performed loop shaping and determine the structure of the feedback controller.

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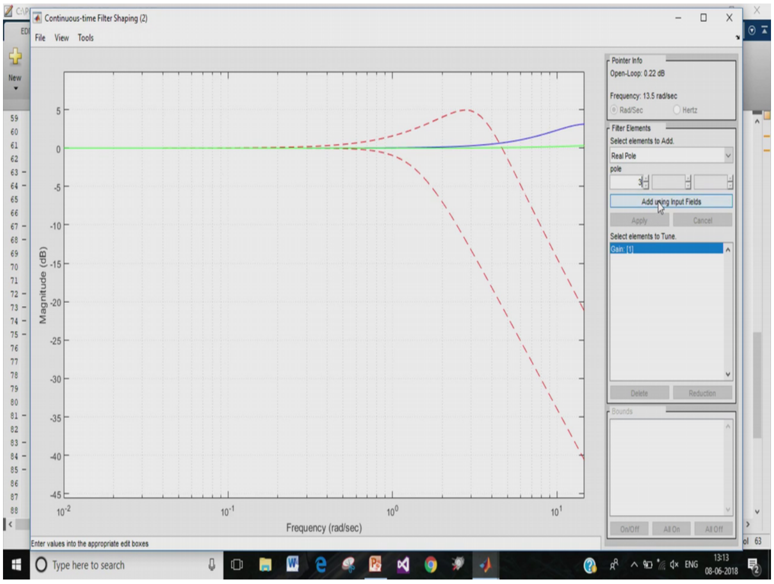

So, once again this here shows the overall transmission function and the prefilter is right now chosen to be just f is equal to 1 or its gain k is equal to 1. And, we can once again play around with the pole and 0 locations of the prefilter and the red dotted lines here indicate the transfer functions t upper and t lower. So, we want the maximum variation of the transmission function to be between these two dotted lines. It so happens that for the controller that we have design the performance specification as far as robust tracking has been met much better than what was expected.

The blue curve here shows the upper limit of the transmission function, while the green curve here shows the lower limit of the transmission function. And, we see that at each of the frequencies the difference between the upper limit and the lower limit is much less than what we desire it to be. And, that was because we choose a feedback controller whose gain had to be so high in order to meet the other performance specifications such as disturbance, rejection of output disturbance and input disturbance; that as far as robust tracking was concerned the high gain resulted in very small variation in the transmission function of the overall system.

However, while the restriction of the variation in the transmission function has been successfully achieved by the feedback controller. The nominal transmission function has to be designed and this is done by using the prefilter. So, we can choose the prefilter to have a real pole for instance and we can choose to place the pole, let us say at 3 radians per second. And, if we do that if we apply it we see that there is an improvement in the extent to which the transmission functions t upper and t lower which are indicated by the blue curve and the green curve here fall between the specified limits indicated by the red dotted lines.

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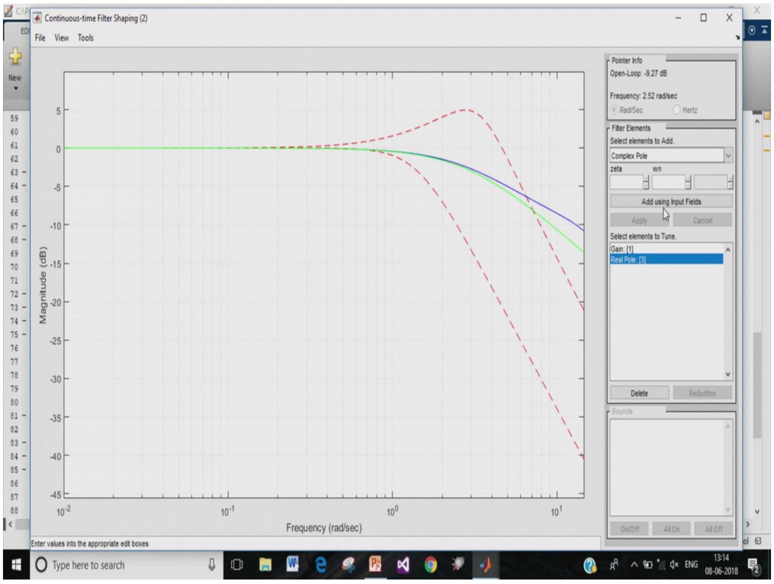

So, we want the dominant poles to be at minus 1.5 plus minus 2 j. So, what we need to do therefore, is to select a complex pole here whose zeta and omega n are chosen in such a manner that the real part of this complex pole is minus 1.5 and the imaginary part of this complex pole is 2 units. I have chosen that to be the structure of our prefilter and with that choice we note that the maximum and the minimum variation in the transmission function of our overall system fall well within the bounds t upper and t lower, that we had specified for ourselves.

With this we have completed the design of the prefilter and the feedback controller. The next step is to check for performance. The first check that we need to do is to ensure that our stability specification has been met.

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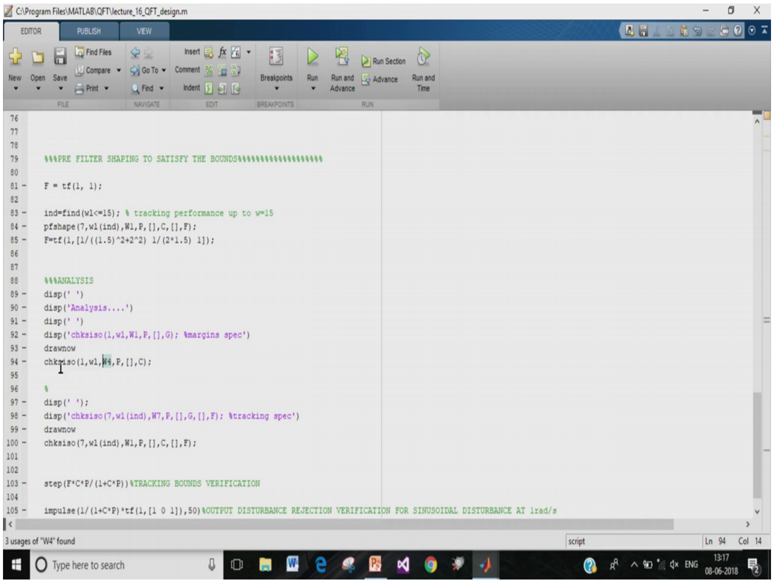

And the function that allows us to check for stability is the function chksiso. The input argument, the first input argument to this function is a problem type and for stability considerations the problem type has to be equal to 1. And, w 1 is the frequency range over which we have to check for the stability specification. W 4 is the specific weight for stability, we wanted t 1 max to be less than or equal to 3.5 dB for essentially in the linear scale above 1.41 units.

And, that essentially becomes W 4, P is of course, the set of uncertain plants and C is the controller. So, if we run this execute this particular line of the code we will be able to check whether at all the frequencies the maximum value of the transmission function is within the specified upper limit of 3 dB or not.

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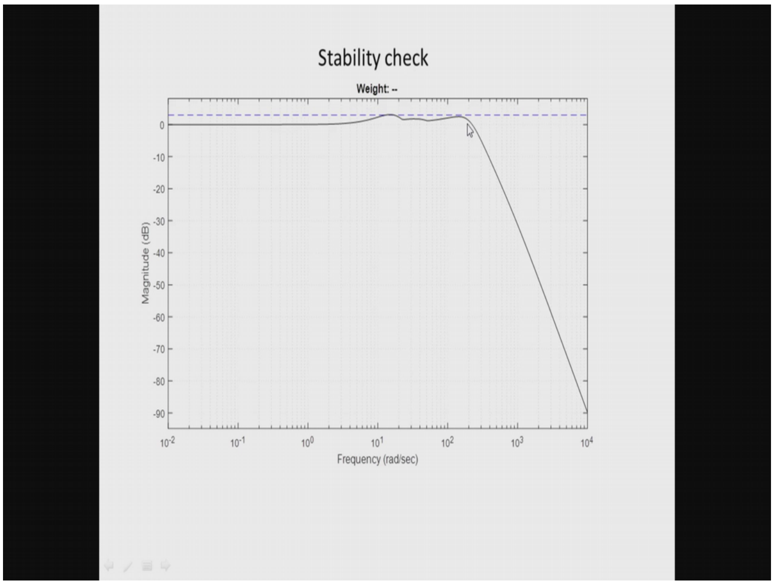

And that has been done here in this slide. So, this slide here shows the transmission function of the feedback part alone. And, the blue dotted curve on the top, which I am now indicating by means of this arrow here indicates the line 3 dB. And, the black curve here shows the maximum value the transmission function at different frequencies. And, we see that at no frequency is a transmission function exceeding 3 dB in magnitude and this in turn implies that the stability specification of our closed loop system has been met. It so happens that the specification that the magnitude of t 1 should be less than or equal to 3 dB, ensures that the phase margin of our closed loop system is at least 40 degrees.

So, the design that we have executed now ensures that our closed loop system has a phase margin of at least 40 degrees, regardless of the uncertainty that we have in the plants parameters. So, having therefore, completed the different steps in the design performed essentially loop shaping to determine the structure of the feedback controller and then design the prefilter. And, also verified the closed loop transmission function t 1 satisfies the specified stability limit. The last step is to verify that the disturbance rejection and robust tracking specifications have been met in the time domain.

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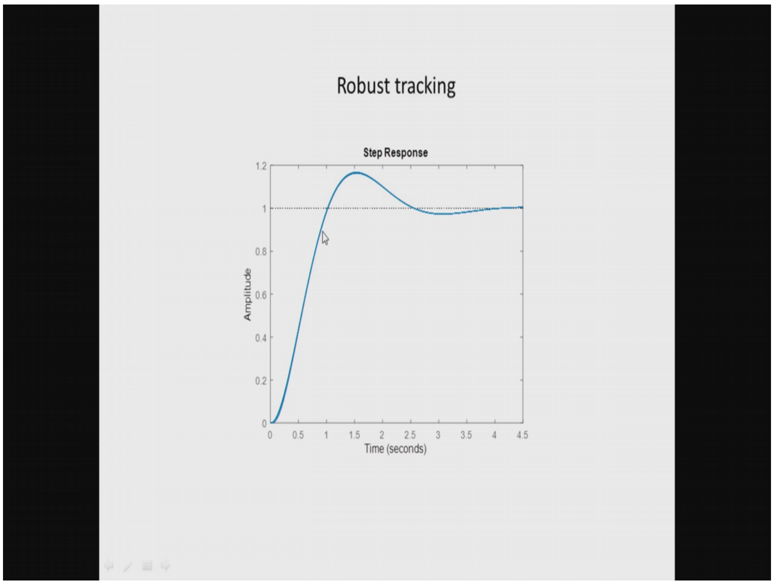

So, what I have done therefore, in the next slide is plotted the step response of the closed loop system for the different parameters of the plant. And, what we see here is that the closed loop system step response for different parameters, almost all sit one on top of another. And, the dominant dynamics of this is decided by the desired closed loop pole locations; namely minus 1.5 plus minus 2 j.

Now, the spread in the response is much lesser than what we were with simply because, the other performance specifications in terms of output disturbance rejection and input disturbance rejection demanded such a high loop gain at the frequencies; where robust tracking had to be performed. But, the variation of the overall transmission function at these frequencies was much smaller than what we were willing to tolerate. As a consequence therefore, the step response of the overall closed loop system for all the different possible transfer functions of the plant can have all sit nearly one on top of another.

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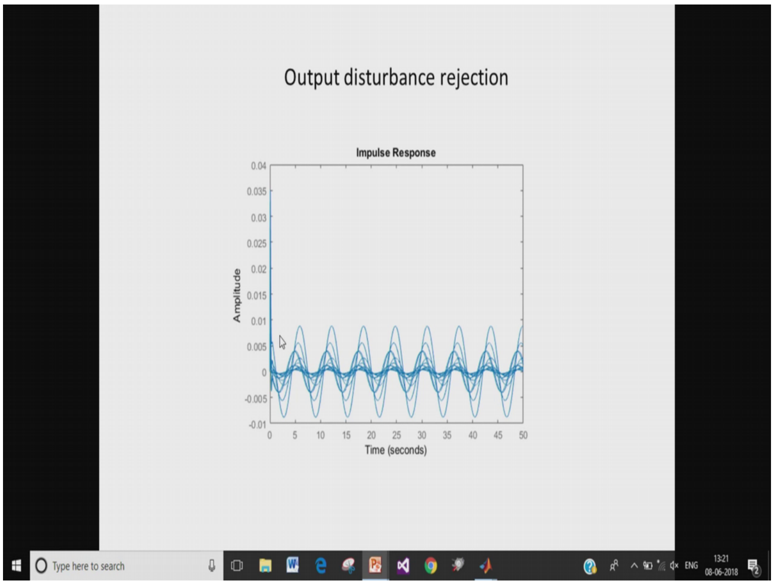

Likewise, we have also plotted the output disturbance rejection performance. We expected the output disturbance should be rejected by 90 percent or the output disturbance should leak by an extent of at most 10 percent to the output of the overall feedback system. And, what we see here is that the amplitude of the disturbance in this case we have chosen the frequency of the disturbance to be 1 radian per second is suppressed to just 1 percent of its initial magnitude.

So, although we desired 10 percent leakage or better we have managed to do much better than what was expected of us and the amount of leakage that we have of the output disturbance to the output is merely 1 percent at 1 radian per second. We can also perform this test of the output disturbance rejection at other frequencies up to 5 radians per second and what we will see is that even at other frequencies the output disturbance has been suppressed by the desired extent.

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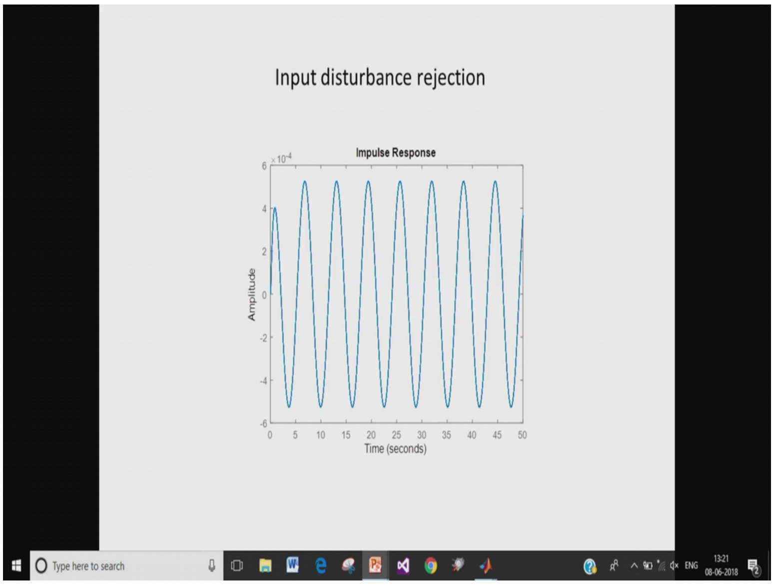

Likewise, when we look at the input disturbance rejection, we note that the amplitude of the response or the output due to input disturbance is nearly 6 into 10 to power minus 4 units, which indicates that we have managed to suppress the input disturbance by an amount it is much larger than what was expected of us. We wanted 1 percent of the input disturbance to appear at the output of our system and what we have succeeded in doing is to suppress it not to just 0.01 units, but actually to 6 into 10 power minus 4 units.

Once again this much larger suppression of input disturbance compared to what was desired was a consequence of coming up with a controller which resulted in a nominal loop gain that was so high, that for such high values of the nominal loop gain the input disturbance got suppress to a very small value. So, with this we come to the end of this section on quantitative feedback theory. And, the goal of this particular clip was to introduce to you the different functions in the QFT toolbox that you can use to firstly, draw the plant templates. Subsequently compute the bounds on different performance and stability specifications plot those bounds, obtain the intersection of these bounds.

And finally, perform loop shaping and prefilter shaping and complete the design. The toolbox also allows us to check for performance, both in the time and the frequency domains and even that has been discussed in this clip. With this we come to the end of our discussion on quantitative feedback theory

Thank you.