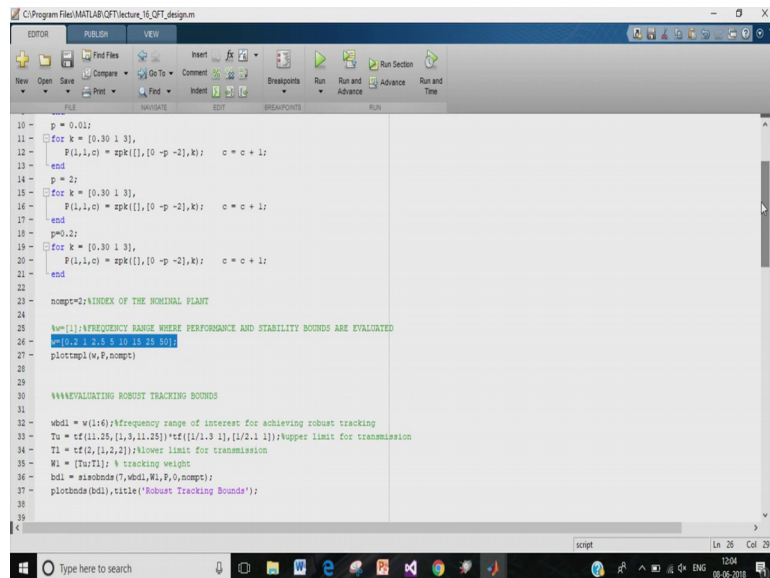


Control System Design
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Lecture – 38
Tutorial on QFT Toolbox software (Part 2/3)

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```
10 - p = 0.01;
11 - for k = [0.30 1 3],
12 -     P(1,1,c) = zpK(1,[0 -p -2],k); c = c + 1;
13 - end
14 - p = 2;
15 - for k = [0.30 1 3],
16 -     P(1,1,c) = zpK(1,[0 -p -2],k); c = c + 1;
17 - end
18 - p=0.2;
19 - for k = [0.30 1 3],
20 -     P(1,1,c) = zpK(1,[0 -p -2],k); c = c + 1;
21 - end
22 -
23 - ncomp=2;%INDEX OF THE NOMINAL PLANT
24 -
25 - w=[1];%FREQUENCY RANGE WHERE PERFORMANCE AND STABILITY BOUNDS ARE EVALUATED
26 - w=[0.2 1 2.5 5 10 15 25 50];
27 - plottpl(V,P,ncomp)
28 -
29 -
30 - %%%EVALUATING ROBUST TRACKING BOUNDS
31 -
32 - wbd1 = w(16);%frequency range of interest for achieving robust tracking
33 - Tu = tf(11.25,[1,3,11.25])*tf(1/[1.3 1],[1/2.1 1]);%upper limit for transmission
34 - Tl = tf(2,[1,2,2]);%lower limit for transmission
35 - Wt = [Tu Tl]; % tracking weight
36 - hdl = slrbdnds(T,wbd1,Wt,P,0,ncomp);
37 - plotbdnds(hdl,'title('Robust Tracking Bounds');
38 -
39 -
```

So what you see here is the code that you would be using for performing control design using the QFT toolbox. The first step as we discussed just a few minutes back is to identify the frequencies of interest to us from the points of view of performance and stability. And that set of frequencies has been indicated by this frequency vector w which I am highlighting right now.

Now, there is a basis for choosing the entries of this frequency vector w . For one thing we are interested in output disturbance rejection up to a frequency of 5 radians per second, we are interested in input disturbance rejection up to a frequency of 10 radians per second. So, if you notice here therefore, we have the numbers 5 and the number 10 both being entries in the frequency vector w for which we will be plotting the plan templates.

But since, we have disturbance up to 5 or 10 radians per second, we also have to pick frequencies that are less than these two numbers. So, I have picked some small

frequencies 0.2 radian per second, 1 radian per second, 2.5 radian per second as some of the other entries at which we would be concerned about both input and output disturbances.

Now, you might ask why did we just pick 3 entries in the frequency range between 0 and 5 radians per second, we could have picked more, we could have picked less, it is just that we picked enough number of them for us to be able to have enough bounds for us to execute the design properly. The more we pick the better it would be but then we have to keep track of many more bounds at many more frequencies and that would make the control design procedure a little bit more confusing.

Hence, I have taken the middle path and chosen just enough frequencies for us to have enough number of bounds for to execute design, but yet not get confused with having too many bounds or too many frequencies. So, that justifies why we have frequencies up to 10 radian per second. The other performance requirement that we have for the control system is on robust tracking.

Now, if you go back to the previous clip, we noted that we are interested in robust tracking up to that frequency at which the upper limit of the transmission function or t upper, which was what we used in the previous clip fell to minus 20 db and it turned out in the previous clip that the magnitude of t upper came down to minus 20 db at 15 radians per second.

Hence, we are interested in robust tracking or in other words we are interested in restricting the variation of the transmission function due to uncertainty in the plant parameters up to 15 radian per second and that is why the number 15 is also one of the members of this frequency vector w .

Now, while all these frequencies up to 15 radian per second are there in order for us to drop bounds in the interest of achieving specified performance specifications made with disturbance rejection or robust tracking, we cannot stop worrying about stability at 15 radian per second because, a loop gain might cross over at frequencies that are greater than 15 radian per second.

In order to make sure that, we have the stability bounds being satisfied at frequencies beyond 15 radian per second also, I have picked two more frequencies namely 25 radian

per second and 50 radian per second where, we would draw only the bounds on based on the stability specification and not on performance specification. It is worth noting, but the two frequencies namely 25 radian per second and 50 radian per second are both significantly greater than the corner frequencies of the plant, which are at 0, 1 and 2 radian per second nominally and hence the plant template at these two frequencies will resemble nearly a straight line.

Therefore, it does not matter whether we pick two frequencies or many more frequencies, the shape of the plant template will be independent of the number of frequencies or the specific frequency we pick, because all of these frequencies are much greater than the corner frequency of the plant. In principle we could have picked only 1 frequency but for the sake of being conservative in design I have picked two entries namely 25 and 50 radians per second.

So, to justify why these entries are there in the frequency vector, we have entries from 0 to 5 radians per second because, it is in this frequency range that we have output disturbance affecting our system. And the number 10 exists here because, the input disturbance exists up to 10 radians per second and in the frequency range 0 to 15 radian per second we are interested in robust tracking and hence the number 15 also exists as part of the frequency vector or the set of frequencies is very interested in performance. And the last 2 entries namely 25 and 50 radians per second or frequencies at which, we are not any longer interested in performance because we do not have disturbance is affecting our system at these frequencies or nor are we interested in robust tracking at these frequencies but stability is still a concern.

Hence, we have these 2 entries at which, we would draw the stability bounds and make sure that our loop gain is within the acceptable region of the Nichols plot, that is outside these stability bounds. So, this is the first step of the control design using QFT toolbox namely, to determine the frequencies of interest to us from the points of view of performance and stability.

The next step is to plot the plant templates at the frequencies that have been identified in the first step; namely to plot the plant templates at each of the frequencies that we have in the frequency vector w . So, let us recall what we mean by a plant template, a plant

template is a set of complex gains that a plant can assume at any particular frequency ω .

If the plant has no uncertainty associated with its parameters, then the plant will have only 1 complex gain at a given frequency; however, if the gain of the plant or its pole location is uncertain then depending on the specific gain and a specific pole location the magnitude of p of $j\omega$ will be different at the same frequency ω and set of different values at p of $j\omega$ can assume or a set of different magnitudes and phases that p of $j\omega$ can have for the different possible combinations of the plants gain k and its pole location p defines the plant template at that particular frequency.

Now, how do we draw the plant template?

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```
3 clear
4 clear all;
5 c = 1; p = 1; %OBTAIN THE SET OF POSSIBLE PLANT TRANSFER FUNCTIONS. NOTE: THE INTEGRATOR IN THE PLANT TRANSFER FUNCTION HAS BEEN REPLACED WITH A POLE AT -0.1
6 %NEEDED LOOP STEP RESPONSES LOOK GREAT
7 for k = (0.30 : 0.3)
8     P(1,1,c) = zpK(1,1,0-p,-2),k); c = c + 1;
9
10 p = 0.01;
11 for k = (0.30 : 0.3)
12     P(1,1,c) = zpK(1,1,0-p,-2),k); c = c + 1;
13
14 p = 0.2;
15 for k = (0.30 : 0.3)
16     P(1,1,c) = zpK(1,1,0-p,-2),k); c = c + 1;
17
18 p = 0.2;
19 for k = (0.30 : 0.3)
20     P(1,1,c) = zpK(1,1,0-p,-2),k); c = c + 1;
21
22 nompc=2; %INDEX OF THE NOMINAL PLANT
23
24 %w=[1]; %FREQUENCY RANGE WHERE PERFORMANCE AND STABILITY BOUNDS ARE EVALUATED
25 %w=[0.2 1 2.5 5 10 15 30];
26 plottempl(w,P,nompc)
27
28
29
30 %%%EVALUATING ROBUST TRACKING BOUNDS
31
```

In order to draw the plant template code I want to first highlight the first part of the where we have defined, 12 different plants. Now, all of these plants have the same structure as the plant that, we have we are concerned about. Namely the uncertain plant p of s is equal to k by s times s plus p times s plus 2. Now, so happens that each of these plants have different gains and different pole positions and therefore, represent the possible transfer functions that our uncertain plant can assume, when its plant parameters vary.

So, for example, if you focus on the first line of the code, in this line we have varied the gain of the plant from 0.3 to 3.00 and that is what I am highlighting now and the pole location p in this line of code has been assumed to be equal to 1. So, that is from the first line which I have highlighted here.

So, this represents 3 plants, all of whom have the same pole location namely at s is equal to minus 1 but each of them have different gains; one has a gain of 0.3, the other has a gain of 1, the third one has a gain of 3. Likewise in the next quote, we have assumed that the pole location is close to 0 because, we know that our value of p in our plant transfer function can vary between 0 and 2.

In this case we have chosen the value p to be close 0, so, here are a set it to be equal to 0.1. And in this line of code we have defined 3 plant transfer functions and in this 3 plant transfer functions once again the value of p is the same, namely 0.01 or close to 0 and the value of gain is allowed to change, it can assume values between 0.3 and 3.

So, the gain k here can take values between 0.3 and 3. It can take 3 values 0.3, 1 and 3. Likewise, we have in the next line we have defined three more transfer functions. In this case the pole location is assumed to be at x is equal to minus 2 and therefore, we have set p is equal to 2 here and we have defined three plant transfer functions, where the pole location is at x is equal to minus 2 and the gain is allowed to assume 3 different values. It is nominal value of 1 and the minimum value of 0.3 and the maximum value of 3 and we have repeated the same exercise at a different value of pole location p is equal to 0.2 and obtain another three set off transfer functions.

So, this entire set of transfer functions, represent the permissible set of transfer functions that the plant can have when its gain and pole locations change. Of course, the gain need not be only 0.3, 1 and 3 and the pole locations need not be just at 0.01, 2, 1 and so on, but in this particular case we have considered only 12 different combinations of the pole locations and the plant gain. We can consider many more locations and get a better plant template if we define more possible transfer functions, but the plant can assume when its gain and its pole locations vary.

So, here therefore, we have 12 transfer functions all of which represent the transfer functions that, our plant can assume for some combination of its open loop pole position and gain. Now in order to plan plot the plan template at any particular frequency, we

have to substitute the value of that frequency for in each of these transfer functions and compute their magnitude and phase and locate them on the Nichols plot.

Now, in order to do that I shall first do it at one single frequency. So, I shall do it at ω equal to 1, so this is the frequency at which I shall do it. I shall not do it at all the other frequencies namely from 0.2 to 50, just for the sake of clarity. So, I shall comment out the line of the code where you are computing it at all the other frequencies, we shall compute the plant template at only one frequency, namely at ω equal to 1.

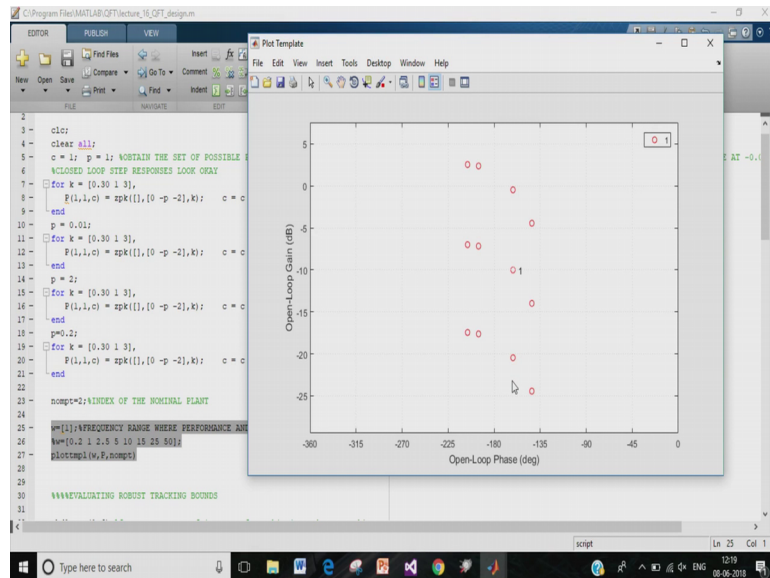
Now, the plant template also has to represent the nominal gain of the plant at this particular frequency, namely 1 radian per second and nominal gain of the plant is the gain of the plant, when its parameters assume the nominal values namely, when p small p is equal to 1 and the gain small k is equal to 1. So, the index of the transfer function at which the plant pole location is p equal to 1 and the gain k is equal to 1 is essentially 2.

So, among the set of different transfer functions that we have defined here which could be the possible transfer functions that our plant can have when its gain and its pole locations change. It is a second transfer function in this set which represents the nominal transfer function of the plant. Therefore, the index of the nominal plant which has been referred to as `nompt` in this code is equal to 2 because among these transfer functions, it is a second transfer function that represents a nominal transfer function of the plant. So, when we plot the plant template, the plant template will also indicate the location of the nominal plant.

So, with this extra information let us proceed to draw the plant template of this particular uncertain plant at ω equal to 1 radian per second. And that brings us to the first function of the `q of tt will box` and that function is what is known as `plot_tm_pl`, which is a short form for plot template and it is intended to plot the plant templates, if you are given an uncertain plant. The input arguments to this function are the frequency vector or the set of frequencies at which, we want to plot the plant templates. Then the second input argument is the set of uncertain plants which is denoted by capital `P` here and a third input argument is the index of the nominal plant. So, we know that the nominal plants index is 2. So, that is the a second third input argument to this function.

So, if we were to run this specific line of code, we will be able to plot the plant template at just 1 radian per second. So, I am executing just that line of code now and in a few seconds time we get to see the plant template, but 1 radian per second.

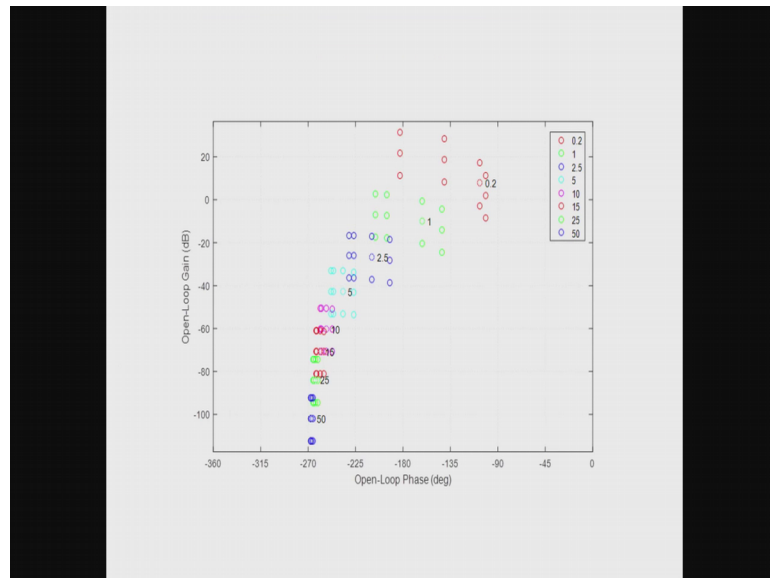
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So, this is how the plant template appears, what you see therefore, is that the software has computed the gain and phase values of p of j omega for the different combinations of the plants gain k and its pole location small p and at the center here which I am indicating by this arrow is a location of the nominal plant.

So, at 1 radian per second the open loop gain of the nominal plant is around minus 10 db and the open loop phase is somewhere between minus 180 degrees and minus 135 degrees what is the location of the nominal plant.

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Now, we can repeat this exercise of other frequencies also, in the frequency vector that we have just defined. And the plant templates at the different frequencies are all plotted together in this graph here. So, they have all been color coded for instance, the red circles at the top represent the plant template at 0.2 radians per second.

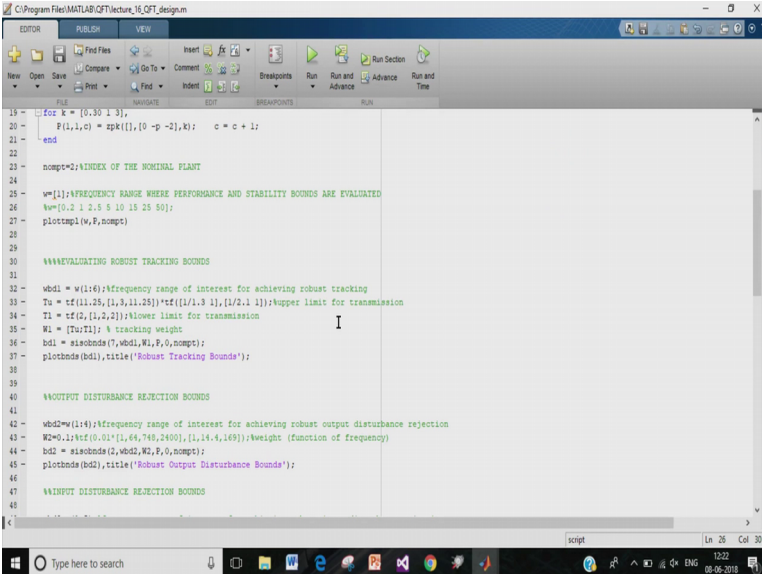
Now, this plant template would have many more points within it, if we had chosen many more values of the plant gain k and its pole location p . In order to define many more set of possible transfer functions that our plant can assume and computed the gains and the phase values of these transfer functions, but each of these frequencies. Here we have chosen 12 transfer functions, so, we have only therefore, represented 12 values of the gain and phase for the uncertain plant.

So, at 0.2 radian per second, this area which I am showing here by means of the arrow represents the plant template at 1 radian per second the green area the area that is, covered by the green circles here green open circles represents the plant template 2.5 radiant per second. The blue area here represents the plant template at 5 radians per second the cyan color area here represents the plant template and so on and so forth. And as you see here, as a frequencies increasing, the width of the plant template is reducing, which effectively means that, the uncertainty associated with the phase of the plant reduces as the frequency increases and what frequencies that are much greater than the

corner frequency of the plant, namely at close to 25 and 50 radians per second. What you see is that the plant template looks almost like a straight line.

The height of each of these plant templates is determined by the uncertainty in the gain that we have. And in all cases, the uncertainty is a factor of 10 and therefore, the height of each of these plant templates is equal to 20 db, which is basically equal to 20 log to the base 10 of 10 units. So, the height of each of the plant templates here at each of the frequencies is the same namely 20 db but the width is dependent on the kind of phase uncertainty that we would have due to the uncertainty in the location of the pole of the plant. And this phase uncertainty reduces with increasing frequency and that frequency is much greater than the corner frequency of the plant. The phase uncertainty is so small at the plant template practically looks like a straight line.

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```
19 = for k = [0.30 1 3],
20 =     F(1,1,0) = spk(1,[0 -p -2],k);   c = c + 1;
21 = end
22
23 ncomp=2; %INDEX OF THE NOMINAL PLANT
24
25 w=[1]; %FREQUENCY RANGE WHERE PERFORMANCE AND STABILITY BOUNDS ARE EVALUATED
26 %w=[0.2 1 2 5 10 15 25 50];
27 plotmag1(w,P,ncomp)
28
29
30 %%%EVALUATING ROBUST TRACKING BOUNDS
31
32 wbd1 = w(1:6); %frequency range of interest for achieving robust tracking
33 Tc = tf(1,1.25,[1,3,11.25])*tf(1/(1.2 1),[12 1 1]); %upper limit for transmission
34 Tl = tf(2,[1,2,2]); %lower limit for transmission
35 Wl = [TcTl]; % tracking weight
36 bdl = sisobnds(7,wbd1,Wl,P,0,ncomp);
37 plotbnds(bdl),title('Robust Tracking Bounds');
38
39
40 %%%OUTPUT DISTURBANCE REJECTION BOUNDS
41
42 wbd2=w(1:6); %frequency range of interest for achieving robust output disturbance rejection
43 W2=0.1*tf(0.01*[1,64,740,2400],[1,14.4,169]); %weight (function of frequency)
44 bdl = sisobnds(2,wbd2,W2,P,0,ncomp);
45 plotbnds(bdl),title('Robust Output Disturbance Bounds');
46
47 %%%INPUT DISTURBANCE REJECTION BOUNDS
48
```

The next step in our design procedure is to compute the bounds on performance and stability, in each of the frequencies where these bounds have to be computed. So, let us first start with robust tracking bounds. We know that we are interested in restricting the variation of the transmission function due to variation in the plant parameters up to 15 radians per second. So, in order to therefore, compute the bounds on the loop gain we choose frequencies up to 15 radian per second. So, we define a new frequency vector w d 1 which has been highlighted in blue color as you see on the screen now. And this frequency vector comprises the first six entries of the original frequency vector w.

So, original frequency vector w had frequencies going from 0.2 to 50 radians per second and the 6th element here is the is 15 radians per second and they are interested in computing the robust tracking bounds up to 15 radians per second because, we are interested in restricting the variation in the transmission function due to uncertainty in the plant only up to 15 radiant per second.

So, this is the frequency vector up to which we are interested in computing the bounds for robust tracking. Next we have also defined here, the transfer functions for the upper and lower limit of the spread in the transmission functions in of the overall system and we called them t upper and t lower when we discussed it in the previous clip. And very similar symbols have been used here, t_u here represents the upper bound for the permissible variation in the transmission functions and t_l lower represents the lower bound for the permissible variation in the transmission functions.

When we allow the dominant closed loop pole to be located anywhere within the 2 rectangles defined by the complex number p is equal to a plus or minus jb where, a can assume values between minus 1 and minus 3 and b can assume values between 1 and 3 And in order to draw the robust tracking bound we need to have an input to the function that draws these bounds and that input is this so called tracking wake, which is essentially a vector comprised of the upper bound t upper and the lower bound t lower and that is what is has been labeled as w_1 here.

So, w_1 is a tracking weight and this information will be used by a function that we will be talking about in a few minutes time. In order to compute the robust tracking bound for all the frequencies that are part of a frequency vector w_{bd} , it is a first entry in this part of the code and i am highlighting that again at this time.

So, the second function in the queue of `tt` will box that I want to introduce to you is this function called SISO Bounds `siso_bnds` is a function that is used to compute the bounds on the loop gain for a variety of different constraints that you might have `siso_bnds` is a short form for single input single output bounds. So, it is included to compute the bounds on the performance and stability of a control system that is included to control a single input single output plant.

Now, if you go to the MATLAB command window and type help siso bnds then you will get information about what the input arguments to this function are and what exactly to the different input arguments represent.

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```

sisobnds Compute Single-Input/Single-Output QFT bounds.
    sisobnds(PTYPE,Ws,P,R,NOM,C,LOC,PHS) computes bounds for the
    closed-loop configuration designated by PTYPE.
W is the set of frequencies designating bounds to compute.
Ws is the performance specification (scalar or LTI/FRD model),
P is the frequency response data of the plant (scalar or LTI/FRD model),
R is the disk radius for non-parametric uncertainty (scalar or LTI/FRD model),
NOM designates the nominal plant and controller array indices.
    LOC specifies location of unknown controller in the loop: 1 for G, 2 for H.
    PHS specifies the phase (degrees) grid for bound computations (0 to -360 every -5 deg is
    default).

    PTYPE specifies the closed-loop configuration.
    PTYPE=1: |FPGH/(1+PGH)| < Ws
    PTYPE=2: |F/(1+PGH)| < Ws
    PTYPE=3: |FP/(1+PGH)| < Ws
    PTYPE=4: |FG/(1+PGH)| < Ws
    PTYPE=5: |FGH/(1+PGH)| < Ws
    PTYPE=6: |FPG/(1+PGH)| < Ws
    PTYPE=7: Ws1 < |FPG/(1+PGH)| < Ws2
    PTYPE=8: |FH/(1+PGH)| < Ws
    PTYPE=9: |FH/(1+PGH)| < Ws

```

So, in this power point slide I have pasted the result of piping help siso bnds in the MATLAB command window and this is the information that you get regarding this particular function. So, it has several input arguments, it is intended to compute the bounds for problems of different types. So, the first input argument to this function is the problem type or p type.

So, the same function is used to compute the bounds for input disturbance rejection output disturbance rejection as well as robust tracking. So, the problem type represents the specific problem for which we are going to be computing the bounds. Now if you come to the bottom, in the indices have all been indicated and the problem types have been listed out. Problem type 1 represents a constraint of the kind that is shown on the right namely, F times PGH divided by 1 plus PGF is less than some weighing function or performance specification. Here I want to indicate that f stands for the transfer function of the pre filter p is a transfer function of the plant, g is a transfer function of the feedback controller that is cascaded with the plant and h is a transfer function of an element that we might have as part of the feedback loop.

In our case we have used unity gain feedback and therefore, in our particular control problems h is going to be equal to 1. So, problem type 1 is intended to draw the bounds when f times PGF divided by $1 + PGH$ magnitude has to be less than a certain specified value W_s . Problem type 2 likewise is for the problem where F by $1 + PGH$ is less than W_s and if you notice this is the kind of specification that we would have when we want to suppress output disturbance because the transfer function that relates the output to the output disturbance is essentially F by $1 + PGH$ problem type 3 is F times P by $1 + PGH$ in magnitude being less than some particular performance specification W_s .

And if you notice this is the problem type that will allow us to plot the bounds on the loop gain when we want to suppress input disturbance because a transfer function for input disturbance is going to be equal to P by $1 + PGH$. Likewise, your a bunch of other different problem types, if you come to problem type 7 you note that the transfer function is f times pg by $1 + PGH$ and that represents the overall transmission function in the event that is H is set equal to 1 between the output and the reference of a 2 degree of freedom control system. And we want to restrict this variation the magnitude of the variation of this transmission function between 2 way in functions $W_s 1$ and $W_s 2$.

So, this is the problem type 7. And if you go back to the specific problem for which we are trying to plot the bounds you will recognize that this is the problem type that we need to use in order to achieve robust tracking because, we want to restrict a transmission function which is essentially given by f times PG by $1 + PG$ to be within 2 limits namely t upper and t lower. So, our $W_s 1$ here will be t lower and $W_s 2$ there will be p upper.

So, the first input to the function `sisobnds` is the problem type and if you are trying to perform robust tracking or in other words restricting the transmission function to within certain limits, owing to variation in the plant parameters our problem type input has to be 7. The second input to this function is the set of frequencies w at which we are expecting to draw the bounds. So, this function will compute a bounds at each of the frequencies that are elements of this frequency vector W .

The third input is W_s which is the performance specification So, when we are trying to achieve robust tracking, W_s as we discussed a few minutes back is comprised of the

transfer functions t_{upper} and t_{lower} . The next input is P , which represents the set of uncertain plants. The input after that is capital R and as you see here when you type help siso bounds what is RRS here stands for the disk radius of nonparametric uncertainty. So, in the problem that we have considered, we have we assume that we know the structure of the plant namely it is a third order plant, it is just that the parameters of this plant are not known but it is also possible that we may not even know the order of the plant.

So, this next input here R captures the uncertainty associated with the structure of the plant itself. In our particular problem in the problem that we are trying to solve now m , we have no uncertainty associated with the structure of the plant, we are quite sure that it is a third order system it is just at the parameters of the system or uncertain. So, in our specific case this particular input r which is the rate disk radius of nonparametric uncertainty t will be set to 0.

The next input is NOM which designates the index of the nominal plant and it is therefore, synonymous with the symbol nom_{pt} which we used a few minutes back when you were trying to plot the plant template at different frequencies. The last 2 inputs have to do with specific location of the unknown controller and the phase grid for bound computation and these are not very relevant to us and therefore, we shall skip describing them in great detail and now we shall return to the code that computes the robust tracking bounds.

So, if we return to the code that computes the robust tracking bounds we see that the bounds have been labeled as bd_1 and that is equal to siso bounds of these particular inputs. I have highlighted that particular line of the code here, as we said the first input argument is the problem type, to draw bounds for robust tracking the problem type is 7. The second input argument is a frequency range over which you want to plot the robust tracking bounds and that is up to 15 radian per second. So, up to the sixth element in the frequency vector and that is been defined as wvd_1 some a few lines earlier.

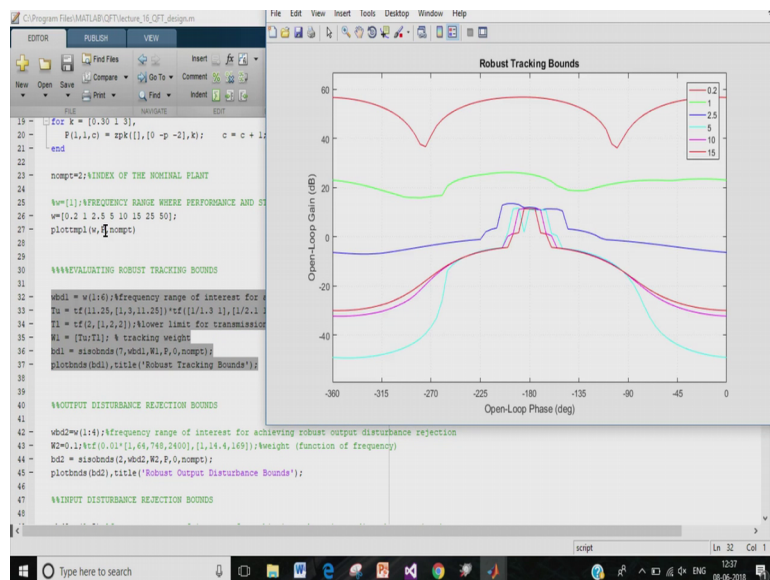
And the third input argument is the tracking weight or the performance specification which is essentially, comprised of the upper limit of the permissible variation t_{upper} and the lower limit of the permissible variation t_{lower} . The fourth input argument is the plant transfer function or a set of uncertain plant which is represented by capital T . The fifth is the nonparametric uncertainty which in our specific case is 0 and therefore, we have

entered it to be 0 and the last input argument is the index of the nominal plant which is which happens to be 2, So, nompt is a last entry input argument to this function siso bnds.

So, once this from this line is executed, the bounds at all the different entries of the frequency vector wbd 1 are computed. And then after computing this bound we would of course, like to be able to view these bounds in the Nichols plot. So, the third function in relation to the QFT toolbox, which will allow us to plot the bounds after computing them at different frequencies, is the function plot bnds. So, this is the function that I am highlighting now. This function allows us to plot the bounds that have been computed by the function siso bnds.

So, let us now run just this part of the code to see how the bounds appear at the different frequencies from between 0.2 and 15 gradient per second. So, I am executing just this part of the code and in a few seconds time you will be able to give the bounds cut the different frequencies.

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So, what you will see therefore, here are the bounds on the loop gain at different frequencies in order for us to achieve robust tracking, these bounds have been color coded. So, the red bound at the top which I am showing by means of the arrow here corresponds to the bound on the loop gain at 0.2 radians per second and what it essentially implies is that in this Nichols plot the loop gain, the nominal loop gain has to

be located above this red colored curve in order for it to satisfy the robust tracking requirement at this particular frequency or in other words in order for the variation in the transmission function to be within the specified limits of t upper and t lower at 0.2 radians per second due to variation in the plants parameters.

So, it has to be located in the region above this red curve likewise at 1 radian per second the bound on the loop gain is in depicted by this green curve here which, essentially again implies that the loop gain has to lie above the green curve in order for the variation in the transmission function at 1 radian per second. To be within t upper and t lower due to variation in the plant parameters and similarly also at other frequencies 2.5, 5, 10 and 15. So, we notice that they are not plotted the bounds at all frequencies, we have plotted them only up to 15 radian per second because, we are not interested in restricting the variation in the transmission function beyond 15 radian per second.

Since this frequency is much greater than the location of the dominant poles of our closed loop system.

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```

24
25 %w=[1];%FREQUENCY RANGE WHERE PERFORMANCE AND STABILITY BOUNDS ARE EVALUATED
26 w=[0.2 1 2.5 5 10 15 25 50];
27 plottp1(v,P,ncmpt)
28
29
30 %%%EVALUATING ROBUST TRACKING BOUNDS
31
32 wbd1=w(1:6);%frequency range of interest for achieving robust tracking
33 Tu = tf(11.25,[1,3,11.25])*tf(1/(1.3 1],[1/2.1 1]);%upper limit for transmission
34 Tl = tf(2,[1,2,2]);%lower limit for transmission
35 Wt = [Tu;Tl]; % tracking weight
36 bdl = slobnds(7,wbd1,Wt,P,0,ncmpt);
37 plothbds(bdl,'title('Robust Tracking Bounds');
38
39
40 %%%OUTPUT DISTURBANCE REJECTION BOUNDS
41
42 wbd2=w(1:4);%frequency range of interest for achieving robust output disturbance rejection
43 Wd=0.1;%weight (function of frequency)
44 bdl = slobnds(2,wbd2,Wd,P,0,ncmpt);
45 plothbds(bdl,'title('Robust Output Disturbance Bounds');
46
47 %%%INPUT DISTURBANCE REJECTION BOUNDS
48
49 wbd3=w(1:5);%frequency range of interest for achieving robust input disturbance rejection
50 Wt = 0.01;% weight
51 bdl = slobnds(3,wbd3,Wt,P,0,ncmpt);
52 plothbds(bdl,'title('Robust Input Disturbance Bounds');
53

```

Having seen how we can block the bounds on robust tracking at the frequencies of interest to us, you can repeat the same exercise for plotting the bounds on output disturbance rejection and input disturbance rejection. As far as output disturbance rejection we note that we are interested in rejecting disturbance only up to 5 radii output disturbance only up to 5 radians per second and the number 5 radians per second

represents the 4th element in the frequency vector which I am highlighting now. And therefore, we define a new frequency vector which comprises only the first four entries of the original frequency vector and we label this new frequency vector as $w_{bd} 2$.

So, it is for the entries of this particular frequency vector $w_{bd} 2$ that we have to compute the performance bounds or disturbance rejection bounds. As far as disturbance rejection performance is concerned as we stated in the beginning, we wanted 90 percent rejection of output disturbance up to 5 radians per second therefore, we have specified the performance or the weight as far as output disturbance rejection is concerned to be 0.1 or other words we want the output disturbance to be rejected to 0.1h or 10 percent of its original magnitude up to 5 radians per second.

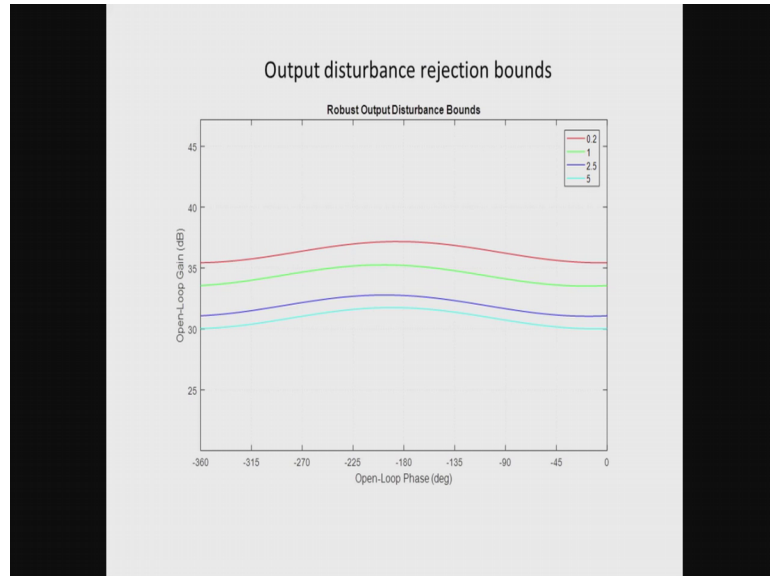
So, it is a constant equal to 0.1. In principle w do can also be a function of frequency, if you want different ex10se of suppression of disturbance at different frequencies it. So, happens that in the problem statement that we have taken, We want constant separation of disturbance at all frequencies namely, we wanted to be suppressed to 10 percent of its original value all the way up to ϕ radians per second. So, $w 2$ is a constant equal to 0.1.

Now in order to plug the bounds, we once again use the function `siso_bnds` but the difference here is in the first input argument. When we had to compute the robust tracking bounds, the problem type was 7. However, if we go back to indices of the different problem types we note that it is the problem type 2 that corresponds to rejection of output disturbance. It is in problem type 2 that we want to ensure that the transfer function relating the output disturbance to the output be the magnitude of this transfer function be less than a certain particular value.

Hence, the first input argument to the function `siso_bnds` in this case when we are trying to compute the output disturbance rejection bound is 2. The second input argument of course, is a set of frequencies where we want to compute the bounds on disturbance output disturbance rejection and that is $w_{bd} 2$, which comprises the first 4 in piece of the frequency vector w namely, up to ϕ radians per second. $W 2$ specifies the weight or the performance specification or the extent to which we expect output disturbance to be rejected and capital P again represents uncertain plant models and 0 represents the magnitude of unstructured parametric consultancy, which in our case is actually equal to 0 and `nom_pt` again represents the index of the nominal plant.

So, once again if we run this code and use the same function plot bnds that we will be able to plot the bounds on output disturbance rejection.

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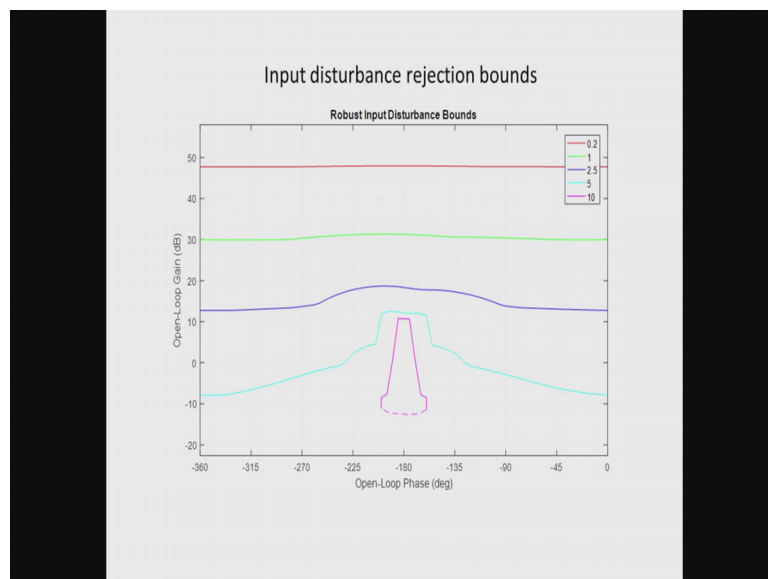
That has been done here and in this slide we show the bounds on output disturbance rejection in the frequency range where the output disturbance will affect our system. So, we have plotted output disturbance at 0.2 radians per second and that has been color coded. So, the red curve here represents the bound on the loop gain at 0.2 radian per second in order for it to reject output disturbance. What this essentially means is that once again the nominal loop gain has to lie in the region above the red curve or on the red curve in order for it to satisfy the output disturbance rejection at 0.2 radians per second.

Likewise the green curve here represents the bound on output disturbance rejection at 1 radian per second or in other words at 1 radian per second the nominal loop gain has to lie above the green curve in order for it to satisfy the output disturbance rejection specification and likewise also for the other 2 frequencies 2.5 radian per second and 5 radian per second. The same exercise has been repeated for input disturbance or the same exercise has been repeated to compute the bounds for input disturbance rejection, the only difference there once again is the problem type.

So, while the problem type had to be 2 in order to draw the bounds for output disturbance rejection, the problem type has to be 3 in order for us to draw the bounds for

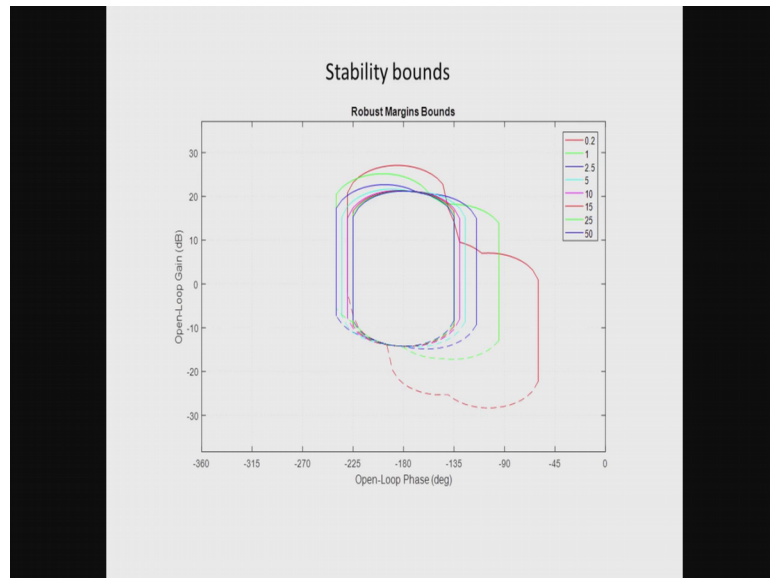
input disturbance rejection. And by changing the first input argument to the function `siso_bnds` to 3 and redefining the frequency vector over which we would be interested to plot the bounds in the case of input disturbance we are interested in rejecting input disturbance up to 10 radians per second. So, we make sure that the frequency vector also has the number 10 as 1 of its entries. We are able to then again run the function `siso_bnds` and use the function `plot_bnds` in order to plot the bounds for input disturbance rejection and that has been shown in this slide here.

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So, after drawing the input disturbance rejection bounds, the output disturbance rejection bounds and the bounds robust tracking what we are left with is to draw the bounds for stability. Now as far as stability is concerned we should compute the stability bounds at all the frequencies that are entries of the original frequency vector w because, we cannot stop worrying about stability only at 15 gradient per second at which point we stop worrying about performance. Our loop gain may cross over higher frequencies and therefore, we need to draw the bounds even at frequencies beyond 15 radian per second, we picked 2 additional frequencies namely 25 and 50 radians per second and we have computed the stability bounds at all frequencies that are entries of the original frequency vector w .

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And that set of stability bounds is shown here. So, if you notice on the right top we have computed the bounds at 0.2 radian per second, 1 radian per second, 2.5 and so on and so forth all the way up to 50 radians per second. So, what this essentially implies is that our nominal loop gain has to lie outside the red curve here at 0.2 radian per second in order for the stability specification to be met at 0.2 radian per second and so, on and so forth.

So, at each of the frequencies if there is certain stability bound our loop gain has to lie outside that bound at best on that bound in order for the stability specification namely that the maximum value of the transmission function at that frequency or being less than 3 db to be met. I just wanted to point to 1 important fact, when we were defining stability bound we defined it in terms of db and said that the maximum value of the feedback part of the transmission function namely, c_p by $1 + c_p$ should be less than or equal to 3 db.

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```

39 %OUTPUT DISTURBANCE REJECTION BOUNDS
40
41 %wb2=w(1:4);%frequency range of interest for achieving robust output disturbance rejection
42 %W2=1;%weight (function of frequency)
43 %bd2 = siso_bnds(2,wb2,W2,F,0,comp1);
44 %plotbnds(bd2),title('Robust Output Disturbance Bounds');
45
46 %INPUT DISTURBANCE REJECTION BOUNDS
47
48 %wb3=w(1:5);%frequency range of interest for achieving robust input disturbance rejection
49 %W3 = 0.01;% weight
50 %bd3 = siso_bnds(3,wb3,W3,F,0,comp1);
51 %plotbnds(bd3),title('Robust Input Disturbance Bounds');
52
53 %STABILITY BOUNDS
54
55 %W4 = 1.41; % robust margins weight
56 %bd4 = siso_bnds(1,w,W4,F,0,comp1);
57 %plotbnds(bd4),title('Robust Margins Bounds');
58
59
60
61 %VIEWING ALL THE BOUNDS TOGETHER
62
63 %all_bnds=mrpbnds(bd1,bd2,bd3,bd4);
64 %plotbnds(all_bnds),title('Margins and Tracking Bounds');
65
66 %INTERSECTION OF STABILITY AND PERFORMANCE BOUNDS
67 %final_bnds=sectbnds(all_bnds);
68 %plotbnds(final_bnds),title('Intersection of Bounds');

```

Now, in this particular quote we have to convert db to the linear scale. So, 3 db essentially come gets converted to 1.41 units in the linear scale. So, the performance specification for stability is w 4 and is equal to 1.41 units, which essentially signifies that the maximum value of our feedback part of the transmission function t 1 should be less than or equal to 3 db. So, subsequent to drawing all the bounds we would wish to view all the different bounds that exist for our loop gain.