

Control System Design
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Lecture – 37
Tutorial on QFT Toolbox software (Part 2/3)

Hello; now in the previous two clips we have seen how we can employ quantitative feedback theory to design two degree of freedom control systems for single input single output plants.

So, in the first example we actually designed a 1 degree of freedom control system. That was intended to reject disturbances. And in the previous clip we designed a 2 degree of freedom control system that ensured that we had a desired dominant dynamics. And we talked about how the tool of quantitative feedback theory allows us to solve the problem even when root locus based design approach fails; on account of the kind of specifications that we have for the permissible variation in the closed loop pole due to uncertainty in the plants model.

In both the cases we used a particular software called QFT toolbox where QFT stands for Quantitative Feedback Theory.

So, we use this particular software in order to execute the design where we had to draw the bounds on performance first and stability next, find the intersection of bounds and having determined the intersection of all the bounds and in the frequencies of interest to us within subsequently did performed loop shaping.

Now, what we will do in this clip is to familiarize the students to this tool of QFT toolbox or qualitative feedback theory toolbox. So, the instructions for installation downloading and installation of the software are provided in the lecture notes that would be distributed to students who have registered for this course and after installing the software how can one use it for executing design of control systems for single input single output plants is what we would be discussing in this lecture.

So, what we shall do in this lecture is to first take a numerical example and subsequently outline the steps that are necessary for executing the design. Then we shall actually use this software and introduce to the viewers the different commands that are of importance

while performing control design and subsequently execute the design and show how one can interactively shape the loop by using this particular toolbox.

So, let us first start with the problem statement.

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QUANTITATIVE FEEDBACK THEORY

Block diagram showing a feedback control system with blocks F, C, and P. The transfer function is given as $P(s) = \frac{K}{s(s+p)(s+2)}$.

Design requirements:

- ① Input disturbance rejection
99% from 0 to 10 rad/s
- ② Output disturbance rejection
90% from 0 to 5 rad/s
- ③ Dominant dynamics $\frac{p^d}{p^y} = -1.5 \pm 2j$
 $p^d = a \pm jb$ $-3 \leq a \leq -1$
 $1 \leq b \leq 3$
- ④ $|T_i|_{\max} \leq 3 \text{ dB}$

Additional parameters:
 $K_{nom} = 1$
 $0.3 \leq K \leq 3$
 $p_{nom} = 1$
 $0 \leq p \leq 2$

Transfer function: $P(s) = \frac{K}{s(s+p)(s+2)}$

Time constant: $T_i = \frac{CP}{T_{FBP}}$

We have a plant with an uncertain plant given by P of s is equal to K by S times S plus p times S plus 2. Where the nominal value of the gain K is 1, but it can vary between the limits 0.3 and 3. So, the plant is for all intents and purposes identical to the ones that we have considered when we were doing root locus based design. So, we have two uncertain parameters in the example we have considered here.

The gain is uncertain it can vary between 0.3 and 3 with a nominal value of 1. And one of the poles is uncertain. So, the pole P nominally assumes a value of 1 or one of the plant poles is at s is equal to minus 1, but it can be uncertain it can lie anywhere between P can assume values anywhere between 0 and 2. So, the plant can have its second pole anywhere between S is equal to 0 and S is equal to minus 2. So, this is the uncertain plant that we are trying to control.

But in the process of controlling this certain plant we are also expected to satisfy some disturbance rejection and robust tracking requirements. So, for instance this plant is assumed to be afflicted by input disturbance. And we are expected to reject the input disturbance by 99 percent in the frequency range 0 to 10 radians per second.

So, in this schematic here I shall include an input disturbance that afflicts of a plant. I shall call that D_{in} . And it so, happens that the frequency content of D_{in} is only up to 10 radian per second, but its magnitude is so large that we are expected to reject it significantly.

So, 99 percent rejection is expected in the frequency range 0 to 10 radians per second for this disturbance. Likewise we also have a specification on output disturbance rejection presumably because this plant is also afflicted by an output disturbance. So, we expect 90 percent rejection of output disturbance in the frequency range 0 to 5 radians per second.

So, this once again might arise from the fact that our experiments might have revealed the output disturbance to have its frequency content only to be significant between 0 and 5 radians per second.

So, we want the output disturbance to be rejected only in this frequency range. And the specified extent of rejection has been indicated to be 90 percent. So, in this schematic again I shall add an output disturbance to indicate that our plant is also affected by output disturbance D_{out} and we are expected to reject it by 90 percent up to 5 radians per second. So, these are the two performance specifications as far as taking care of uncertainties in the environment is concerned.

So, because we have the two disturbances afflicting our plant furthermore we want a certain extent of robustness to variation in the plant's parameters. So, we want the nominal dominant dynamics to be decided by the pole P_{c1} and P_{c1}^* which is given to be $-1.5 \pm 2j$. So, we want the dominant poles of our closed loop system to be located at $-1.5 \pm 2j$.

However when the plant's parameters change so, when the gain of the plant is different from its nominal value and the pole location of the plant is different from its nominal value, of course, we expect the closed loop poles to also change their position and what has been specified here is that the closed loop pole has to be of the form $a \pm jb$ where the real part of the closed loop pole a should be within the limits -3 and -1 and the imaginary part b has to be within the limits 1 and 3 .

So, when our plant parameters change we are with the closed loop pole pandering within the rectangle given by $a \pm jb$ where a is between the limits -3 and -1 and b is between the limits 1 and 3 .

So, this is our robustness requirement. And the last requirement the fourth requirement that has been given here which is in the interest of stability of the feedback part of our system is that the maximum magnitude of our feedback transmission function T_1 which is given by T_1 is equal to $C P$ by $1 + C P$.

So, the maximum magnitude of that should be less than or equal to 3 dB. As we discussed in the previous two clips the maximum magnitude of the transmission function for the feedback part of our control system is indicative of the extent of damping that we have for our closed loop system.

So, in place of specifying a phase margin for the open loop system in this design procedure we choose to specify the maximum allowable value for the transmission function for the feedback part.

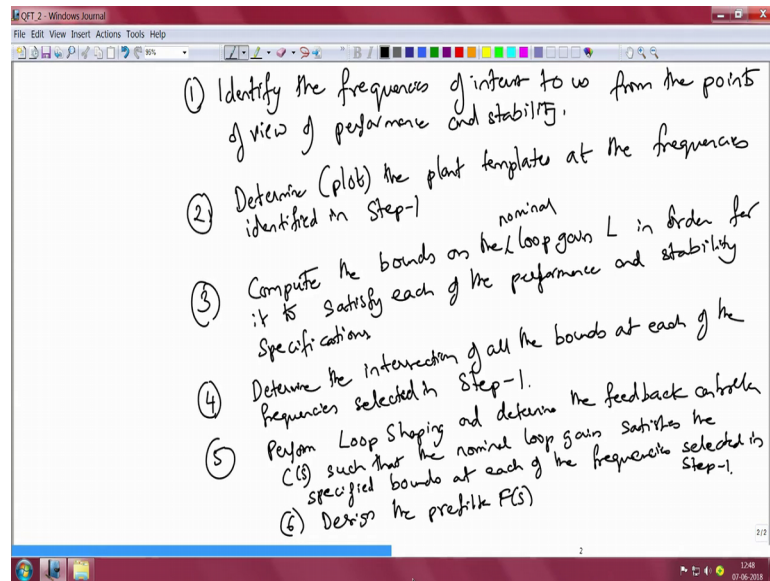
And the larger is this value the lesser is the closed loop damping and a closer is our closed loop system to instability. So, in this case we have specified that the maximum value of E_1 should be less than or equal to 3 dB. So, these are the problem specifications that have been given to us.

So, what we would do next is to outline the steps that we would adopt in attacking this feedback control problem. And subsequently we will go over and fire up QFT toolbox and introduce to the listeners the different commands that would be used in plotting the bounds importing the plant template in executing the feedback controller design in executing the pre filter design and so on and so forth for the specific problem that has been given to us.

Once again the goal of this exercise is to also familiarize the viewers with this particular toolbox with the intention that they would themselves be able to use it later on.

So, let us first outline the different steps that are necessary for executing the design given the kind of problem specifications that we just discussed.

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So, the first step in solving this problem is to identify the frequencies of interest to us from the points of view of performance and stability.

So, from the point of view of performance we know that there are two disturbances input disturbance and output disturbance input disturbance affects our system up to a 10 radians per second and output disturbance affects our system up to a 5 radians per second. So, we have to pick a few frequencies between 0 and 10 and 0 and 5 radians per second and compute the performance bounds for disturbance rejection at these frequencies in the interest of coming up with a control system that rejects these disturbances by a specified amount.

There is another performance requirement that has been given to us and that has to do with the robustness of our closed loop system to variation in plant parameters. And that has been specified in terms of the permissible variation of the closed loop pole position.

So, in the previous clip we saw how this could be translated to a corresponding frequency up to which we would be interested in restricting the variation of the closed loop transmission function and that is going to be another frequency that would be of interest to us from the point of view of performance.

Finally as far as stability is concerned we have to worry about stability for frequencies well beyond the ones at which we are interested in performance because our loop gain

can cross over at a frequency that is much larger than the frequency at which we are expecting performance. So, what we do therefore is to take a few additional frequencies where the plant template does not change its shape. and we draw the stability bounds we compute the stability bounds at these particular frequencies and do not worry about the performance at these frequencies.

So having determined the frequencies of interest to us from the point of view of performance and stability the next step is to determine the plant templates at these different frequencies. So, determine or in other words plot the plant templates at the frequencies identified in step 1.

So, this is the second step. The next step is to plot the performance bounds either plot or compute the performance bounds using this plant template at the frequencies of interest to us.

So, the third step therefore, is to compute the bounds on the loop gain L in order for it to satisfy each of the performance and stability specifications; so, this is a third step. So, we have to compute the bounds on the loop gain and I want to underscore here that when I talk of the loop gain I am talking of the nominal loop gain.

So, I shall insert the word nominal here. Compute the bounds on the nominal loop gain L in order for it to satisfy each of the performance and stability specifications. So, in this problem we have three performance specifications one is for input disturbance rejection, other is for output disturbance rejection and the third is for robust tracking or robustness of the closed loop system to variation in plant parameters. And we have one stability specification and that has to do with the maximum permissible value for the transmission function of the feedback part of our control system.

So, at each of the frequencies of interest to us we have to determine the bounds on the nominal loop gain in order for it to satisfy the specifications that have been provided for each of these four cases namely input disturbance rejection output disturbance rejection robust tracking and stability.

So, once we have computed the bounds on the nominal loop gain the next step the fourth step is to determine the intersection of all these bonds. So, determine the intersection of all the bounds at each of the frequencies selected in step 1.

So, when we determine the intersection of these bounds then this curve will reveal to us the permissible region in the complex plane or in the Nichols plot where our nominal loop gain can lie for it to simultaneously satisfy the different requirements that have been specified to us. So, it will specify the region in which the loop gain can lie for it to reject both the input disturbance the output disturbance achieve the desired amount of robustness to plant parameter variations and achieve a desired stability specification at each frequency of interest to us which has been already decided as part of step 1.

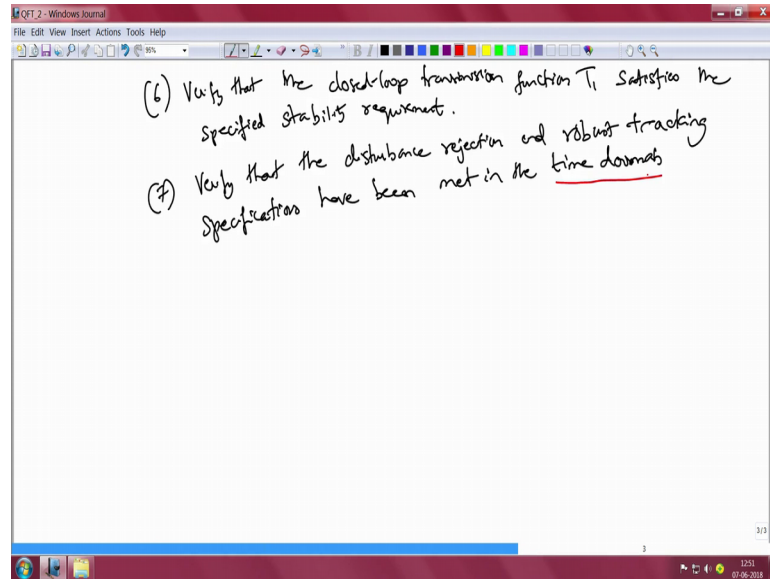
So, now that we have determined the intersection of all the bounds at each of the frequencies of interest to us. The next step is to execute loop shaping of the feedback controller C of s . So, perform loop shaping and determine the feedback controller C of s such that the nominal loop gain satisfies the specified bounds at each of the frequencies of interest at each of the frequencies selected in step 1.

So, the fifth step therefore, is to perform loop shaping or in other words determine the structure of the feedback controller C of s which ensures that the nominal loop gain lies within the permissible regions at each of the frequencies of interest to us in the Nichol's plot. And the last step as far as the design is concerned is to design the pre filter. So, design the pre F of s and the due the pre filter is designed to make sure that we get the desired transmission function between the reference and the output.

So, the nominal transmission function from the reference to the output is decided by the pre filter F of s . While the variation of the transmission function is constrained by the feedback controller C of s . So, these are the steps that are involved in the design of a 2 degree of freedom control system using quantitative feedback theory, but all of these steps are performed in the frequency domain and we necessarily have to check for performance in the time domain after completing the design.

So, after completing the determination of the structure of the feedback controller C of s and the pre filter F of s there will be a two more steps.

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So, the sixth step therefore, would be to verify that the closed loop transmission function T_1 satisfies the specified stability requirement.

So, we have to check for stability of the closed loop system by ensuring that the magnitude of T_1 the maximum magnitude of T_1 is less than or equal to 3 dB as has been specified to us.

And the last step is to check for the time domain performance of our feedback control system. So, having determined the pre filter and the feedback controller we have the entire transfer function of our control system. Now we apply disturbances at the input to the plant or at the output of the plant and then verify whether our feedback controller is able to reject these disturbance by the specified amounts. And subsequently we also change the structure of the plant and see whether the closed loop dominant dynamics does not wander outside that rectangle $a \pm jb$ that has been specified to us.

So, the last step is to verify that the disturbance rejection and robust tracking specifications have been met in the time domain. Since there are approximations that one makes in going from frequency domain to time domain, but ultimately it is a time domain where we have to check for performance. The last step is to verify that all the specifications that have been given to us have been met also in the time domain.

If the specifications are met then our design comes to an end. If they are not met then we have to go back and fine tune our feedback controller and pre filter until the specifications are met. So, what we shall do next is therefore, fire up QFT toolbox and go through each of these 7 steps in the design of our feedback control system using quantitative feedback theory.

Thank you.