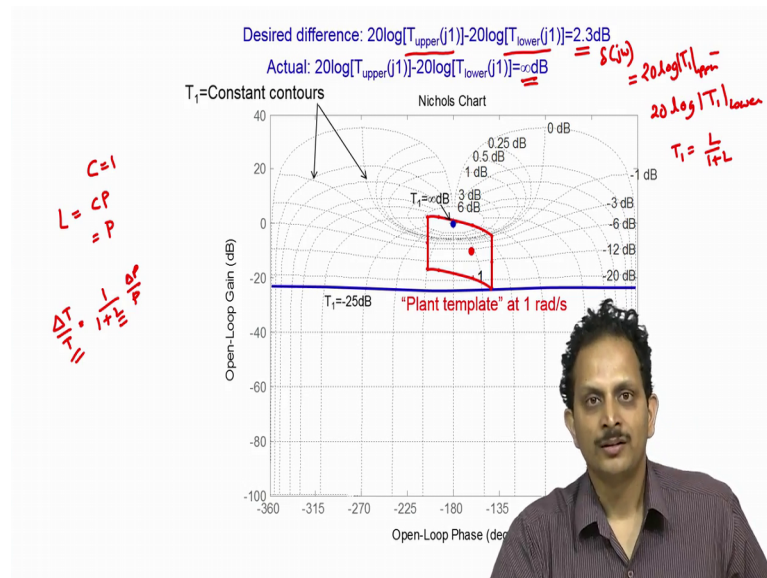


Control System Design
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Lecture - 36
Robust control design using Quantitative feedback theory (Part 2/2)

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So, we have the plant template at 1 radian per second to have this particular shape. And in the first cut of design we note that our controller at the start we assume is C is equal to 1, because you have not yet designed the full controller, so in which case we would have the loop gain L to be equal to C times P or simply equal to P itself. Therefore, at the very start of our design, we note that the loop gain also occupies the same area in the Nichols plot as the plant does.

Now, the question is, is this an acceptable area for the loop gain to occupy in the complex plane? To answer this question what we need to note is that the desired difference between T upper at 1 radian per second and T lower at 1 radian per second in the logarithmic scale, we find from the Bode plots that we had drawn a couple of slides back. We note that the desired difference Δ , which is the spread permissible spread in the transmission function at 1 radian per second, if we find it numerically, to be equal to 2.3 dB.

So, to answer the question about whether this is an acceptable location for the loop gain at 1 radian per second what we need to check for is whether the difference in the logarithmic scale between the maximum magnitude of the transmission function and the minimum magnitude of the transmission function namely T_{upper} and T_{lower} is equal to 2.3 dB or less than 2.3 dB. If it is not, then we know that this is not an acceptable location for the loop gain to lie.

Now, how do we compute the value of transmission function T_{upper} and T_{lower} ? Firstly, we note that the difference between T_{upper} and T_{lower} is essentially equal to the difference between $T1_{upper}$ and $T1_{lower}$, because the transmission function T is equal to the pre filter times $T1$. Therefore, in the logarithmic scale, when we take the difference between T_{upper} and T_{lower} , the pre filter terms get cancelled out and we will be left with only the transmission function for the feedback control parts. So, this is going to be equal to $20 \log$ magnitude of $T1_{upper}$ minus $20 \log$ magnitude of $T1_{lower}$.

Now, since we know that our loop gain lies inside this particular area we can compute $T1$ at each of these locations, because we have the equation $T1$ is equal to the loop gain L by $1 + L$. So, each of the locations we can compute $T1$ and hence we can determine what is the maximum magnitude of $T1$ and what is the minimum magnitude of $T1$ if the loop gain L , lies within this particular area.

So, in this particular case we discover that the minimum magnitude of $T1$ happens to be minus 25 dB and that is at this right extreme corner of the plant template. And the maximum magnitude occurs inside the plant template, because the plant template encloses the point 0 dB comma minus 180 degrees or in other words the critical point. And at the critical point the transmission function assumes a value of infinity dB.

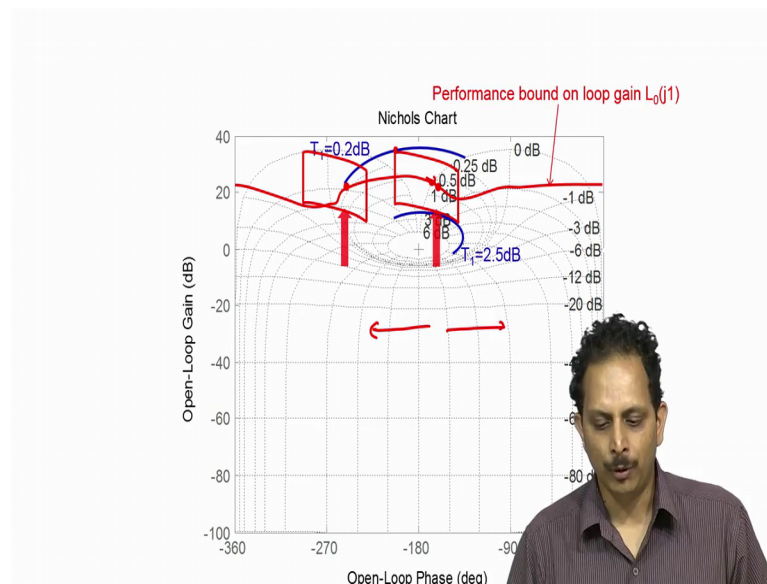
So, while our specification is that the difference between $T1_{upper}$ and $T1_{lower}$ in the logarithmic scale should be 2.3 dB, in practice the upper value is infinity and lower value is minus 25 dB. And hence the difference between the two is actually infinity. So, clearly this location is not an acceptable location for our loop gain to lie in order for it to satisfy our robust tracking requirements at this particular frequency. So, what do we do?

To understand what needs to be done let us go back to the sensitivity equation that relates the uncertainty in the overall transmission function to the uncertainty in the plant transmission function. This was derived a few clips back and to refresh your memory the

equation was that $\Delta T/T$ is equal to $1/(1 + L \Delta p/P)$. What has happened in this case is that for the controller C is equal to 1 our loop gain L has ended up becoming so low that our $\Delta T/T$ has become a huge number, in this particular case it is infinity dB.

So, in order to minimize $\Delta T/T$ we should choose a higher value for loop gain. And what does that mean? We should push up the plant template. We should choose a controller that adds gain and that controller will in turn result in displacing the plant template upwards.

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So, suppose the plant template gets displaced upwards, then as we move the plant template up, so, when we say that we are moving the plant template up without moving it sideways, we are essentially multiplying the plant with a proportional controller. And hence the overall loop gain increases in magnitude, but does not change in phase. Hence, the plant template stays put at the same X position in the Nichols plot, but gets displaced along the Y axis. So, it moves up when the controller gain is increased and at each location as it is moving up we can check for the maximum T_1 upper and T_1 lower at each location within the plant template. And determine whether it is now within the specified limit, namely 2.3 dB or not.

It so happens that when the plant template moves to this particular location in the Nichols plot we discover that the minimum value happens to be 0.2 dB, which occurs at

the left top extreme of the plant template. And the maximum value happens to be 2.5 dB, which occurs close to the bottom corner of the plant template. So, at this particular location of the plant template we would have the maximum difference between T between the transmission functions for all possible values that the loop gain can assume within this region to be utmost 2.3 dB.

Hence, if we locate our loop gain at this particular point, we can ensure that our transmission function T changes by utmost 2.3 dB, when the plant parameters namely its gain and its pole location change within the specified limits. However, we are not guaranteed that our controller eventually is going to be a proportional controller. So, at 1 radian per second our controller phase may not be exactly 0 and therefore it may not necessarily just push the plant template up or down.

It may also result in displace in the plant template laterally. So, if the controller adds a phase lag, then the plant gain will move to the left in the Nichols plot, because the overall phase lag of the loop gain will increase. On the other hand, if the controller adds phase lead, it will move to the right in the Nichols plot, because the overall phase of the loop gain will decrease.

So, since we do not know whether our controller is likely to add phase lead or phase lag what we need to do is that at each of the angular positions which are the potential locations, where the plant template can sit depending on how much phase lag or lead the controller can add. We have to determine the extent by which the plant template needs to be pushed up in order for the difference in the transmission functions within the plant template to be within 2.3 dB.

So, in this case, for instance, when the plant template is at this particular location, it turns out that it has to be pushed to this particular location, so that at this particular location of the nominal loop gain the difference between T upper and T lower will be within 2.3 dB. Now, this activity has to be repeated at a range of different angular values. In fact, at every possible angular position and this is clearly a tedious task to perform manually.

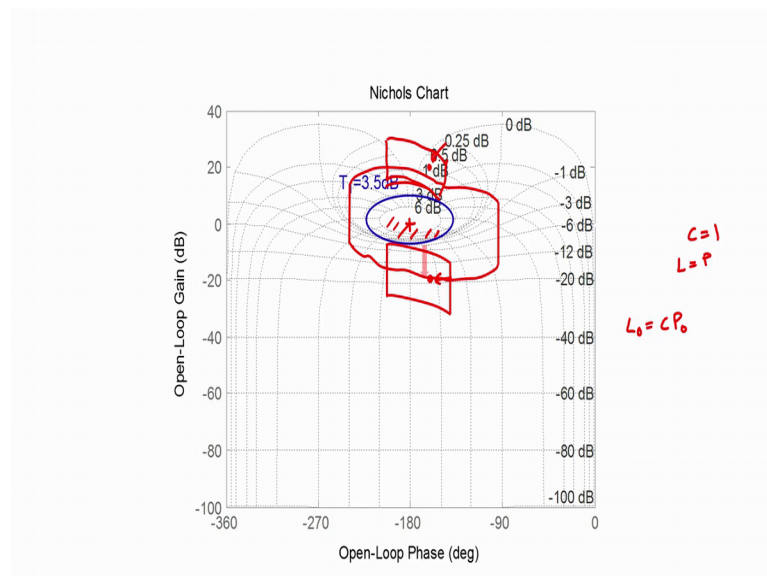
And hence there is a software called QFT toolbox, which automatically determines the bound for performance specification, when it comes to robust tracking. So, in this case, we have employed this software and determine the locations on which the nominal loop

gain has to lie in order for the difference between T_1 upper and T_1 lower in the logarithmic scale to be exactly equal to 2.3 dB.

So, now if the loop gain is chosen to be above the region that has been demarcated by this red curve, we would notice that the variation in the transmission function will be less than 2.3 dB. So, on the other hand, if the loop gain were to be in the region that is below this red curve, then the variation in the transmission function would be greater than 2.3 dB. So, this line therefore represents a boundary above, which the loop gain is allowed to lie in order for T_1 upper minus T_1 lower in the logarithmic scale to be within 2.3 dB. And hence this marks the performance bound on the loop gain at 1 radian per second

Now, we do not know whether we would have stability issues at 1 radian per second or not. So, it is useful to also determine the stability bound at 1 radian per second and subsequently determine the intersection of these bounds at 1 radian per second. So, in order to determine the stability bound let us return to the original location of the plant template.

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So, this was where the plant template was located for the controller C is equal to 1. So, in other words, when the loop gain L was simply equal to the plant transfer function and the plant transfer function assumed values within this curvy linear rectangle. So, we note that this rectangle is partially inside the contour magnitude of T_1 is equal to 3.5 dB.

So, this contour divides the complex plane into two parts. If the loop gain assumes values within this closed contour then for those particular values of loop gain if we compute the transmission function T_1 , we would see that its magnitude is greater than 3.5 dB. And for regions outside this contour, if we compute the transfer function T_1 , we would find that its magnitude is less than 3.5 dB.

In this particular case, we see that this plant template is sitting partially inside this closed contour, which means that for some combination of plant parameters, namely the plants uncertain gain and its pole location in particular those combination of plant parameters as result in the plant transfer function being in this particular region of the Nichols plot. We would have the transmission function being greater than 3.5 dB. Indeed, since we already saw in the previous slide that this plant template encloses the critical point namely 0 dB and minus 180 degrees at that particular location the transmission function will have a magnitude of infinity dB.

So, clearly whatever controller it is that we design it should make sure that no part of the plant template is inside this contour, namely T_1 is equal to 3.5 dB. So, one possible way to do it is to have a controller that attenuates the gain of the overall open loop system or in other words such a controller will push down the plant template. So, if the plant template is pushed down, then we need to push it down until every point within the plant template lies either on or outside the constant transmission function T_1 is equal to 3.5 dB contour. So, it is at this particular location for the nominal loop gain that every point within that plant template will lie outside the contour magnitude of T_1 is equal to 3.5 dB.

Now, if you were to use a controller that amplifies the gain, then we need to choose such a gain for the controller that the plant template will be somewhere higher up here in this particular manner. And once again no part of the plant template will be inside the contour T_1 is equal to 3.5 dB, so in that case this will be the location at which the loop gain has to lie in order for it to satisfy the stability specification for all possible combinations of the plants gain and its pole location.

Now, we are not guaranteed once again that we would be using a controller that adds phase lead or phase lag. Therefore, it is not necessary that the plant template just gets moved up or down. It can also get moved sideways just as we saw in the previous slide.

Therefore, we need to repeat this exercise at different angular positions of the plant template, because these are the angular positions that it can potentially assume depending on the particular structure of the controller.

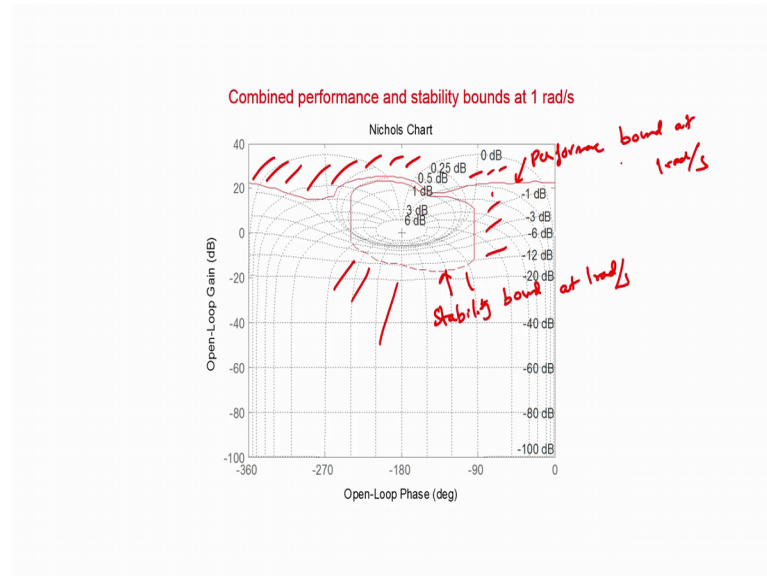
If the controller provides phase lead, then the plant template will be to the right of its location, when the controller C is equal to 1. If the controller adds phase lag, then the plant template will be to the left of that position, when the controller C is equal to 1. And at each of these locations we need to determine what where the nominal loop gain has to be in order for every point within the plant template to be outside the contour T_1 is equal to 3.5dB.

Once again this can be either done graphically or it can be done with the help of software and by using QFT toolbox we can automatically obtain. The locations where the loop gain has to lie in order for the worst case plant to utmost have a magnitude of 3.5 dB. And if you use software to plot this curve, it would look something like this. So, for all possible locations of the loop gain on this particular closed curve, we would have the worst case plant to have a maximum value of T_1 being equal to 3.5 dB.

Hence, for values of the nominal loop gain L_{naught} , which is equal to the controller times the nominal plant p_{naught} , which are outside this closed contour the red closed contour, we would have the worst case transmission function T_1 to have a maximum value of less than 3.5 dB. Likewise, for values of the nominal loop gain L_{naught} that are inside this red curve, the worst case transmission function T_1 or in other words the maximum value of the transmission function will be greater than 3.5 dB. And hence our stability specification will be violated

So, once again this closed contour divides the Nichols plot into two parts. The part region, which the nominal loop gain is allowed to lie for the closed loop system to have a T_1 max of at least 3.5 dB. And a part within which it is not allowed to lie. Now, just as we had the performance specification divide the Nichol's plot into two regions, one where the loop gain of the nominal system is allowed to lie in order for it to satisfy the robustness requirement and the other where it is not allowed to lie. So, also we have in case of stability two regions in the Nichol's plot, one where the loop gain allowed to lie and the other where it is not allowed to lie

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So, what I have therefore shown in this particular slide are the two bounds. This is the performance bound at 1 radian per second and this here is a stability bound at 1 radian per second. So, in order for the closed loop system to be stable the loop gain, the nominal loop gain L_{naught} should lie in the region that is outside the stability bound and in order for the robustness requirement to be satisfied or in other words in order for the difference between $T_{1 upper}$ and $T_{1 lower}$ to be within 2.3 dB at 1 radian per second. The nominal loop gain has to lie above the performance bound

So, in order for both of them to be met it should lie in the region that is the intersection of these two bounds. And in this particular case at this particular frequency that region simply ends up being bounded by the performance bound itself, because the performance bound and the stability bound do not intersect in this particular case. Now, we can repeat this exercise at other frequencies of interest to us.

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So, I have repeated this, now at 2.5 radians per second. So, a 2.5 radians per second the plant template is situated at this particular location in the Nichol's plot. And the green dot at the center indicates the nominal plant transfer function p naught or the value the magnitude and the phase of the plant transfer function for the nominal gain k naught equal to 1 and the nominal pole location p naught equal to 1. And when the gains k and p changes the plant transfer function assumes a range of other complex numbers and that is represented by this curvy linear rectangle. So, this is the plant template at 2.5 radians per second.

Now, we can once again compute the performance bound on the loop gain at 2.5 radians per second. So, we can look at what the permissible variation δ is but 2.5 radians per second. So, δ j of 2.5 can be found out from the bode plot that we draw at the beginning of this clip. And then we can determine where the loop gain has to lie in order for the difference between T_1 upper and T_1 lower in the logarithmic scale to be within the number δ of at 2.5 radians per second.

And the boundary of that region is given by this green curve here. And it represents the performance bound at 2.5 radians per second. Likewise, we cannot forget about stability at 2.5 radians per second. So, assuming that the gain crosses over a 2.5 radians per second we want to make sure that the maximum value of T_1 or magnitude of T_1 max is within the specified stability limit or in other words it is within 3.5 dB.

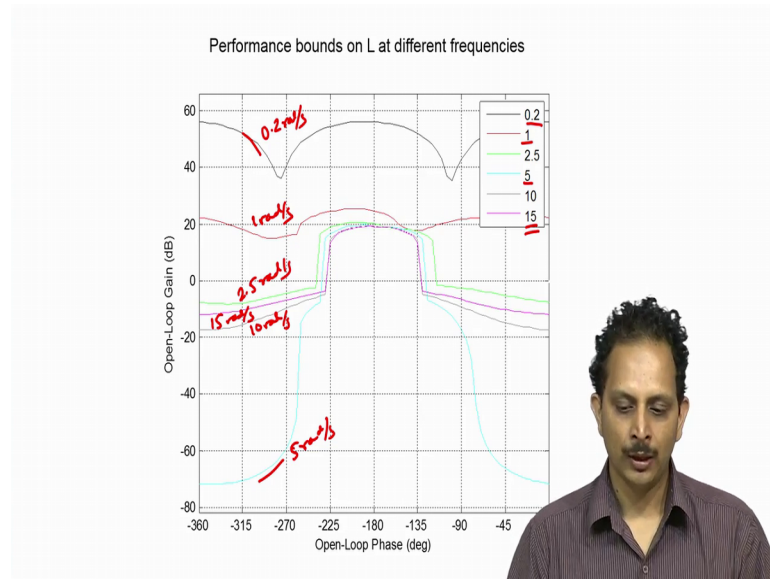
So, we have also computed the stability bound at 2.5 radians per second. And the stability bound is another closed curve that is shown here. So, in this case for the closed loop systems to be to be stable and the maximum value of T_1 to be within 3.5 dB, we need the loop gain to lie outside this green closed curve. And for the performance requirements to be met it should be above the green curve, which is the performance bound at 2.5 radians per second.

So, the stability bound is this closed curve and the loop gain should lie. Outside of this closed curve for the worst case loop gain to result in a transmission function T_1 whose magnitude is within 3.5 dB. And the nominal loop gain should lie above the performance bound for the robustness requirement or in other words the difference in the logarithmic scale between T_1 upper and T_1 lower at 2.5 radians per second to be less than or equal to the specified amount Δ of $j 2.5$

So, in order for the loop gain to satisfy both these requirements simultaneously it should lie in the region that is the intersection of these two permissible regions and that region is bounded by the intersection of both these bounds. And this overall green curve here, which is which represents the boundary of the intersection of these permissible regions as far as performance and stability specification is concerned represents the overall bound on the loop gain at 2.5 radians per second.

In other words, the nominal loop gain has to lie about this particular green bound in order for the closed loop system to both achieve the specified amount of robustness to plant parameter variations in the overall transmission function and also achieve the specified amount of stability for the closed loop system. So, this is repeated at other frequencies of interest. We discussed that we would limit restricting the performance to within 15 radians per second.

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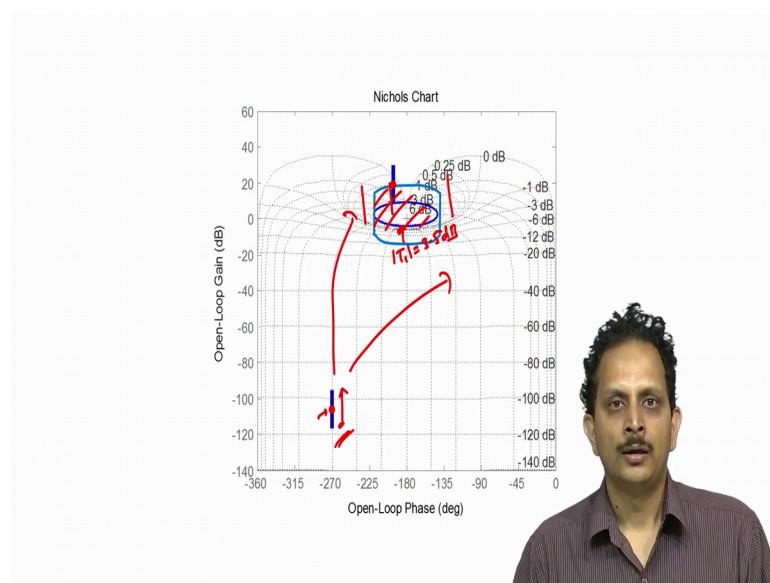


So, I have taken a set of other frequencies. So, I have for example, considered a very low frequency 0.2 radians per second and at 0.2 radians per second the overall bound on L is given by this black curve here, this is at 0.2 radians per second. So, this software QFT toolbox color codes the bounds, so at 1 radian per second we would have this red curve, which represents the overall bounds the intersection of the performance bound as well as stability bound at 1 radian per second. And the green curve is the overall bound at 2.5 radians per second. The cyan colored curve here represents the overall bounds at 5 radians per second. The black curve here is for 10 radians per second and the purple colored curve here is for 15 radians per second.

Now, as you can see these curves are rather non intuitive in their appearance and their shape. And hence it is not easy to describe their shape using a simple mathematical expression. It is therefore best to employ software such as QFT toolbox in order to plot all these different bounds, now that we have plotted the performance bounds. We are ready to undertake the design of the controller that makes sure that at each of these particular frequencies the loop gain is located within the permissible areas at those respective frequencies. But, before we do that we need to still worry about stability at frequencies beyond 15 radians per second. Although our performance specification stopped at 15 radian per second because we found that the transmission function T upper had fallen to minus 20 dB at 15 radian per second.

And therefore we were not expecting any tracking specifications beyond this particular frequency. We cannot stop worrying about stability at the same frequency. It is quite possible that the loop gain might cross over at a frequency that is greater than 15 gradient per second. And if that happens, then we need to worry about stability at the frequency at which the loop gain crosses over. Therefore, what we need to do is to continue to worry about stability at frequencies beyond which we have stopped worrying about performance. So, we need to take a few more frequencies compute the plant template at those frequencies and simply plot the stability bounds at those higher frequencies.

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So, in this case I have taken 50 radians per second as one of those frequencies. And as we discussed two clips back at frequencies that are sufficiently high, the plant template essentially reduces to a straight line in its appearance. So and the red dot here indicates the nominal value of the plant transfer function at 50 radians per second. The reason that it reduces to a straight line is because the at very high frequencies the phase lag added by all the terms of the transfer function end up reaching their ultimate values

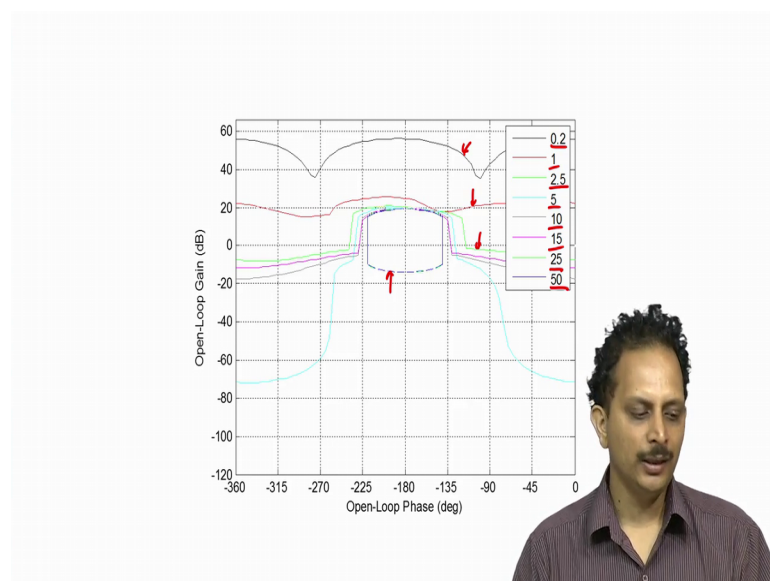
So, when we design a controller, this plant template will no longer be located, where it is situated now, but it could move anywhere in the complex plane. So, it could move here for instance, it could move there for instance, it could move here and so on and so forth. It depends on the gain of the overall open loop system at that particular frequency but whatever it is that the controller does and wherever it is that the controller moves the

overall plant template. We should make sure that no part of the plant template enters this forbidden circle, which is magnitude of T_1 is equal to 3.5 dB at this particular frequency.

Hence, what we do is we have to locate the plant template such that only the extremity of the plant template is sitting is making contact with this curve. And allow it to move around this closed curve or in other words allow the loop gain to assume a range of phase values and determine the magnitude of the loop gain at which the worst case plant would have the magnitude of T_1 to be equal to 3.5 dB. And if we do that we get this curve, which essentially represents the curve on which the nominal loop gain is allowed to lie. In order for the worst case loop gain to result in a transmission function T_1 whose magnitude is less than or equal to 3.5 dB.

So, this curve once again divides the Nichol's plot into two parts. The region inside the curve is the region where the loop gain is not supposed to lie. The nominal loop gain is not supposed to lie in order for the overall transmission function T_1 to be less than or equal to 3.5 dB. So, the region outside this closed curve is a region within which the loop gain is allowed to lie and this guarantees that the worst case transmission function T_1 will always be less than or equal to 3.5 dB. So, having computed the stability bound alone at 50 radian per second, I have included that along with the other bounds that we had computed.

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So, we have computed bounds at a 0.2 radian per second, 1 radian per second, 2.5, 5, 10 and 15. In addition to 15 I have computed it at two other frequencies at a 50 radian per second, which is what I discussed just a few minutes back. And also at another intermediate frequency, which is once again higher than the frequency at which we are expecting performance. In this case I have chosen to compute the performance bound at 25 radians per second

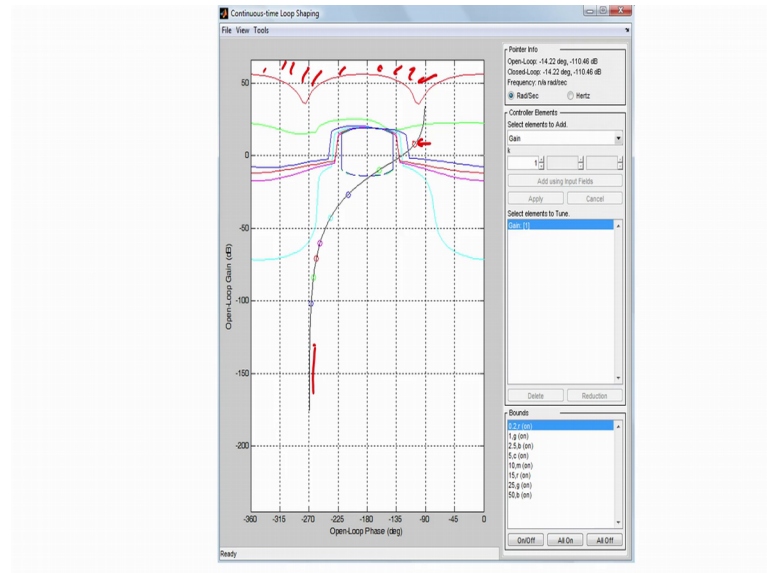
What you will notice is that the performance bounds at 25 radians per second and at a 50 radian per second are almost coincident and that is because the plant template does not change its shape at frequencies that are significantly greater than the corner frequencies of the plant. In this case the corner frequencies of the plant happen to be around 1 or 2 radians per second and since 25 and 50 radians per second are significantly higher than the corner frequencies of the plant, the plant template will not change its shape from 25 to 50 radians per second. And hence the stability bounds at these two frequencies remain essentially the same.

So, now that we have determined the bounds on performance and stability at all the different frequencies of interest to us. We are ready to execute the design. So, when we are trying to execute the design, what we are trying to do is to come up with a controller structure that ensures that the loop gain, at each of the frequencies is within the permissible regions at those particular frequencies. For instance, at 0.2 radians per second the loop gain should be above the black bound that has been indicated here. At 1 radian per second the loop gain should be above the red bound, at 2.5 radian per second it should be above the green bound and so on and so forth.

If the loop gain is exactly on these bounds, then the performance specifications will be exactly met. On the other hand, if it is above these bounds, then the performance specification would be met better than what is expected or in other words the variation in the transmission function will be regulated to a magnitude that is less than what we had wanted it to be regulated to. And that would result in a slightly conservative design, which is still acceptable, because, we have ended up doing better than what we were asked to do.

So, now that we have computed the bounds. We need to perform loop shaping and to perform loop shaping once again we take the help of the dialogue box that comes with the QFT tool box.

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And this dialog box essentially plots the bounds and super poses it with the Nichol's plot of the plant itself. So, the Nichols plot of the plant has been shown by this black curve here. And the color coded circles here represent the loop gains at each of the frequencies that are of interest to us. So, the red circle here represents the loop gain at the smallest frequency namely 0.2 radian per second. The green represents the loop gain at 1 radian per second and so on and so forth.

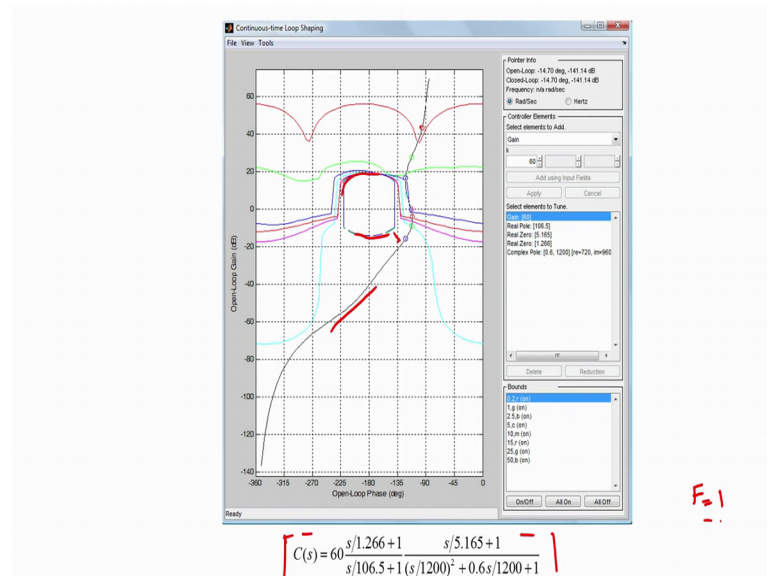
And if we compare the location of the loop gains at each of these frequencies with the corresponding bounds, we note that they are all outside the specified bounds. We wanted the red circle for instance to be either on or above the red bound. So, we wanted it to be inside this particular region, but this is where it is currently located. We wanted the green circle, to be above the green bound, the blue circle, to be above the blue bound and so on and so forth.

Since it is not being met we have to now choose an appropriate controller structure that will allow us to place these loop gains within the permitted regions in the complex plane at each of these frequencies. And I discussed that this can be done in an interactive manner using this dialog box. For instance, we can start by using a proportional

controller and that proportional controller can be used to push up the overall gain, so that to start with the red circle is within the red bound, but then you might end up with a closed loop system that is unstable. And hence we can choose to pick 0 and locate the 0 such that the Nichol's plot of the overall loop gain avoids the forbidden circle and so on and so forth.

Some of these steps were discussed by me in the previous design that we undertook, where we did one degree of freedom control design that took care of disturbance specifications. So, I have repeated the very similar steps in this particular case. And at the end I have obtained the loop gain to look something like this. So, you notice that for the particular controller that we have designed. And the controller structure is shown at the bottom of this slide here.

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So, for this particular controller the red circle is within the red bound, the green circle is actually above the green bound, the blue circle is exactly on top of the blue bound and so on and so forth. And if you look carefully, we also notice that this Nichol's plot of the loop gain, which is shown by the black curve here, is avoiding the forbidden circle. And this in turn implies that our closed loop system is going to be stable.

And it is the maximum value of the transmission function is going to be actually much less than 3.5 dB. If this loop just touched this forbidden circle then for some particular combination of the plants gain and pole locations. We would have the maximum value of

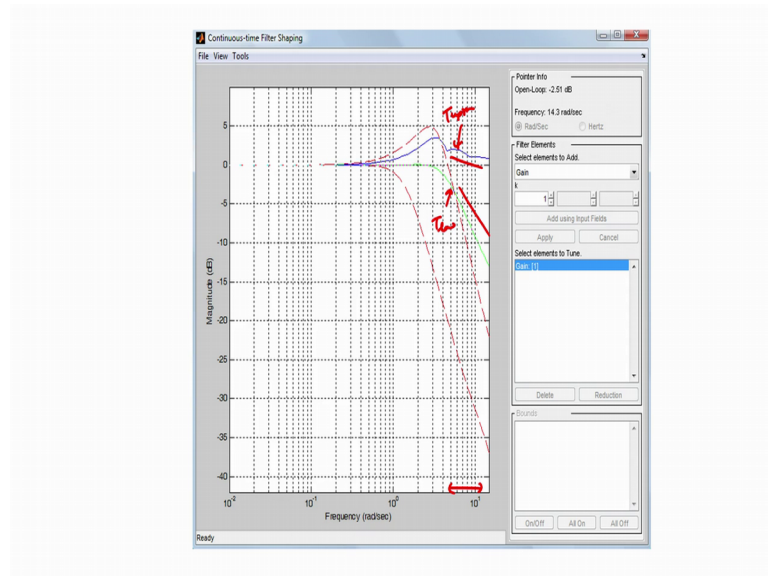
the transmission function to be exactly 3.5 dB. Since it is not touching, but it is actually avoiding the forbidden circle, namely this stability bound by a certain distance. The maximum value of the transmission function T_1 for the worst case combination of the plants gain and pole locations will be actually less than 3.5 dB.

So, we have, therefore now designed a closed loop system that is actually somewhat conservative, because in each at each of the frequencies the loop gains are well within the bounds that have been specified. If they were sitting exactly on top of the bounds that was specified and the design would exactly satisfy the specifications that were set. If they are within the bounds that were set, then it means that our closed loop system will do better than what we were asked to do.

The variation that we are going to permit for the transmission function would be less than what we have been allowed or in other words the dominant poles of the closed loop system will wander in a region that is smaller than the rectangular region that was specified to us at the outset.

So, this completes the design of the feedback controller and this feedback control structure, which I said we could obtain by trial and error using the interactive dialogue box that is available as part of the QFT software gives us the performance necessary to restrict the variation of the transmission function due to uncertainty in the plant parameters. And at the same time ensure that our feedback system is stable by the specified amount, but we are not yet done with the pre filter design. So, at the moment our pre filter is F is equal to 1, because we have not yet started with a design of the pre filter. And the software QFT toolbox also allows us to design the pre filter.

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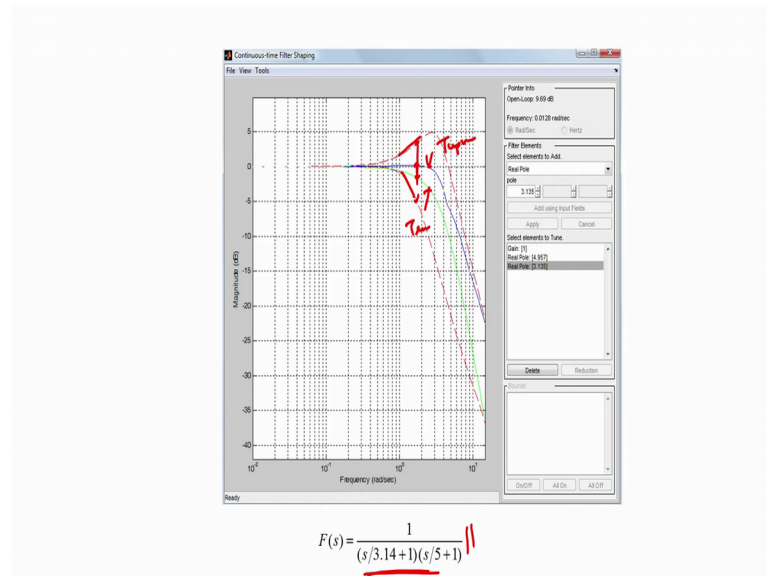


So, when the pre filter F is equal to 1, the software plots T_{upper} and T_{lower} so that is plotted by the blue curve here. This is T_{upper} and this is T_{lower} . The green curve here is T_{upper} T_{lower} . And the dotted red traces represent the actual bounds for T_{upper} and T_{lower} , which was specified by us based on the fact that the closed loop poles p_{cl} should lie within that is particular rectangle.

So, these two curves were obtained by us at the outset of our design procedure. And we note that when we do not have a pre filter, the T_{upper} and T_{lower} violate these bounds. So, parts of T_{upper} and parts of T_{lower} are outside the bounds in a in certain frequency ranges.

So, once again this dialog box allows us to pick the pre filter structure that makes sure that the transmission functions T_{upper} and T_{lower} are within the specified bounds. So, we can choose the pre filter to have certain gains and certain pole locations and certain 0 locations that together make sure that this T_{upper} and T_{lower} are within the specified bounds. And I have used the software to determine the structure of the pre filter and the pre filter structure is shown here.

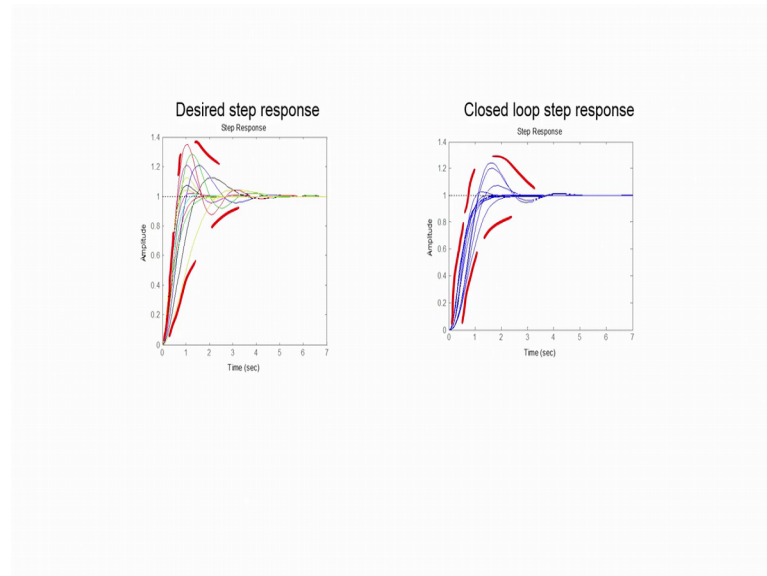
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We note that exactly as in the case of the root locus design the pre filter has unity DC gain. And for this particular pre filter structure we note that both T upper and T lower are within the permissible limits as specified by these upper and lower red curves. So, since the spread delta of j omega between T upper and T lower is significantly less than what we were permitted.

Our design in this case has been a little conservative or in other words as a consequence of this particular controller and this pre filter we are permitting our closed loop pole to wander in a region that is smaller in area compared to what it would actually vary within namely the rectangle that we had specified at the start of the design procedure. So, having completed the design it is only prudent to check for the performance of the overall control system in the time domain.

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So, I have plotted here, the closed loop step response of the overall system for different combinations of the plants gain and pole locations. And we see that we have considerably greater spread in the response than what we could achieve using the root locus. For reference I have also plotted here the desired set of step responses. And we see that the spread that was allowed in the desired set of step responses is actually greater than the spread that we have accomplished with our particular controller and pre filter combination.

And this as I discussed was a result of our conservative design, where we ended up choosing such a controller structure that the loop gain was well within the permitted bounds at each of the frequencies of interest to us from the point of view of performance and stability. However, since we have ended up with a closed loop system whose performance is better than what was specified to us, at this point we can conclude the design.

If on the other hand, if this spread that we see here in the closed loop step response for this particular controller is found to be too conservative, then we can go back to the controller structure and fine tune it a little bit in an interactive manner using the QFT toolbox so that the loop gains at each of the frequencies lie almost exactly on the particular bounds at those frequencies. And hence we would have a closed loop system that just first satisfies the desired performance specifications.

Thank you.