Control System Design
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Lecture – 34
Feedback control design using Nichols plot

Hello, in the previous clip we introduced a new design tool which was called as the Nichols plot, in order to represent the constraint imposed on the loop gain by the various performance specifications that would be given to us. The motivation for choosing this plot arose from the in adequacies of the other plot that we have seen so, far. So, we could rule out the root locus very early on because performance specifications cannot be depicted using the root locus.

And the other plot was the Bode plot and we spent some time talking about the difficulties in indicating the performance specifications in a Bode plot especially, when the performance requirements are not extremely stringent. And also the difficulty that arise when we plot the boded plot of a loop gain. When we have a uncertain plant, in which case we would have a bundle of loop gains each of which correspond to a certain combination of uncertain parameters of the plant.

And hence, the question of which one of the loop gains within this bundle should be designed our overall control system becomes problematic. And it was in this context that we discovered that ironically the Nyquist plot is better at representing the constraints imposed on the loop gain by a different performance specifications, simply because a directly plots the imaginary part of the loop gain was as the real part.

So, in this complex plane of the loop gain it becomes possible for us to depict the permissible areas in the complex plane where the loop gain is allowed to lie. So, we saw how this could be employed to depict the permissible areas after in which the loop gain can lie to reject output disturbance, to reject input disturbance, to achieve also stability, where we introduced a new way of depicting the stability of the close loop system in terms of the maximum value that the transmission function can be allowed to assume.

The final drawback though with the Nyquist plot was that when we have an uncertain plant, we have in area in the complex plane of the Nyquist plot where the plant transfer function P of j omega can assume its values. Now, when we multiply the plant with the controller then the area gets magnified and rotated.

And since we would have designed the controller at the outset it becomes difficult for us to proceed with a design. Simply because, a controller changes the shape as well as the location of the area within which the loop gain L of j omega can assume its values depending on the plant uncertainty P of j omega and the controller transfer function C of j omega. And this became the motivation for us to introduce the Nichols plot which is for all practical purposes a distorted version of the Nyquist plot. While in the Nyquist plot we plot the real part of L versus the imaginary part of L. In the Nichols plot, we plot the angle of L versus the magnitude of L in the logarithmic scale.

And the effect of this is that if we have an uncertain plant and when we multiply this plant transfer function with a controller transfer function, the particular choice of axis for the Nichols plot results in the area within which the plant transfer function P of j omega assumes values to neither change its shape nor change its size when multiplied with a controller transfer function

We shall visit this again in the next clip, but in this particular clip we shall familiarize ourselves with the design steps that are necessary for executing feedback control design using the Nichols plot. For the sake of simplicity, let us undertake one degree of freedom control design using the Nichols plot and I have indicated here the plant that we would be trying to control.

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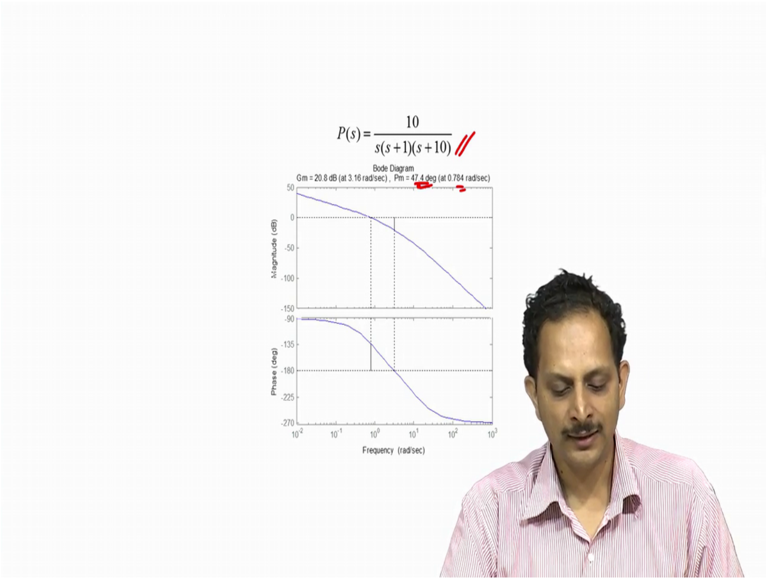

So, indicated here the plant that we would be trying to control. The plant is given by P of s is equal to 10 by s times s plus 1 times s plus 10. For the sake of familiarity I have indicated the Bode plot of the plant here and we see that the plant has a phase margin of about 50 degrees and a gain cross over frequency of around 0.784 radians per second. Now, this plant is assumed to be affected by both input and output disturbances.

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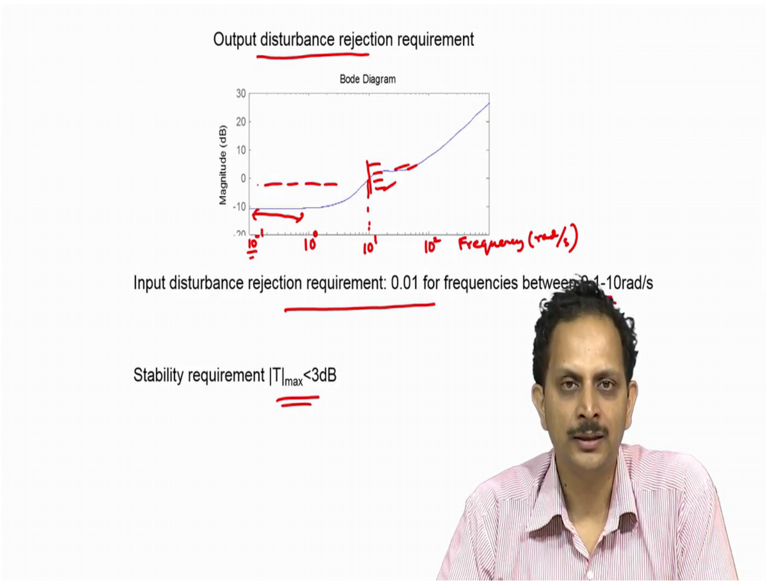

And the specifications on the rejection of the disturbances has been outlined in the next slide. As far as output disturbance rejection is concerned, the specification is that between 0.1 radian per second and 1 radian per second or in other words in this particular frequency region we want a magnitude of the output disturbance to be suppressed by minus 10 decibels. And at 10 radian per second, we want its magnitude to be suppressed by 50 percent and we do not care about the performance as far as how to disturbances concerned beyond 10 radians per second.

So, we are concerned about output disturbance, rejection, performance only up to 10 radian per second. Presumably because our output disturbance in this particular case has it is frequency contained only up to 10 radians per second. And in the low frequency range or in the frequency much less than 10 radians per second namely between 0.1 and 1 radian per second we wanted to be suppressed by minus 10 dB.

And at 10 radian per second we want it to be suppressed by minus 6 dB or in other words suppressed by 50 percent and we do not care about the performance at higher frequencies. This is the output disturbance rejection requirement that has been specified to us by the people who want us to design this control system. Now it so happens that the control system is also afflicted by input disturbance and in this case we want the input disturbance to be uniformly rejected by 99 percent or rejected to 0.01 of its actual magnitude for frequencies between 0.1 and 1 and 10 radians per second.

So, the frequency range within which the output disturbance and the input disturbance affects our plant is identical. It is between 0.1 and 10 radian per second, but the extent to which each of these disturbances how to be rejected are different. In case of output disturbance, there is a frequency dependent demand on the extent to which it needs to be rejected. At low frequencies it has to be suppressed by 10 dB and at 10 radian per second it has to be suppressed by 6 dB and other frequencies and higher frequencies from care about suppressing it at all.

Whereas, the input disturbance is to be suppressed by a factor of by 99 percent or by a factor of 100 over the entire frequency range between 0.1 and 10 radian per second; so, these are the specifications as far as a performance is concerned. And in terms of stability, let us say we want our maximum value of the transmission function to be less than or equal to 3 dB. This will ensure that the corresponding phase margin for the open loop transfer function is going to be greater than 40 degrees.

Hence, for stability we want T max to be less than or equal to 3 dB. The question now is how do we execute a design using the Nichols plot for this particular performance and stability specifications? So, I shall now write down a different steps that are involved in the design of a feedback control system in the Nichols plot to satisfy these performance specifications.

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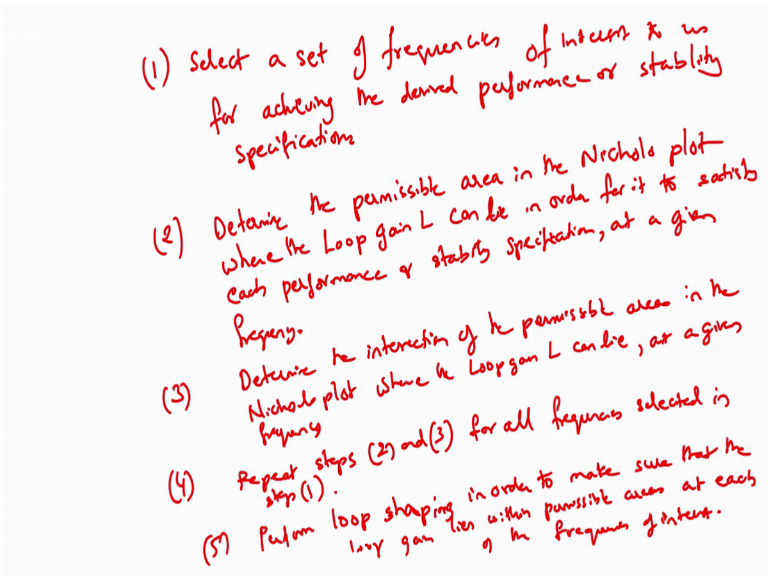

The first step is to select a set of frequencies of interest to us for achieving the desired performance or stability specifications. So, in this particular problem for instance, we know that we are not concerned about performance as far as disturbance rejection is concerned beyond 10 radian per second. So, we need to pick a few sample frequencies between 0 and 10 radian per second, which are of interest to us because these are the frequencies at which the closed loop system is going to get affected by both input and output disturbances.

The second step in the design is to determine the permissible area in the Nichols plot, where the loop gain L can lie in order for it to satisfy each performance or stability specification at a given frequency. So, if we pick one particular frequency, we to determine what is the permissible values of the phase of L and the what is the permissible values of the magnitude of L which will satisfy the input disturbance specification.

Similarly, what are the permissible values of the phase of L in the magnitude of L that will satisfy the output disturbance specification and likewise also the permissible values of magnitude and phase that satisfy the stability specification. This in affect boils down to indicating the area in the Nichols plot because, the Nichols plot essentially plots the phase of L versus the magnitude of L in the logarithmic scale. It boils down to indicating the permissible area in the Nichols plot where the loop gain can lie for it to satisfy each of these performance and or stability specifications.

The third step is to determine the intersection of these permissible areas because, for the control system to simultaneously satisfy the multiple performance specifications, the area in which the loop gain can be permitted to lie would be the intersection of the permissible areas and which it can lie for to satisfy each of the individual performance specifications at a given frequency.

So, determine the intersection of the permissible areas in the Nichols plot where the loop gain L can lie at a given frequency. The fourth step is once we have determined a permissible region in which the loop gain in lie at a particular frequency in order for it to simultaneously satisfy the performance and stability specifications. We repeat the exercise at other frequencies also.

So, we have already selected a set of frequencies has part of our first step. So, we and each of those frequencies that we have selected we determine the permissible area in the Nichols plot in which the loop gain can lie in order for it to simultaneously satisfy all the specifications at that frequency. So, repeat steps 2 and 3 for all frequencies selected in step 1.

And the last step is once we have clearly identified the permissible areas in the complex plane or in the Nichols plot where the loop gain can lie at each of the frequencies of interest to us be. Then come up with a controller that will make sure that the actual loop gain of the overall open loop system assumes values within this permissible areas and this is part of a loop shaping exercise.

So, the last step is to perform loop shaping in order to make sure that the loop gain lies within permissible areas at each of the frequencies of interest. So, these are the 5 steps that are involved in execute in design in the Nichols plot.

The first step is to select a set of frequencies of interest to us for achieving the desired performance of stability specification. The second step is that at each frequency that we have picked we determine the permissible area in which the loop gain can lie for it to satisfy every one of these performance specifications or stability specifications.

And the third step is to determine the intersection of all these permissible areas and that would represent the area in the Nichols plot where the loop gain can lie, which guarantees that it will simultaneously satisfy all the performance of stability specifications at a given frequency. And the fourth step is to repeat this identification of permissible area for all the frequencies that we have selected in the first step. And finally, having determined the permissible areas at each of the frequencies of interest to us we then execute the design where we determine a controller that makes sure that the overall loop gain assumes values within the permissible areas at each of the frequencies of interest to us.

Now, let us undertake an actual design where we illustrate the each of the individual steps that we have discussed here. So, in the particular problem that we just outlined the notice that we had a certain input disturbance specification which have to be suppressed by 99 percent over the frequency range from 0.1 to 10 radian per second. And output disturbance rejection specification which was frequency dependent and a certain stability specification.

So, we pick as a as part of the first step a few frequencies of interest to us in terms of output disturbance rejection and input disturbance rejection and these frequencies therefore, how to be between 0 and 10 radians per second. As far as stability is concerned we will have to pick a few more frequencies that are outside 10 radians per second for reasons that we will get to in a minute. So, let us first start with one candidate frequency, namely 5 radiance per second.

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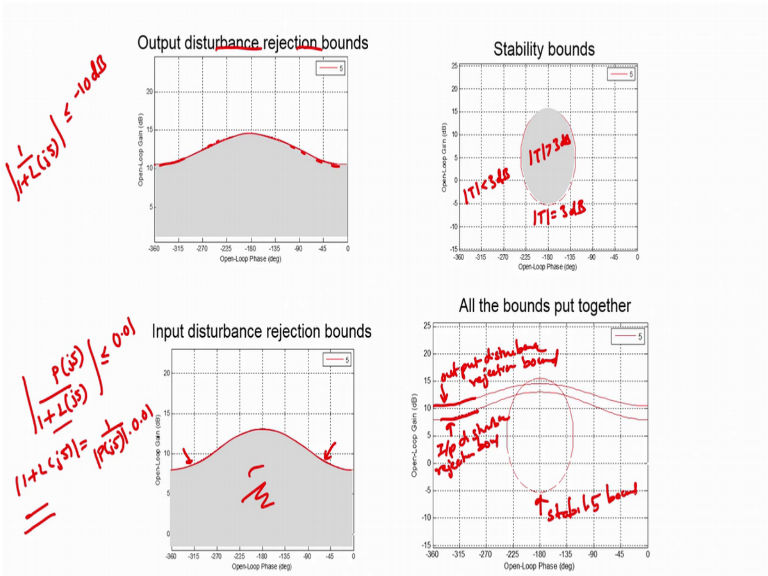

So, in order for us to reject output disturbance the transfer function at relates the output disturbance and the output is given by 1 by 1 plus L. Now, at 5 radians per second, the value of this transfer function is 1 by 1 plus L of j 5 and the magnitude of this has to be less than or equal to 10 dB minus 10 dB in order for us to reject the output disturbance.

And, as we discussed in the previous clip, this equation essentially represents a certain region in the Nyquist plot or equivalently in the Nichols plot where the loop gain L can lie. And the boundary of that region is given by the red curve that has been plotted here. Now, such a boundary can be plotted using what is known as a QFT toolbox. So, by using this software we can determine the permissible values of phase and the permissible values of gain for which we can achieve the specified performance requirements as far as output disturbance rejection is concerned.

And this boundary is called as the output disturbance rejection bound. So, this red curve is known as the output disturbance rejection bound and it divides the complex plane of the loop gain L into 2 halves. One is the area which has been shaded by this gray color, here which is the area within which a loop gain should not lie for it to satisfy the particular performance specification as far as output disturbance is concerned.

Therefore, for it to reject output disturbance by 10 dB the loop gain has to assume values in the area above the shaded part. And the boundary of that shaded part is what is known as the output disturbance rejection bound. Now likewise, we also have an input disturbance rejection bound. We know that at 5 radians per second we want the input disturbance to be rejected by 99 percent and the transfer function that relates the input disturbance to the output is given by P by 1 plus L.

So, if you were to evaluate this transfer function at 5 radians per second or in other words compute p of j 5 divided by 1 plus L of j 5 and the magnitude of this. This magnitude should be less than or equal to 0.01. Now once again, this is an inequality at represents a certain region in either the Nichols plot or the Nyquist plot which essentially tells us the permissible values of the magnitude and the phase of the loop gain that allows for the particular inequality to be satisfied.

So, if we were to plot the Nichols plot here, this red curve once again indicates the boundary of this permissible region. In other words if loop gain lies anywhere in this part that has been shaded gray, then this inequality will not be met. Hence, this red curve here essentially represents the boundary of the region which ensures that this particular performance specification is just met.

And this red curve demarcates a complex plane into 2 halves once again, the gray half which is the region in which the loop gain should not lie in order for it to satisfy the output disturbance rejection specification. And the white half in which the loop gain is allowed to lie in order for it to satisfy the input disturbance rejection specification.

The third specification is of stability, we are interested in stability at all frequencies that we might where we are interested in performance as well. And the stability specification namely that the maximum value of the transmission function should be less than or equal to 3 dB results in what is known as the stability bound. Just as we had a performance bound either for output disturbance rejection or for input disturbance rejection which were the 2 red curves in their respective plots. We have a stability bound which represents the curve on which the magnitude of the transmission function is exactly equal to 3 dB.

And this closed curve divides once again the Nichols plot into 2 parts, one is the part inside the curve where the magnitude of T is greater than 3 dB and the other is the part outside this curve where the magnitude of T is less than 3 dB.

Hence, for us to satisfy the stability specification at this particular frequency namely 5 radian per second, the white area is the area in which the loop gain is allowed to lie. If we put all these bounds together we get a graph that looks something like this. The first curve here corresponds to the output disturbance rejection bound. The second curve here represents the input disturbance rejection bound and the closed circular curve here represents this stability bound.

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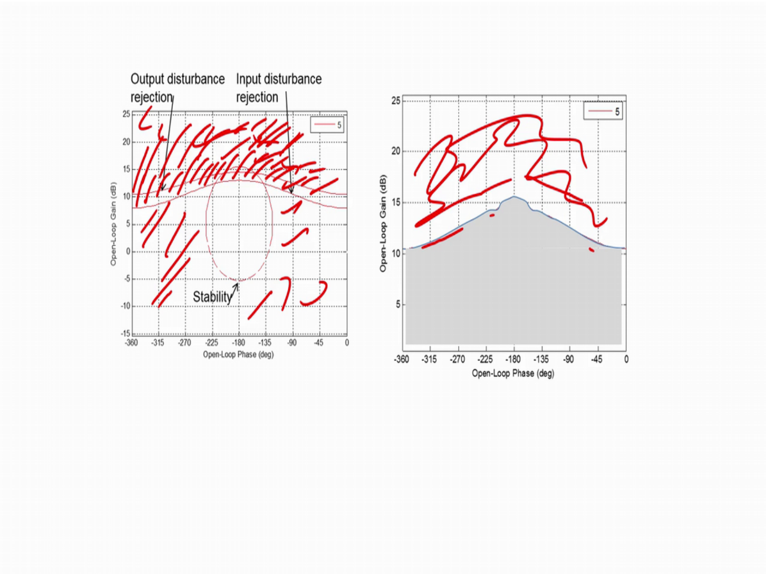

Now, the same has also been plotted in the graph on the left, the output disturbance of rejection bound, the input disturbance of rejection bound and the stability bound have all been indicated. Now, in order for the output disturbance rejection to be met the loop gain should lie in this particular area. In order for the input disturbance rejection specification to be met the loop gain should lie in this particular area that I am currently shading which includes significant parts of the area where the loop gain can lie in order to satisfy the output disturbance specification.

In order for the stability specification to be met, the loop gain can lie in the area that is outside of this circle which also once again overlaps partially with the area where the loop gain can lie for it to satisfy the input and output disturbance rejection specifications. In order for the loop gain to simultaneous satisfy all these 3 specifications, it should essentially lie in an area that is the intersection of the 3 areas namely the area, the permissible area for it to satisfy input disturbance rejection spec, the permissible area for it to satisfy the output disturbance rejection spec and the permissible area for it to satisfy the stability specification.

And that area is obtained by taking the intersection of these 3 bounds and that is indicated by this new red curve that is shown here. And this new red curve once again divides up the complex plane into 2 parts, one is the plane within which the loop gain should not lie, in order for it to simultaneously satisfy all the 3 specifications. And other this is the region in which the loop gain is allowed to lie and this ensures that it will simultaneously satisfy all the 3 specifications of 2 output disturbance and the stability specification at 5 radians per second.

Now, we have undertaken computation of the bounds at one particular frequency. Determination of it is intersection at the particular frequency and hence determination of the permissible area in the Nichols plot, where the loop gain can lie at that particular frequency. We have to now repeat this at other frequencies of interest to us within the frequency range that the disturbance is affect our system.

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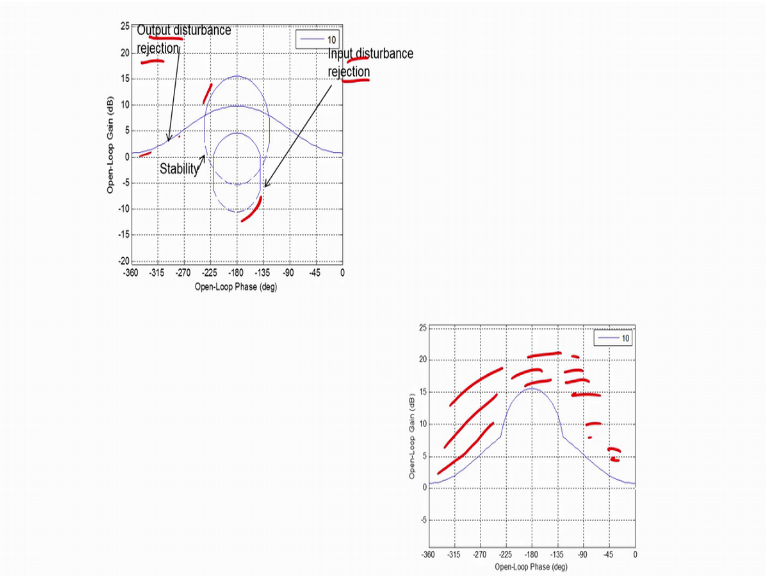

So, this has now been repeated at 10 radians per second and the 3 bounds have been indicated here. The output disturbance rejection bound is given by this blue curve, the input disturbance rejection bound is given by this closed curve and the stability bound has been given by this other closed curve.

And the permissible area within which the loop gain can lie which obtained by first taking the intersection of all these curves that divides the complex plane once again into 2 parts. The part above this curve is the part where the loop gain can lie so, that the loop gain simultaneously satisfies all the 3 specifications. And the part below it is a part where it should not lie for it to satisfy simultaneously all these specifications.

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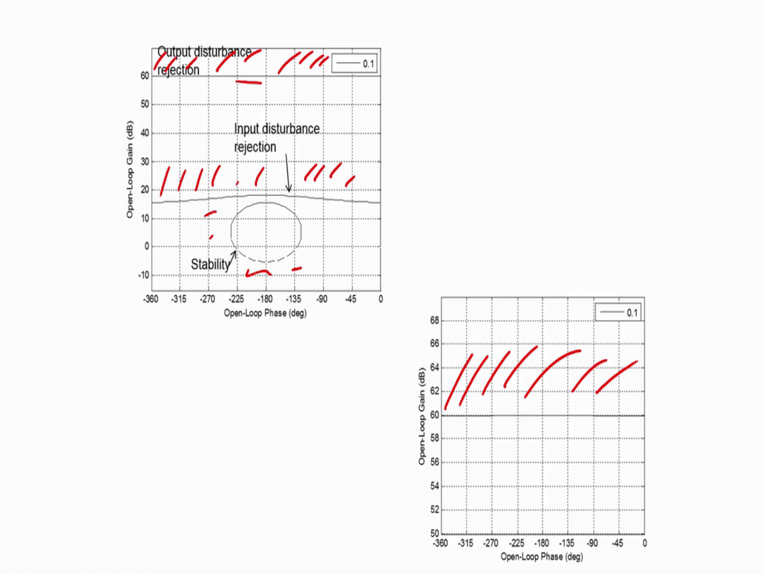

This has also been repeated at one lesser frequency, namely at 0.1 radian per second. Now, at this frequency the output disturbance rejection specifications given by this black curve here; so, the loop gain should lie above this black curve for it to reject the output disturbance by the specified amount at that frequency. The input disturbance rejection specification has been given by this other black curve here. So, the loop gain should lie above this curve for it to satisfy input disturbance rejection spec.

And for it to satisfy a stability specification it should lie outside this circle and you see that there is no real intersection between these 3 curves. And the permissible area within which the loop gain can lie at 0.1 radian per second is essentially the area that is above the highest of these curves which is essentially the bound set by the output disturbance rejection specification. And that has been shown in this graph here.

So, the area above 60 dB is the area in which the loop gain can lie at 0.1 radian per second for it to simultaneously satisfy all the 3 specifications at this particular frequency. Now, our requirements on performance stop at 10 radian per second, but there is no guarantee that we can stop worrying about stability also at 10 radian per second because, the gain might cross over at a frequency well beyond 10 radian per second. And hence, to be sure that at that frequency where the gain crosses over we have our particular stability specification being met.

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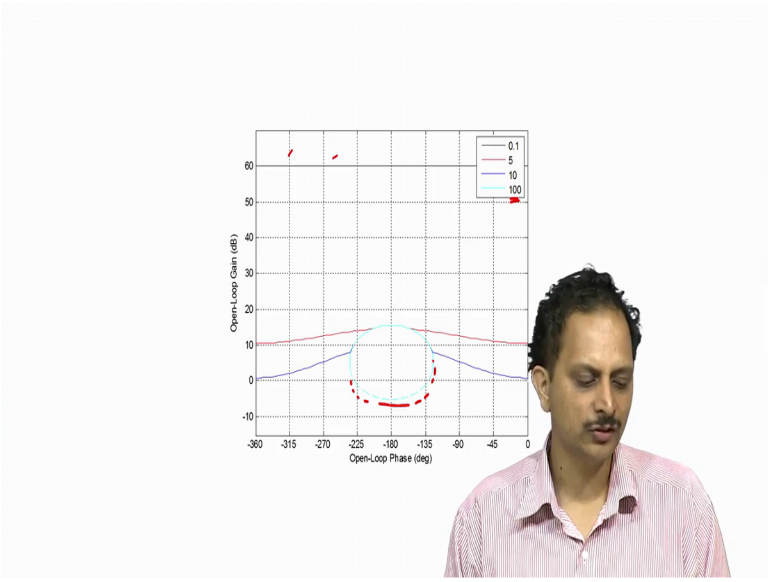

We pick one more frequency which is much bigger than a frequency at which we are interested in performance. So, we have picked one more frequency namely 100 radians per second and at this frequency we are not concerned about performance at all, we are concerned only about stability. So, we have therefore, indicated that is stability consideration by means of this cyan colored closed curve.

So, to summarize at 0.1 radians per second the loop gain should lie above the black color curve in order for it to simultaneously satisfy the disturbance rejection specs as well as the stability spec. At 5 radian per second, it should lie the loop gain should lie above the red curve for it to satisfy stability spec as well as disturbance rejection specs. At 10 radian per second, it should lie above the blue color curve sorry to satisfy once again the stability and disturbance rejection requirements.

At 100 radians per second, there is no performance requirement because we are not concerned about disturbance rejection beyond 10 radian per second, but we are still concerned about stability. Now this number 100 was rather arbitrary, it could be any number that is beyond 10 radian per second and at that particular frequency we are concerned only about the stability bounds. And it does not matter what number we pick because, the shape of the bound will be independent of the number that we pick when we have no uncertainty associated with the plant.

And in this case we have picked 100 radian per second and that this frequency were concerned only about stability. So, for the system to be stable at this high frequency whatever number it might be it should lie outside this cyan colored bound. The next step for us is to execute our control design or in other words determine the structure of the controller that ensures that the loop gain is within these particular regions at each of these frequencies.

In order to execute the design we once again use what is known as the QFT toolbox which is a priced software that runs along with MATLAB, but if you have access to this toolbox then you can open a dialog box.

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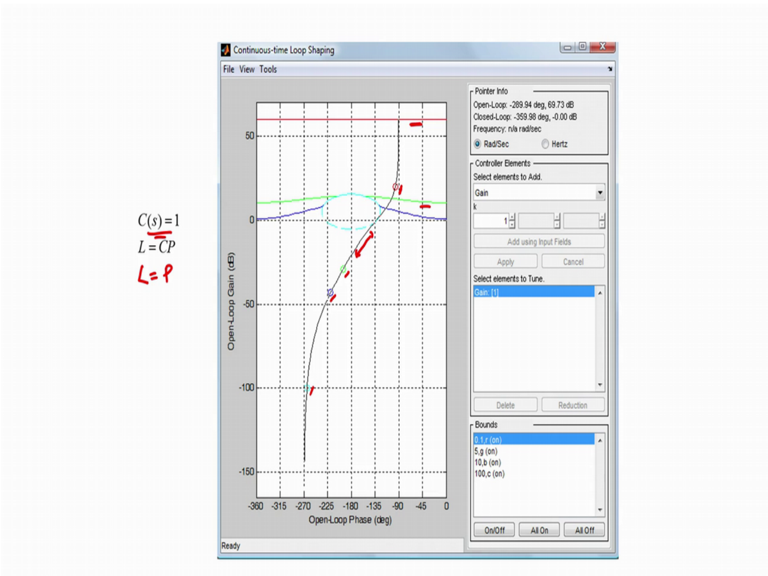

Where the same bounds are represented and on top of these bounds we have also super post the Nichols plot of the plant itself. The software indicates through appropriately colored circles, the loop gains at each of the frequencies of interest to us. For instance this red circle indicates the loop gain at 0.1 radian per second and the red bound here corresponds to the bound that 0.1 radian per second. The green circle indicates the loop gain at 5 radians per second, while the green bound indicates the bound on the collective performance and stability at 5 radians per second. The blue circle here indicates the bound the value of the loop gain at 10 radians per second and the cyan colored circle indicates the value of the loop gain at 100 radians per second.

Now, we see that almost every single specification is being violated by just the plant alone and in particular the red circle which is a loop gain at 0.1 radian per second is well below the red bound and hence in the area that is not permissible at 0.1 radian per second. The green circle is well below the green bound, the blue circle is well below the blue bound and so, on and so, forth. This is the situation when our controller C of s is equal to 1 in which case our loop gain is simply be equal to the plant transfer function.

Now, the software allows us to interactively change the gain of the controller and add different elements to the controller namely poles and zeros including complex poles and complex zeros that will allow us to change the appearance of the loop shape.

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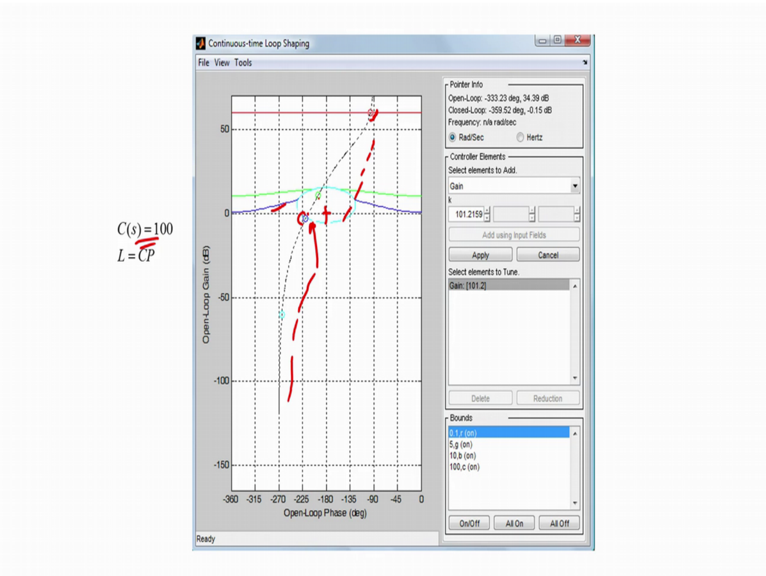

For instance, if we increase the gain of the controller by a factor of 100, the whole of the Nichols plot gets 2 stop. So, if you remember the Nichols plot earlier look something like this and it got pushed up by 40 decibels as a consequence of choosing a controller of this particular structure. Now, if we choose this we note that our red circle which represents a loop gain at 0.1 radian per second will sit exactly on the red bound. And therefore, will just satisfied the specifications at 0.1 radian per second. The green circle; however, still will be below the green bound, the blue circle will be below the blue bound.

And what is worse? When we look at the Nichols plot we see that it is encircling the critical point namely 0 dB and minus 180 degrees in the wrong direction. On other words, at the gain cross over frequency the phase lag is greater than minus 180 degrees and therefore, our close loop system is going to be unstable with a simple proportional controller of gain 100.

So, the first thing that we do is to ensures stability and to do that we move this we try to move the entire Nichols plot of the loop gain to the right. We know that since a 0 adds phase lead, we choose a 0 for the controller and that will shift the entire Nichols plot to the right.

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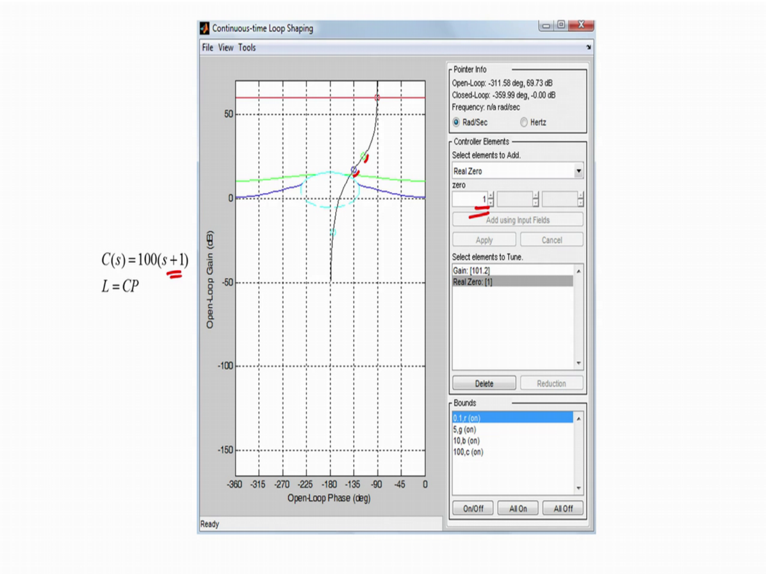

So, once we choose a 0 at s is equal to plus 1 so, this can be done in an interactive manner. So, we have chosen a 0 at s is equal to plus 1. We see that a Nichols plot has moved to the right, but it still passes through this forbidden circle within which the loop gain should not pass in order for the transmission functions maximum value to be less than or equal to 3 dB.

Hence, we are still not at stable enough for our design to be concluded; however, this 0 has had the beneficial effect of pushing the green circle which represents the loop gain at 5 radians per second above the green bound. And hence at 5 radians per second the loop gain is within the permissible area at that particular frequency. It has also pushed the blue circle above the blue bound which indicates that at 10 radians per second the loop gain is within the permissible area at that particular frequency.

The only concern now is with stability because, the loop now still passes through the forbidden circle. And which indicates that our transmission function will assume a value will assume a magnitude it is greater than 3 dB. And hence our close loop system will have a much smaller phase margin and what we had initially planned for. To address this issue the try to move the Nichols plot further to the right which essentially means that we have to add a phase lead. So, we choose to multiply our controller transfer function with another 0.

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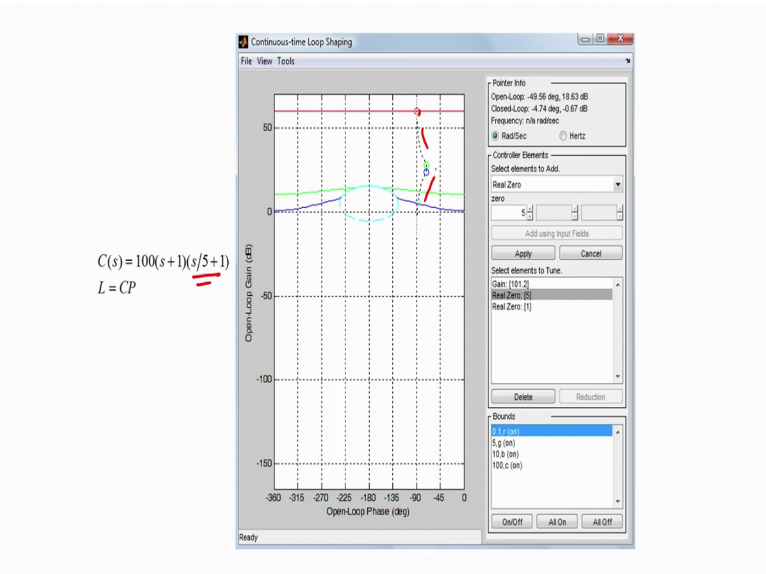

So, this time the 0 has been chosen to be at 5 radians per second and as a consequence we note that our Nichols plot gets transformed to the curve that has been shown by the dotted line here. Now, when we examine this curve we see that all the performance specifications are met.

The red circle which is the loop gain at 0.1 radian per second is exactly on the red bound. The green and blue circles are above their respective bounds and hence the loop gains are higher than what they need to be in order for all the specifications to be met at those particular frequencies. And this curve also is not inside the forbidden circle which is necessary for stability to be satisfied.

So, in principle this controller satisfies all the specifications, but the problem with it is that it is a non causal transfer function. So, we only have 2 zeros and hence a numerator polynomial will be of degree 2 and we have no denominator polynomial at all and hence you have to add to controller poles for the sake of causality. We should make sure that we add these poles in such a manner that they do not affect the performance at the frequencies of interest to us. And since the frequencies of interest as far as performance are up to 10 radians per second we choose to add the pole for the controller at a much higher frequency.

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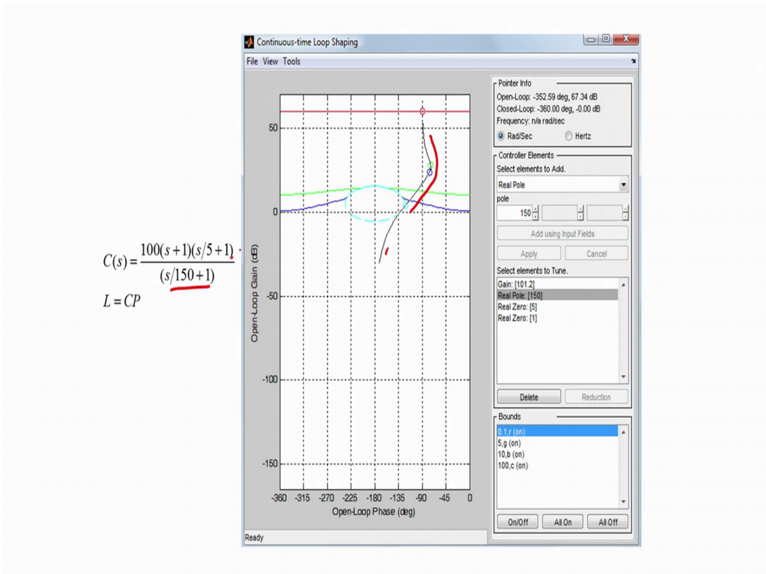

In this case, the controller pole has one controller pole has chosen to be added at 150 radians per second and as a consequence the Nichols plot changes to curve that has been shown here. Once again we see that the loop gain is within the permissible regions in for all the frequencies that we have considered and even at 100 radians per second the loop gain is outside the forbidden circle.

But, still adding a single pole for the controller does not make the controller transfer function completely causal because, the numerator is of degree 2 you have to add one more pole. And since far away poles do not significantly affect the magnitude or phase characteristics at lesser frequencies, we choose to add a second pole at 500 radians per second.

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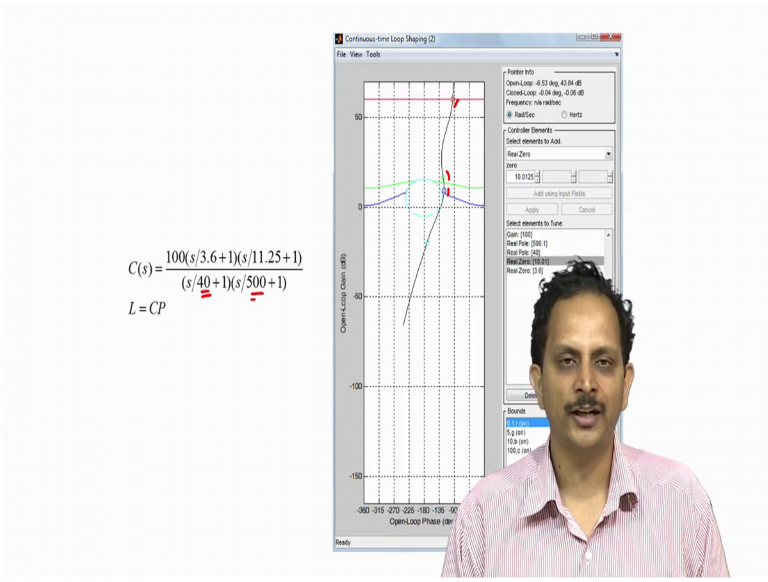

And, we fine tune the position of the first pole we initially had placed it at 150 radians per second and a software here allows us to interactively change its position in order to fine tune the loop shape. So, each time the location of this pole is changed, the software will reveal the manner in which a loop shape changes. And by interactively doing this I discover that the pole at s is equal to 40, at omega equal to 40 and the pole at omega equal to 500 ensure that all the performance specifications have been met and the forbidden circle which is required for stability is just avoided by the loop shape.

So, with this we are done with the control design using the Nichols plot because, this particular controller is firstly, not only causal. But, also places the loop gain at each of the frequencies of interest towards namely 0.1 radian per second, 5 radian per second, 10 radian per second and 100 radian per second in the regions that are permissible at each of these frequencies. In the next clip we shall take a look at performing control design in the case of an uncertain plant. In such a case we would need to design a 2 degree of freedom control system using the Nichols plot.

Thank you.