Control System Design
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Lecture – 33
Introduction to Nichols plot

Hello in the previous clips, we have taken a look at a one degree of freedom control and this was performed primarily using a bode plots. Although root locus was also employed to look at stability issues during the design of one degree of freedom control systems and subsequently we looked at 2 degree of freedom control and in this, we employed the root locus technique in order to design robust control systems, that ensure that the variation in the transient response of a closed loop system is within specified limits owing to variation in the or owing to variation or uncertainty in the model of the plant.

However we discussed that, as elegant and intuitive as root locus based design approach was, it is not possible to represent the constraints imposed on the closed loop system by specifications such as input disturbance rejection, output disturbance rejection and so on, when one performs design using the root locus. Hence, the topic of a few clips starting from the present clip, is to perform 2 degree of freedom control design, where one is not concerned only about achieving robustness of the overall transmission function to variation in plant parameters, although this would also be one of the specifications.

But in addition to that there are other specifications as well. For instance, I have drawn the block diagram of the control system that, we would be interested in over the next few clips.

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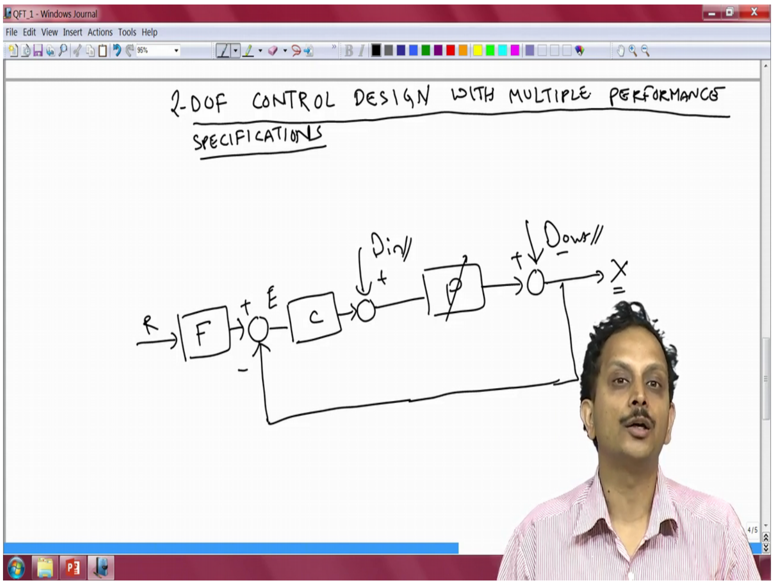

If you look at this feedback block diagram, you see that the control system is afflicted by output disturbance D out, input disturbance D in and then there is also a certain reference that needs to be tracked and the frequency content of D out and D in and the reference, all are may all be very different and the expected tracking performance in tracking the reference or the rejection performances for rejecting D in or D out in their particular frequency ranges might also be quite different.

And on top of all of this we may have an uncertain plant. So, a plant whose, pole locations and the gains and the zero locations might be uncertain. So, the plant model itself has some uncertainty associated with it. So, despite all of this, despite the presence of plant uncertainty, despite the presence of an output disturbance, despite the presence of an input disturbance, we want our output X of the overall control system to reject these disturbances by the specified amounts and track this reference by with the specified degree of accuracy.

So, how does one design the feedback controller and the pre filter in order to solve a problem of this kind? So, the first step in our attempt to address problems of this degree of complexity is to come up with a suitable design tool that will show us in a very clear and transparent manner the constraints imposed by each of these performance specifications on the performance of the overall control system. So, if you recollect the performance of the control system is intimately tied to the loop gain, which is the product of the controller in the plant transfer functions of our open loop system.

The higher the loop gain, the better is our performance, in terms of rejection of disturbances and achieving robustness to variations in plant parameters. And the loop gain is in general a complex number. It has a magnitude and it has a phase. Now each of the performance specifications namely, the rejection of output disturbances and the rejection of input disturbances and the robust tracking requirements, impose constraints on the loop gain or in other words, the magnitude and the phase of the loop gain, how to assume values only in certain specified ranges in order for them to reject disturbances or track references according to the specifications.

So, what one wishes therefore, is a tool that will allow us to clearly depict the constraints imposed on the loop gain of our feedback system by the different performance specifications that have been given to us. Now, what are the different design tools that we have looked at so, far? We have looked at the Nyquist plot and then subsequently, we looked at the bode plot and the third design tool that we have looked at is the root locus.

Now, right away we can disqualify root locus as a good candidate for depicting these restrictions on the loop gain imposed by the performance specifications, because the root locus is not a good tool for depicting performance specifications. So, what that leaves us with is the Nyquist plot and the bode plot, it turns out that despite the fact that the bode plot has significant advantages over the Nyquist plot in terms of the appearance of the loop gain in a bode plot namely, that the asymptotic loop gain looks like a bunch of broken straight lines and the product of the controller and the plant gets converted to a summation operation, when one uses the bode plot for doing design.

It turns out ironically that, when it comes to representing the constraints imposed on the loop gain by the different performance specifications. The Nyquist plot is actually better than the bode plot. But it also turns out the despite the superiority of the Nyquist plot compared to the bode plot, in terms of depicting the constraints though, the loop shape itself might be non intuitive in it is appearance, the constraints can be depicted in a transparent and clear manner in a Nyquist plot compared to a bode plot.

But despite this advantage, what you would see is that the clarity that, one can get with a Nyquist plot is still not adequate enough for us to execute our control design in the presence of multiple performance requirements. In terms of multiple disturbance rejections and achieving robustness to plant parameter variations and also achieving robust tracking of certain reference signals within certain specified ranges.

So, what we shall therefore, introduce in this clip is a new tool which is called as a Nichols plot, which allows us to address some of the important limitations of the nyquist plots and therefore, enables us to represent the in constraints imposed by the performance specifications in a transparent manner.

To get started let us first revisit the bode plot and try to understand, why the bode plot is not that is good of a tool, when it comes to representing the constraints imposed on the loop gain by the different performance specifications.

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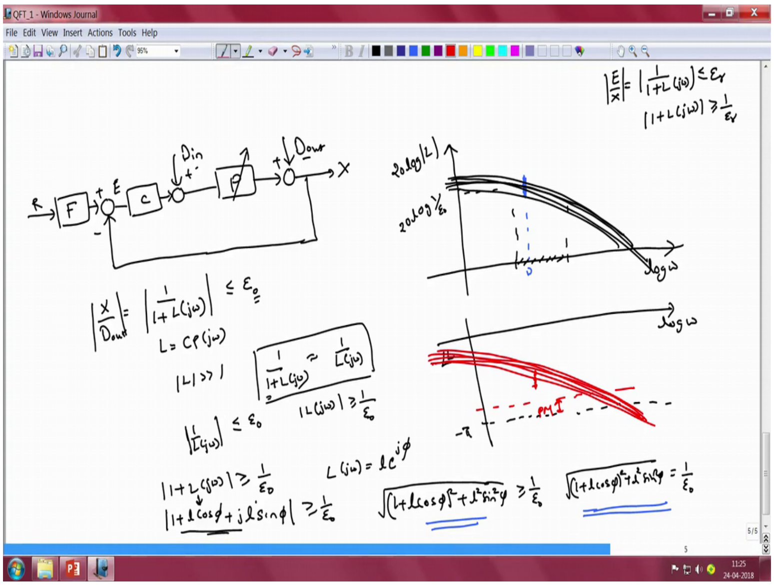

So, I have shown here the same block diagram on the left and on the right, I have plotted the bode plot of the open loop system. Now let us take one particular performance specification for instance we are let us say interested in output disturbance rejection.

Now, the transfer function that relates the output X to the output disturbance D out is given by X by D out is equal to 1 by 1 plus L of j omega, where L is equal to C times P of j omega. So, if we want to reject the output disturbance by a certain amount then, what we generally insist, is that the magnitude of X by D out, should be which is in a sense equal to the magnitude of 1 by 1 plus L of j omega, should be less than or equal to some small number epsilon o, the subscript o is intended to indicate that, this is the output disturbance that we are talking about.

Now, when we undertook control design using the, bode plots several lectures back, what we assumed was that this number epsilon o was a very small number. If the number epsilon 0 is a very small number then in order for 1 by 1 plus L to be a very small number, we need to have the magnitude of L to be much greater than 1. Now in this limit, we can approximate the term 1 by 1 plus L of j omega to be approximately equal to 1 by L of j omega.

And we can ignore the term, 1 in relation to the magnitude of the loop thing L and hence make this approximation, if the result that you would have the corresponding performance constraint on the loop gain to be that, 1 by L of j omega magnitude of 1 by L of j omega should be less than or equal to epsilon 0 or equivalently the magnitude of L of j omega should be greater than or equal to 1 by epsilon 0. Now this performance constraint can be very readily shown on the bode plot.

So, if the frequency range of interest is within this shaded region, along the X axis of the bode plot, then within the shaded region, we need to have the magnitude of the loop gain be greater than or equal to 1 by epsilon 0. So, if I were to draw a horizontal line here, which corresponds to the magnitude being equal to 20 log of 1 by epsilon 0, then in this frequency range, we need the loop gain to assume a magnitude that is greater than the shaded region here.

And if that is done, that we will be able to reject the output disturbance in this frequency range by the desired extract now this approximation works fine as long as we assume epsilon 0 to be a very small number much smaller than unity, but suppose epsilon 0 is a small number, but not exceedingly small. So, let us say instead of epsilon 0 being 0.01 or 0.001, we are with epsilon 0 being 0.5 or 0.4 or in other words, some small number that is still comparable in magnitude 2 unity, then you can easily appreciate that this particular approximation cannot be made.

This approximation works only under the condition that, the term 1 plus L is much greater than unity, but that is not going to be the case, when our performance specification is such that epsilon 0 is not a very small number. So, in such a case the actual constraint on the loop gain is given by the equation magnitude of 1 plus L of j omega should be greater than or equal to 1 by epsilon 0. This is the correct constraint that the loop gain has to satisfy, in order for it to reject a disturbance by the desired specified amount epsilon 0.

Now, this equation has multiple values for the magnitude and a phase of the loop gain that can satisfy it. To see this, let us first substitute the loop gain L of j omega as some magnitude L and some phase e power j phi. So, let phi be the phase of the loop gain and small l be the magnitude, in which case you would have 1 plus l of j omega to be equal to 1 plus l cos phi plus j times l sin phi and the magnitude of this entire term, which is essentially 1 plus capital L of j omega should be greater than or equal to 1 by epsilon 0.

So, the magnitude of the term given here is essentially given by square root of 1 plus l cos phi the square plus l square sin square phi, it is essentially square root of the real part square of the real part plus the square of the imaginary part and that should be greater than or equal to 1 by epsilon 0. Now let us first consider the equality in order to determine the minimum permissible values of the loop gain, that satisfy the specified performance requirement.

So, if you consider only the equality and not the inequality, we would have square root of 1 plus l cos phi the square plus l square sin square phi to be equal to 1 by epsilon 0. Now if you are given an equation of this kind, we note that for each particular value of phi, there exist a particular value of small l that satisfies this equation. Hence, if epsilon 0 happens to not be a very small number, but a number that is comparable to unity, then for each particular value of phase, which I which, I shall indicate by a dot here in the complex plane, there exists a certain magnitude, which is indicated by the blue dot here beyond which the performance specifications are met.

For a different value of phase, which are indicate by a different red dot in the phase plot of the bode plot. There exists a different magnitude at which beyond which the performance requirements are met at a particular frequency omega, where we are interested in rejecting the disturbance. Now for different possible values of phase, which are indicate by a different circular dots, there exist different magnitudes for which the equation that I have indicated here and I now, underlining with the blue marker is going to be met.

Now, I have a continuum of angles and corresponding to that, I have a continuum of magnitudes at which this equation is met and beyond which this, inequality that are now underlining with the blue marker is going to be met. So, all these blue dots are going to essentially merged together, when we are trying to represent them together and they are going to end up (Refer Time: 15:30) and becoming a band and likewise all these red dots are going to merge together and end up looking like a band.

And this one is to one relationship between the magnitude and a phase that together satisfy the equation that, we have at the right extreme gets lost, when you are trying to represent the constraint imposed by this disturbance rejection specification on the loop gain of the system. Hence we cannot independently point out, the magnitude for a specified phase at which this equality is valid and beyond which, this inequality becomes valid and this loss of information happens, because we are not able to separately indicate the desired magnitude at a specified phase beyond which, the disturbance rejection performance is going to be met using the bode plot.

Now, the same problem also exists with the, bode plot when we are trying to specify robust tracking requirements. So, for instance the tracking error E is related to the output X by the transfer function, which is identical to that of the output disturbance namely 1 by 1 plus L of j omega. So, if the tracking errors magnitude has to be within a certain limit which, I shall call as epsilon r, then corresponding to that, I will have 1 by 1 plus L of j omega, should be greater than or equal to 1 by epsilon r.

Now, once again for each particular value of the phase of L, there exists a magnitude of L beyond which its inequality is met. And this one to one relationship between the magnitude that corresponds to a certain phase of the loop gain beyond which, this equation is valid is lost, when we are trying to depict this inequality in the case of a bode plot.

All these problems actually become far worse, when we are confronted with a plant that has uncertain dynamics. So, if there is a uncertainty associated with the gain of the plant or the locations of the poles of the zeros of the plant then, we do not have a single transfer function to represent the dynamics of the plant. If you have a single transfer function for the plant then, we have a single curve for the loop gain and let us say, I shall draw the magnitude curve here to indicate the magnitude characteristics of the loop gain for a plant which has no uncertainty.

Now, suppose we have a plant that is uncertain, then for a certain specific value of gain. The curve that, I have drawn would be a valid for a higher gain, the curve would be higher up, for a lower gain, the curve would be lower down for uncertainty associated with the location of the poles of the plant, then we would have a different curve and for uncertainty associated with the zeroes of the plant we would have another different curve. So, in essence therefore, you would have a bundle of loop gains with each a curve within that bundle, corresponding to the loop gain for a certain specific combination of parameters of the uncertain plant.

The same thing also happens, when we are looking at the phase characteristics. So, the nominal phase characteristics of the plant might have a single curve, but when there is uncertainty associated with either the pole location or the 0 location of the plant then, these phase responses will also show a certain spread. So, for each particular permissible location for the pole of the plant, you will have a certain curve and similarly for each particular location for the permissible location for the 0 of the plant, you will have a certain curve.

So, once again the phase characteristics get transformed from a single curve to a bundle of curves. Now the question is how do we perform the design when we are given a bundle of loop gains in order to meet both the performance specification as well as a stability specification. Now a stability specification, now becomes slightly more difficult to impose because even if, we were to go with a certain phase margin the question still remains as to which one of the curves within this bundle of phase curves do we apply the phase margin? Because each of the curves within this bundle has a different phase margin and the worst case phase characteristic, need not necessarily correspond to the worst case magnitude characteristic.

Hence it is not possible for us to extract just a single curve from the magnitude characteristic and the corresponding phase characteristic of that loop gain and just design our feedback control system for just that curve alone and hope that our design is going to be valid for all other curves. If even if, it is likely to work, it might result in a overly conservative design because, the worst case magnitude characteristic need not necessarily correspond to the worst case phase characteristic.

Hence you begin to now appreciate the difficulties in representing the performance specifications for example, in output disturbance rejection or for reference tracking, when we are using bode plots and also the difficulty in representing the different possible values of the loop gain, that the overall system can assume owing to uncertainty in the plant dynamics, because in the presence of plant uncertainty we do not have a single curve for the magnitude and a phase characteristics of the loop gain.

But rather we have a bundle of curves with each curve corresponding to a particular combination of the parameters of the plant and the question us to, which one of these curves do we take in order to perform our design or do we or which is the so called worst case curve for which we, we need to perform the design? All become difficult to resolve, when one adopts the bode plot for performing design in the case of uncertain systems, where multiple performance specifications have been provided.

Now, let us briefly revisit the plot which in the beginning, we argued was a little inferior to the, bode plots, when it came to performing control design and that is the Nyquist plot, what we will see ironically is that despite the Nyquist plot of a certain transfer function being non intuitive and despite the fact that the Nyquist plot of the controller times, the plant may not be easily visualized, if you have given the Nyquist plot of the controller and the plant separately.

When it comes to depicting the constraints imposed on the loop gain with a different performance specifications, the Nyquist plot out does the bode plot in terms of it is ability to depict these, constraints in a transparent manner. Let us see that by considering for instance the constraints on rejection of output disturbance, exactly in the manner as we have done here.

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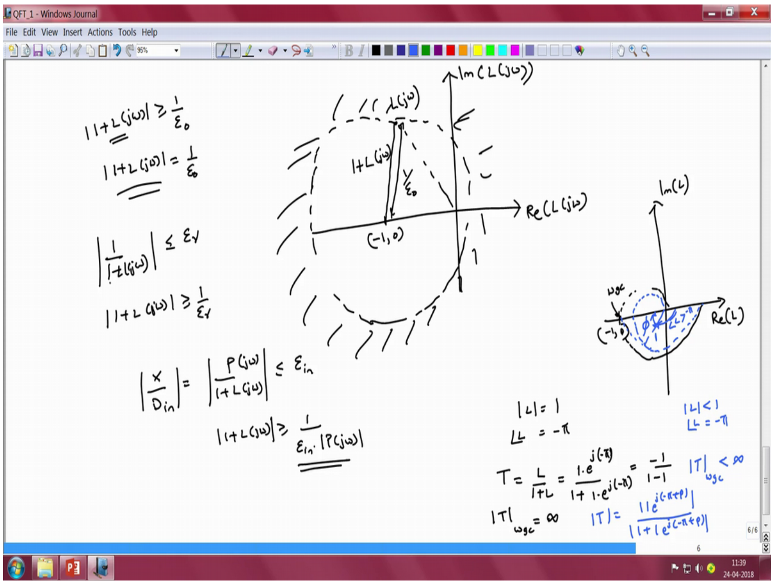

So, I shall first plot the Nyquist plot, it is essentially plots, the real part of the loop gain L of j omega with respect to the imaginary part of the loop gain L of j omega.

Now, any arbitrary point in this complex plane of imaginary part of L versus the real part of L, essentially represents the complex number L of j omega. Now if we, revisit the requirement on the loop gain in order for it to satisfy our disturbance rejection specification, we had that the magnitude of 1 plus L of j omega, has to be greater than or equal to 1 by epsilon 0. This was the specification and we had difficulties in representing the specification in a transparent manner in the bode plot, because this 1 is to 1 correspondence between the magnitude of the loop gain and the phase of the loop gain at which, the equation 1 plus magnitude of 1 plus L equal to 1 by epsilon 0 is met was not possible.

This problem; however, goes away when we used the Nyquist plot, because if this dotted line represents a complex number L of j omega, we know that a critical point in the Nyquist plot is the point minus 1 comma 0, we note that the complex number that starts from minus 1 comma 0 and ends at the point l of j omega essentially represents, the complex number 1 plus L or in other words 1 plus L of j omega. Now the magnitude of 1 plus L refers to the length of this line segment, the length of this line segment is given by the magnitude of the complex number 1 plus L of j omega.

Now, let us first focus on the equation magnitude of 1 plus L of j omega is equal to 1 by epsilon 0. Now for this equation, we know that the magnitude of this complex number 1 plus L has to be a constant equal to 1 by epsilon 0. So, if I were to draw a circle here centered at the point minus 1 comma 0 and of radius equal to 1 by epsilon 0, then all the points on the boundary of the circle on the periphery of the circle are, points that satisfy this particular equation.

So, we are now able to clearly indicate the magnitude and the phase combinations of the loop gain, that satisfy the equation magnitude of 1 plus L is equal to 1 by epsilon 0, how about the inequality? That we are interested in namely magnitude of 1 plus L is greater than or equal to 1 by epsilon 0 that essentially refers to, all the complex numbers that lie outside of the circle. So, the points on the circle along with the points that lie outside the circle are the points that satisfy, the inequality magnitude of 1 plus L greater than or equal to 1 by epsilon 0.

So, we see therefore, that we can very clearly depict the combination of magnitude and phase values of loop gains that are permissible for us to be able to reject an output disturbance by the amount epsilon 0. The same argument also holds, when we are trying to minimize the tracking error to a certain amount epsilon r, now the transfer function that relates a tracking error to the output as we discussed in the previous slide is once again given by 1 by 1 plus L of j omega. If you want the magnitude of this will be less than or equal to epsilon r, where epsilon r is another small number, but not so small that, we can ignore unity in comparison with L of j omega.

Then we would have the magnitude of 1 plus L of j omega should be greater than or equal to 1 by epsilon r. Now this inequality represents the region in the complex plane of the Nyquist plot, that is outside of a circle of radius 1 by epsilon r therefore, in order for us to track references with error less than or equal to epsilon r, we need to choose our loop gain to be some point in the Nyquist plot, that is within this region that is outside of the circle of radius 1 by epsilon r.

In a similar manner, if you want to suppress the output disturbance to a factor of epsilon 0, we should choose a loop gain that assumes values that are outside of a circle of radius 1 by epsilon 0 and with a center being a minus 1 comma 0. Now if you want, if you want to consider the input disturbance specification the transfer function, it relates to the input disturbance is given by X by D in is equal to magnitude of P by 1 plus L. So, P of j omega divided by 1 plus L of j omega and we might want this to be less than or equal to a certain other small number epsilon in, where the subscript in represents the input disturbance the rejection requirement.

So, once again if we have no uncertainty associated with the plant, then we can write this as magnitude of 1 plus L of j omega, should be greater than or equal to 1 by epsilon in times magnitude of P of j omega. So, in the frequency range omega of interest to us, where the input disturbance affects our plant, we need to choose the loop gain to lie in a region that is outside of a circle of radius given by 1 by epsilon in times magnitude of P of j omega and centered at a point minus 1 comma 0.

So, if the loop gain assumes values outside the circle, we are then guarantee to be able to reject the input disturbance by the specified amount epsilon in. Now as far as depicting stability is concerned that, 2 can be done in a fairly straightforward manner using the Nyquist plot. In fact, the notion of the phase margin and the gain margin essentially arose from the Nyquist plot and the points at which the, Nyquist plot of a certain open loop system crosses the negative real axis.

But there is one other alternative way, whereby we can represent the stability specification in a Nyquist plot. So, if we go back and look at the threshold of instability in a Nyquist plot so, I am drawing the Nyquist plane here, it is the real part of L versus the imaginary part of L and the critical point minus 1 comma 0 is located here if a Nyquist plot of the loop gain, it just passes through the point minus 1 comma 0, then our closed loop system is on the threshold of instability.

So, at this frequency which I shall call as omega gain crossover frequency, we would have the magnitude of L to be equal to 1, because it is passing through the point minus 1 comma 0 and the phase of L will be equal to minus pi radians. Now what this indicates? Therefore, is that at the point omega gc at the frequency omega gc the overall transmission function t which relates the output to the reference which is given by the expression T, is equal to L by 1 plus L and this in turn is given by 1 times e to the power j times minus pi, because the phase of L at the gain crossover frequency is equal to minus pi divided by 1 plus 1 times e to the power j minus pi.

Now, we know that e to the power j minus pi is essentially equal to minus 1. So, the transmission function will be equal to minus 1 by 1 minus 1. Hence the magnitude of T at omega gc, when the Nyquist plot of the loop gain, exactly passes through the point minus 1 comma 0 will be equal to infinity, if on the other hand our Nyquist plot was such that, it did not really result in our closed loop system being on the threshold of instability, but it was actually a stable system in which case, the Nyquist plot looks the way it has been depicted in the blue dotted curve here.

We would have the magnitude of L to be less than 1, then the angle of L is equal to minus pi, hence at the gain cross over frequency, which is the frequency at which the magnitude of the loop gain, becomes equal to 1 unit the phase of L will not be exactly equal to minus 180 degrees, but the phase of L which is given by this angle here, will be actually greater than minus pi radians and in such a case the magnitude of T at the gain cross over frequency will be less than infinity and it is evident from this Nyquist plot that, the angle made by the loop gain, when it is magnitude is 1 with the negative real axis is essentially given by the phase margin phi.

So, if we compute magnitude of T at the gain crossover frequency, you would have it to be equal to the magnitude of 1 e power j times minus pi plus phi divided by 1 plus magnitude of 1 plus 1 e power j times minus pi plus phi. Now we can show that, when the phase margin phi increases, the magnitude of the transmission function drops. So, when phi is equal to 0, we saw that the magnitude of the transmission function becomes equal to infinity.

And when the phase margin phi increases the magnitude of the transmission function progressively drops, hence one alternate way of a specifying the stability of the closed loop system, that is different from the notions of the phase margin in the gain margin is the magnitude of the transmission function. The maximum magnitude of the transmission function, that we are willing to allow, if we prevent the transmission function from assuming values greater than a certain specified amount, we are essentially placing a lower limit on the permissible phase margin of our system, because if our phase margin is larger then, our transmission function is going to be lesser in it is magnitude.

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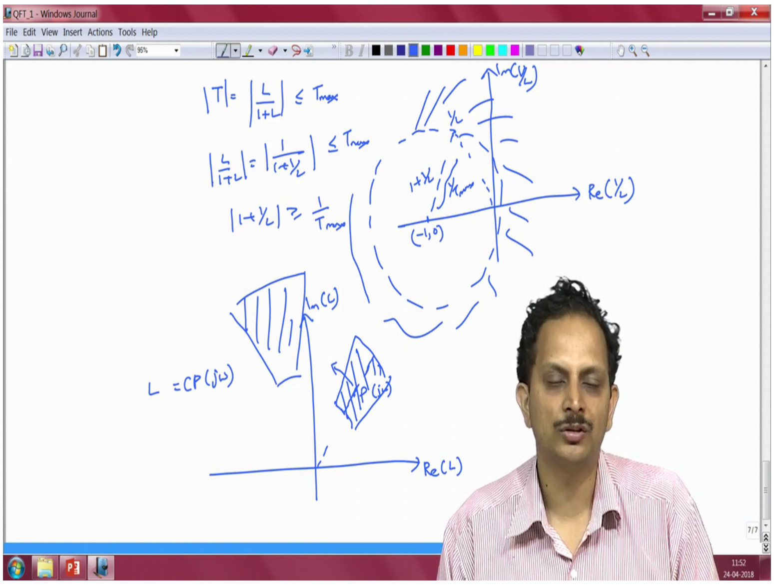

And hence we are going to be guaranteeing a stability of our closed loop system. Now, the relation between the transmission function T and the loop gain is given by T is equal to L by 1 plus L and in the interest of stability, we wish to specify that the magnitude of L by 1 plus L, which is the magnitude of T should be less than or equal to T max and we see that this performance specification also can be represented as a certain permissible region in the Nyquist plot, where the loop gain can be allowed to lie.

To do that let us first rearrange, the left hand side of this equation as magnitude of L by 1 plus L being equal to magnitude of 1 by 1 plus 1 by L and that should be less than or equal to T max and this in turn implies, that the magnitude of 1 plus 1 by L should be greater than or equal to 1 by T max. Now if we were to draw the inverse of the Nyquist plot or in other words, the real part of 1 by L versus the imaginary part of 1 by L, then some point in this complex plane represents the complex number 1 by L.

And hence the phasor connecting the point minus 1 comma 0, to the point 1 by L represents the complex number 1 plus 1 by L and if we want the magnitude of 1 plus 1 by L to be greater than or equal to the magnet of 1 by T max, what we are essentially saying? Is that, we want the complex number 1 by L to lie outside of a circle centered at the point minus 1 comma 0 and of radius given by 1 by T max. So, if I were to draw this circle then the permissible values of 1 by L are the values that lie outside the circle in order for our closed loop system to be stable by the specified amount.

Now, the permissible values of 1 by L, that are the values that are outside or on the periphery of this circle here, correspond to certain permissible set of values of the loop gain and those permissible set of values can be depicted in the Nyquist plot, just as we depicted the permissible values from the loop gain for it to, either reject disturbances or track references.

Hence we see that the Nyquist plot can allow us to clearly represent the performance specifications, in terms of disturbance rejection or in terms of robust tracking or in terms of stability by indicating the permissible area within the Nyquist plot, where the loop gain can lie in order for it to satisfy each of those particular requirements.

The last part that we would be interested as part of 2 degree of freedom control design is to design a control system that achieved a certain amount of robustness to plant parameter variations. So, if you have an uncertain plant, P of j omega, then we would have a magnitude of the plant and the phase of the plant being uncertain either or account of the fact that the gain of the plant is uncertain or it is pole locations are uncertain or it is 0 locations are uncertain.

Now, let us see whether this uncertainty can be represented effectively in the Nyquist plot. If we were to plot the Nyquist plot of the loop gain, you would have the real part of L versus the imaginary part of L. Suppose at the outset our controller was C equal to 1, then initially our loop gain will be simply equal to P of j omega. Now P of j omega represents a certain complex number in the Nyquist plot, now when the magnitude of the complex number varies, because of uncertainty in the gain of the plant this complex number P of j omega either, increases or decreases in magnitude.

Now, when the phase of the complex number P of j omega changes, because of uncertainty associated with the poles or the zeroes of the plant then, the orientation of this complex number changes, hence the set of complex numbers P of j omega, that the plant can assume due to uncertainties in it is structure is essentially depicted by this area in the complex plane. So, these are the possible locations, where the complex number P of j omega can lie. Now when we are about to design a controller C, we would be multiplying the complex number P with C.

Now, since C is a certain fixed complex number with a certain fixed phase and magnitude the effect of multiplying the plant complex number with that, of the controller is to increase the size of the magnitude of P by the amount given by the magnitude of C and change the phase of P by the amount given by the angle of C. So, in other words, our area which represents, the locations where our complex number P of j omega can lie, we will get distorted because of multiplication of P of j omega with C of j omega.

So, it will get magnified and will get notated by a certain amount dependent on the phase of the magnet of C and the angle of C. Now the fact that, the shape of this region within which the values P of j omega can lie can change on account of multiplying P of j omega with the controller transfer function C of j omega, is the one thing that makes the Nyquist plot unattractive as far as dealing with uncertain plants is concerned. Apart from that, when it comes to representing the constraints imposed on the loop gain by the stability specifications or the performance requirements, the Nyquist plot is decidedly better than the bode plot, because it clearly indicates the area in the complex plane, where the loop gain l is allowed to lie.

The only issue with it; however, is that when we have an uncertain plant, then the uncertain plant results in an area in the Nyquist plot, where the plant transfer function we have a complex number P of j omega, can lie and when this area gets multiplied by the controller transfer function C of j omega, the area gets expanded and rotated and now assumes a different orientation and a different size compared to the size of the original area and this variation in size that occurs, because of our controller transfer function is what makes the nyquist plot and unattractive plot, when we are dealing with uncertain plants, because we have not yet design the controller and we do not know exactly by what amount it would amplify? Or rotate the area that is occupied by P of j omega in the complex plane.

And hence in order to address this, one important problem, when we are trying to design a control system for uncertain plants, what we would therefore use? Is essentially a distorted Nyquist plot, where instead of plotting the imaginary part of L versus the real part of L, we would plot 20 times a logarithm of the magnitude of L versus the phase of L.

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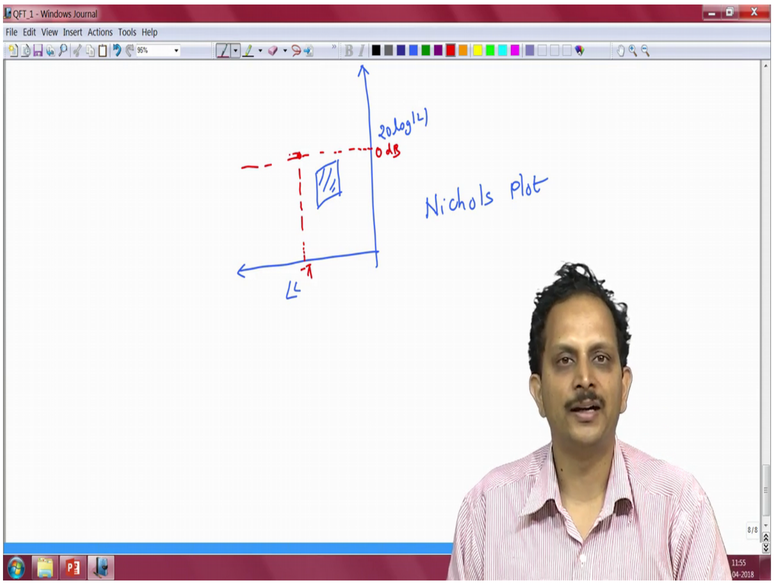

So, the X axis of the new plot would be the angle of L and the Y axis of the new plot would be the magnitude of L, but in a logarithmic scale.

So, 20 times logarithm of magnitude of L. So, if we were to choose this as the plane in which, we represent our performance specifications, we can show that the area within which, the plant transfer function P of j omega can lie, when we are dealing with a uncertain plant. We will not change either it is size or it is orientation, when we multiply it with a controller transfer function, it just gets translated in this plane, we shall return to this point 2 clips down the line, but at this juncture it suffices to mention that the important motivation for us to distort the Nyquist plot.

And plot the angle of L, what system magnitude of L in the logarithmic scale? Instead of plotting the real part of L versus the imaginary part of L, had to do with the fact that the area within which, the complex number P of j omega can lie, when we are dealing with an uncertain plant does not change, it is shape or does not change, it is orientation, when multiplied with the controller transfer function. When it is represented in this particular plot and this plot has a special name it is called the Nichols plot.

So, just as the bode plot plots the magnitude of the transfer function L in the logarithmic scale versus log of frequency and the phase of L versus log of frequency. This plot plots, the magnitude of L in the logarithmic scale along the Y axis and the phase of L, along the X axis so once again, we will discard the frequency information just, as we did in the case of the Nyquist plot and just plot the magnitude versus phase. So, let us now briefly look at the Nichols plots of some common transfer functions.

The important points in a Nichols plot correspond to the point at which, the magnitude of L becomes equal to 0 dB. So, that is indicated by this horizontal line here and the point at which, the phase of L becomes equal to minus pi radians. So, the phase being equal to minus pi and the magnitude being equal to 0 dB, corresponds to a certain point here and this is the critical point, the way in which the Nichols plot of a certain loop gain encircles, this critical point will determine, whether our closed loop system is going to be stable or unstable.

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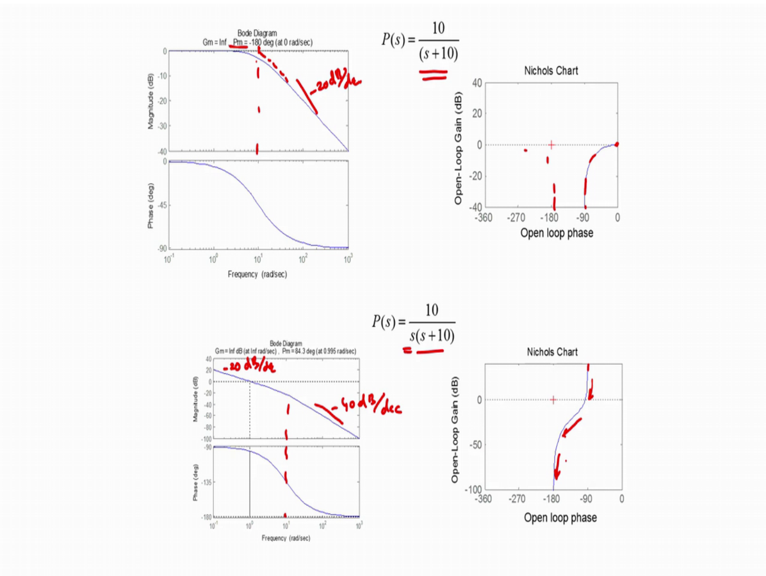

Now, let us look at the Nichols plots of some common transfer functions. In order to help built familiarity, I have first plotted in the bode plot of this transfer function and alongside it, I would plot it is corresponding Nichols plot. So, a first transfer function, we are considering is P of s is equal to 10 by s plus 10 and we note that, the bode plot starts with at with a gain of 0 dB and that the cornel frequency of 10 radian per second, the bode plot will start reducing in magnitude at the rate of minus 20 dB per decade.

So, the asymptotic bode plot would look something like this, where the corner frequency is going to be 10 radians per second, the phase plot of course, starts at 0 radians and asymptotically approaches minus 90 degrees or minus pi by 2 radians, because this is a first order plant, now if one way to plot the Nichols plot of this plant 1 is trying to essentially plot the magnitude characteristic versus the phase characteristic and the Nichols plot looks something like this.

When the loop gain is close to 0 dB, the phase is also close to 0 and as the loop gain reduces in magnitude the phase also becomes more negative and as the loop gain tends to minus infinity the phase tends to minus 90 degrees and asymptotically approaches it and we note that, the critical point namely 0 dB and minus 180 degrees is quite some distance away from the Nichols plot and hence a first order system can never be destabilized by employing proportional controllers.

The second example, I have considered is a second order system namely P of s is equal to 10 by s times s plus 10. So, in the low frequency end, the bode plot of this plant will have a minus 20 dB per decade role of on account of the presence of the integrating term here and at the frequency corner frequency of 10 radian per second. The characteristic will roll off at the rate of minus 40 decibels per decade as far as the phase of this characteristic is concerned you see that, it starts at minus 90 degrees, because of the presence of the integrator.

And asymptotically approaches minus 180 degrees, because a term 1 by s plus 10 changes it is phase from 0 to minus 90 degrees. If one were to plot the Nichols plot of this transfer function then, we note that at very low frequencies and the magnitude is very high, the phase is closed to minus pi by 2 and as the frequency is increasing, the magnitude starts to drop and a phase lag starts to increase and the phase lag asymptotically approaches minus 180 degrees.

So, just as in the case of the Nyquist plot, the frequency information is marked out on the Nichols plot of the transfer function.

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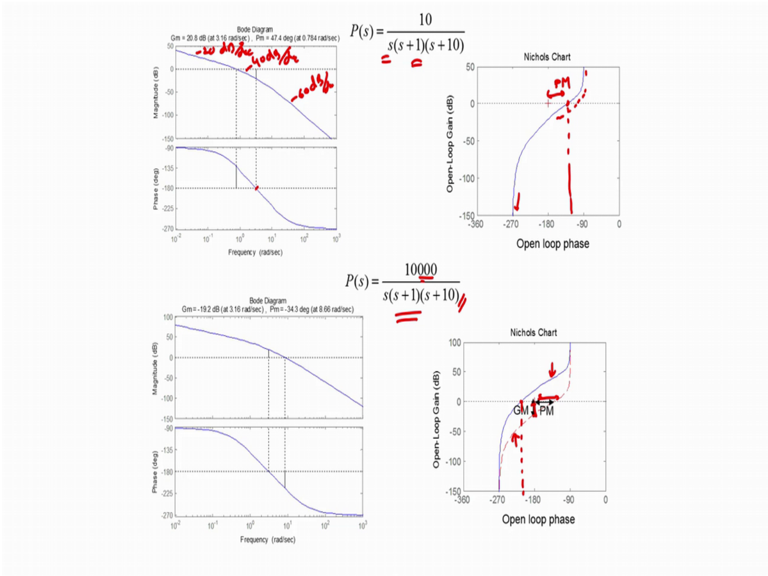

If you were to now consider, another transfer function. In this case, it is a third order system P of s is equal to 10 by s times s plus 1 times s plus 10, we see that, we have 2 corner frequencies namely s is equal to 1 and s is equal to 10. So, the initially the magnitude characteristic rolls off at minus 20 dB per decade on account of the presence of the integrator.

And in between 1 and 10 radian per second, the role off is minus 40 dB per decade and beyond 10 radian per second, the role off is minus 60 dB per decade and a phase characteristics starts at minus 90 degrees and crosses over at some particular frequency and finally, asymptotically approaches minus 270 degrees, because we have 3 open loop poles for the plant.

The corresponding Nichols plot looks as shown here, at very low frequencies, the magnitude is very high and a phase is close to minus 90 degrees and as the frequency is increased the magnitude characteristic drops and at some particular frequency. The magnitude becomes equal to 0 dB and at that frequency, we see that the phase lag is not exactly minus 180 degrees is actually greater than minus 180 degrees and hence, we have a positive phase margin.

And as the frequency is increased the magnitude characteristic drops further and the angle, the phase lag increases and ultimately asymptotically approaches minus 270 degrees and if you use a same transfer function, but this time, multiply the numerator by a factor of 1000, then we see that this entire, Nichols plot of the plant gets shifted up by 60 dB, because of the multiplication of the numerator by a factor of 10 to the power 3 and as a consequence, the Nichols plot of the new transfer function, which is given here P of s is equal to 10000 divided by s times s plus 1 times s plus 10 is shown by a loop trace here.

For reference, we have plotted in dotted red curve, the Nichols plot of the original transfer function P of s is equal to 10 by s times s plus 1 times s plus 10, we note that in the first case, we had a positive phase margin and we had a positive gain margin; however, when the gain of the open loop system is increased by a factor of 10 to the power 3, we note that at the frequency at which the gain crosses over namely, when the gain becomes equal to 0 dB, the phase lag is larger than 180 degrees and hence, the phase margin is essentially negative and so is the gain margin negative, as a consequence a closed loop system, with this as the open loop transfer function is unstable.

So, the gain margin and the phase margin information can be gleaned from a Nichols plot, just as we can glean it from the Nyquist plot or the bode plot and by looking at the way in which, the Nichols plot of a plant passes pass to the critical point 0 dB comma minus pi radians, it is possible to judge, whether the closed loop system is going to be stable or unstable?

What we shall do in the next clip is to employ the Nichols plot to execute a design, where we have multiple performance specifications namely in terms of input disturbance rejection and output disturbance rejection.

Thank you.