Control System Design
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Lecture – 31
2-Degree of freedom robust control design for plants with multiple uncertainties in their structure

Hello, in the previous clip we looked at how to execute 2 Degree of freedom of control design in order to achieve robustness against variation of the plants pole. And that design procedure followed along the same lines as the design procedure we had discussed two clips back where we had discussed the design of a robust control system that is intended to achieve the specified amount of robustness to variation in the plants gain. The only difference in the previous clip was the kind of trajectory along which the close loop pole varies when the plants pole location changes.

This trajectory was different from the trajectory that the close loop pole took when the plants gain changed. So, the only thing that we had to do afresh was to derive the trajectory along which the close loop pole changes due to variation in the plants pole. Subsequent to that we could adopt the same steps that we had adopted earlier in order to determine the location of the controller 0s and poles. And then complete the design of the feedback controller and subsequently also the pre filter.

Now, the design strategy that we have discussing over the last two clips is more general than just being able to handle uncertainties associated with just the gain or just the pole location of a plant. In fact, it can handle simultaneous uncertainties associated with both the gain as well as the pole position and in the uncertainties associated not just with one pole, but also with other poles of the plant.

So, what we shall see in today’s clip is to how to undertake design. When you have uncertainties in multiple parameters in your plant and achieve a robust control system whose dominant dynamics does not change by more than a specified amount when the plants uncertain parameters vary within the bounds that have been specified to them.

So, let us take a look at the problem that we would be handling today that has been stated.

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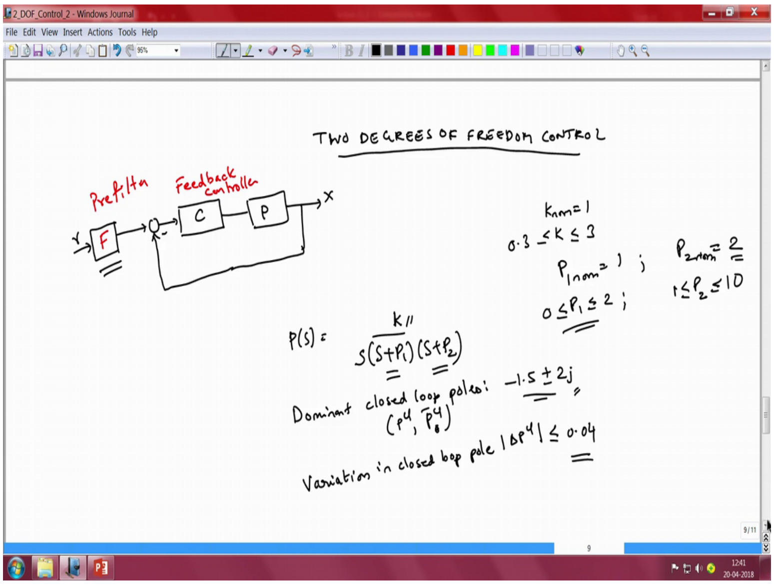

On the slide we are going to stick to the same plant that we have been discussing over the last several lectures. The plant is P of s is equal to K by S times s plus p times s plus 2. In this case the gain of the plant K is assumed to be uncertain unlike in the previous clip where we assumed K to be equal to 1. Here it is nominal value is equal to 1, but it is actual value could be anywhere between 0.3 and 3.

So, the gain of the plant can vary by an order or magnitude. So, there is uncertainty associated with the gain of the plant. And this reminds us of the example that we considered two clips back where we had considered only uncertainty in the gain of the plant. And this was precisely the extent of uncertainty that was considered in that example. Now in addition to uncertainty in the gain of the plant we also have uncertainty in it is pole location.

So, the nominal location of the pole P is equal to one in other words it is situated at S is equal to minus 1, but the pole could be located anywhere between S is equal to 0. And S is equal to minus 2 in other words. The variable P in this equation can assume values anywhere between 0 and 2. Now in this example we have a plant whose gain K and whose pole location P can independently vary between the limits that have been specified here. They have a certain nominal values in this case a nominal values of the gain K and the pole location are both equal to 1. And have a certain variation a range of variation in the values about the nominal value. The dominant close loop poles of the control system that we are about to design once again is required to be at minus 1.5 plus minus 2 j.

So, this is exactly the same specification that we have borrowed from the previous two examples that we considered. And the third specification is that the variation in the close loop pole which would necessarily happen when the plants model changes either due to uncertainty in the gain or due to uncertainty in the pole location that should be within 0.04 units. And this specification is identical to the one that we had considered in the previous lecture when we have uncertainty in the pole position.

So, once again I want to remind you that such a set of specifications cannot be realized with a 1 degree of freedom control system because the control system can position the close loop pole nominally at the location where we desire it to be. Namely, minus 1.5 plus minus 2 j, but it cannot guarantee that the variation in the close loop pole due to uncertainties associated with the plant are going to be within the specified limit of 0.04 units. Hence we need a 2 degree of freedom control architecture where we have a pre filter as well as a feedback controller. As far as the design of the feedback controller and the pre filter are concerned. We have looked at some standard grade lines that are associated with their design.

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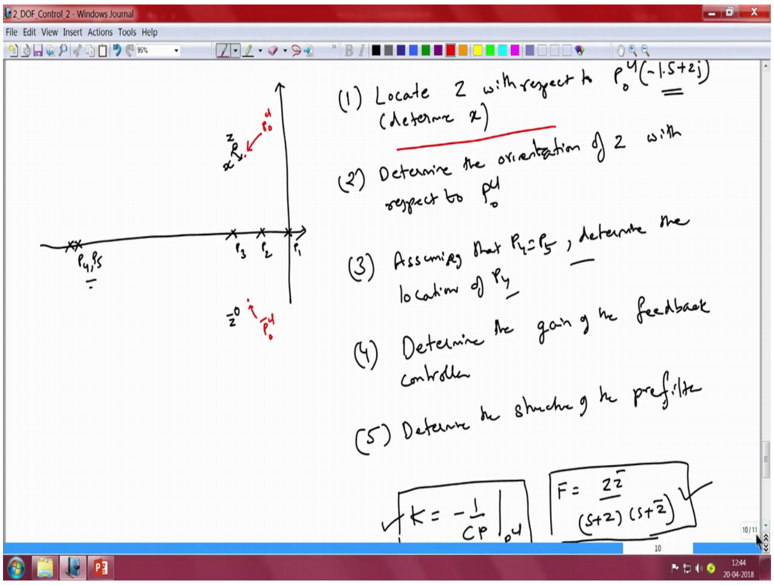

In particular we know that our feedback controller needs to have a pair of 0s that are located very close to the pair of dominant close loop poles which we have in this case called P c l naught. These are essentially the poles minus 1.5 plus 2 j and minus 1.5 minus 2 j.

So, we have to place the two 0s very close to these poles. And these 0s have the effect of pinning these close loop poles and arresting the variation of the close loop pole due to the changes in the plants parameters, but because we have two 0s of the controller the controller needs to also have two poles.

So, we have to choose to place the poles at some distance and we assume that we have placed quite some distance away from the origin. And the design steps essentially have been outlined here. Now that we have the structure for the feedback controller namely that it has two 0s each of them are placed close to the location where we want our dominant dynamics to lie and to not vary too much and then we have two poles placed farther away.

So, the first step is to locate the distance of the 0 from the point P c l naught from the point where we want the dominant nominal dominant poles to lie namely minus 1.5plus 2 j and minus 1.5 minus 2 j. Now this is the first step. The second step is to determine the orientation of the 0 with respect to this point. The third step is to determine the positions of the two controller poles that we have placed in the interest of causality in the realization of the controller. And here if we assume that these two poles are coincident then we have to determine the location of either one of these two poles. And we discussed in the first clip that this location is determined by stability considerations because these two poles become responsible for other branches of the root locus which could potentially destabilize our control system when the plants parameter change after determining the position of these far away poles of the controller.

The next step is to complete the feedback controller design and that is by determining the gain of the feedback controller. So, if we know the structure of the controller the gain of the controller can be determined in a very straightforward manner by using the equation that has been given at the bottom here. K is equal to minus 1 by CP. Where this transfer function C and transfer function P are both evaluated at the point P c l naught the nominal position of the close loop pole.

The last step is to determine the structure of the pre filter because the previous four steps were devoted exclusively to determining the structure of the feedback controller. With the fourth step we would have the gain of the controller as well as it is pole locations pole and 0 pole positions. So, the final step is to determine the structure of the pre filter. And as we have discussed in the past the primary purpose of the pre filter is to get it to cancel the 0s of the feedback controller. And thereby get the dominant dynamics of the overall transfer function that relates the output to the reference to be determined by the points P c l naught and P c l naught bar in other words minus 1.5 plus minus 2 j.

So, that cancellation is done by choosing a pre filter that has the structure that has been shown here. F is equal to Z Z bar by s plus z times s plus z bar. The numerator of the pre filter has been chosen to be equal to Z Z bar because we want the pre filter to have unity this gain.

So, if we revisit the five steps that we have in the design just as we discussed in the previous clip the steps 2 to 5 are all very similar in their approach to the steps that one needed to adopt in the first clip. Where we had to worry about where we undertook the design to determine the orientation of the 0 with and the position of the far away poles of the controller and subsequently the gain of the feedback controller as well as the pre filter structure.

So, the steps the procedure is going to be no different from what was adopted earlier in the first clip and the same procedure was also borrowed in the second clip. And in this clip as well we shall do the same thing. We shall assume that these steps are things that we can routinely undertake. Once we are able to determine the position or the distance where this 0 z has to be located with respect to the point P c l naught.

So, it is the first step alone that is unique to the problem that we have posed for ourselves where we have simultaneous uncertainties in two parameters. Namely the gain of the plant as well as it is pole position. So, in this clip therefore, we would focus only on the first part of the design and we shall not discuss the other steps in great detail. We shall simply borrow the procedure from the previous clips and execute the design without describing in detail what is done in each of these cases.

So, let us therefore, get down to determining the distance of the 0 namely the distance x from the point P c l naught in order to prevent the variation of the close loop pole P c l from being more than 0.04 units when the plants gain and it is pole location changes.

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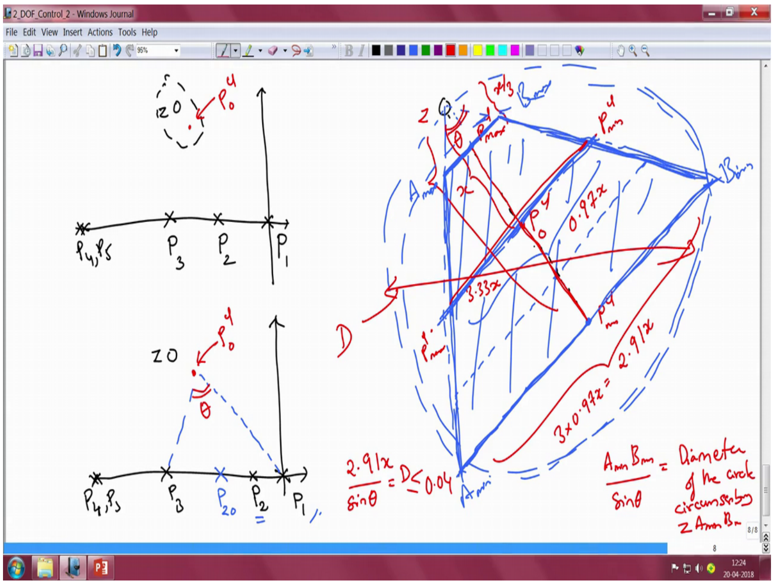

In order to undertake the first step in the design we shall look at the effect of each of these uncertainties separately. So, we shall look at the effect of change of gain of the plant and subsequently look at the change of the pole location of the plant.

So, in order to undertake that I have drawn here the complex plane where in the first graph I have located the points P 1 P 2 P 3 which are the nominal positions of the plant poles along with the pole locations P 4 and P 5 of the controller. And I have located the point z in the upper quadrant of the complex plane. There is going to be a point Z bar, but that has not been shown here for the sake of convenience and the point P c l naught where we want the nominal close loop pole to be located has also been indicated.

So, the second graph here shows the exact same thing as the first graph with the exception that in the second case we assume that the pole location P 2 can vary. So, nominally it is located at the point P 2 naught which is indicated by this blue cross here, but that is just it is nominal position the actual position of P 2 can be anywhere between P 1 and P 3 because we assume in our specification that our plant pole can be anywhere between s is equal to 0 and S is equal to minus 2.

So, if we return to the first graph and we zoom in to the area near the point Z and the point P c l naught I shall draw it in a magnified manner here. So, I shall indicate the point P c l naught here. And I shall indicate the point Z there. In the first case we assume that we have only uncertainty associated with the gain of the plant. Now if we have uncertainty associated with the gain of the plant and the plants gain can increase by a factor of 3 from it is nominal value of 1 it can go up to a value of 3 or it can reduce to 30 percent of it is initial value. Then two clips back we learnt that the close loop pole will vary along a straight line that connects the point P c l naught to Z.

In particular we learnt that when the gain increases by a factor of 3 the point will move to the location P c l max which is going to be one third of the distance from the point Z as the point P c l naught S. Likewise we also discovered that when the gain drops by a factor of three the point P c l will move along the same line in the other direction to the point P c l min which is 3 times farther away 3.33 times farther away from the point z as the point P c l naught s.

So, if the distance from the point P c l naught to the point z if you were to call that distance to be x. Then we learnt two clips back that when the gain increases by a factor of 3 the close loop pole comes to the point P c l max which is at a distance of x by 3. And when the gain drops to 30 percent of it is nominal value. The close loop pole most of the point P c l min which is 3.33 times father away from the point z as the point P c l naught s.

So, the trajectory of the close loop pole is going to be a straight line in the complex plane given by this red line here. So, this is the line along which the close loop pole would vary when the plants gain alone were to change. Now if we come to the second case where you have uncertainty associated with the plants pole location. In this case let us for a minute assume that there is no uncertainty associated with the gain. Let us assume for a minute therefore, that we have uncertainty only in the location of the pole of the plant.

Now, in the previous clip we looked at how the close loop pole position changes when the plants pole location changes. In particular we discovered that if the plant pole position P 2 changes between the points P 1 and P 3. Then the close loop pole P c l changes between the points P c l min and P c l max. In such a manner that the triangle described by the points P c l naught P 1 and P 3 this triangle is similar to the triangle described by the points z P c l min and P c l max.

So, if the plant pole position alone were to change then the close loop pole will move along this blue colored solid straight line as shown here which passes through the point P c l naught in such a manner that the extremities of this line when connected to the point z form a triangle which is similar to the triangle formed when the extremities of the point P 2 are connected to the point P c l naught or in other words the triangle P c l naught P 1 P 3 is similar to the triangle z P c l min P c l max.

So, this is the line along which the close loop pole is going to change it is position when we have uncertainty only in the pole location of the plant. Now let us take another case where our plants gain is not at it is nominal value, but is at it is maximum value. Now when the plants gain is at it is maximum value and it is pole location is at it is nominal position we know from the analysis two lectures back the out close loop pole is going to be located at this point P c l max at the point that is closest to the 0 which I have highlighted with a blue circle.

Now, suppose the gain is at it is maximum value. And we allow for uncertainty in the position of the pole of the plant. Then we know that the close loop pole is going to vary along a straight line of this kind which is going to be parallel to the straight line along which it varied when the plants gain was at it is nominal value. Why is it parallel? It is parallel simply because we still need the extremities of the position of the close loop pole when we have uncertainty associated with the plants pole to form a triangle that is similar to the triangle that we have here namely P c l naught P 1 and P 3.

Since that similarity still needs to be there these needs to move along a straight line that describes the triangles similar to the triangle that we have drawn here namely P c l naught P 1 and P 3. And that forces this straight line along which the close loop pole is going to vary which I am highlighting now to be parallel to the straight line along which the close loop pole will vary when the gain was at it is nominal value. What is the magnitude of this straight line from similar triangles we can see that if z P c l max which is this distance here indicated by the blue line is Z by 3 and z P c l naught is x then the shorter blue line is going to be one third in length compared to the bigger blue line.

So, this is going to be the line along which the close loop pole is going to vary. Now if we take the other case when the gain has assumed it is smallest value namely when the gain of the plant has become 0.3 then we know that our close loop pole is going to be located at this location which I am indicating by the blue circle which is going to be at the point P c l min.

Now, if we permit for variation in the pole P 2 then we know that this close loop pole should move on a straight line which I am drawing now in such a manner that the extremities of this straight line when connected to the point Z forms a triangle that is similar to the triangle that is formed when the extremities of the point P 2 are connected to the point P c l naught. Or in other words the extremities when connected to the points Z forms a triangle that is once again similar to the triangle P c l naught P 1 and P 3.

So, this internally therefore, implies that this line along which the close loop pole varies due to the variation in the open loop pole position of the plant is going to be once again parallel to the two other lines along which the close loop pole varies when the plants gain are at different values. In the in the one case the plant gain was assumed to be at it is nominal value. In the other case the plant gain was assumed to be at it is maximum value.

Now, continuing this argument further we can see that for some other value of the gain of the plant which is in between the values 0.3 and 3 our closed loop pole position would be at some other location on this red straight line it would be at some other point here which I will indicate it here by this green point in the absence of uncertainty in the pole position. However, if we now assume that the gain was at this particular value for the plant, but it is pole became uncertain and it could wander back and forth between the points S is equal to 0 and S is equal to minus 2. Then the corresponding close loop pole would vary along a straight line that is parallel to the other straight lines that I have sketched in this particular manner.

In such a way that it once again describes it is extremities once again for my triangle that is with the point z that is similar to the triangle that the point P 2 forms extremities of the point P 2 forms with the respect to the point P c l naught. Hence if we have independent variations in the pole location of the plant and the gain of the plant it is not difficult to see that in general the close loop pole will vary within this particular quadrilateral.

So, for the maximum gain of the plant the close loop pole will vary along that boundary when the plants pole location changes. For the minimum gain of the plant the plants poles will vary along this edge when the plants pole location changes. And likewise for the particular pole location namely at S is equal to minus 2 when the plants gain changes the close loop pole will vary along this particular straight line which I am highlighting here with blue color. And likewise when the plant pole is at S is equal to 0. And it is gain changes by a factor of either increases by a factor of 3 or reduces to 30 percent of it is nominal value the close loop pole is going to vary along the straight line that I am highlighting now.

So, these four straight lines form the edges of the region within which the close loop pole would vary when the plants gain and pole changes. So, for any other intermediate combination of plants gain and pole location the close loop pole will be located somewhere inside this area. So, this area is there for the region within which the close loop pole would vary when the plant and plant gain and pole locations assume different values within the ranges that have been specified to us.

Now, our goal as control engineers is to choose the location of the 0 with respect to the point P c l naught or in other words determine the distance x. So, that when the close loop pole varies within this area. The maximum distance that it can traverse is no more than 0.4 units. So, that is what we are supposed to do. So, for the sake of geometric calculations let us name the corners of this square as A max B max A min B min.

So, one way to approach this problem of restricting the variation of the close loop pole to within 0.04 units is to determine which of the diagonals of this quadrilateral is the longest. And that longest diagonal would be some constant times this distance x because this entire figure scales with this distance x. And we should insist that this diagonal should be less than or equal to 0.04 units in order to determine at what position x we have to place 0s at with respect to the point P c l naught.

So, this is one way in which the problem can be solved the other way in which this problem can be solved which is a more approximate, but a slightly easier root is to find the diameter of the circle that circumscribes this quadrilateral. The quadrilateral A min B min A max B max and insist that the diameter of the circle be less than or equal to 0.04 units which is the maximum permissible variation in the close loop pole. However, if you notice this quadrilateral it is first of all a parallelogram because the sides A max B max is parallel to the side A min B min. And unless a parallelogram is an isosceles parallelogram it is not possible to exactly circumscribe this parallelogram. And the circle that best encloses this parallelogram is once again slightly difficult to compute. So, if you want an easy way out to quickly estimate the approximate distance at which they have to place the 0 with respect to the point P c l naught. We can instead insist that the entire triangle Z A min B min should be enclosed within a circle.

Now, this circle which encloses which circumscribes the triangle z A min B min will be approximately of the same diameter as the circle which encloses the quadrilateral A min B min A max B max because the point A max and B max are located fairly close to the point z, but this circle is going to be slightly bigger. So, therefore, our design is going to be slightly conservative.

Now, the reason we undertook this step was because there are ready made expressions in trigonometry that will allow us to determine the diameter of the circle that circumscribes the triangle Z A min B min. In particular if the distance from the point Z to the point P c l naught is x. We noted from our discussions in the previous clip that the point P c l max P c l min this distance therefore, is going to be equal to 0.097 x. And since A min B min is going to be 3 times the length of P c l max P c l min which is the red curve that I just highlighted now. We would have this distance A min B min to be 3 times 0.97 x which essentially is going to be equal to 2.91 x that is the length of the line A min B min.

Now, the triangle Z A min B min has a apex angle which I shall call as theta which is known to us because that angle is essentially going to be the same as the angle that is formed by extremities of the pole P 2 of the plant with the point P c l naught. So, from trigonometry I can obtain the angle theta from the triangle P 1 P 3 P c l naught and the same angle is going to be the one that is going to be formed between the points A min Z and B min. Now from the sin rule which we learnt in our high school we know that the distance A min B min divided by sin theta is going to be equal to the diameter of the circle that circumscribes the triangle Z A min B min circumscribing Z A min B min.

Now, here we know that A in B min is of length 2.91 x and we know the angle theta from trigonometry. And we can show that this angle for the particular locations of the points P 1 and P 3 that angle is closed to 60 degrees. So, 2.91 x divided by sin theta is going to be equal to the diameter of this circle which circumscribes the triangle. So, I shall indicate the diameter by the symbol D here that going to be equal to B. And since we want the close loop pole to not vary by more than 0.04 units we have to insist that the diameter D of this circle should be less than or equal to 0.04 units.

Now, this equation will help us to determine exact distance x of the 0 from the point P c l naught. With that the first step of our 2 degree of freedom control design gets done. The next step is to determine the orientation of the 0 with respect to the point P c l naught. And to do this we just apply the angle criterion at the point P c l naught namely the angle subtended by the different open loop poles and 0s of the plant and the controller should add up to minus 180 degrees and in doing. So, we ignore the angle contributions from the points P 4 and P 5 because they are very far away from the origin. And assume that the angle that the point z bar subtends at the point p\P c l naught is approximately 90 degrees because the points z and z bar are placed very close to the points P c l naught and P c l naught bar. And with this assumption we can obtain the angle of this orientation of the 0 with respect to the point P c l naught.

The third step is to determine the positions of the points P 4 and P 5 and the 3 plant poles near the origin with a single effective pole. And use the resulting simplified root locus to determine the gain at which the close loop pole one of the one pair of the close loop poles cross over from the left half of the complex plane and to the right half of the complex plane. And determine the locations where P 4 and P 5 have to be located have to be situated. Such that this gain is less than the maximum gain that the plant can assume. With that we would completely determine the structure of our controller it would have 2 poles P 4 and P 5 whose locations we would have determined. It would have two Os whose locations once again we would have determined.

Finally, we determine the gain of the controller and would be done with the feedback controller design. The last step is to determine the structure of the pre filter and that is simply given by F F of S is equal to Z Z bar by S plus Z times S plus Z bar where Z and Z bar are the locations of our controller feedback controller zeros and with that our design will complete.

I have undertaken this design and I wish to share the results of some simulations that have been performed with this controller design.

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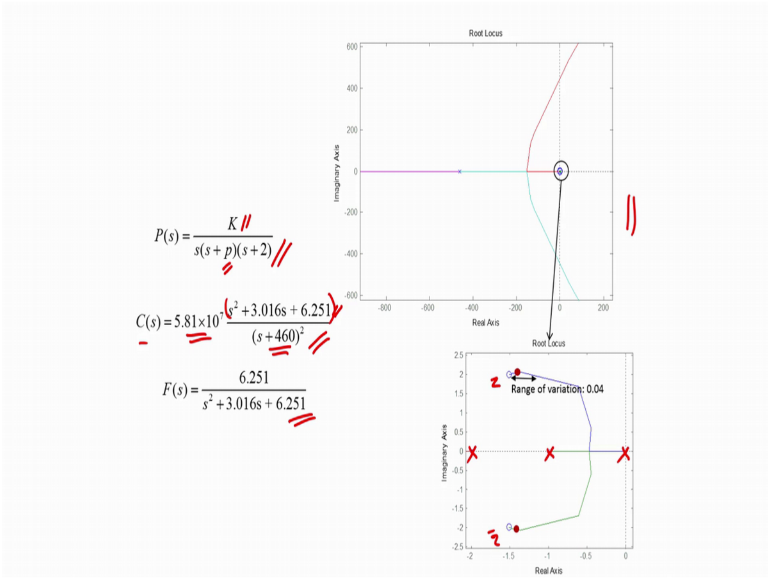

So, this slide shows the plant structure that we have designed we see that there is uncertainty associated with the gain of the plant as well as it is one of it is pole positions the gain can vary from 0.3 to 3 and the pole P can change from 0 to 2. And associated with this plant we had certain specifications on the desired robustness to these changes in the plants parameters. And to achieve those specifications the controller structure and some p something like this it has 2 zeros whose distance we determining whose distance we discussed in this clip and if you undertake the rest of the design for the feedback controller we can obtain the locations of the points P 4 and P 5 as well as the gain of the feedback controller.

This completes the first four steps in the design. The last step is to determine the pre filter and the pre filter is intended essentially to cancel the controller feedback controller zeros and thereby alone the dominant poles of the overall transmission function to be at the points minus 1.5 plus minus 2 j. So, with this feedback controller the root locus of the feedback system looks as it has been shown here. So, the poles of the plant and the zeros of the controller are situated very close to the origin are not visible in this particular graph. We only have the far away poles of the plant that are visible at the point S is equal to minus 460. In order to view the root locus in the vicinity of the origin we zoom into the area near the origin where we can locate the points P 1 P 2 and P 3 of the plant and the points Z and Z bar. And the root locus is appears the way it is shown in this figure.

Now, when we change the gains of the plant within the limits that have been specified and the pole locations of the plant within the limits that have been specified. We can verify from root locus that the variation of the close loop pole is actually within 0.04 units. So, now, that we are done with the design we can now look at a step response of the overall close loop system that has been plotted in this graph here.

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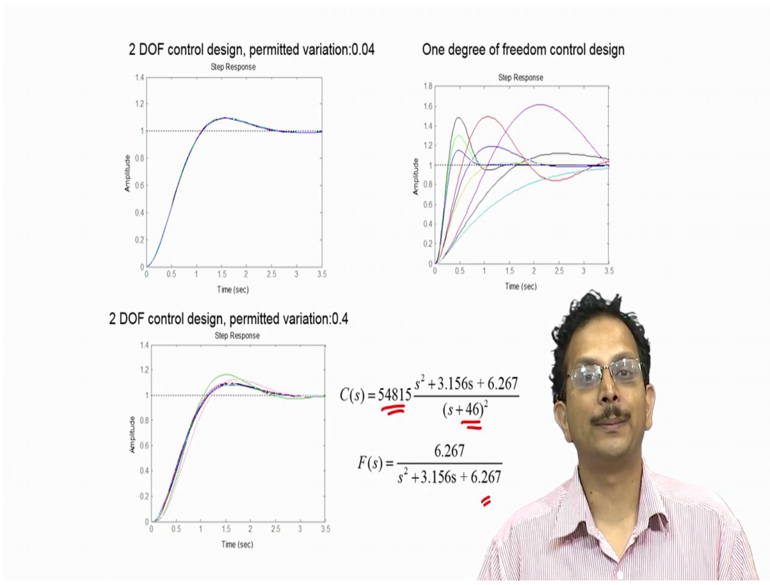

What I have plotted here is a step response of a 1 degree of freedom control system. So, we do not have the pre filter here. We just have a feedback controller which ensures that the nominal positions of the close loop poles re at minus 1.5 plus minus 2 j. However, when the plants gain changes or when the plants pole locations changes the controller cannot restrict the variation of the close loop pole about it is nominal position. As a consequence what we see here is huge spread in the transient response of the close loop system to your step input for different possible values of the gain of the plant and it is pole location.

In contrast if one were to plot the step response for the same uncertain plant, but this time with a 2 degree of freedom controller. One can see that for all the different possible combinations of the plants gain and it is pole location the response step response of the close loop system almost exactly sit one on top of another with particularly no perceptible variation between them. This once again underscores the power of a 2 degree of freedom control strategy to achieve robustness or insensitivity to variation in the plants model.

In the previous two clips we looked at uncertainties associated with only one of the parameters in one case it was the gain in another case it was the pole location. In this case we have assumed that both the gain as well as the pole position can change, but despite these changes we are able to design a control system that ensures robustness to variation of both these parameters independently

Now, the reason that we have got such a good response in this case was because we restricted the variation of the close loop pole to just 0.04 units when the plants pole or gain were to change. If we were to relax this constraint a little bit and if we were to permit the variation in the close loop pole to be within 0.4 units instead of 0.04 units we can allow for slightly greater spread in the variation of the transient response of the close loop system when the plants parameters change.

So, when we do that we see that the spread is slightly higher, but still the performance of the 2 degree of freedom control system is significantly better than the performance of a 1 degree of freedom control system. The benefit of relaxing this variation is that we end up with the controller whose pole is at s is equal to minus 46 instead of S is equals to minus 460. And also a controller whose gain is much smaller than that of the gain of a controller which has to restrict the variation of the close loop pole to within just 0.04 units the pre filter however, has a very similar structure as that of the pre filter in the previous case.

Hence we have seen in this clip that it is possible for us to address the problem of independent variation of two parameters of the plant. And design a control system that achieves a specified amount of robustness in the phase of independent variation of these two parameters what we shall do next is look at how we can take up a slightly more challenging problem where we have more than two parameters of the plant that could potentially be varying. In particular let us consider that one of the other poles of the plant which was nominally at x is equal to minus 2 also has some uncertainty associated with it. Let us assume in this case that this pole can vary anywhere between the points S is equal to minus 1 and S is equal to minus 10.

So, let me formally state the problem and briefly sketch the approach that we would adopt in order to solve the problem. The steps however, for this case are very similar to the steps that we have adopted for the three examples that we have considered. So, far and in particular the first step of the design is very similar to the case that we considered just now in this clip where we had independent variation of the gain and the pole location. So, we shall focus only on the first step once again. And I shall briefly say how we can extend the procedure that we have adopted in this clip to address the problem where we have uncertainty associated with two poles or perhaps even more poles of the plant.

So, we shall return to this slide where we state the problem the plant is very similar to the plant that we have been looking at in all the previous examples, but with the exception that this plant has three parameters that are uncertain. The gain k of the plant is uncertain it is nominal value is 1, but it can change between 0.3 and 3. The pole location P 1 is uncertain it is nominal value is one, but it can change between 0 and 2. And the second pole also P 2. Which is nominally at s is equal to minus 2 is also assumed to be uncertain.

So, it can vary between S is equal 0 minus 1 and s is equal to minus 10. So, there is uncertainty associated with one pole another pole as well as the gain. Now how do we systematically design a 2 degree of freedom in a control system that achieves the specified amount of robustness in the phase of the uncertainties have been specified for the plant structure. The nominal position of the close loop pole is still expected to be close loop minus 1.5 plus minus 2 j.

So, if we look at the steps in the design the steps are all exactly similar to the ones that we have been looking at. So, far we have to have 2 controller zeros place placed near the point P c l naught and P c l naught bar and for the sake of causality we are going to introduce two controller poles P 4 and P 5. And we assume that these poles are located very far away. The first step is the design is to determine the distance x of the point Z from the point P c l naught. The second is to determine the orientation of z with respect to P c l naught. And subsequently if you assume P 4 and P 5 are coincident then determine the location of P 4 in the interest of stability. And then finally, finish the feedback controller design by determining the gain of the feedback controller by using a root locus equation it has been shown here. The last step is to determine the structure of the pre filter and that is given by this expression here.

So, if you have fixed the location of the feedback controller 0 z and z bar. Then the controller feed then the pre filter structure automatically gets fixed. So, once again we shall not undertake the design for this particular case. We shall just focus on the first step which is to locate the 0 z with respect to point P c l naught. So, we saw that in order to undertake the analysis for the first step in the design.

What we need to do is determine the shape of the curve or the area within which the close loop pole would vary when the plants parameters change. When the plants gain alone change we noted that this the close loop pole varies along a straight line the passes through the point z. In the second case when the plants pole location change we noted that the close loop pole varied along a straight line that is different from the straight line in the first case, but the extremities of this straight line resulted in a triangle that was with respect to the point z that was similar to the triangle that the extremities of the open loop the plant pole formed with the point P c l naught.

So, that allowed us once again to determine the distance at which we had to locate the 0 with respect to the point P c l naught. So, likewise in the in the third case when both the plant gain as well as it is pole position were uncertain we noticed that the close loop pole could vary within an area. This time it was not a single curve it is not a straight line it was the area within which the close loop pole could vary. Hence our challenge in this new problem is to just determine the shape of the area within which the close loop pole would vary when the plants two poles are different in position compared to the nominal values. And when the plants gain is also different compared to it is nominal value of gain.

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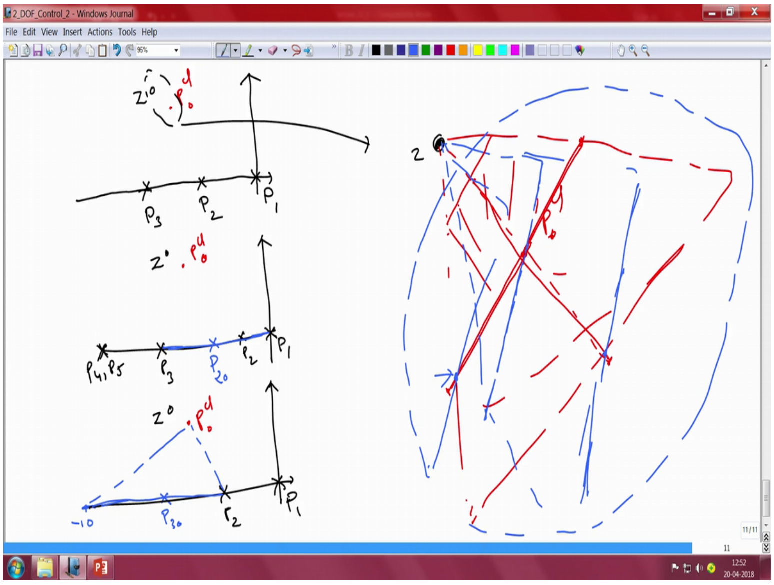

In order to solve this problem I have drawn three different complex planes on the left the first complex plane indicates the location of the poles P 1 P 2 and P 3 of the plant which are assumed at in this case to be located at the nominal positions. And we consider only the variation in the gain of the plant.

In order to understand how the close loop pole position changes when the gain of the plant varies. I shall zoom into this region near the point z and P c l naught and separately draw it here exactly as we had done in the previous example. So, this is the point z and our the close loop pole position P c l naught is somewhere here.

Now we know that when the gain of the plant changes the close loop pole varies along this particular straight line to between the points P c l max and P c l min which are one third of the distance in the point z to the point P c l naught and 3 times the distance of the point z to the point P c l naught respectively.

So, this is the line along which the close loop pole varies. If you have uncertainty only in the gain of the plant and no uncertainty whatsoever in the either of the pole positions. Now if you had uncertainty in the pole position P 1 we notice that the close loop pole will vary along a different straight line which we sketched out earlier in this clip. So, these were the limits it would vary along this straight line within the limits that describe a triangle which is similar to the triangle described by the limits of the point P 2 with respect to the point P c l naught.

So, that was the second case. Now if both the gain and the pole position P 1 varied. We noticed that it would form a quadrilateral. And from this and the close look pole would vary would be located somewhere within this quadrilateral. So, this was our discussion just a few minutes back. Now suppose we have the other pole also varying. So, suppose we have the pole P 3 also varying as has been stated in this problem it is nominally located at P 3 naught which is at s is ex is equals to minus 2, but it can vary from s is equal to minus 1, but which point it will coincide with the point P 2 and s is equal to minus 10.

So, n the third graph here. Have indicated with this blue line the range over which the the plant pole location can change. Now the problem of determining the area within which the close loop pole varies when the second loop pole plant also changes it is position is not very different from the cases that we have considered before this.

So, for the moment let us assume that the plant gain and the first pole are both at it is nominal positions. In which case the close loop pole would be located at the point P c l naught. If the third pole were to wander about were to change it is position with respect to it is nominal value which is P 3 naught. Then we know from the analysis that we have performed that when one of the poles of the plant were to change then the close loop pole pole varies in such a manner that the triangle described by the extremities of the close loop pole positions with respect to the points z is similar to the triangle described by the extremities of the point P 3 naught with respect to the point P c l naught.

So, this triangle would be similar to the triangle described by the point P c l when the pole location P 3 were to change within the specified limits. Now this is the line along which the close loop pole P c l will vary if there was no uncertainty either in the gain of the plant or in the pole location P 2 of the plant. Now, suppose we also had uncertainty associated with the gain of the plant. So, if the gain was lesser by a certain amounts.

So, let us say the gain was at it is minimum value of 0.3 then the close loop pole would be located somewhere here when the open loop poles of the plant both the open loop poles of the plant are located at the nominal positions. However, if the once again the pole P 3 were to vary between the specified limits. Then we would have the close loop pole varying along this particular straight line. In such a manner that it describes another triangle which is similar to the triangle described by the extremities of the point P 3 with respect to the point P c l naught.

So, for a let us say we have now uncertainty associated with the pole location P 2 as well. So, for the case when the pole P 2 is located at the point s is equal to minus 2 and the pole P 3 is at it is nominal position. And the gain is at it is nominal value our close loop pole would be located somewhere here.

Now, if you permit for variation in the pole P 3 of the plant then our close loop pole would vary along a straight line that would describe a triangle that is similar to the triangle described by the extremities of the point P 3 with respect to the point P c l naught. So, what we essentially do therefore is we consider the variation of one parameter at a time. And determine the curve along which the close loop pole would vary when this particular parameter varies. And then we subsequently as allow for the second parameter to vary and then the third parameter would vary and finally, determine the entire overall region in which the close loop pole can possibly vary when one or more of these parameters change.

Now, we determine the maximum variation of the close loop pole within this region interims of the distance x between the point z and the point P c l naught. And we insist that this maximum variation has to be within the specified limit. Which in this case is 0.04 units. And that will allow us to determine the distance at which the 0 has to be located from the point P c l naught in order to restrict the variation of the close loop poles due to uncertainties in the plant. In this case we have two poles are uncertain. And the gain that is uncertain, but yet we can systematically determine the location of the 0 of the controller from the point P c l naught that restricts the variation of the close loop pole to within the specified limit, the exact shape of this curve.

However, maybe more complicated than the one that we considered in the previous examples. And hence one can even take the help of a computer in order to obtain the area within which the close loop pole will vary when we have each of these parameters of the plant being uncertain, but with that additional step taken into account one can in a very straight forward manner undertake the rest of the design.

Determine the numerical value of x and subsequently undertake the remaining steps in the design where you determine the angle of the point z with respect to the point P c l naught and hence easy to determine the exact position of z and sub then determine the location of the far away poles P 4 and P 5 and then the gain of the feedback controller and as a last step the structure of the pre filter.

Since these step are identical to the once that we undertook in the first clip where we did 2 degree of freedom control design I shall not discuss the details of these steps in this clip. So, with this we come to the end of our discussion regarding achieving robustness for the overall feedback control system to uncertainties associated with the plant and the examples that I have considered here. Reveal to you that this control strategy is quite general and can handle uncertainties of fairly large magnitudes and of different parameters in the plant.