Control System Design
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Lecture – 30
2 - Degree of freedom robust control design for plants with uncertain pole

Hello. In the previous clip, we undertook two degree of freedom control design to achieve robustness to variation of plant parameters. In the example that we considered, we assume that the gain of the plant was uncertain and it could increase by a factor of 3 or drop 230 percent of it is nominal volume. In other words, the gain could change by a factor of 10 from 0.3 all the way to 3 while it is nominal value was assumed to be 1.

So, we saw how we could go about designing a 2 degree of freedom control system that could achieve a specified amount of robustness to the variation of the plants gain. At the heart of this approach is to locate a pair of 0's near the place where the dominant closed loop poles of the overall close loop system is supposed to be located and these zeros have the effect of restricting or pinning the variation of the close loop poles when the plants gain changes. But then, when one places 0's in order to achieve causality, we also have to have control of poles that need to be placed and we saw that we could not place these controller poles anywhere. There were stability concerns that arose based on the position where we chose to locate the controller poles and we found that that to be located at least some distance away from the imaginary axis from the origin of our complex plane in order for the closed loop system to be stable or for the close loop poles and the other branches of the root locus to not have the poles on the right half of the complex plane.

So, although we considered only the variation in gain of the plant in our design in the previous discussion, in principle the same approach can be adopted to address variation of several other parameters of the plant. For instance, if the plants pole location is uncertain it is possible to design a robust control system whose overall response does not change significantly in response to the uncertainty associated with the pole location of the plant. Likewise if the pole and the gain are both uncertain once again, we can come up with a systematic design strategy that will allow us to achieve the specified amount of robustness to variation independent variation of both the gain of the plant as well as it is pole location.

Now, if the plant has multiple poles and each of them are uncertain and even the gain of the plant is uncertain, then the control design strategies that we would be talking about allows us to handle even such fairly sophisticated and challenging problems using the techniques that we talked about in the previous lecture.

So, what we shall do today is to look at one of these problems namely uncertainty associated with the pole location of the plant. Having solved this kind of a problem in the past, in the previous clip we looked at how we attack the problem associated with designing a robust control system that achieves that minimizes the sensitivity of the dominant pole to the variations in the gain of the plant, the steps that would be adopted in this design are also quite similar and many of the steps that we would undertake would follow exactly along the same lines the ones that we talked about in the previous clip.

So, what we shall do? First is we shall state the problem, then we shall underscore the important steps involved in the design and rather than go through each of these individual steps in detail exactly as we did in the previous clip, we shall focus on what is new and what is it that needs special attention and discuss only those aspects of the design that are new to the new problem specification. And the other steps might remain the same. So, we shall briefly touch upon what the other steps are, but we shall not get into the details of solving the other design steps of the problem.

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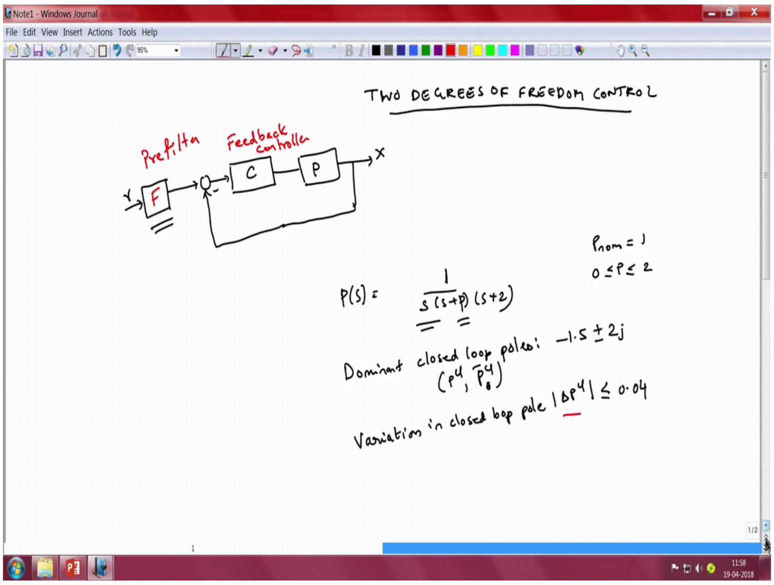

So, if we look at the problem that we are considering in today's in this clip, we once again are asked to design a two degree of freedom control system to achieve robustness to variation in plant parameters. So, the plant that we have considered is exactly the same plant that we looked at in the previous clip P of S is equal to 1 by S times S plus 1 times S plus 2 that was the plant, but in this example we have assume that the pole at P equal to at s is equal to minus 1 is uncertain.

So, the nominal value of P in this expression here is equal to 1. So, nominally the plant has one pole at the location S is equal to minus 1, but this pole can lie anywhere between 0 and 2. In other words, the pole can lie at the origin or it can go all the way to S is equal to minus 2. So, it can lie anywhere on the real axis of the complex plane between s is equals to 0 and s is equals to minus 2.

So, once again here we have a plant that is uncertain. In this case there is no uncertainty associated with the gain unlike the previous example that we considered in the last clip. So, the gain is always equal to 1. So, there is no uncertainty associated with that, but the pole location is uncertain and the location of the dominant closed loop poles is expected to be the same as what we had in the previous clip; they are expected to be at minus 1.5 plus minus 2 j. This as we discussed in the absence of uncertainty in the plants model, we can use one degree of freedom control design to locate the close loop poles at this particular pair of points, but the problem arises when one wants to restrict the variation of the dominant closed loop poles which are nominally located at minus 1.5 plus minus 2 j when the plants parameter in this particular case when the plants pole changes or when there is uncertainty in the plants pole.

This problem cannot be handled using one degree of freedom control design then one has to adopt a two degree of freedom control design. Now, what has been specified as far as the variation in the dominant pole is concerned due to the variation in the plant pole location is that we want the variation in the closed loop pole at the point minus 1.5 plus minus 2 j to be utmost 0.04 units. So, it might wander about in the complex plane, but if one computes the maximum distance that it travels, then if that distance should be less than or equal to 0.04 units.

So, this is the problem specification and as we have been discussing, the trick that one adopts in order to restrict the variation of the close loop pole at 1.5 plus minus 2 j due to the variation in the plants location is to place a controller 0 near the place where the dominant pole is expected to be located.

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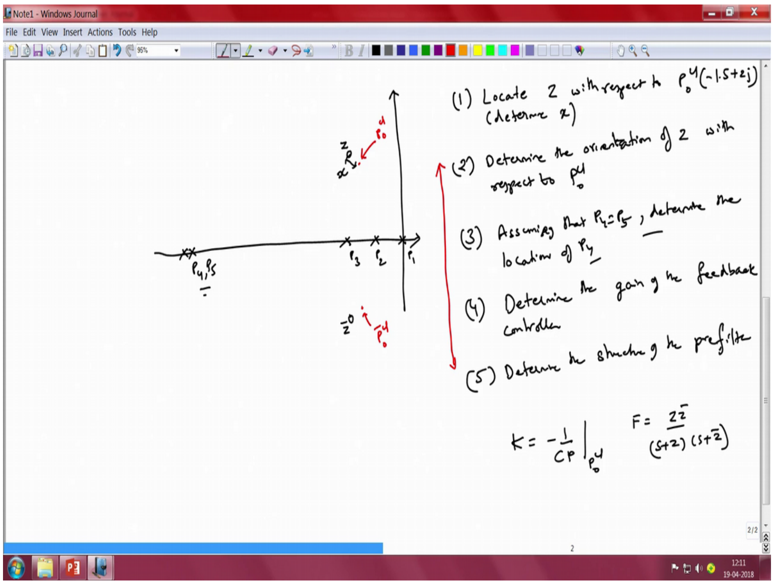

So, we have the dominant pole at minus 1.5 plus 2 j another pole at minus 1.5 minus 2 j. So, we locate two controller 0's; z and z bar quite close to these the locations where P C L naught and P C L naught bar are located. And our first step in the design is to determine the exact position of the 0 in the complex plane or equivalently we can try to determine the distance of the 0 from the point P C L naught whose coordinated we know very well and the angle of 0 with respect to the point P C L naught. This will be the first two steps in the design.

Next, we have since we have had two 0's for the controller. In order for the controller to be a causal transfer function, the denominator polynomial of the controller should also have at least the same degree as the numerator polynomial which demands that our controller should have at least two poles and these poles we have called P4 and P 5 and we assume that these poles have been placed very far away from the origin.

So, the second step in the design after locating the 0 is to determine the positions of these poles p 4 and p 5 because these poles result in extra branches to the root locus of the system of the open loop system and the poles that lie on one of these pair of branches could potentially become unstable when the gains are very high.

So, one has to be mindfull of the location of these poles and ensure through appropriate design that the controller poles are placed sufficiently far away that for the kind of variation in the plant parameters that we would have in this case, the closed loop poles on these branches will not become unstable. The third step is to complete the feedback controller design and that is to determine the gain of the feedback controller. So, we will be able to, the first two steps will allow us to determine the structure of this controller. It would have two 0's z and z bar, the specific 0's of this position would be identified. It would have two poles assuming that these poles are coincident, then we would have the locations of these poles determined in the second step.

The next step is to determine the gain of the controller if one does that one is done with the design of the feedback controller C. The final step is to determine the prefilters structure. The prefilter exist in order to cancel the poles so that the overall transmission function relating the output to the reference will not have the terms s plus z and s plus z bar in the numerator of this of the transfer function and that will allow the dominant dynamics to be determined by the points P C L naught and P C L naught bar.

So, let us write down the different steps the first step is to locate the distance of the 0 from the point P C L naught. Let me call that distance as x. So, locate z with respect to P C L naught which is the location of the dominant close loop pole in this case it is minus 1.5 plus 2 j.

So, in other words we have to determine the distance x. That is the first step in the design. The second step in the design is to determine the orientation of z with respect to the point P C L naught that is the second step. The third step is to determine the positions of the poles P 4 and P 5. So, assuming that P 4 is equal to P 5, so you have therefore, coincident poles for the controller determine the location of p 4. The fourth step is to determine the gain of the controller, feedback controller and this would complete the design of the feedback controller. The next step is to determine the structure of the pre filter and that would complete the design of the robust control system.

Now, among these different steps, we note that in order to determine the distance of the 0 z from the point P C L naught or equivalently determine the distance x, we need to know exactly how the close loop pole P C L wanders about when the plants parameter changes. In this case the plants parameter is it is pole location. So, there is one pole that is whose location is uncertain.

So, when this uncertain poles position changes in what particular manner does the close loop pole position vary? This is something that needs to be determined and this is something that is specific to the particular parameter that is varying. So, we have to undertake this design step in detail once again and we cannot borrow this from our discussion in the previous clip. When we come to the second step here determination of the orientation of z with respect to pc naught, you will notice that you can directly borrow the steps that we adopted from the previous example.

So, in the previous example to remind you we assume that the points p 4 and p 5 are very far away from the origin and we assume therefore, that they do not contribute significantly to the angle at the point P C L naught. Hence, we computed the angles subtended by the other poles and zeros of the open loop system namely p 1, p 2, p 3 z and z bar and equated that to minus 180 degrees. This allowed us to get the angle that the 0 z had to subtend at the point P C L naught for the point P C L naught to be a point on the root locus.

Now, this exact same procedure can once again be adopted without any modification for this new design as well. So, even when we have this other parameter that is varying, the point P C L naught is going to be a point on the root locus and hence the angle criterion has to be met at the point P C L naught which means that the sum of the angles subtended at P C L naught by p 1, p 2, p 3, z and z bar should add up to minus 180 degrees. The only unknown here will be the angles subtended by the point z at the point P C L naught because the point the angle subtended by the point z bar at the point P C L naught is approximately plus 90 degrees.

So, assuming once again that the angle subtended by p 4 and p 5 at the point P C L naught is negligible, we can determine the angle subtended by z at the point P C L naught. In the first step of the design if we are successful in determining the distance x and in the second step if you know the angle of this point z with respect to P C L naught, then the 2 together will allow us to fix the point z. If the point z gets fixed, the point z bar also gets fixed.

So, the second step of the design is essentially going to follow the same logic and arguments as the corresponding step that we undertook in our previous design example when we had uncertainty in the plants gain. So, in this clip therefore, we shall not look at this in great detail we shall just assume that we can apply the same steps and get the answer for this second part.

Coming to the third part, in the previous example when we assume that p 4 and p 5 are coincident poles, we then drew a big picture root locus where we noticed that we could lump the three poles of the plant and the two zeros of the controller that were near the origin as one equivalent single pole and therefore, we had a simplified root locus in this big picture where we had 2 controller poles far away at the point p 4 and p 5 which we assume to be equal to be coincident and a single pole near the origin. And using this picture, we could determine the gain at which the root locus crossed the imaginary axis and we could ensure that this gain was less than the gain of the maximum gain at the overall open loop system could assume and the same steps can in principle, can again be adopted in this case as well. Hence we shall not discuss in great detail about determination of the points p 4 and p 5 because the same steps shall be adopted as what we had done in the previous clip.

Likewise for the fourth and the fifth point also, once we have determined the locations of the points p 4, p 5 and z and z bar then we know the structure of the controller. The structure of the plant is already given to us therefore, we can determine the gain k of the controller by using the equation k is equal to minus one by cp evaluated at P C L naught and with that our feedback controller design would be done. And the final step is to determine the structure of the pre filter and we notice that in the previous example our pre filter had the structure f is equal to z bar divided by s plus z times s plus z bar and this structure ensured that the pre filter transfer function cancelled the zeros of the transfer function for the feedback part of the system alone and that ensure that the points P C L naught and P C L naught bar were the dominant poles of our close loop system.

So, steps 4 and 5 therefore, are also identical to the steps that we undertook in the first example. Hence, the first step alone is something that is new and that has to be investigated for this particular case where we have the plants pole to be uncertain instead of it is gain while the other steps are exactly identical to the steps that we undertook in the previous design which we discussed in the previous clip.

So, viewers of this clip are urged to look at the design steps in the previous clip in order to understand how to undertake the steps 2 to 5 in this particular design example. Although the numerical values might be different in each of these steps, the procedure that one employs is exactly the same; the procedure and the logic are no different from what we discussed in the first example.

So, in this clip therefore, we shall focus only on the first step of the design namely the technique that one might adopt in order to locate the point z with respect to the point P C L naught.

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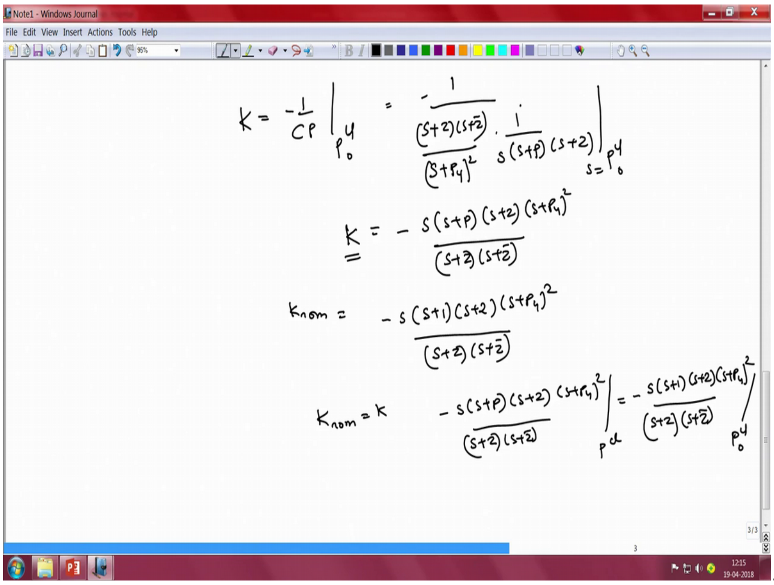

Since the point P C L naught is a point on the root locus, we would have the root locus equation to be valid at the point P C L naught. In other words, k will be equal to minus 1 by C P at the point P C L naught and we know that this is going to be equal to minus 1 by we know that we have a controller with two zeros which is s plus z and s plus z bar and it has two poles and we have assumed that these poles are coincident. So, s plus p 4 the square and our plant is given by 1 by s times s plus p times s plus 2 and this entire thing is evaluated at P C L naught s is equal to P C L naught.

So, if we were to undertake some simplification, this would be given by k is equal to minus of s times s plus p times s plus 2 times s plus p 4 the square divided by s plus z times s plus z bar. In the nominal case, we would have k nominal to be equal to minus of s times s plus 1 because the nominal value of the plant pole is at s is equal to minus 1 times s plus 2 times s plus p 4 the square divided by s plus z times s plus z bar.

Now, we note that when the plants pole location changes, it has absolutely no effect on this gain k because the gain k is independent of the pole location of the plant. Hence, we would have k nominal to always be equal to k regardless of where the actual plants pole is between the limits that we have indicated, between the limits s is equal to 0 and s is equals to minus 2. So, regardless of where between these two limits the point s is equals to minus p is located, the root locus equation will be valid with the same gain k and hence, we would have that minus of s times s plus p times s plus 2 times s plus p 4 the square divided by s plus z times s plus z bar and this would be evaluated at the location where the dominant close loop pole would be located and when p is not equal to it is nominal value of 1, then the close loop pole will not be located at P C L naught. It will be located at some point P C L.

So, this is going to be equal to minus of s times s plus 1 times s plus 2 times s plus p 4 the square divided by s plus z times S plus z bar and this is evaluated at the point P C L naught. In order to make further progress we shall depend upon the geometric interpretation of the terms S, S plus P, S plus 2 and so on evaluated at the point P C L naught.

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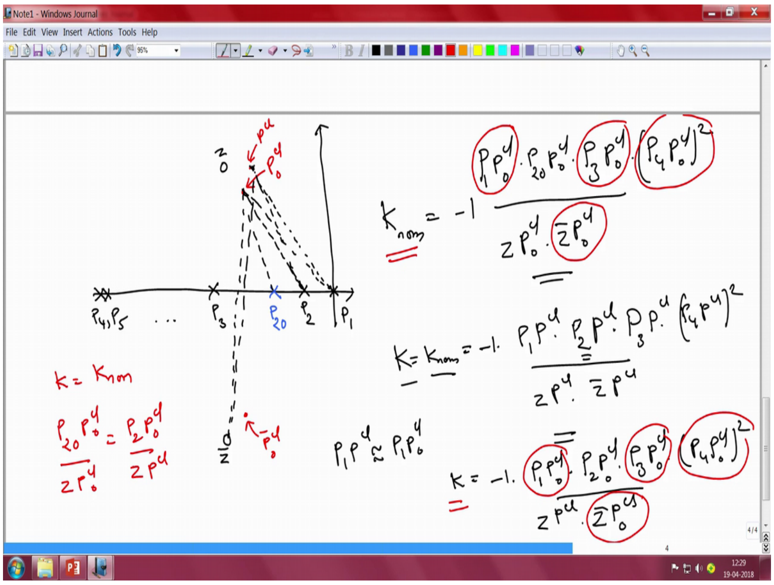

Now, if we look at the term s evaluated at the point P C L naught, it essentially represents the complex number starting at the point P 1 ending at the point P C L naught and hence it would be P 1 P C L naught. So, this is the term s.

Similarly, the term s plus one evaluated at the point P C L naught will essentially be the complex number starting at the point p 2 naught which is the nominal location of the pole that is wandering about and that nominal location is at s is equal to minus 1. So, this distance p 2 naught P C L naught represents the complex number s plus 1 evaluated at the point P C L naught.

So, the second term is going to be p 2 naught P C L naught. The third term is s plus 2 evaluated at the point P C L naught and the point S is equal to minus 2 is essentially the point p 3 here. Therefore, once again with the same logic we can conclude that the term s plus 2 is nothing but the complex number that starts at the point p 3 and ends at the point P C L naught.

So, the third term is p 3 P C L naught. Similarly, s plus p 4 and s plus p 5 since p 4 and p 5 are coincident poles will essentially be s plus p 4 the square and evaluated at the point P C L naught will essentially be give me p 4 P C L naught the square. This divided by z P C L naught which represents s plus z evaluated at the point P C L naught and z bar P C L naught which represents the point S plus z bar evaluated at the point P C L naught times minus 1 gives me the nominal gain k nom.

Now, likewise I would have the actual gain of the system which is going to be equal to the nominal gain to be equal to minus 1 times p 1 P C L because when the plants pole location changes, the close loop pole will no longer be located at the point P C L naught it will be located at some other location we shall call that location as P C L.

Now, when this plant pole is at this location, let me call this actual position of the plant pole to be p 2 while p 2 naught is it is nominal position when it is at p 2 let us assume that the close loop pole will be at the point P C L. So, nominally when it is as p 2 naught namely at s is equal to minus 1, the close loop pole will be at P C L naught.

So, since the point P C L would also be a point on the root locus we would have the root locus equation to be valid at this point P C L and hence the value of s, the number the complex number s evaluated at the point P C L is nothing but the complex number p 1, P C L and that is what has been written here. Similarly, the point the complex number s plus P evaluated at the point P C L where p is the point p 2 here is essentially going to be p 2 P C L. So, this is going to be the second complex number and the third complex number is going to be p 3 P C L which essentially represents the complex number s plus 2 evaluated at the point P C L times p 4 P C L the square divided by z P C L times z bar P C L.

So, since the plant gain or the controller gain does not change when the plant pole location changes, k will be equal to k nominal. So, this equation here will be equal to the equation there. Now, we will make one simplification in the second equation. If we look at the point p 1 P C L, it represents as I said the complex number joining the point p one to the point P C L.

Now, if we look at the point p 1 P C L naught that represents the complex number joining, the point p 1 to the point P C L naught. Now, from our specifications we know that the two points p P C L naught and P C L should not be more than 0.04 units away from one another. If we compare the kind of distances that we have between the point p 1 and P C L naught or the point p p 1 and P C L, these distances are significantly bigger than the maximum permissible distance between the point P C L naught and P C L because the point p 1 is at s is equal to 0, p 2 is at s is equal to minus 1 nominally, p 3 s is equal to minus 2 and z will be located close to the dominant closed loop pole location minus 1.5 plus 2 j.

So, these are all distances whose magnitude is on the order of unity. On the other hand, the distance between P C L and P C L naught is expected to be just less that 0.04 units. So, which is almost two orders of magnitude lesser than the typical distances between the other open loop poles and zeros of the plant and the controller. Hence, we can conclude that p 1 P C L is approximately equal to p 1 P C L naught.

So, what I can do therefore, is the second equation I can write it as K is equal to minus 1 times instead of writing it as p 1 P C L P C L, I shall write it as p 1 P C L naught and the second term is p 2 P C L and that represents the distance of the point p 2 from the point P C L and I am indicating that by this dotted curve here.

So, if you notice once again, the complex number p 2 P C L is approximately equal in magnitude as well as orientation to the complex number p 2 P C L naught and that is because the points p P C L and P C L naught are very close to one another, just 0.04 units away from one another whereas, the distance of the point p 2 from both these complex numbers is quite large. It is on the order of unity. Hence, I can replace the second term in this expression which is p 2 P C L with the term p 2 P C L naught. Likewise, p 3 P C L can be replaced by p 3 P C L naught and p 4 P C L can be replaced by p 4 P C L naught the square and in the denominator we would have z P C L. Now, we cannot replace this with z P C L naught because by choice we have to place a 0 very close to the point P C L naught. So, close that the distance between the 0 and the point P C L naught would be comparable to the permissible variation in the closed loop pole location namely 0.04 units.

Hence when this point P C L naught wanders to the new location P C L due to uncertainty in the plants pole location, we cannot assume that the complex number z P C L would be very nearly the same as the complex number z P C L naught. Hence, we shall preserve the complex number in the denominator as z P C L and the final complex number here is z bar P C L and z bar P C L represents the complex number that connects the points z bar to the point P C L and you can see from the schematic here that this complex number is for all practical purposes identical to the complex number that connects the point z bar to the point P C L naught simply because once again the point z bar is very far away from both P C L as well P C L naught. Hence, I can replace the last term with the term z bar P C L naught.

So, now we are in a position to make some meaningful simplification to the expressions that we have written in this slide. If we compare the terms k nominal and the terms k, we see that the first term in k nominal is p 1 P C L naught and the first term in k is also p 1 P C L naught after making this appropriate small simplification hence let me circle the terms which are equal to one; another p 1 P C L naught is existing both in k nominal as well as in k; the second term in k nominal is p 2 naught P C L naught and the second term in k is p 2 P C L naught; these two are not necessarily equal to one another.

So, I cannot cancel them out circle them out. The third term is p 3 P C L naught in the first expression and it is also p 3 P C L naught in the final expression. The fourth term likewise is p 4 P C L naught the square here also it is p 4 P C L naught the square. So, these also are the same in the two expressions; one for k nominal and other for k. In the denominator, I cannot remove I cannot circle z P C L naught and z P C L because these two are different.

So, I have z bar P C L naught as the second term in the denominator of the first expression and I have the same term also repeated in the final expression. Therefore, when I equate k to be equal to k nominal, I discover that all these terms simply cancel one another and go away from the equation and I am left with just a few terms in particular I would have p 2 naught P C L naught divided by z P C L naught to be equal to p 2 P C L naught divided by z P C L. Now, what is the geometric interpretation or meaning of this equation?

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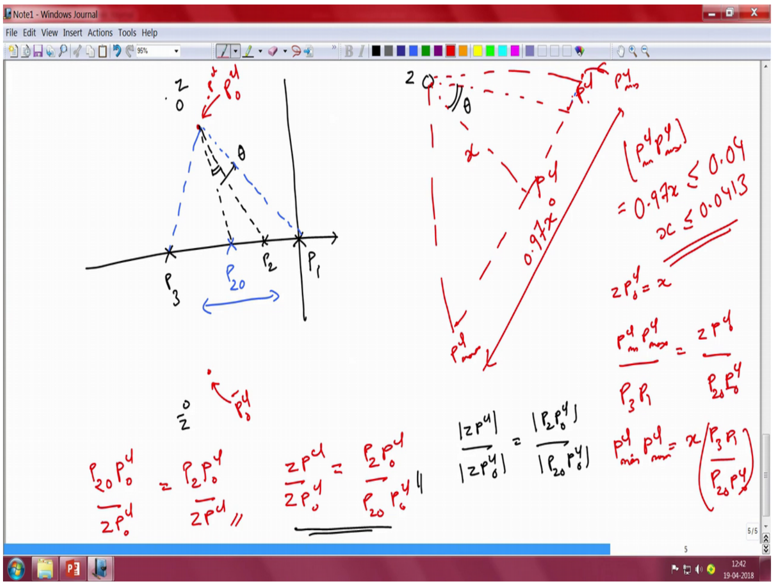

To understand the geometric meaning of this equation, I have written down this expression once again and alongside it above it I have once again the complex plane displaying the locations of the different open loop poles and the desired dominant pole location on the right hand side, I have separately zoomed in to the area near the 0 z and indicated the point z in relation to the point P C L naught.

So, let us return to this equation p 2 naught P C L naught divided by z P C L naught is equal to p 2 P C L divided by z P C L. Let me rearrange it a little bit. I would have z P C L divided by z P C L naught to be equal to p 2 P C L naught divided by p 2 naught P C L naught. So, this is the equation. Let us look at the geometric interpretation of this equation. So, the point P C L would be somewhere here that corresponds to the close loop pole position when my point when the when the plant pole is actually at the point p 2 and p 2 naught represents the nominal position of the plant pole which is s is equal to minus 1 and for this nominal position we know that the close loop pole will be located at P C L naught namely minus 1.5 plus 2 j.

So, in this zoomed in view also I shall locate the point P C L. So, it would be located let us say somewhere here now the term z P C L essentially refers to the complex number that connects the point z to the point P C L, that is the complex number. The term z P C L naught represents the complex number that connects the point z to the point P C L naught. Now, on the right hand side we have p 2 P C L naught and p 2 naught P C L naught the term p 2 P C L naught represents the complex number that connects the point p 2 to the point P C L naught it is this complex number likewise the point p 2 naught P C L naught represents the complex number that connects the point p 2 naught to the point p P C L naught which is the complex number and what this equation here says is that the ratio of these two complex numbers z P C L and z P C L naught is equal to the ratio of those two complex numbers namely p 2 P C L naught and p 2 naught P C L naught.

Now, when the ratio of two complex numbers is equal to the ratio of 2 other complex numbers, what we can essentially say is that the ratios of the magnitudes of these two complex numbers are equal or in the other words we can say that the magnitude to z P C L divided by the magnitude of z P C L naught is equal to the magnitude of p 2 P C L naught divided by the magnitude of p 2 naught P C L naught. So, this is one statement that we can make directly from the expression that has been given here. Likewise, what we can also say if the ratio of these two complex numbers is equal to the ratio of the other 2 complex numbers is that the angle between the complex number z P C L and z P C L naught should be equal to the angle between the complex numbers p 2 P C L naught and p 2 naught P C L naught.

So, what we have in our hands are four complex numbers z P C L and z P C L naught. The ratio of the lengths of z P C L and z P C L naught is equal to the ratio of the length p 2 P C L naught and p 2 naught P C L naught and the angle between z P C L and z P C L naught which is given by this angle theta here is also equal to the angle between p 2 P C L naught and p 2 naught P C L naught. So, even this angle is theta.

So, what these two statements imply is that the triangle for by the points P C L z and P C L naught is similar to the triangle formed by the points p 2 P C L naught and p 2 naught. That is because the ratios of the adjacent sides of one triangle is equal to the ratios of the corresponding adjacent sides on of the other triangle and the included angle in the in one triangle is equal to the included in the other triangle. Hence as the point p 2 wanders about along the real axis from the point s is equal to 0 to the point s is equal to minus 2. So, it describes a triangle that is bounded by the point p 1 and the point p 3 because the point p 3 corresponds to s is equal to minus 2 and the point p 1 corresponds to the point s is equals to 0.

So, the point p 2 can vary anywhere between the points p one and p 3 and therefore, it describes the triangle P C L naught p 1, p 3. Now, as this point p 2 wanders about, the close loop pole P C L wander about in such a manner that the triangle that it describes with respect to the point z. Let me call this point as P C L max and the other point the other extreme as P C L min. So, P C L min corresponds to the close loop pole location when the plant pole is at s is equal to 0, P C L max represents the close loop pole location when the plant pole z is equal to s is equal to minus 2 or coincident with the point p 3 in this schematic here based on what we have discovered now namely that the triangles described by the point P C L with respect to the point z and the point P C L naught is similar to the triangle described by the point p 2 with respect to the points p 2 naught and P C L naught. We can conclude that when the plant pole wanders about between the points p 1 and p 3, the close loop pole wanders about between the points P C L max and P C L min in such a reaction that the triangle z P C L min P C L max is similar to the triangle P C L naught p 1 and p 3.

Now, since we know the coordinated of the point P C L naught and the coordinated of point p 1 and p 3, you know everything about the triangle P C L naught p one p 3. What we wish to know is the variation of the close loop pole that is something that we wish to know and we wish to constrain it to within 0.04 units as has been given to us by the specification. Now, if the distance from the 0 said to the point P C L naught is x then from the similar triangles that we just drew this distance x is proportional to the distance P C L naught, p 2 naught and the distance P C L max P C L min which is given by this distance here is proportional to the distance p 3 p 1. Hence, if we have z P C L naught to be equal to x we would have P C L min P C L max divided by p 3 p 1 to be equal to z P C L naught divided by p 2 naught P C L naught.

In other words, since z P C L naught we assume is equal to x, we would have P C L max P C L min, P C L max to be equal to x times p 3 p 1 divided by p 2 naught P C L naught and if you compute this ratio you find that for this particular triangle that we have here P C L naught, p 1 and p 3 this ratio p 1, p 3, the length of p 3, p 1 divided by the length of p 2 naught P C L naught is equal to 0.97. Hence, this overall distance here is going to be equal to 0.97 x.

Now, we have discovered that when our plant pole varies along a straight line on the real axis, our close loop pole dominant pole which is nominally at the point P C L naught also varies along a straight line which is given by a straight line on which the points P C L max and P C L min lies and if the distance of the 0 z from the point P C L naught is designated as x, the line along which the close loop pole varies is given by 0.97 x now we have been specified that this distance namely P C L min P C L max the magnitude of this which is equal to 0.97 x should be less than or equal to 0.04 units, this is our specification. We want the close loop pole variation to be restricted to within 0.04 units and this automatically implies therefore, that x should be less than or equal to 0.0413 units.

So, this equation essentially tells us at what distance we have to position the 0 in order for us to restrict the variation of the close loop pole to within 0.04 units. It tells us in particular it has to be located within a distance of 0.0413 units from the point P C L naught. So, this completes the first step in our design.

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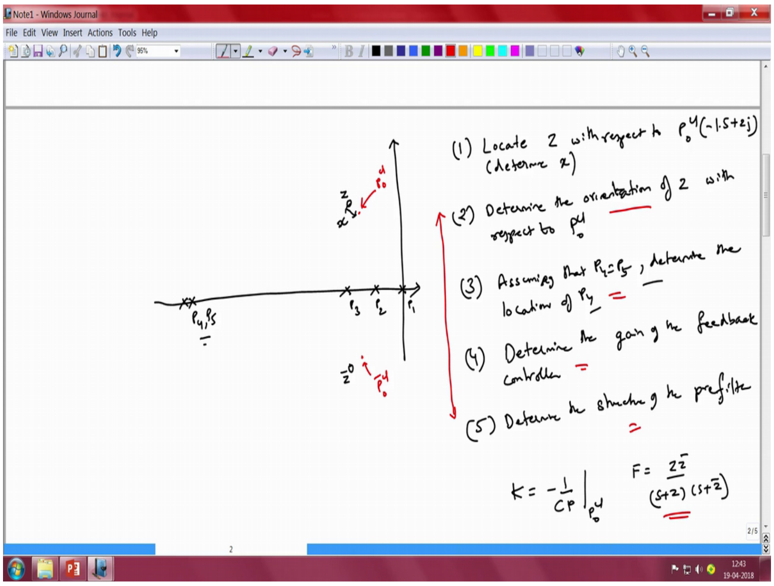

Now, if we come back to the other steps determination of the orientation of the point z with respect to the point P C L naught can be done by applying the angle criterion at the point P C L naught, I urge the viewers of this video clip to look at the previous video clip to see that arguments that go behind the determination of the orientation of z with respect to P C L naught and subsequently determination of the position of the far away poles of the controller namely p 4 and p 5 once again can be looked up in the previous clip because the steps are exactly the same as what we had in previous clip. The same also applies for determination of the gain of the feedback controller as well as the structure of the pre filter. The structure of the pre filter is already been written out here if we determine the orientation of the 0 with respect to P C L naught and the distance x automatically the structure of the pre filter gets fixed.

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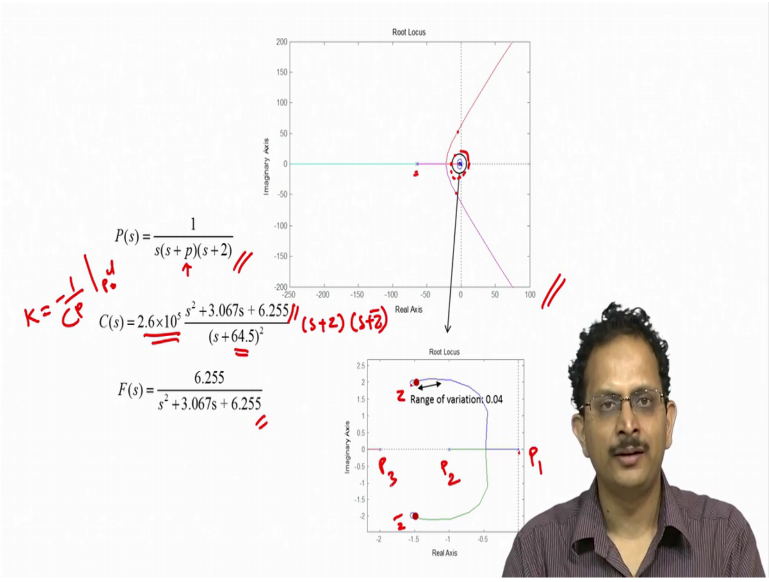

So, I have computed the locations of the 0's, z and z bar for the controller and the far away pole locations for the plant that we just talked about the plant where the pole location of the plant was uncertain nominally it was at s is equal to minus 1. In other words, here nominally the value of p was 1, but it could change from 0 to 2 and using the design steps that we discussed we conclude that the controller would be of this kind. The feedback controller would have two zeros, z and z bar the design steps that we just discussed allowed us to determine the distance x of the 0 from the point P C L naught, but the angle criterion can be borrowed from the previous clip and using that we can determine the exact position of 0 z and if you compute s plus z times s plus z bar, what you get is essentially this expression here.

The next step is to determine the location of the far away poles of the controller and this was done using the stability considerations. Once again, I urge the viewers to look at the previous clip to understand the different steps and we discover that the 2 poles p 4 and p 5 assuming they are coincident have to be placed at the point s is equal to minus 64.5 or beyond that in order for the close loop system to be stable. Next, in the final step in the determination of structure of the feedback controller is to determine it is gain which is given here and this was determined by using the expression k is equal to minus 1 by c p at the point P C L naught. By applying this condition, we obtain the gain k. This completes the design of the feedback controller.

Now, since we know the locations of the zeros z and z bar, the pre filter structure is quite straight forward as we discussed a little while ago in this clip and that is given by this particular structure. The overall root locus of this system is given here all the poles of the plant and the controller are bunched somewhere near the origin and the point minus 64.5 is located here. So, we have two branches of the root locus which are which arise because of these two controller poles at s is equal to minus 64.5, but by design we have made sure that the closed loop poles that are located on these branches do not cross over from the left half of the complex plane when our plant pole varies about. If you zoom into the area near the origin of the root locus, we can look at the three plant poles s is equal to 0 s is equal to minus 1, s is equal to minus 2 the 2 locations of the 0 s is equal to z and z bar.

So, this is the point p 1, this is the point p 2, this is the point p 3.

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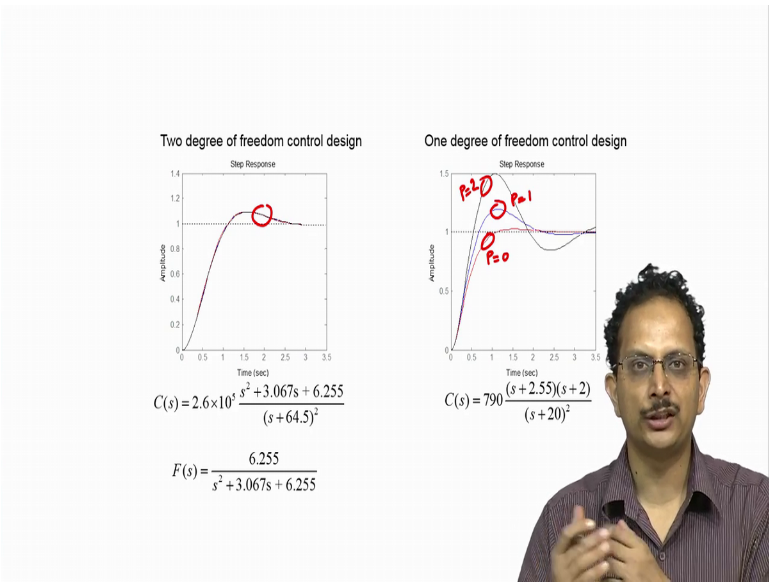

And what I have done on the right hand side is I have shown the step response of a one degree of freedom close loop system; in other words, the close loop system that does not have a pre filter due to the different possible positions of the plant pole. So, the blue curve here is the close loop step response of the system when the plant pole is at it is nominal position namely s is equal to minus 1. The red curve here represents the step response of the close loop system when the plant pole is at s is equal to 0 and the black curve here represents the.

So, this for the case when p is equal to 0, this is for the case when p is equal to 1 and the black curve represents the step response of the close loop system when the point p is at 2 or in other words the pole of the plant is located at s is equal to minus 2. And as you can see, the one degree of freedom control does a poor job in restricting the variation of the transient response due to the variation in the plants pole location or equivalently the dominant pole of the close loop system wanders about by unexpectedly large amounts when the plants pole changes. On the other hand, when one plots the step responses of the 2 degree of freedom control system one sees that all these three curves the nominal curve which is indicated by blue the curve when the plant pole is located at s is equal to 0 and the curve the black curve when the plant pole is located at s is equal to minus 2, all of them sit almost nearly one on top of another and this is because the 2 degree of freedom control system has successfully restricted the variation in the close loop pole P C L naught to less than or equal to 0.04 units when the plants pole varies.

Hence this again showcases the power of a 2 degree of freedom control architecture in order to achieve robustness in the response of the close loop system to variation in plant parameters. What we shall do next is to see how we can design a feedback controller with once again 2 degree of freedom control configuration that will allow for achieving robustness against variation of more than one parameter of the plant. For instance, in the first case we shall consider the variation of the plants gain as well as it is pole position and in the next case we shall look at the variation of the gain along with the variation of more than one poles of the plant.

So, if you have uncertainty in more than one pole locations of the plant as well as in it is gain and you still want a robust control system whose dominant pole is located at a certain point and varies by just a certain specified amount in response to variation of these plant parameters, we shall see how we can use root locus to undertake robust control design. These we shall do in the next clip.

Thank you