Control System Design
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Lecture – 28
2- Degree of freedom robust control design for plants with gain uncertainty (Part 1/2)

Hello, in the previous clip we saw how we can put 2 degree of control architecture for a control system to good use. In particular we saw how it could be employed to minimize the effect of measurement noise, while not sacrificing on the overall closed loop bandwidth as far as tracking of references is concerned; that was the first example that we took. And the second example that we took was to demonstrate the ability to achieve robustness in the closed loop response of our system to variation in plant parameters. In particular we considered the case of a first order plant and we insisted that it have 0 steady state error to dc references and have its dominant dynamics at a particular location.

In the numerical example that we considered it was a s is equal to minus 20. What was; however, unique about this problem was that we assumed that this plant has a huge uncertainty associated with its gain. So, we assumed that the gain could reduce by a factor of 5 or increase by a factor of 5. So, overall there was a 25 fold variation or uncertainty associated with the gain of the plant and what we wanted to do was to design a control system, whose dominant dynamics stays put at s is equals to minus 20 even when you have variation in the gain of the plant.

Now, to we saw firstly, that this was not possible to achieve with a 1 degree of freedom control architecture, but with a 2 degree of freedom control architecture it became possible. And, the there were 3 important steps that we had to adopt; first we had to place the open loop 0 of our feedback system, near the location where we wanted our closed loop pole to lie. Now, this open loop 0 is intended to pin the variation or restrict the variation of the closed loop pole, because of the changes in the plants gain, but just placing the 0 there does not guarantee to us that the closed loop pole would be located wherever we want it to be located.

If we place an open loop 0 close to where we want our close loop pole to be located, we also need to choose a very high gain for our open loop system, in order to make sure that the closed loop pole would be located in the vicinity of the 0. So, these are the first two steps first is to place a 0, near where we want the closed loop dominant pole to be located. The second point is to increase the gain of our open loop system so, that our closed loop pole is going to be located where it is intended to be located and the third step that we had to undertake was to choose a pre filter which cancels the effect of this 0.

So, left to itself 0 that we have placed will end up suppressing the dominant dynamics of our closed loop system, because they are placed very close to one another. But, by choosing a pre filter in such a manner that it cancels the 0 of our feedback system, we can ensure that the overall transmission function would have the desired dominant dynamics. And, because this desired dominant dynamics is located close to the open loop zero its position will change by a negligible amount when our open loop plants gain varies by a factor of 25. And we saw in the previous clip spectacular results very impressive result that showed that the overall variation in the transient response reduce dramatically, because of adopting this technique in comparison with 1 degree of freedom control design.

The problem that we considered however, in the previous clip was a rather simple one. We had a simple first order plant and our overall open loop system was chosen to be of such a kind that we have only two branches to the root locus and our control system could never become unstable. Our close loop poles would always lie to the left half of the complex plane and quite some distance away from the imaginary axis.

So, that was one simplifying factor that was there in the previous problem that we considered. And, the second simplifying factor was that we did not really specify how much of variation in the closed loop pole was tolerable when we had a change in the gain of the plant. Now as engineers it is only prudent for us to be able to design to the specification. So, if we want a certain specified variation in the closed loop poles location in response to the uncertainty that already exists in the plants model, then how do we choose the open loop gain of our plant. And, how do we choose the location of the 0 so, that this precise variation in the closed loop pole occurs when our plants gain or other dynamics were to vary.

So, both these questions remain unanswered. The issue of instability was never encountered and the issue of quantitative design, where we design our controller in order to meet specified requirements in terms of variation of the dominant dynamics in response to variation or uncertainty in the plant model; both these questions were not addressed in the previous clip.

So, what we shall do in this clip is to take a look at another control design, where both these issues have been specified and one has to deal with both the stability or instability issue as well as design the controller, to a specified variation in the closed loop pole. So, I have written down the problem statement here.

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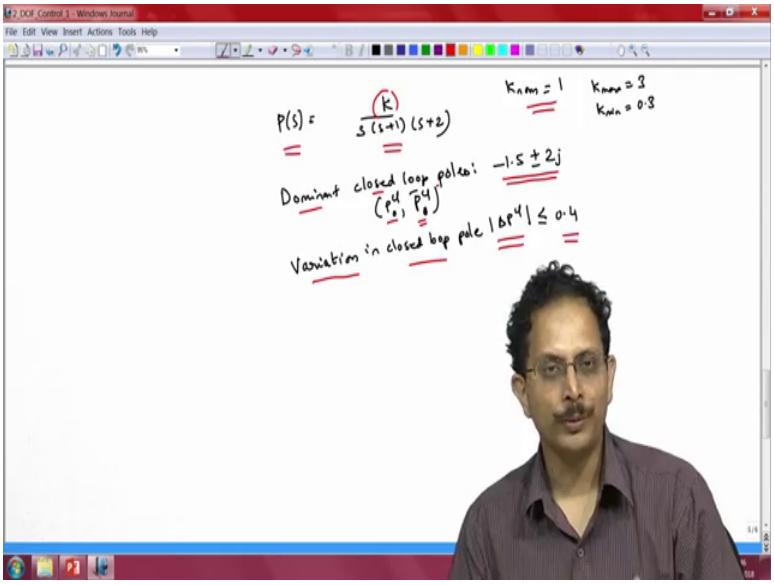

So, we shall undertake the design by means of this particular numerical example, we have been given a plant P of s equal to K by s times s plus 1 times s plus 2. So, this plant has 3 open loop poles namely at s is equal to 0 s is equal to minus 1 s is equal to minus 2. However, the gain k of the plant is assumed to be uncertain its nominal value is assumed to be 1. So, nominally the plant is going to be 1 by s times s plus 1 times s plus 2; however, the gain k can assume the value of 3 or can assume the value of 0.3 or any other value in between these limits. So, the gain k can vary between the limits of 0.3 and 3 although its nominal value is going to be equal to 1.

So, this is another example of an uncertain plant; a plant whose gain is uncertain. So, though nominally we know its gain for this particular plant at this instant of time we are not sure what its value might be. We can only say that it would this value whatever it is lies between the limits of 0.3 and 3. So, this is the plant that we have been given and now we are asked to design a closed loop system a feedback control system, whose dominant dynamics is located at these two particular positions at minus 1.5 plus 2 j and minus 1.5 minus 2 j.

So, this is the location of the dominant closed loop poles. So, nominally when the plant gain is equal to 1, we want the closed loop poles to be located exactly at this particular position namely minus 1.5 plus minus 2 j. And we have called this location by the symbols P cl naught and P cl naught bar where the bar represents complex conjugate. P cl represents the closed loop pole position and the subscript naught is intended to indicate this is the nominal position of the closed loop pole.

Now, when our plant parameters change; so, when we are having a plant with a different gain compared to what its nominal value would be, naturally we would expect the closed loop pole to not lie exactly at minus 1.5 plus minus 2 j, but to lie at a slightly different location. Now what we have now also been specified is that, we want the variation in the closed loop pole due to uncertainty in the plant which has been indicated by the term delta P cl; the variation the magnitude of delta P cl which is the magnitude of variation in the closed loop pole due to the change uncertainty associated with the plants gain, we have insisted that this should be less than or equal to 0.4 units.

So, all these are arbitrary numbers which can be changed, but they have been picked in order to illustrate the steps in the design. So, the first thing that we want to see is whether all these performance specifications can be met by using a 1 degree of freedom control system. If you have a 1 degree of freedom control system in principle it is possible by choosing a suitable controller to have the dominant poles be located wherever you wanted to be located namely at minus 1.5 plus minus 2 j in this particular case.

So, to do this we just follow the steps for performing control design using the root locus that we discussed a few clips back, when we talked about root locus space feedback control design. But what is not possible for this controller to accomplish is also to restrict the variation in the closed loop pole. So, this is something that we discover in the previous example to be not possible to accomplish using a 1 degree of freedom controller, and the problem we expect would continue to persist even with a 1 degree freedom controller one might decide to satisfy just the first specification.

So, let us first convince ourselves that a 1 degree of freedom controller does not do the job so, that we would have motivation to undertake the design using a 2 degree of freedom controller.

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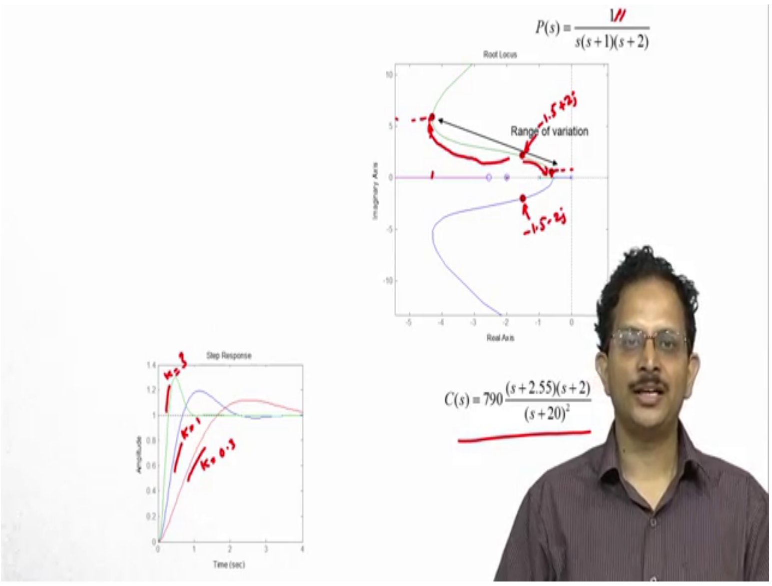

So, what I have plotted on this graph here is a root locus for a 1 degree of freedom control system, whose dominant poles are located nominally at minus 1.5 plus 2 j and minus 1.5 minus 2 j. The steps that have been adopted in coming up with this controller structure is exactly identical to the steps that we undertook a few clips back when we did the design using root locus based techniques.

So, essentially what we do is we compute the phase of the overall open loop system at the desired closed loop pole positions, and we come up with a controller that ensures the net angle subtended by all the open loop poles and zeros at the desired closed loop pole position is minus 180 or its slightly equivalence. By undertaking that these steps of design we end up with a controller that looks something like this.

Now, with this controller we have managed you have succeeded in getting the dominant closed loop poles could be exactly where we want it to be, but this controller does not guarantee to us that when the plants model changes. So, when the plant gain here which has been assumed to be equal to 1 or to reduce to 0.3 or to increase to 3. So, when it fluctuates by a factor of 10 there is no guarantee that this controller is going to restrict the variation in the closed loop pole to within 0.4 units.

Indeed when one looks at what happens when the gain were to fall by a factor of 3 from 1 to 0.3, we see that the closed loop pole now comes closer to the imaginary axis. So, it reduces in magnitude and comes down along the root locus as shown in this arrow; likewise when the gain increases by a factor of 3. So, when it when it goes from 1 to 3 the closed loop pole position travels up along the root locus to this new location here. Now, if one were to look at the real and imaginary parts of the closed loop pole when the gain of the plant is 3 and when the gain of plant is 0.3, we see that we see that the difference in that positions is significantly greater than 0.4 in fact, it is more than 4 units. So, it is off by an alter of magnitude.

So, our 1 degree of freedom controller will not help us in restricting the variation of the closed loop pole due to uncertainty associated with the plant model. And this is manifested in the corresponding huge fluctuation in the transient response of our closed loop system. So, the blue trace here plots the step response of the close loop system, when the plant has assumed its nominal gain value of 1 and this step response is in alignment with what we would expect it to be when the dominant poles are located at the two positions that we have indicated namely minus 1.5 plus minus 2 j.

However when the gain falls by a factor of 3 even if it reduces from one to 0.3 the step response looks as shown by the red curve here and when the gain increases by a factor of 3 the response is shown by the green curve. So, this is for the case when the plant gain is 1, this is for the case when the plant gain is 0.3 that is for the case when the plant gain is 3. And we see that there is a huge variation in the transient response of our closed loop system due to the corresponding huge variation in the location of the dominant closed loop poles of our system. And hence we cannot employ 1 degree of freedom control to achieve what we have set out to achieve namely to restrict the variation in the dominant poles to within 0.4 units. We will therefore, have to undertake this design using 2 degree of freedom control design.

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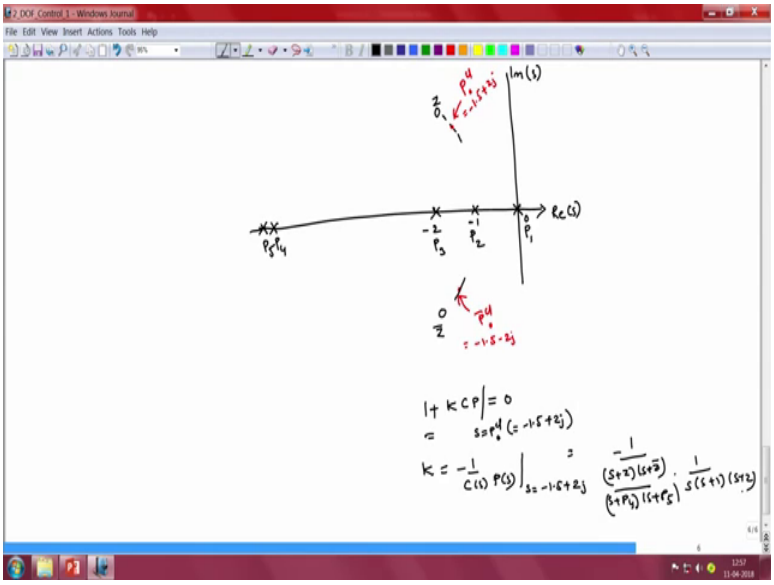

In this graph here, I have located on the complex plane the positions of the 3 open loop poles of our plant. So, we have a pole at s equal to 0, we have a pole at s is equal to minus 1, we have a pole at s is equal to minus 2. I shall in the same graph also indicate where we want our dominant close loop poles to be located. I shall indicate it by a red dot we want one of the dominant poles to be at minus 1.5 plus 2 j which will be somewhere here, and the other pole to be at minus 1.5 minus 2 j which would be at the reflection of that first point about the real axis.

So, this point which we have called P cl naught is going to be equal to minus 1.5 plus 2 j and this point which we have called P cl naught bar the complex conjugate at P cl naught is going to be equal to minus 1.5 minus 2 j. Now if we want our dominant poles to lie here and it should not vary significantly because of the tenfold variation in the plants gain, our previous example revealed that we have to position an open lop controller 0, in the vicinity of this pole P cl naught as well as P cl naught bar.

So, we have to position a 0 z and its complex conjugate z bar, in the vicinity of where we want our dominant closed loop poles to lie and we have to ensure that the gain of our open loop system is adequately high in order to make sure that our close loop pole lies exactly at P cl naught and P cl naught bar.

Now, that we have chosen our controller to have two zeros, in the interest of (Refer Time: 16:52) you know in order to physically realize is controller we need to have at least two poles for this controller so, that the relative degree of the controller is at least equal to 0 and we shall choose to place these two poles very far away. So, that they do not play a significant role in determining the dominant dynamics of your close loop system. So, these are the two poles, I shall now label these 5 poles that we have for the open loop system the first pole the open loop pole I shall call P 1 the second pole P 2 the third pole P 3 the fourth and the fifth poles as P 4 and P 5 respectively.

So, our open loop system therefore, has two 0's z and z bar and 5 poles the 3 plant poles P 1, P 2, P 3 and the two controller poles P 4 and P 5 that have been added for the sake of (Refer Time: 17:47) So, our control design will essentially comprise 3 steps; the first step is to locate the position of the 0 z and z bar with respect to P cl naught in other words we have to determine its distance from P cl naught and its inclination with respect to P cl naught. So, that is going to be the first part of the design, subsequent to that we have to locate the positions of the poles of the controller.

So, these poles have to be located judiciously with the stability of the closed loop system in mind and finally, we have to decide the pre filter. So, that the effect of these two zeros z and z bar on the overall transmission function relating the output to the reference is suppressed. So, let us undertake the design in a step by step manner and address the challenges in each of these steps.

So, the first step is to determine the location of the 0 or in other words its position and its orientation in relation to the point P cl naught and P cl naught pi. So, since the point P cl naught is a point on the root locus. So, by it is necessary for the root locus to pass though P cl naught in order for the closed loop pole to lie at that particular position by definition therefore, at the point P cl naught, we would have the root locus equation to be varying or in other words 1 plus k times C times P to be equal to 0. At the point s is equal to P cl naught or equivalently is equal to minus 1.5 plus 2 j. The same thing is going to be valid also at P cl naught bar, but since any analysis that we might perform at the point P cl naught is going to be almost identically applicable to the P cl naught bar, we shall not talk about P cl naught bar from this point forward.

We shall just focus on the point P cl naught. So, what this indicates what this equation indicates is that, we would have the open loop gain k of our system which is the product of the plant gain and the controller gain to be equal to minus 1 by C of s times P of s where our s is going to be equal to minus 1.5 plus 2 j now we already know that our plant has 3 poles. So, this is going to be of the kind 1 by C of s. C of s is a controller which would be of the kind s plus z times s plus z bar divided by S plus P 4 times S plus P 5 thats going to be the structure of our controller our plant would be 1 by S times S plus 1 times S plus 2.

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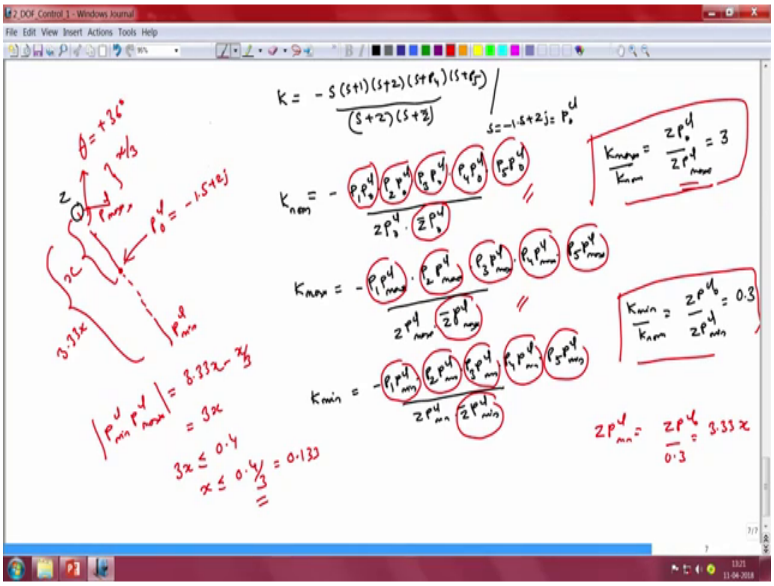

If we rearrange this equation we would have a gain K to be equal to minus of s times s plus 1 times s plus 2 times s plus P 4 times s plus P 5 divided by s plus z times s plus z bar and this is evaluated at the point s is equal to minus 1.5 plus 2 j.

Now, this entire expression here has a very neat geometrical interpretation. If you come back to the complex plain where we have plotted the locations of the open loop poles and the desired closed loop pole position the desired closed loop pole position is P cl naught and we are evaluating the terms s and s plus 1 and s plus 2 and so, on and so, forth at the point P cl naught. So, if you notice the term s evaluated at the point P cl naught essentially refers to this particular complex number which connects the origin to the point P cl naught.

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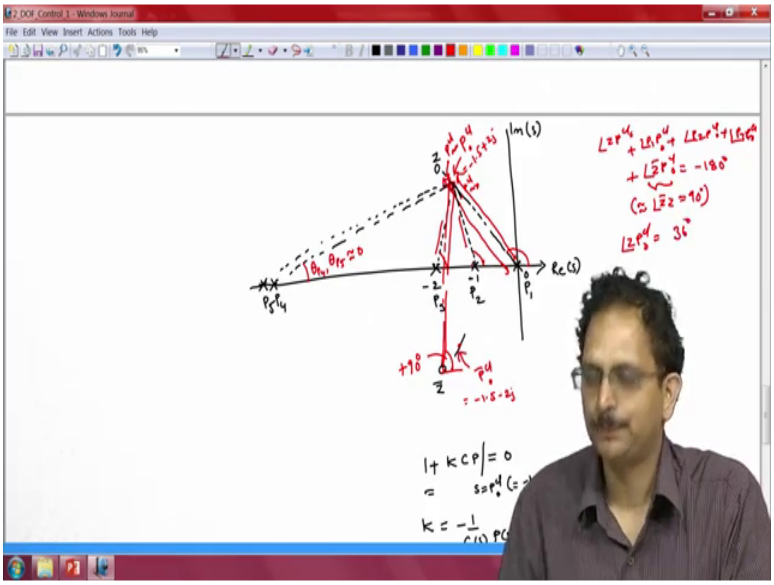

Similarly the term s plus 1 refers to this complex number, which connects the point P cl naught to the point s is equal to minus 1. Likewise the term s plus 2 evaluated at the point P cl naught essentially refers to the complex number that connects the point s is equal to minus 2 to the point P cl naught. And s plus P 4 once again refers to the complex number that connects the point P 4 to P cl naught and s plus P 5 represents the complex number that connects the point s is equal to minus P 5 to the point P cl naught.

So, there is a neat geometrical interpretation of the product s times s plus 1 times s plus 2 times s plus P 4 times s plus P 5. So, essentially therefore, this is going to be equal to minus of P 1 P cl naught, which is the complex number that connects the point P 1 the location of the first open loop pole which happens to be in this case s is equals to 0 to the point P cl naught, which is where we are evaluating this gain k. So, P cl naught is minus 1.5 plus 2 j. So, s evaluated at minus 1.5 plus 2 j essentially is P 1 P cl naught.

Likewise s plus 1 is P 2 P cl naught. So, the complex number starting from P 2 and ending with P cl naught has been represented by this line segment P 2 P cl naught. S plus 2 likewise will be P 3 P cl naught and s plus P 4 will be P 4 P cl naught. So, the terms P 1 P cl naught represents the vector starting from P 1 ending at P cl naught and so, on and so, forth and s plus P 5 likewise will be P 5 P cl naught. And in the denominator we would have s plus z to be the complex number z P cl naught and s plus z bar to be the complex number z bar P cl naught the complex number that connects z bar to the point P cl naught.

So, this is going to be an equation that would be valid at the point s is equal to P cl naught; now this is true when we have the gain of our plant to be its nominal value. So, when the gain is at its nominal value the closed loop pole will be located where you wanted to be located namely at P cl naught, and this equation is going to be valid. Now when the gain of the plant increases to its maximum value; so, when it goes from 1 to 3 the open loop gain which is the product of the controller gain and the plant gain will also go up by a factor of 3 because, the controller has a fixed structure its gain is going to be the same regardless of what is the plants parameters might be.

So, our overall open loop gain which I shall call as K max will be 3 times higher than the value of the nominal gain. And when the plant has assumed its maximum gain, the closed loop pole can no longer be located at the same position P cl naught it would have drifted a little bit away from the desired location. So, let us call the new location as P cl max in which case you would have K max to be equal to minus P 1 P cl max, times P 2 P cl max, times P 3 P cl max, times P 4 P cl max, times P 5 P cl max divided by z P cl max times z bar P cl max.

In a similar manner when our gain reduces from 1 to 0.3, our overall open loop gain will also reduce by a to 30 percent of its initial value and when the gain reduces once again the closed loop pole can no longer stay at P cl naught, it will once again drift away from that point. So, let us say the new location is P cl min, we would have K min to be equal to minus P 1 P cl min, times P 2 P cl min, times P 3 P cl min, times P 4 P cl min, times P 5 P cl min divided by z P cl min times z bar P cl min. Now, let us look at the significance of each of these terms. To do that let us once again come back to the complex plain where we have indicated the locations of the different open loop poles and zeros as well as the desired closed loop pole position.

So, when our gain increases our close loop pole is going to move closer to the 0. So, this is going to be where we are going to have P cl max and when the gain reduces its going to move farther away from the 0. So, somewhere here we are going to have P cl min. And we notice that this variation between the location of P cl max and P cl min has been specified to be 0.4 units.

Now, if you compare the complex number P 1 P cl naught that is given by this complex number here with the complex number P 1 P cl max, which is going to be this new complex number that connects the point P 1 to the point P cl max. You see that these two complex numbers are very nearly identical in magnitude as well as direction why is that so? Because, we have insisted that the variation in the close loop pole be just 0.4 units: which is a small number compared to the distances between the different open loop poles and the 0 of the open loop system. Since we have insisted that our P cl max lie fairly close to P cl naught when our open loop gain were to change the complex number P 1 P cl naught and P 1 P cl max will be very nearly equal to one another.

Likewise we would have P 2 P cl naught and P 2 P cl max to once again be very nearly equal to one another, because the point P cl max is located close to the point P cl naught. So, this complex number that we have written down here, which represents P 2 P cl naught is going to be very nearly the same in magnitude and angle with respect to P 2 P cl max. We can undertake the same analysis for P 3 P cl naught be equal to P 3 P cl max likewise P 4 P cl max P 4 P cl naught and P 4 P cl max are nearly identical and the same is going to be true for the point P 5 also.

When we come to the point z and z bar, we see that the point z bar is also very far away from the point P cl naught. Therefore, when the point P cl naught moves just a little bit away due to the increase in gain to the point P cl max, the complex number z bar P cl naught and the complex number z bar P cl max which are indicated by a two red curves that I just drew here are very nearly equal in magnitude as well as orientation therefore, these two terms will also be nearly equal to one another.

When we come to the last term here z P cl naught and z P cl max, we see that we cannot make this approximation. That is because the point z is by design located very close to the point P cl naught. So, the distance is already very small. So, when our point P cl naught changes to the new location P cl max, which is close to P cl naught we cannot assume that z P cl max and z P cl naught are nearly equal to one another in fact, they could be substantially different from one another.

With this insight in place if we now return to the equation that we had written, we have K nominal to be given by this expression and we have K maximum to be given by this expression. And from the analysis that we just undertook the approximate analysis, we discover that the term P 1 P cl naught and the term P 1 P cl max are nearly equal to one another because P cl max and P cl naught are situated fairly close to one another by virtue of the specification, that P cl max and P cl min should not vary the distance should not vary by more than 0.4 units which is small compared to the typical distances that we have between the different open loop poles and zeros of our system. Likewise P 2 P cl naught and P 2 P cl max were found to be very nearly the same, P 3 P cl naught and P 3 P cl max were found to be nearly the same and so, also where P 4 P cl naught P 5 P cl naught and P 4 P cl max and P 5 P cl max respectively.

We also found that z bar P cl naught is very nearly identical to z bar P cl max, because the point z bar is located very far away from the points P cl naught and P cl max. As a consequence all these terms are equal to one another very nearly and therefore, we would have k maximum divided by K nominal to be equal to z P cl naught divided by z P cl max.

Now, we can undertake a very similar argument even in case of the other location of the close loop pole P cl min, when the plant gain drops by a factor of 3. So, we would once again have the point P cl min to be located very close to the point P cl naught and since the points P 1 P 2 P 3 P 4 P 5 and z bar are all located very far away from P cl naught and P cl min, we can assume that P cl P 1 P cl min is very nearly equal to P 1 P cl naught P 2 P cl min is very nearly equal to P 2 P cl naught and so, on and so, forth.

So, P 4 P cl min is very nearly equal to P 4 P cl naught P 5 P cl min is very nearly equal to P 5 P cl naught and z bar P cl min is nearly equal to z bar P cl naught. And that what that allows us to do is to write down the second equation namely k min by k norm is equal to z P cl naught by z P cl min.

Now, we know that in our particular problem, the plant gain can increase from 12 3. So, that the ratio K maximum by K nominal is therefore, going to be equal to 3. Likewise our gain can drop from 1 to 0.3 therefore, the ratio K minimum by K nominal is going to be equal to 0.3. Now if you look t these two equations they give us clues about how the closed loop pole position P cl changes when the plants gain varies. What the first equation says is that the ratio of z P cl naught and z P cl max is equal to 3.

Now, let us on the left hand side indicate where our 0 is the 0 in the complex plane is going to be located somewhere here let us say, and the point P cl naught is going to be located somewhere here. So, this is going to be the point P cl naught. So, this is a part of the complex plane, that we have zoomed in and our focused on in order to understand how the closed loop pole varies when our plants parameter changes. So, the point P cl naught which is minus 1.5 plus 2 j let us say it is here, and let us say our 0 is somewhere there then z P cl naught represents this particular complex number and let us say we have another point here which represents P cl max then z P cl max represents that complex number.

Now, what the first equation says is that the ratio of z P cl naught and z P cl max is a constant a real constant equal to 3. Now what this tells us about these two complex numbers is that these two complex numbers are firstly, collinear because the ratio of two complex numbers is equal to a real number only when the two complex numbers are collinear.

So, the point P cl max therefore, essentially lies on the line that connects the point P cl naught to the point z; so, P cl max lies somewhere here. Likewise when we come to the second equation the second equation says that the ratio of z P cl naught and z P cl min is 0.3. This once again indicates that the point P cl min lies on the same line that connects the point P cl naught to z, because we have two complex numbers z P cl naught and z P cl min whose ratio is a real number.

So, for that to happen P cl min should lie on the same line as the one that connects the point P cl naught to z and that ratio is given to be 0.3. So, what that indicates is that if the distance of the 0 from the point P cl naught which is something that we have not determined yet if we were to call the distance to be x, then the first equation indicates that the distance of the point P cl max from the point z is going to be equal to x by 3. Because z P cl max is equal to z P cl naught divided by 3 from the first equation here and if our z P cl that has been assumed to be x units the z P cl max will be x by 3 units.

Likewise from the second equation we would have z P cl min to be equal to z P cl naught divided by 0.3 or essentially 3.33 times z P cl naught and since we have assumed that z P cl naught to be equal to x is going to be equal to 3.33 x. So, this overall distance from P cl min to z is going to be equal to 3.33 x.

So, the overall variation of the closed loop pole would be from the point P cl max to the point P cl min, we see that this variation is going to be along a straight line approximately and this straight line passes through the point z, and passes through also the point P cl naught. And the overall variation is going to be given by P cl min P cl max that is going to be the overall variation, the magnitude of that essentially is going to be equal to 3.33 x minus x by 3 that is essentially equal to 3 times x. And from our problem specification we know that 3 times x has to be less than or equal to 0.4 units which implies the x has to be less than or equal to 0.4 divided by 3 units or in other words we have to place the 0 at a distance, which is utmost equal to 0.4 by 3 units or in other words equal to 0.133 units away from the location P cl naught

So, what we have succeeded through this analysis is we have succeeded in determining the distance of the 0 from the point P cl naught. We have not yet determined the orientation of the 0 with respect to the point P cl naught. If we were to do that then we would determine the exact position of the point z with respect to the point P cl naught. In order to determine the orientation of the 0 with respect to the point P cl naught or in other words the angle theta which I am plotting here, that the 0 subtends at the point P cl naught, we can employ the angle criterion of the root locus. Since the point P cl naught is a point on the root locus we should have the net angle subtended by all the open loop poles and zeros of our system to be equal to minus 180 degrees at the point P cl naught.

Now, if we go back to the location of the different open loop poles and zeros of our system, we can compute the angle subtended by the first open loop pole P 1 and the second open loop pole P 2 and the third open loop pole directly from geometry. As far as the angle subtended by the point z bar at the point P cl naught is concerned, we have some uncertainty because we have not yet fixed the location of z bar. However, since the point z bar is a reflection of the point z and the point z is located very close to the point P cl naught. In fact, it is just 0.133 units away from the point P cl naught we can therefore, conclude that the angle subtended by the point z bar at the point P cl naught is going to be very similar nearly equal to the angle subtended by the same point z bar at the point z and that angle essentially is going to be close to 90 degrees. So, the point z bar subtends an angle of close to plus 90 degrees at the point P cl naught.

How about the points P 4 and P 5? Since we have not yet fixed their locations we do not yet know what angles they subtend at the point P cl naught. However, we are assuming that these two points are very far away from the other poles and zeros. So, we shall assume that whatever angle it is which we do not know at this point these angles theta P 4 and theta P 5 are very close to 0.

So, with this assumption in place we would have that the angle of z P cl naught plus the angle of P 1 P cl naught plus the angle of P 2 P cl naught plus the angle of P 3 P cl naught plus the angle of z bar P cl naught should together be equal to minus 180 degrees. And we found that the angle z bar P cl naught is approximately equal to the angle of z bar z that is equal to 90 degrees. So, from this equation and by using trigonometry to determine the angles P 1 P cl naught P 2 P cl naught and P 3 P cl naught we will get that the angle of z P cl naught has to be equal to 36 degrees or in other words this angle theta has to be equal to 36 degrees plus 36 degrees.

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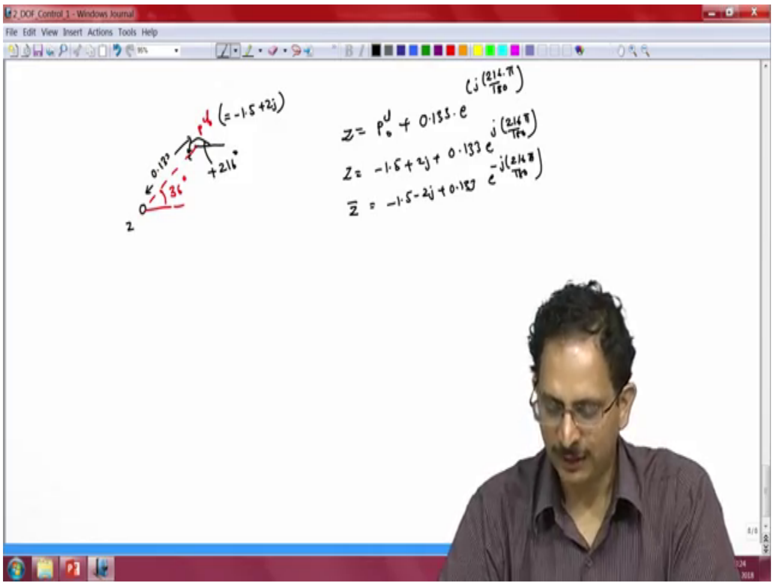

So, the schematic that we have drawn here is not entirely accurate in order for the 0 to subtend an angle of plus 36 degrees at the point P cl naught the 0 has to be located at this particular position and this should be plus 36 degrees. So, the 0 is located at a distance of 0.133 units from the point P cl naught we know the location of the point P cl naught it is minus 1.5 plus 2 j and it subtends an angle of plus 36 degrees at the point P cl naught with these two bits of information, we can we can obtain the position of the 0.

The 0 position therefore, is given by z is equal to P cl naught which is the location of the dominant closed loop pole plus this particular complex number, which is going to be given by 0.133 times e to the power j. The angle subtended by this complex number that connects the point P cl naught to z with respect to the real axis is going to be given by this angle it is going to be equal to plus 216 degrees and therefore, z location the location of 0 is given by z is equal to P cl naught plus 0.133, e to the power j 216 and since 216 is in degrees we have to convert it to radians.

So, I multiply it with pi and divide it by 180 and this is going to be the location of the 0. And the location of the 0 therefore, is going to become minus 1.5 plus 2 j plus 0.133 e to the power j 216 pi divided by 180. This can be simplified to obtain its exact location and the location of the 0 other 0 is given by z bar, which is going to be equal to minus 1.5 minus 2 j plus 0.133 times e to the power minus j times 216 pi by 180.

So, we have now finished the first step in the design, we have determined the locations of z and z bar.

Thank you.