Control System Design
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Lecture – 27
Introduction to 2-degree of freedom control

Hello, in the clips that we have in the previous clips we have looked at one degree of freedom control of physical systems and in the previous clip in particular, we took a look at some of the drawbacks of one degree of freedom control and I highlighted 2 important drawbacks. One was the fact that the control system is a rendered more sensitivity measurement noise because we cannot independently control the bandwidth for measurement noise and that for reference tracking.

The second problem was the fact that, we cannot engineer the sensitivity function and the transmission function independently; if the consequence that the transient response of the closed loop system is rendered sensitive to variation in plant parameters.

So, therefore, we discussed that having one more controller in our feedback system, would potentially help to resolve these problems. So, what we shall do starting from this lecture or starting from this clip is to look at how adding one more controller to the feedback loop, would help resolve the problems that we talked about and also helped to achieve other things, whose which will did not discuss in the previous clip. So, in the schematic here I have shown the one degree of freedom control system that we have been looking at all this time.

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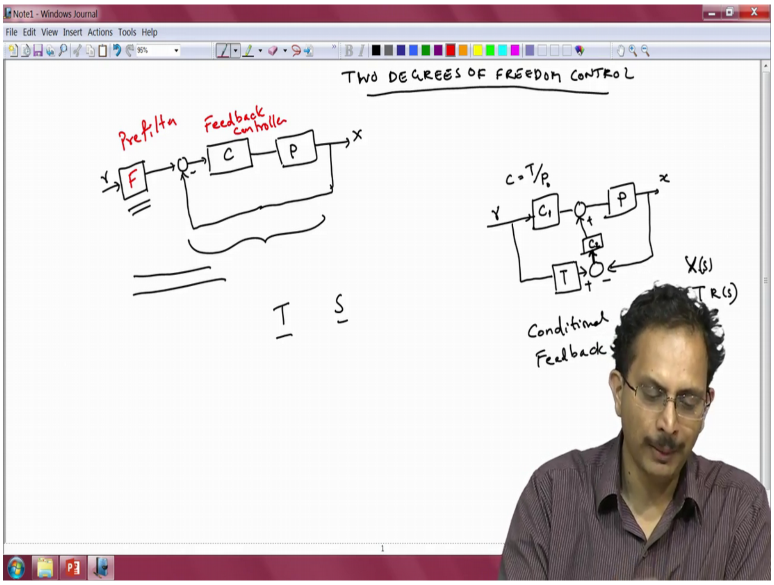

So, we have a single controller C. Now, we discovered that this single controller has significant advantages. So, it allows us to reject disturbances tracked references and so on and so forth but it there are there is scope for improvement, and the problems were the ones that we talked about in the previous clip.

Now, a two degree of freedom control system essentially has one more controller as part of the feedback system. So, this would be just one controller there would be one more controller with which we can achieve performance specifications that we could not achieve with a single controller. Now the first question that one needs to answer when one thinks of adding another controller in the feedback loop is where to position it in this feedback loop.

There are several locations where we can have the other controller; for instance we can have it as part of the feedback path and I shall do that now. I shall remove this feedback, I shall take the output x feed it through feed it to another controller C 2, and the output of that controller I shall feed to my I shall compare with the reference r and the error is fed to the controller C 1 as is shown here.

So, this is one location mainly the feedback path where we can insert the second controller, but that is not necessarily the only location where we can insert the second controller. One other possibility I shall draw now. So, in the feedback path let us say we do not we choose not to insert this controller, but instead we insert it before the reference, but insert it between the feedback system namely this part here and the reference r. So, let us say this is the location where we insert our second controller C 2. So, this is one other configuration that is possible.

In fact, people have looked at other configurations far more clever configurations also. So, for instance there is one other control strategy, which is called conditional control where you have the plant P under you have a controller C. The controller C has been chosen to be equal to C equal to T by P naught, where P naught is the nominal model for the plant P and T is the overall transmission function that we desired.

Now, if one were to apply a reference r and one, but simply feed the output of the controller to the plant, the overall transfer function relating the output x to the reference r will be the transmission function T. But then it is possible that the plant model P and the model P naught are not identical to one another. So, it is possible that you have uncertainty in the plants dynamics. Now, what is done in this case is we take the output x and compare it with T times r. Remember that if the plant P at the exact same model as P naught, then x of s will be equal to T times R of s. Therefore, in the absence of uncertainty in the plant model T times R will be exactly equal to x. So, if you were to take the difference between these 2 this difference will actually be 0.

However if there is a difference between P and P naught, then these 2 signals namely T times R and x will be different from one another and that difference is going to manifest as a non 0 signal in this path here, and that is going to be manipulated by another transfer function, which we shall now call C 2 and this will become the first controller C 1 and that is fed back to the plant. So, you notice in this case that you have this feedback path operational, only when there is an error between the actual plant model and its nominal model hence this kind of control strategy is called conditional feedback.

Here to we use 2 controllers to accomplish our feedback objective of achieving robustness in tracking this reference r even in the presence of plant uncertainty in other words the difference between P and P naught. Now, there are other fairly clever configurations of C 1 and C 2 the people have come up with and that have been reported in literature and the first question that we need to resolve in this clip before we can go any further is which of these multiple configurations do we adopt for our particular requirement.

Now, to answer this question what one needs to recognize is that, regardless of which configuration we use, regardless of whether C 2 is acting as a pre filter and appears immediately after the reference r and before the feedback part of the system or whether it is part of this feedback conditional feedback system. Or whether it is in the feedback path as we first drew, ultimately there will be only 2 transfer functions associated with the overall feedback system, that we can independently tune. And those 2 transfer functions will be the transmission function T and a sensitivity function S. So, regardless of which of these structures we use, we will ultimately have the ability to tune only 2 functions independently.

Even if we insert more controllers let us say we have C 3 in the feedback path here or we have other controllers elsewhere, we will see that they will their effects will all get lump together and at the end of the day, you will have the ability to independently design only 2 functions, mainly the transmission function and the sensitivity function. So, exactly which one of these different configurations we adopt is nearly a matter of choice. From a fundamental prospective, it does not matter what can be accomplished with one configuration can in principle be also accomplish with a second a different configuration. The difference though might arise from the point of view of practical realization, but that is outside the scope of this course, because in this course we have focused on the theory of feedback control.

So, anyone of these multiple configurations would be suitable for us. In this clip and in future clips we shall adopt this particular configuration, the one that I have shown to the left. And in this case we shall not call this C 1 and C 2, but we shall call the transfer function C 2 as F or in other words the pre filter. And the transfer function C 1 we shall rename it as C, which was what we have been using all alone and we shall call it the feedback controller.

So, the reason for choosing this particular names are obvious, the feedback controller is the controller it is part of the feedback loop, the pre filter is one that is not part of the feedback loop. But is cascaded with the feedback loop and appears in between the reference r, in other words it filters the reference r and feeds this filtered signal to the feedback system and hence it is called as a pre filter.

In all our future design examples, we shall stick with this particular configuration for the 2 degree of freedom control system. Having talked about the particular configuration that we would adopt for doing feedback control design, we shall now take a look at the use of 2 degree of freedom control. If we go back to the previous clip, we saw that there were 2 problems that we highlighted. The first problem that we highlighted was our inability to reject measurement noise while achieving high bandwidth for reference tracking that is one problem and the second problem was our inability to design the sensitivity function and the transmission function independently.

So, what we shall do here you see, how the 2 degree of freedom control structure will allow us to address the first problem. So, we shall first see that and subsequently we will move to the second problem. So, the first problem is one of measurement noise; I shall draw the feedback block diagram here.

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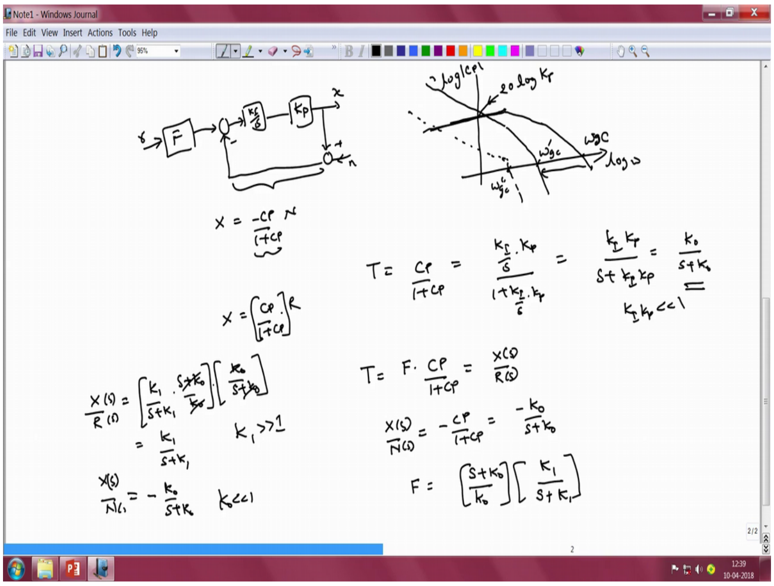

So, we have the controller C, we have the plant P the output of the plant P is x, and when you are measuring this output inevitably you will end up adding this noise n and the overall signal which is the x which is the noise plus the signal x is fed back and compared with the reference r. And, the difference between r and the signal is the error which is fed to the controller c. So, this is the standard one degree of freedom control configuration.

Now, for the sake of simplicity, let us assume that our plant has a certain disagree. So, the bode plot of our plant which is log of magnitude of P versus log of omega, let us say it look something like this. So, magnitude plot has a certain flat characteristic up to your certain frequency and then the corner frequencies of the plant come in to the picture. So, the it will roll off at minus 20 dB per decade or its multiples. So, this is approximately the bode plot of the plant. Now, suppose we want this plant to have 0 tracking error to DC references, then we need to achieve infinite loop gain, but DC and one can easily see that an integrator thus precisely this. So, if you have an integrator as part of our controller structure, then we would have let us replace C here with the term K I divided by s.

So, because we have an integrator we would have the low frequency characteristics to be rolling off at minus 20 dB per decade and the roll off will only increase with when we come to the corner frequency. So, the plant and generally the bandwidth of the closed loop system gets reduced. So, this is actually preferable from the point of view of minimizing the effect of noise, especially if this noise signal here is a wide band signal.

Typically the kind of noise that we come across or that we encounter are wide band signals typically noises of the noises like Gaussian white noise affect our measurement signals and therefore, the only way for us to minimize the effect of noise is to minimize the close loop bandwidth of our system. That is because the transfer function with relates the output x to the noise N is given by x is equal to minus CP by 1 plus CP times N. And therefore, if we make the close loop bandwidth of the transmission function which is CP by 1 plus CP very small, then we will be minimizing the amount of noise we are letting in to our system and getting it to affect the output.

Now, how do we minimize the bandwidth? What I have plotted here is essentially the bode plot of our open loop system or in other words log of magnitude of CP versus log of omega, and this is the plot that we would have when we have a integral controller as our as part of the controller structure. Now, if we want to minimize the effect of noise, we have only one way forward and that is to minimize the closed loop bandwidth further. And we know that there is a relationship between the open loop gain cross over frequency and the close loop band width. So, these 2 number are typically very close to one another. So, one can minimize the close loop band width by equivalently minimizing the gain cross over frequency.

Now, how does one minimize the gain cross over frequency? The only way we can do it if you are stuck with an integral controller is to reduce the integral controllers gain. So, if we were to reduce the integral controllers gain, then our close loop, the bode plot of our open loop system would look something like this. And our gain cross over frequency it was initially the plants gain cross over frequency was somewhere there, that became something smaller with the use of the integral controller and if you attenuate the gain K I of the integral controller, further it becomes even smaller. So, omega g c that we will try.

And a smaller is the gain of the integral controller, the better it is as far as rejection of noise is concerned, but what is the price we are paying for reducing this bandwidth of the close loop system to such low values? The price is obvious; we will not be able to track fast changing reference at all because our close loop bandwidth when it is very small cannot track references, that are out whose frequency content is outside the bandwidth of the overall closed loop system.

And with the one degree of freedom control structure there is really nothing we can do about this. Now if our gain has come down to a very low value, in the frequency range of interest to us control engineers, which is essentially up to the gain cross over frequency. We notice that the plant has nearly flat gain characteristics or the plant can therefore, be modeled approximately as a proportional system. So, I can remake remove the plant transfer function assume that it does not it does not exhibit dynamics, in the frequency range of interest to us namely the frequency range up to omega g c double prime here. And up to the frequency I can model the plant a simple proportionality gain K P. And K P is the DC gain of the plant. So, the gain of this flat characteristic will essentially be 20 times log of K P.

So, if this is the approximate model of our plant and we have a integral controller then the approximate transmission function T which is given by C times P by 1 plus C times P is given by T is equal to K I by s times K P divided by 1 plus K I by s times K P which essentially is equal to K I K P divided by S plus K I K P, now I shall call K I K P as a new constant K naught. So, I shall call this as K naught by S plus K naught.

Now, in general we want K I K P to be much less than 1 to be a very small number for us to achieve an adequately no close loop bandwidth that filters out to a large extent white noise that might affect the measurement signal this as I discussed as the problem that the transfer function with relates to a reference to the output is also given by the same transmission function x is equal to CP by 1 plus CP times R. And hence having a very low bandwidth for the overall close loop system, then severely restrict the range of signals that we can track with our close loop system, and that is a big problem for us as control engineers.

I sketched out this problem using 1 degree of freedom control system, to highlight the fact that there is no simple resolution to the problem of measurement noise if one were to use only a single controller as part of our feedback system. However, if one were to allow another controller to come into the picture, in this case it is going to be the pre filter, then we do not have to make this compromise between the achievable close loop band width as far as packing reference is concerned and the bandwidth necessary to restrict the effect of measurement noise how do we do that? So, if you are using a 2 degree of freedom control system, we would have a pre filter F also cascaded with our feedback system and our reference would be filtered by the pre filter F.

Therefore our overall transmission function, they no longer be equal to CP by 1 plus CP, but it will be equal to F times CP divided by 1 plus CP. Because this overall feedback system as a transmission function of CP by 1 plus CP and that gets multiplied with F and therefore, this is going to be the transmission function that relates X of s to the reference R of s. Now however, as far as the noise is concerned, because the noise does not in any way pass through this pre filter F you would have the transfer function relating the noise to the output namely X of S by N of S to still be equal to minus CP by 1 plus CP.

And what we saw in this particular example that we considered, was this was equal to minus K naught by S plus K naught which is a first order low pass filter or very low bandwidth because we have chosen K naught which is equal to K I times K p to be to be a very small number.

Now, how do we engineer this pre filter F in order not to limit the bandwidth of the close loop system to a very small value, we can see that if we choose the pre filter F to be of the kind F is equal to S plus K naught by K naught in other words we are essentially cancelling the dynamics of our feedback system. And replacing it with the desired dynamics let me call that K 1 and therefore, replacing it with the term of the kind K 1 by S plus K 1 then we would have the overall transmission function from X from R to X namely X of s by R of s to be equal to K 1 by S plus K 1 times S plus K naught by K naught which is the transfer function of our pre filter multiplied by the approximate transfer function of our feedback system, which we have obtained to be K naught by S plus K naught. Now, cancelling K naught and S plus K naught from this expression we would get X of s by R of s to be equal to K 1 by S plus K 1.

On the other hand the relationship between the output X and the noise N is still simply be equal to minus K naught by S plus K naught, where K naught is much less than 1. Now we see that as the result of incorporating a second controller as part of our feedback system, we can get 2 different expressions for the transfer functions that relate the output to the reference and the output to the noise.

Now, if we want our references to be tracked adequately well, even for relatively fast changing references all we need to do is choose K 1 to be a very large number. So, if K 1 is made adequately large, then we can track adequately fast changing references using our system. However, the same control system will still be able to minimize the effect of measurement noise, because the transfer function relating the measurement noise to the output is not decided by K 1, but is instead decided by K naught which by design we have chosen to be a very small number.

So, this ability to independently pick the transfer functions relating the noise to the output and the reference to the output, and choosing the transfer function relating the relating the reference to the output to have a very large bandwidth, while having while choosing the transfer function that relates the noise to the output to have a very small bandwidth highlights the advantage of using 2 degree of freedom control to suppress the effect of noise, while simultaneously being able to track fast changing references. Now that we have seen, how we can exploit the second controller? In order to accomplish what was impossible to accomplish using the 1 degree of freedom control architecture, let us take another example where we show how we can achieve robustness of the overall closed loop systems response to variation in plant parameters.

So, this essentially addresses the second drawback we pointed out, in connection with one degree of freedom control. Namely our inability to independently engineer our transmission function and the sensitivity function and a consequence of that was that our transient response of the close loop system was affected significantly, when our plant parameters change. Now, let us see how that can also be addressed by using 2 degree of freedom control, let us do it with the help of a numerical example.

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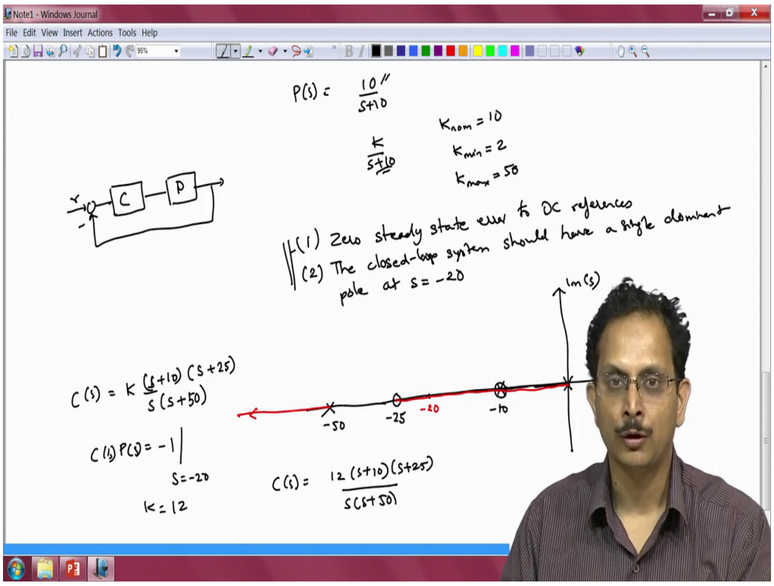

Suppose we are given a plant P of s given by P of s is equal to 10 by s plus 10. Now the nominal gain of the plant is 10. However, we assume that there is enormous uncertainty associated with the gain of the plant. In particular we shall assume that our plant can in general be written as K by s plus 10 where the nominal value of the gain K is 10 exactly as we have written out here, but K can assume a value as low as 2 and as high as 50. In other words the gain can fall by a factor of 5 or can increase by a factor of 5. So, what it indicates is that here is a plant that is that is extremely uncertain there is a 25 fold variation in the minimum and the maximum values of the gain that this plant can assume.

Of course you might ask very legitimately if there are examples of plants which can whose gains can change by a factor of 25 during their operation or from one plant to another. Perhaps there is no such plant, but the objective of assuming such a large variation in the plant parameters, is to showcase the capability of the control strategy that we would be discussing now in order to handle even such large variation in plant parameters.

So, suppose we have a plant of this kind which has a huge amount of uncertainty associated with it, let us assume that we want this plant to have 0 steady state error to DC references. So, we want to design a feedback system for this plant with the following 2 specifications. The first specification is 0 steady state error to DC references, that is one requirement that this close loop system should have. A second requirement is that the close loop system should have a dominant pole a single dominant pole, but S is equal to minus 20. So, the close loop system should have a single dominant pole at S is equal to minus 20.

So, what we shall do now is, first assume that this problem can be tackled using one degree of freedom control design and we will see that it is possible to accomplish both the objectives that we have laid down in this case for this nominal plant P of s is equal to 10 by S plus 10.

However when the plants parameters vary one degree of freedom controller does not permit us to control the variation of the closed loop pole, the dominant close loop pole of our system, and hence when the plants gain varies by a factor of 25, when it drops from 10 to 2 or when it increases from 10 to, from 10 to 50. Then we will see that there is going to be corresponding huge variation in a transient response of our closed loop system which we cannot do anything about if one was stuck with a one degree of freedom control structure.

So, let us first undertake one degree of freedom control design, to do that let us use root locus as our design tool of choice. So, the root locus plots the real part of s versus the imaginary part of s and we know that the plant has a pole at S is equal to minus 10, and one of the requirements of the close loop system is that it should have 0 steady state error to DC references, and this in turn implies that we need to include an integrator as part of our controller.

So, if we have a one degree of freedom controller here, then the controller should include an integrator as well. So, for the moment we are assuming that we have a one degree of freedom control structure, just to understand the fact that it is not possible to make sure that these specifications are met in the presence of plant uncertainty.

So, when we have an uncertain plant or a plant whose gain is not known and can vary anywhere between the limits of 2 and 50, then these particular specifications cannot be met. So, for the nominal case however, we have the pole at minus 10, we need to include an integrator; if we simply include an integrator and have a pole at minus 10, it is clear that the second specification cannot be met that is because the root locus of for these 2 open loop poles would look something like this, it would go off into the imaginary axis in between these 2 poles and it will not even pass through the point S is equal to minus 20, which is where we want our dominant close loop pole to light. So, how do we address this issue?

There are multiple ways in which this problem can be approached, I shall adopt one technique. The first thing that we need to make sure in order for the dominant close loop pole to lie at S is equal to minus 20, is to ensure that the root locus passes through the point S is equal to minus 20. At the moment the root locus is not even passing through the point S is equal to minus 20 therefore, what I shall do is, I shall first cancel the plants pole at x is equal to minus 10 with a controller 0. So, I shall place a controller 0 at S is equal to minus 10. So, our interim structure for the controller C of s is it has an integrator and it has a 0 S is equal to minus 10. So, it has a term of the kind S plus 10. Now that will allow the root locus to pass through the point S is equal to minus 20 and I shall place a 0 slightly to the left of S is equal to minus 20 into which the root locus can sink.

So, this shall be the location where I shall place the 0, I shall place as this is equal to minus 25. So, our interim control structure would look something like this, it would have a 0 of S is equal to minus 10, a 0 what S is equal to minus 25. And as a consequence of this our root locus would start from the open loop pole at S is equal to 0 and go all the way to the open loop 0 at S is equal to minus 25 and along the way pass through the point S is equal to minus 20, which is where we want our close loop dominant pole to like.

Now, if you look at this structure for the controller, we notice that it is non causal because the numerator polynomial has a degree 2 which is greater than the degree of a denominator polynomial. So, we have to add one more pole in order to make sure that we have a causal controller. And I shall choose to add that pole to the left of the 0 at S is equal to minus 25 and in particular I shall choose to add it at S is equal to minus 50. To we have another open loop pole at S is equal to minus 50. As a consequence of this pole we would have another branch of the root locus that will be going towards s minus infinity starting from S is equal to minus 40.

Hence we would have 2 poles for the close loop system, one pole would lie on the first branch which is between S is equal to 0 and S is equal to minus 25 and a second pole would lie on the second branch which is between S is equal to minus 50 and S is equal to minus infinity. Somewhere to the left of S is equal to minus 50 and since the close loop pole in the first branch is closer to the origin compared to the close loop pole on the second branch, this pole is going to be the dominant pole of our system. Now, if you want that pole to be exactly at S is equal to minus 20, then we need to choose the gain of the controller K in such a manner that C of s times P of s is equal to minus 1 at the point S is equal to minus 20.

Now, if we were to compute that gain, we would get the gain K to be equal to 12. So, if we choose our controller C of s to be equal to 12 times s plus 10 times s plus 25 divided by S times s plus 50 then we would have the dominant pole of the close loop system to lie at the point S is equal to minus 20, the second pole of the close loop system will lie somewhere to the left of S is equal to minus 50 and therefore, will not be a dominant pole of our system. Hence both these specifications can be met using one degree of freedom control, but the problem arises when we have to take into account the variation in the plants model.

What we have done here is perform the design for the nominal plant. When plants gain changes when it reduces to a value that is up to a factor of 5 from its nominal value or when it increases by a factor of 5 from its nominal value then the close loop response with this particular controller can change dramatically.

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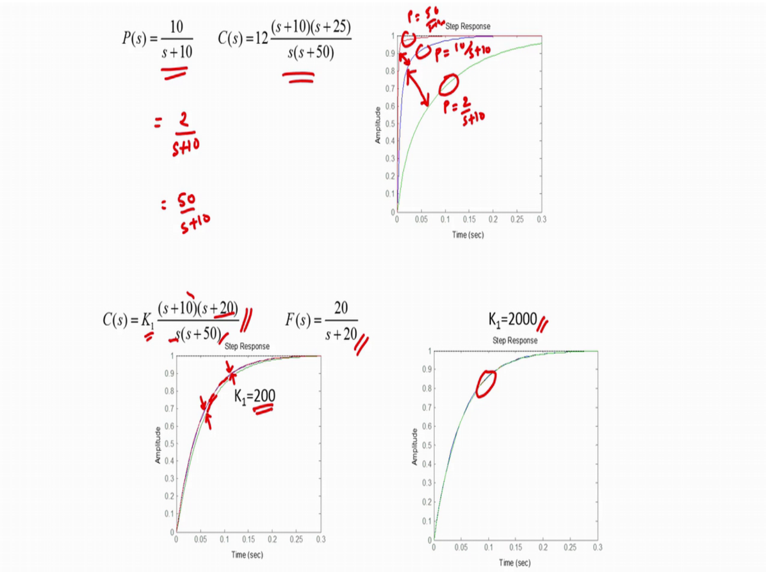

And that is precisely what has been plotted in this slide. So, what you see here? As the blue response is a response of the close loop system, for the nominal plant P which is equal to 10 by s plus 10. So, this is the close loop response for this particular controller that we just design for the nominal plant.

If the plants gain were to drop by a factor of 5 in other words if P of s were to become equal to 2 by s plus 10, then if we were to plot the step response of the close loop system namely the step response of C times P by 1 plus C times P, then what we would get this is green curve here. So, this is a curve that we would get when P is equal to 2 by s plus 10. The blue curve as I said is when P is equal to 10 by s plus 10. So, the gain is its nominal value likewise when the gain of the plant were to increase by a factor of 5 or when the plant has a transfer function of the kind 50 by s plus 10, then our close loop response will look as shown by the red trace here. So, this is for the case when P is equal to 50 by s plus 10.

As you can see from these responses there is a wide variation in the response of the close loop system, when the plants parameters change. It is only for the nominal gain of the plant that the response shows its dominant dynamics that S is equal to minus 20. For other gains of the plant the close loop poles would be would not any longer be at S is equal to minus 20 and therefore, the dominant dynamics for the other gains of the plant are significantly different than what it was for the nominal case. Now how do we address this problem? As we discussed there is nothing much we can do if we were to stick with one degree of freedom control.

However this problem can be effectively addressed by employing 2 degree of freedom control. The insight to solve this problem using 2 degree of freedom control design is comes primarily from root locus based design approach. So, let us come back to the root locus of our system.

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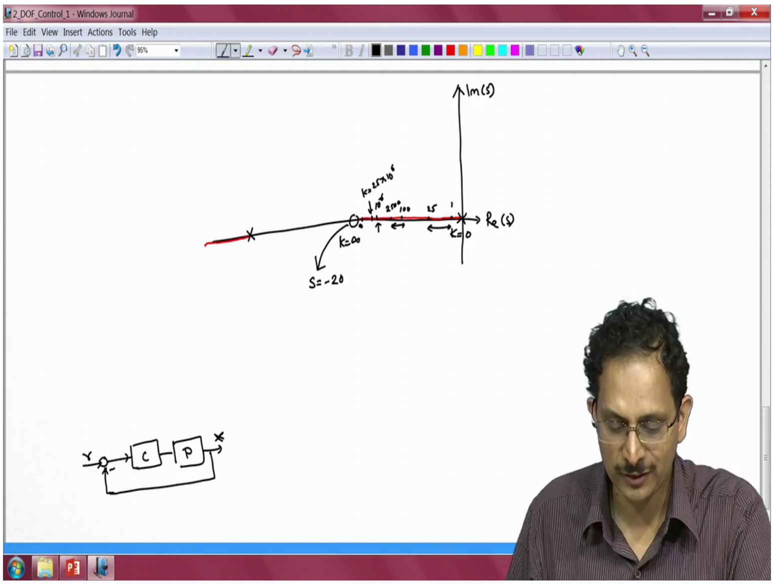

So, we have an integrator and we have cancelled the pole that we had at S is equal to minus 10 with a 0. So, that has not been represented in the root locus and then there is a pole at there is a 0 at S is equal to minus 25 for the moment, I will not indicate exactly the précised position of the 0 although in the previous design we assumed it to be at S is equal to minus 25 let us leave it as it is for the moment.

And of course, there is another pole here that S is equal to minus 50 and that contributes to a branch of the root locus to the left of that pole, and that is anyway going to result in stable dynamic which is not going to dominate the dynamics of the overall system. So, let us not talk about that either at the moment. So, let us now focus on how we can employ root locus to minimize the variation in the close loop pole position due to variation in the plants gain. So, although nominally we could get the close loop pole to be located at S is equal to minus 20, when the plants model changed there was nothing at one degree of freedom control could do, it resulted in a corresponding change in the close loop pole location and that in turn translated to a variation in the transient response of the overall close loop system.

Now, how do we minimize a variation of this close loop pole? If you can minimize the variation of the close loop pole, then we can be sure that even in when the plants gain were to vary by a factor of 25 namely if it were to fall by a factor of 5 or to increase by a factor of 5, there is going to be minimal effect of this variation on the dominant dynamics of our close loop system.

So, let us first try to understand what trick what insight we can adopt in order to minimize the variation of the close loop pole. To do that what I want to first underscore is that the root locus starts at an open loop pole and ends in an open loop 0. And the gain of the root locus of the open loop system near the open loop pole will be 0. So, the gain near the open loop pole is 0 and as the gain is increased the closed loop pole moves away from the open loop pole position and towards the open loop 0. So, at the open loop 0 the gain K is infinity. So, you have this wide variation in gain from K going from 0 to infinity, compressed within this finite space in between the open loop pole and open loop 0.

Now, when our plants gain changes, the open loop systems gain also changes by a factor of 25. Because our plant gain can reduce by a factor of 5 or increase by a factor of 5 and there is therefore, a 25 fold variation in the plant gain that translates to a corresponding 25 fold variation, in the overall loop gain of our system mainly that of the plant times the controller. Now, what happens if our close loop pole was located close to the open loop 0?

So, let us say at this position my gain K was 1. If the plants gain were to increase by a factor of 25, the close loop pole will move to a new position somewhere here. So, this is the position at which the close loop the gain of the open loop system is 25. And if you notice if you are close to the open loop pole there is going to be a huge change in the close loop pole position, when there is a change in the gain by a factor of 25; however, if you come closer to the 0 the gain would have increased. So, if I am somewhere here for instance let us say the gain at this location the open loop gain K was 100 at this location hypothetically. If the gain were to increase by a factor of 25 and become let us say 2500, the close loop pole position will change from this location, where it is where the gain is 100 to another location where the gain is 2500.

But if you notice the extent by which the closed loop pole changed its position, due to the same factor change in the gain namely a factor of 25 is now much lesser then what it was when the gain changed from 1 to 25. So, the variation of the close loop pole when the gain changed from 1 to 25 actually is larger than the variation of the close loop pole, when the gain change from 100 to 2500. The factor of change is the same, but the extent the amount of variation of the close loop pole has reduced because you are now closer to the open loop 0.

Now, if we are even closer to the open loop 0, in which case we are assuming that the initial gain is very high let us say our initial gain is 10 power 6 1 million, then if our gain were to change by a factor of 25 from 1 million to 25 million, our close loop pole will move from this initial position indicated by this arrow here to another position which is not going to be far away from its initial position. So, this is going to be the position at which the gain K will be 25 into 10 power 6. So, what you see therefore, is that if our open loop gain K is high, then the same factor of 25 variation in the gain will result in correspondingly smaller variation in the location of the close loop pole.

Hence, if we are very close to the 0 in our words if our gain is extremely large 1 billon or 10 billon or something like that hypothetically, then a variation of that gain by a factor of 25 will result in a practically negligible variation in the location of the close loop pole. Because this scale, in this scale the gain K is going from 0 to infinity. So, as you approach the open loop 0 the a huge range of gains K get compressed within a very narrow region of this root locus and hence even fairly large changes in gain K will result in correspondingly very small variation in the close loop pole position.

And this therefore, gives us the clue as to what we need to do in order to restrict the variation of the close loop pole, the answer is straight forward. Based on our analysis here we have to make sure that our open loop gain is adequately high and our 0 is placed close to the location where we want our dominant dynamics to lie.

So, therefore, if we choose to place the 0 at the location S is equal to minus 20, because this is the location where we want the dominant pole to lie then and we increase the open loop gain of our system to a very large value, then what we are ensuring is that our close loop pole because the gain is very high is going to be situated very close to the open loop 0. So, it is going to be somewhere here for instance and because this is so, close to the open loop 0 even a large change in the gain by a factor of 25, we will result in a negligible change in the location of the close loop pole. It will become it will come a little bit closer to the open loop 0, but not by a significant amount.

Hence the trick for restricting the variation of the close loop pole, because of variation in the plants gain is to simply add a 0 very close to the location where you want the dominant close loop pole to lie number 1 and ensure that the open loop gain is adequately high that the close loop pole is located very close to the 0. Now, any variation in gain to result in a negligible difference in the position of this close loop pole in reference to the position of the 0. So, this is the trick that we would be adopting in this case as well.

Hence in this design we shall choose not to place the 0 at S is equal to minus 25 as we did in the previous case, but instead we shall choose to position the 0 at the location S is equal to minus 20; now if we were to do that. So, we would have this open loop 0 at S is equal to minus 20 our controller transfer function C of s will have the term s plus 20 in the numerator and you will have the term s plus 50 in the denominator and you have an integrator.

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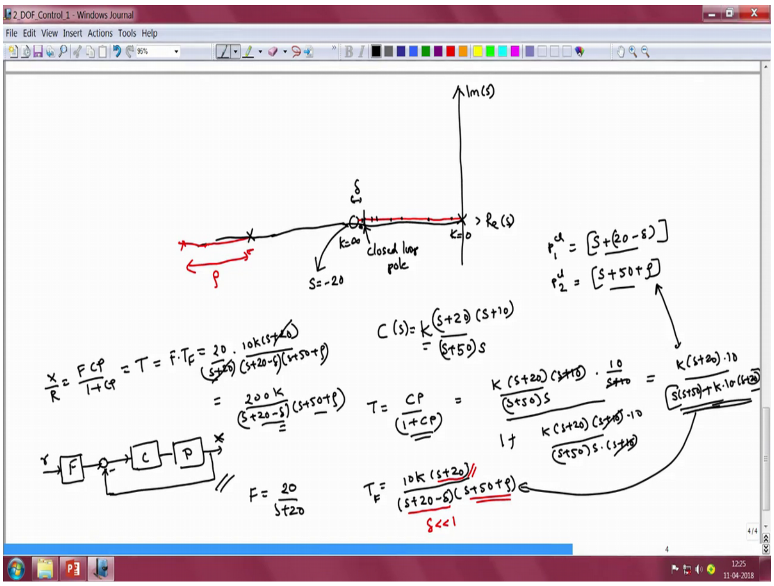

So, you have s also and since we have cancelled the plant pole by means of a 0 you will have the term s plus 10 also in the numerator and we have to make sure that the controller gain K is very high so, that the close loop pole is going to be located very close to the point S is equal to minus 20, and when the open loop gain of the plant changes by a factor of 25, it would result in a very small migration of this close loop pole about the nominal position.

Now, this works very well with the exception that there is a small problem in this control design. So, if this is our controller structure and we have chosen our gain K to be adequately high. So, that our close loop pole is located at a very small distance delta away from the open loop zeros position. So, let us say the difference between the position of the open loop 0 and the close loop pole is delta. So, this is the location at which our close loop pole is situated. Then what we would have as one of the poles of our close loop system would be a term of the kind S plus 20 minus delta. So, this would be one of the close loop pole, I shall call it p c l 1 this is a first close loop pole.

Now, since we would have 2 close loop poles because we have 2 branches to the root locus, the second close loop pole would be located on the second branch which is to the left of the point S is equal to minus 50. So, it would be located somewhere here. So, let me call the distance of that close loop pole from the point S is equal to minus 50 as rho, in which case our second close loop pole will be of the kind p c l 2 is equal to S plus 50 plus rho. So, we would have 2 close loop poles for our system. Now our overall transmission function if one were to stick with one degree of freedom control design would be given by T is equal to C times P divided by 1 plus C times P.

Now, we note that 1 plus C times P is a transfer function, whose 2 zeros are going to be given by definition by p c l 1 and p c l 2. These are the zeroes of the transfer function 1 plus C times P or equivalently they are the close loop poles of our system. Now the transmission function therefore, will look in this particular manner since we have C times P in the numerator of this transmission function, the zeroes of the transmission function will also be the zeroes of the controller and the plant.

So, we have our controller to be of the form K times S plus 20, times S plus 10 divided by S plus 50 times S and we have our plant to nominally be of the kind 10 by S plus 10, and our denominator of the transmission function would be 1 plus the same term 1 plus K times S plus 20 times S plus 10 times 10 divided by s plus 50 times S times s plus 10.

So, upon simplification, you see that the term s plus 10 gets cancelled from the numerator and the denominator. So, the overall transmission function would look something like this. It would be K times s plus 20 times 10 divided by once again the denominator you would have the term s plus 10 getting cancelled you would have S times s plus 50 plus K times 10 times s plus 20. So, you would have a second degree polynomial as the denominator polynomial of our close loop system and this denominator polynomial would have p c l 1 and p c l 2 as the 2 terms as the 2 as its 2 factors. So, we can therefore, write our transmission function T as K times S plus 20 times ten. So, I shall bring the 10 onto the left hand side. So, 10 K times s plus 20 would be a numerator polynomial of a transmission function and the denominator polynomial would essentially be s plus 20 minus delta times s plus 50 plus rho.

So, because we know that our close loop poles are going to be located at these particular locations on the 2 branches of the root locus, I have directly written down the factors of the denominator polynomial of the transmission function and in this particular manner. Now, this expression here reveals an important problem.

If you notice the numerator of the transmission function you have s plus 20 in the numerator that is because the 0 of an open loop system will also be the 0 of the closed loop system. And then you have that term s plus 20 minus delta in the denominator where by design by choosing our open loop gain to be very high, we have made sure that this term delta is much less than 1. So, it is located very close now close loop pole is going to be located very close to the open loop 0, and hence its (Refer Time: 52:56) not vary too much. We have therefore, guarantee that the dominant pole which is the pole at in the vicinity of S is equal to minus 20 does not vary too much, but then this particular expression reveals a serious problem it reveals the problem that the numerator of this transmission function, almost exactly cancels the denominator of the transmission function.

In the numerator you have the term s plus 20, in the denominator you have the term s plus 20 minus delta where, delta is a very small number. So, these 2 almost exactly cancel one another. With the consequence that although we have made sure that the close loop pole at the point S is equal to minus 20 does not vary too much by locating the open loop 0 near this pole, the consequence a tragic consequence of this cancellation is that the overall dynamics is going to be determined by the pole that is far away at the location S is equal to minus of 50 plus rho.

So, we have therefore, despite our effort in restricting the variation of our close loop pole one degree of freedom control system does not allow us to get that to be the dominant dynamics and does not allow us to restrict the variation of the dominant dynamics. So, what is the way forward? Of course, this problem cannot be solved using one degree of freedom control design, if you look at this expression you see that the real villain in this story is this 0, yes s plus 20 that appears in the numerator of the transmission function. It is this 0 that cancels the dominant dynamics that we have designed, very carefully and ensure that the dominant pole does not vary too much. And if the 0 were not there, then we would have the term S is equal to minus 20 plus delta to be the location of the dominant pole of our close loop system and this by design would not change too much when our plants gain were to change by a factor of 25.

So, how do we do that? To do that we add a pre filter to our feedback system. So, instead of providing the reference directly to the feedback system, we provide the reference as an input to the pre filter F. And this pre filter is chosen precisely to address the problem that is been contributed by this 0. If the 0 were not there then this problem of suppression of dominant dynamics will go away, hence we choose the pre filter to cancel this 0, in other words we choose the pre filter to have the form 1 by S plus 20. So, that the overall transmission function from the reference all the way to the output is going to be given by X by R is equal to F times CP divided by 1 plus CP. Now F we have chosen it to have a form 1 by S plus 20, but unfortunately the term 1 by S plus 20 has a DC gain of 1 by 20, and we want our close loop system to have a DC gain close to unity.

Hence in order to make sure that this term has a DC gain of unity, we use we replace the one in the numerator by 20. So, that when omega is close to 0 the DC gain of the pre filter is 1; the DC gain of the feedback controller will also be 1 because you have an integrator as part of our feedback control system and therefore, our DC gain for the overall system is going to be close to 1. So, if you were to now multiply F and transmission function. So, this is the if we have a 2 degree of freedom control system they should be the transmission function for only the feedback part and actually therefore, represented as T F. So, this is going to be equal to T this is a overall transmission function and that is going to be equal to F times the transmission function for the feedback part which is T F.

And if you were to substitute the expressions look at this to be 20 by S plus 20 times 10 K times S plus 20 divided by S plus 20 minus delta times S plus 50 plus rho. Now, we see that in this overall transmission function we would have the 0 of T F being cancelled by the pole of the pre filter F. So, that the overall transmission function would be 200 times K divided by s plus 20 minus delta times s plus 50 plus rho.

Now, we have 2 just 2 poles for the close loop system one is at minus 20 plus delta and the other is at minus 50 minus rho, and between the 2 since the first pole is closer to the origin that will be the dominant pole and since this pole has been placed very close to the open loop 0 this is equal to minus 20, when the open loop gain of the plant changes by a factor of 25, this change in delta will be exceedingly small. So, this is the trick that we can employ in order to restrict the variation in the dominant dynamics of the close loop system by using 2 degree of freedom control architecture.

So, we place a 0 at the location close to where we want our dominant pole to be located, and we ensure that the open loop system with adequately high gain that our close loop pole will be located exactly in the vicinity of the 0, and when this is done variations in the gain of the plant will result in negligible variation in the location of the close loop pole, and hence location of hence of dominant dynamics.

The problem; however, as we discussed is that with a one degree of freedom control structure this dominant pole will be cancelled almost exactly by the open loop 0 that we have placed and therefore, it will sees to be the dominant dynamics of the system. To address this problem we use the pre filter in order to cancel the 0 that we have placed and that allows us to retain the dominant dynamics of our overall system exactly where we wanted it to be.

Now, this has been implemented so, the same. So, this control system has now been changed to a 2 degree of freedom control system. So, we have chosen the pre filter to be of the chi F of s is equal to 20 by s plus 20, and our feedback controller as we discussed is gonna have a 0 at s plus 20 at of the term of the kind s plus 20, we have the term s plus 10 which cancels the plants pole, we have the integrator which is intended to achieve 0 steady state error for tracking DC references, and then we have the term S plus 50 in order for us to have a causal control system.

Now, I have chosen an adequately high gain K 1 for this controller in this case I have chosen K 1 to be 200 and when you choose such a high gain for the controller we see that when the plants gain varies by a factor of 25 in particular, when the plants gain falls from its nominal value of 10 to 2 the close loop response changes from this blue trace which is given here to the red the green trace which is given there. So, you might want to compare the variation that you have in the close loop transient response in case of one degree of freedom control, its a huge variation as I have pointed out here.

In contrast the variation that you have between the close loop response with the nominal plant and the close loop response with the plant of reduced gain is exceedingly small. The results are even more impressive for the case when the gain increases by a factor of 5 and goes to 50. So, for the one degree of freedom control case even here there is a huge variation in the transient response, the red trace in the first graph is very different from the blue trace.

On the other hand by using 2 degree of freedom control you can see that there is practically negligible variation in the transient response of the close loop system, when the gain has increased by a factor of 5 from its nominal value. That is because the red trace that you see in this curve, and the blue trace that you see these 2 are sitting almost exactly one on top of another.

Now, we can expect to do even better if we have higher gain, anyway we have a system with only 2 close loop poles and the root locus analysis indicates that the system can never be destabilized and therefore, if we were to pick at even higher gain for our controller. In this case I have chosen a gain of 2000 you can see that all the 3 responses the response of the close loop system when the plants gain is 10, when the plants gain is 2 and when the plants gain is 50 all exactly overlapped.

So, it is as though the close loop system does not recognize the fact that, there is such huge variation of uncertainty associated with the plants dynamics. So, what this example illustrates is that, even though you could have very large variation in the plants dynamics in this particular case we have considered the gain of the plant, by adopting this 2 degree of freedom based control strategy, it is possible for the close loop system to be robust to not be sensitive to a variation in the close loop, in the parameters of the open loop system in particular the uncertainty associated with the gain of the plant.

Thank you.