Control System Design

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Lecture – 26
Limitations of 1-degreeof freedom control

Hello. In all the previous clips where we discussed control system design we have focused on design of 1 degree of freedom control systems, or control systems where we have only one controller available for us to tune.

Now, we have discussed a variety of the shelf controllers that will do the job for us given disturbance rejection, specifications, robustness, requirements, and so on. We have also looked at some special controller such as the internal model controller which we can exploit when we know something particular, such as a frequency of a disturbance or the reference we want to track and so on and so forth.

But, at the end of the day 1 degree of freedom control systems are not without their limitations, and what we shall do in this clip is to take a look at a couple of important limitations of 1 degree of freedom control systems, which would subsequently set the stage for us to discuss slightly more sophisticated control systems; namely 2 degree of freedom control systems, where we would have 2 controllers available to us for tuning.

Now, the most obvious and important drawback of a 1 degree of freedom control configuration is the fact that, it is sensitive to measurement noise. This is something that we looked at fairly early on in this course. And to make that point again I have drawn this block diagram of a control system, where we have a single controller C and the plant P.

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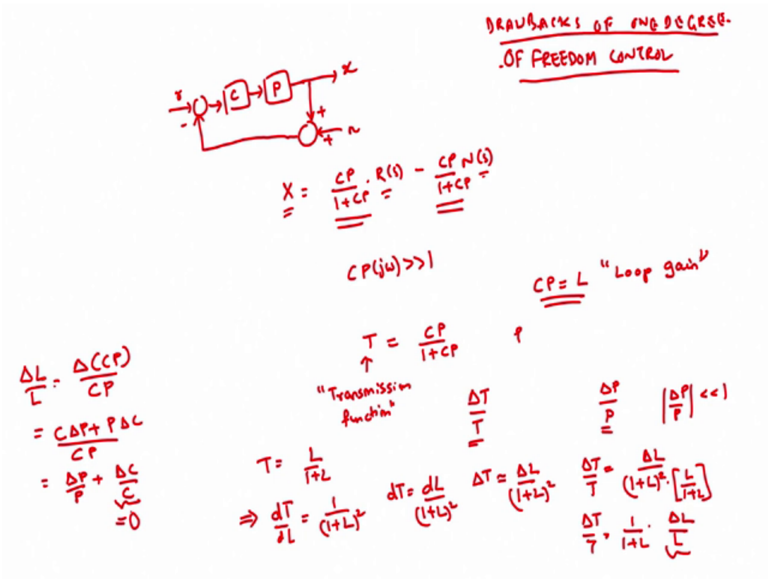

The output of the plant is of course, measured by a certain measurement system and is fed back and compared with the reference R. But invariably when we are making a measurement we cannot avoid some measurement noise n keeping in and affecting the actual measurement.

So, what are essentially therefore, feeding back is not the exact measurement x, but we are feeding back x plus n. Now if we were to write down the transfer function that relates the output X to the reference R and the noise N what we see is that, the transfer function for both of them is exactly the same namely CP by 1 plus CP with the exception that there is a negative sign, that multiplies this transfer function when we are looking at the relationship between the output and the noise.

But magnitude wise they are both the same and what is the disadvantage of this particular fact? It is that as control engineers we want our loop gain to be high within the frequency range there we want references to be tracked. In other words our CP has to be CP of j omega has to be much greater than 1 within the frequency range where we want to track references and reject disturbances and so on and so forth.

But in our effort to track references and reject disturbances, we also end up letting in noise within the same frequency range. And as you can see there is nothing you can do about a independently suppressing the effect of noise in comparison to the reference, in this 1 degree of freedom control architecture. So, over the entire frequency range where we are interested in tracking references we are also going to be letting in measurement noise. And that is going to affect our output and that is something that we do not desire and we do not wish for it to happen, but there is nothing we can do about it as far as 1 degree of freedom control structure is concerned.

The second drawback has to do with the sensitivity of the transient response of this ho closed loop system, to variation in plant parameters. So, to remind you the relationship between the overall transmission function T, and the open loop transfer function C times P is given by T is equal to CP by 1 plus CP. And in the previous clips I have introduced the term CP the product of the controller transfer function and the plant transfer function as what we call as L or in other words a loop gain. And the term T of course, we have introduced as the transmission function.

Now, an important question that one might ask is whether this transmission function will remain the same, when we have an uncertain plant on our hands. So, when the plant P is uncertain or it has parameters whose exact numerical values we do not know, and are therefore, different from the nominal values if we might have assumed during design or they have parameters that may be changing slowly with time. Then it is obvious that our transmission function will also change when the plant model changes. The question that we want to answer is, how much relative change do you have in the transmission function which I have denoted by that term delta T by T, how is this relative change or fractional change in the transmission function, how is that related to a small fractional change in the plant transfer function.

Now, this will allow us to identify one other problem associated with 1 degree of freedom control design; namely the sensitivity of the transient response of the closed loop system to changes in plant parameters. So, let us first try to derive the relationship between delta T by T, which is a fractional change in the transmission function and delta P by T, which is a fractional change in the plant transfer function for small changes in the plant transmission function. Other words when delta P by P is much less than 1 what is the relationship between delta T by T and delta P by P that is what we are out to do.

So, to do this it is a little bit of algebra. So, we know that T is going to be given by L by 1 plus L. So, what this implies is that dT by dL is going to be equal to 1 by 1 plus L the whole square or in other words dT is equal to dL by 1 plus L the square. Now we can replace dT with delta P for small changes in the transmission function T, and write the delta T is approximately equal to delta L by 1 plus L the whole square for corresponding small changes in the loop gain L.

Now, therefore, delta T by T which is the fractional change in the transmission function is going to be equal to delta L by 1 plus L the whole square times T, and in place of T I shall write L by 1 plus L, because that is a definition for T. And what that gives us is that delta T by T is going to be equal to 1 by 1 plus L times delta L by L.

Now, let us focus on this term delta L by L, I shall write this separately on the left hand side delta L by L is going to be equal to delta of CP divided by CP. And this in turn is going to be equal to C times delta of P plus P times delta of C divided by CP or in other words it is going to be equal to delta P by P plus delta C by C. Now if focus on these 2 terms, we note that since a controller is a system that we have designed as engineers, there is really no uncertainty associated with the controller structure. Hence we would have that in all cases delta C by C would be equal to 0.

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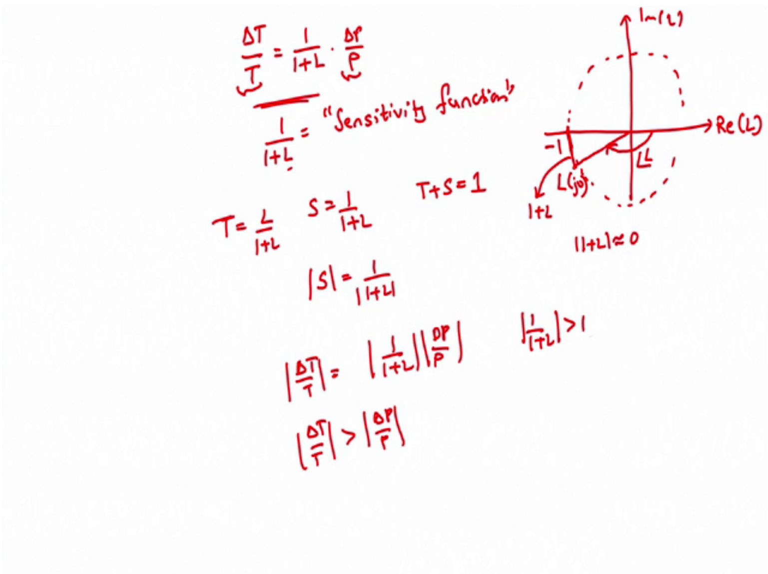

With the consequence therefore that the relationship between the uncertainty in the transmission function delta T by T and the uncertainty in the plant delta P by P is given by delta T by T is equal to 1 by 1 plus L times delta P by P. So, delta P by P represents the fractional uncertainty in the plants model, and delta T by T represents the corresponding fractional uncertainty in the transmission function. And the two are related by that term 1 by 1 plus L.

So, the term 1 by 1 plus L has a special name, because of the relationship that it establishes between the variation in the plants model or the uncertainty in the plants model and the uncertainty in the transmission function. So it is therefore, called as a sensitivity function. The reason it is called a sensitivity function is evident from the equation that I have written here, it is the sensitivity of the overall transmission function to variation in plant parameters.

Now, we note that given the expression for a sensitivity function, which incidentally also happens to be the same transfer function that relates the output of our closed loop system to output disturbances. We note that the actual transmission function which is P which is given by L by 1 plus L and the transmission function S which is given by 1 by L plus L have a special relationship namely that P plus S is always going to be equal to 1 no matter what.

So, if we design the transmission function, then the sensitivity function automatically gets fixed and vice versa. Now how does the sensitivity function yes look as function of frequency, we note that in the low frequency and the mid frequency range of our open loop system, our loop gain L is generally going to be very high, because we would have some performance specification to satisfy in these frequency ranges, either in the form of disturbance rejection or in in terms of robust tracking or some such specification.

Hence in these frequency ranges the loop gain L is going to be very large, with the consequence that a sensitivity function S which is 1 by 1 plus L is going to be a very small number in this frequency range. And if we take the logarithm of the sensitivity function, because if one were to try to try to draw the Bode plot of the sensitivity function. Then we would note that in this frequency range in the frequency range where we are where we are expecting control performance. The logarithm of the sensitivity function is going to be a large negative number, because the sensitivity function itself is going to be a number that is going to be very close to 0.

However, there will be a frequency at which the loop gain will start to reduce in magnitude and at the gain crossover frequency; the loop gains magnitude will be exactly equal to 1. So, in the neighborhood of the gain crossover frequency the loop gain will change its magnitude from some value greater than 0dB to some value less than 0 dB, and what about the phase in the vicinity of the gain crossover frequency? In the vicinity of the gain crossover frequency the phase lag of the open loop system is going to be close to 180 degrees of course, it will not be exactly equal to 180 degrees because there will be a certain phase margin for our open loop system, but it is going to be close to 180 degrees.

So, what will the magnitude of 1 plus L be; therefore, at frequencies that are close to the gain crossover frequency. To answer this question let us draw the Nyquist plot for the loop gain. So, the x axis has of course, a real part of L, and the y axis is the imaginary part of L and the critical point minus 1 is located here. So, when we are near the gain crossover frequency, our loop gain will be located somewhere here, because the phase lag associated with the loop gain will be some negative angle which is going to be still be greater than minus 180 degrees and the magnitude will be close to 1. So, it will be the radius vector for the loop gain L of j omega would have a magnitude close to that of unity.

So, if this is the unit circle, then the loop gain L of j omega will lie somewhere close to the circumference of this unit circle. So, given the location of L of j omega in the nyquist plot, for frequencies omega close to the gain crossover frequency what can we say about the magnitude of 1 plus L? We note that if this is L, then this complex number which starts at the point minus 1 comma 0 and ends at the loop gain is the complex number 1 plus L. And we note that if the angle of L of j omega is close to 180 degrees and its magnitude is close to 1, then the magnitude of 1 plus L will also be close to 0 and what does that imply? It implies then that the sensitivity function the magnitude of the sensitivity function, near the gain crossover frequency which is going to be equal to 1 by the magnitude of 1 plus L is going to be a number greater than one near the gain crossover frequency.

And what does that imply given the definition of a sensitivity function? What that implies then is that, our closed loop system is actually more sensitive to variation in plant parameters in this frequency range; in the frequency range where the magnitude of 1 plus L is less than unity, than the actual plant itself. Because delta T by T is going to be equal to 1 by 1 plus L delta P by P which in turn implies that the magnitude of delta T by T is going to be equal to the magnitude of 1 by 1 plus L times the magnitude of delta P by P and if the magnitude of 1 by 1 plus L is greater than 1, then we would have that the magnitude of delta T by T would be greater than the magnitude of delta P by P.

So, in this frequency range we are actually doing the opposite of what we as control engineers wish to do. In other words, we would end up with a closed loop system, whose response is more sensitive to plant parameter variations in this frequency range than that of the open loop uncertain plant itself indeed.

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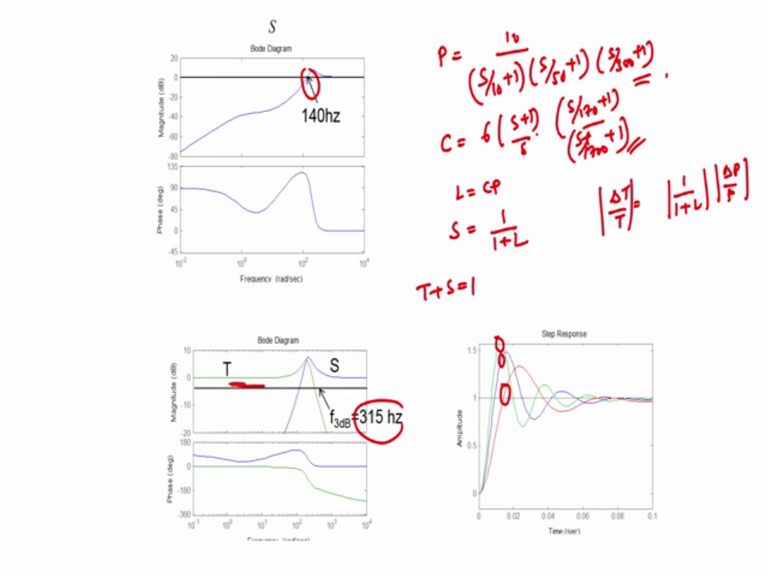

I have in the next slide plotted a sensitivity function for the overall closed loop system that we had, when we when we undertook the design using Bode plots. So, to remind you we had the plant P to be equal to 10 by S by 10 plus 1 times S by 50 plus 1 times S by 300 plus 1 this was the plant transfer function.

And the controller transfer function which was the P I D controller that we ended up designing towards the end of the clip was C was equal to 6 times S plus 1 by S times S by 170 plus 1 divided by S by 1700 plus 1.

Now, for L is equal to C times P, I have plotted the sensitivity function S is equal to 1 by 1 plus L and the magnitude of S as function of frequency and the phase of S with function of frequency is shown here. What is important is the magnitude of S, because that is what is going to determine our control performance, and as you can see it is a very small number in the frequency range where were interested in performance or when it expressed in decibels it is a large negative value minus 60 dB minus 80 dB and so on and so, forth.

A frequency increases the sensitivity also increases, and there is a particular frequency namely 140 hertz at which the sensitivity becomes greater than 0 dB. Now what is implies when we go back to the expression delta T by T is equal to magnitude of delta T by T is equal to magnitude of 1 by 1 plus L times magnitude of delta P by P. What is implies. Therefore is that, for frequencies beyond 140 hertz the fractional change of the overall transmission function is more than the fractional change of the plant itself. Therefore, the overall closed loop system is more sensitive to variations in the plant parameters than the case when we did not have a feedback at all in place.

Now, this might be provided the frequency 140 hertz happens to be well outside the closed loop bandwidth of our transmit of our closed loop system or in other words. If 140 hertz is a frequency it is significantly to the right of the overall bandwidth of T of S then we would not have any problems, because even if our closed loop system is more sensitive to variation in parameters in that frequency range we are anyway not interested in tracking performance or disturbance reduction in the frequency range. So, that sensitivity will not affect our system in any particular manner, but unfortunately that is not the case.

If we were to plot the transmission function T, that has been shown by the curve here we see that its bandwidth is 315 hertz. So, from 140 hertz all the way to 315 hertz, we have a sensitivity that is greater than 0 dB or our closed loop system is more sensitive to variation in the plant parameters, than an open loop system without feedback would be.

And there is nothing we can do about this because T plus S is equal to 1. So, we cannot independently engineer our transmission function to have a much smaller bandwidth than 140 hertz or equivalently we cannot engineer a sensitivity function, to have a much higher frequency at which it reaches 0 dB than 315 hertz. Because of this identity namely T plus S is equal to 1 and there is nothing we can do about it if we are stuck with 1 degree of freedom control architecture.

Hence, the inability to independently design the sensitivity function yes and a transmission function T happens to be an important drawback of 1 degree of freedom control architecture. And the consequence of that is that we are going to have significant sensitivity of the overall closed loop system in the frequency range beyond 140 hertz to changes in the plant parameters. And that in turn manifests itself as large variation in the transient response of our closed tube system when our planned parameters change. So for example: in this case the blue curve here plots the overall closed loop systems transient response step response, for the kind of plant that we have picked and filed for the kind of controller that we have chosen here, this is a nominal plant and this is the controller.

Now, if the plants gain were to increase by 20 percent, then there is a huge change in the step response. So, the green curve here shows the step response of the closed loop system when the plants gain has increased by 20 percent. Likewise, if the plants gain were to drop by 20 percent then the overall transient response of the closed loop system follows the red curve that is shown here. So you see therefore, that when the plants model changes a little bit. In this case its gain as either increase by 20 percent or decrease by 20 percent the transient response of the closed loop system changes dramatically, it goes from the nominal response which is shown by the blue curve to the green curve when the gain increases or to the red curve when the gain decreases and there is a dramatic change in the response.

The question now is, how can we minimize the variation in the transient response of our closed loop system when we have changes in the plant parameters or in other words how can we achieve robustness in the overall response of our closed loop system to variation in plant parameters. It turns out that it is not something that we can effectively accomplish by using 1 degree of freedom control architecture, and in its place therefore, we would have to employ 2 degree of freedom control architecture, where we would have 2 transfer functions available for tuning.

So, both these examples that. I just talked about the sensitivity of the 1 degree of freedom control architecture to noise and our inability to suppress noise in the frequency range where we want to track a certain reference number 1. And number 2 the inability to independently design the transmission function and a sensitivity function, which leads us to this excessive sensitivity of our closed loop systems transient response to variations in plant parameters, turns out to be two important drawbacks of the 1 degree of freedom control architecture. What we will see now is how we can address this issue by having one more controller available for us to tune.

Thank you.