Control System Design
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Lecture – 25
Control of systems with some known parameters (Part 2/2)

Hello. In the previous clip we saw that if we knew something about the disturbance that is affecting our plant we can actually do better than a regular controller in rejecting the disturbance. Or, in a similar manner, if you know something about the reference that we are intending to track and we can do something; we can do much better than what a conventional controller does.

And the tool that allowed us to do it was what we called as a internal model principle. And the central aspect of internal model principle is to include the denominat polynomial of either the disturbance or the reference, that we want to reject or track respectively as per of the denominator polynomial of the controller itself.

Now, just as we could do a little bit better. In fact, we could reject disturbances perfectly in steady state or track reference is perfectly in steady state, if you happen to know their frequency, one would therefore, expect that we should be in a position to do better if we happen to know something more about the plant as well. So for instance, if we have no uncertainty associated with the plant, in other words the plant model is known very well and it is also known not to change significantly with time, then what benefits would accrue as far as control design is concerned.

One straightforward benefit is; that instead of going from closed loop frequency domain, to open loop frequency domain, and performing control design using bode plots or root locus or any of the other techniques and then coming back to the closed loop frequency domain and then looking at the response in the time domain, one can directly synthesize the closed loop transfer function of the overall control system.

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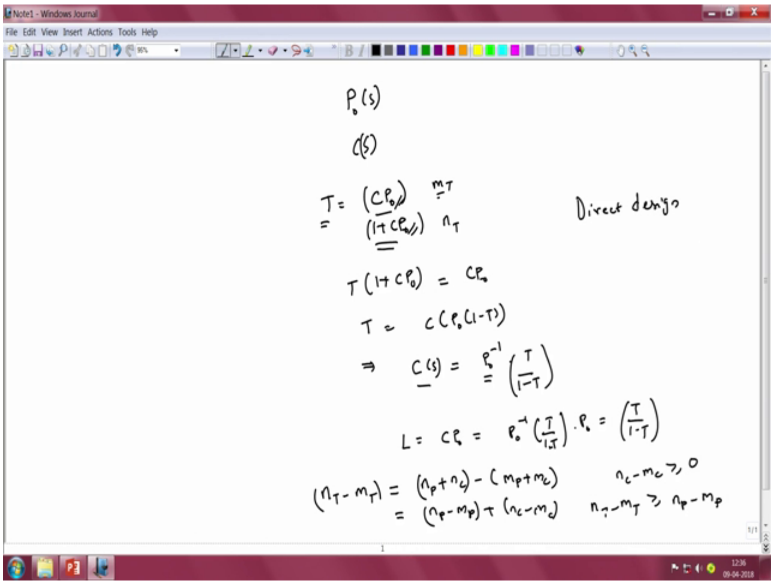

So, for instance if we happen to know the plant model P naught of s very well, then we do not really need to go to the open loop frequency domain, but instead directly obtain that controller C of s which gives us the desired transmission function T; T equal to C times P naught by 1 plus C times P naught. So, what is therefore allow to do; is if we happen to know the plant model P naught very well and we also know that this plant model is not changing with time. So, it is not having one set of parameters now another set of parameters some time from now and so on. Then we can directly synthesize the transmission function T and find that controller C which gives us this transmission function T.

Now, if we rearrange this equation we would have T times 1 plus C times P naught to be equal to C times P naught or in other words P is equal to C times P naught times 1 minus T. And this implies therefore, that the feedback controller C of s which gives us this transmission function P of s is given by C of s is equal to P naught inverse times T by 1 minus T.

So, this simple strategy of synthesizing the feedback controller, by assuming that we know what transmission function T of s we already want is called direct design. And what is quite unsurprising here is to note that our controller transfer function C of s has P naught inverse, has one of the terms and that multiplies T by 1 minus T. So, in effect therefore, we are cancelling the plant dynamics from the overall feedback loop. And replacing it with whatever dynamics we want in the open loop system so, that the closed loop system has the desired transmission function T. Because our open loop gain is going to be given by L is equal to C times P naught, and if I were to substitute for C from this equation above it would be equal to P naught inverse times P by 1 minus T times P naught and that essentially equal to P by 1 minus T.

So, in the forward path therefore, we have cancelled the plan dynamics altogether and replaced it with whatever dynamics, we want to give us the desired transmission function T for the closed loop system. Now as simple as this technique looks, there are if you cover ups. So, for one thing we want our controller C of s to be physically realizable, and if we choose a transmission function whose relative degree is less than the relative degree of the plant, then we will end up with a controller which is not physically realizable.

To see this all we need to look all we need to do is look at this equation here. From this equation we see that if n T is the degree of the denominator polynomial and m T is the de nom degree of the numerator polynomial then the relative degree of the transmission function T is given by n T minus m T. And we see that the degree of the numerator polynomial of T is essentially the sum of the degrees of the numerator polynomial of C and the degree of the numerator polynomial of P naught. Likewise, the degree of the denominator polynomial of T is going to be equal to the sum of the degree of the denominator polynomial of C and the degree of the denominator polynomial of P naught.

So therefore, n T minus m T is therefore, going to be equal to n p plus n c where n p and n c are the degrees of the denominator polynomials of the plant and the controller respectively minus m p plus m c where m p and m c are the degrees of the numerator polynomials of the plant and the controller respectively.

Now, we know that each of these physical each of these systems need to be physically realizable. So, n T minus m p is equal tont minus m p plus n c minus m c. Now since each of these systems have to be physically realizable we want the relative degree of the controller namely n c minus m c to be greater than or equal to 0 and what implies therefore, is that n T minus m T which is the relative degree of the overall closed loop system, should be greater than or equal to n p minus m p which is a relative degree of the plant.

Our closed loop transmission function T has to be chosen to be at least a third order transfer function. Likewise if our plant is a second order system or it has its relative degree is 2 then we have to chose choose our transmission function to at least have a relative degree of 2, in order for the controller to be a physically realizable causal controller.

Now, given this necessary condition for determination of T; we are still left with considerable flexibility and freedom as far as the exact structure of T is concerned. So, how might one want to choose the transmission function T?

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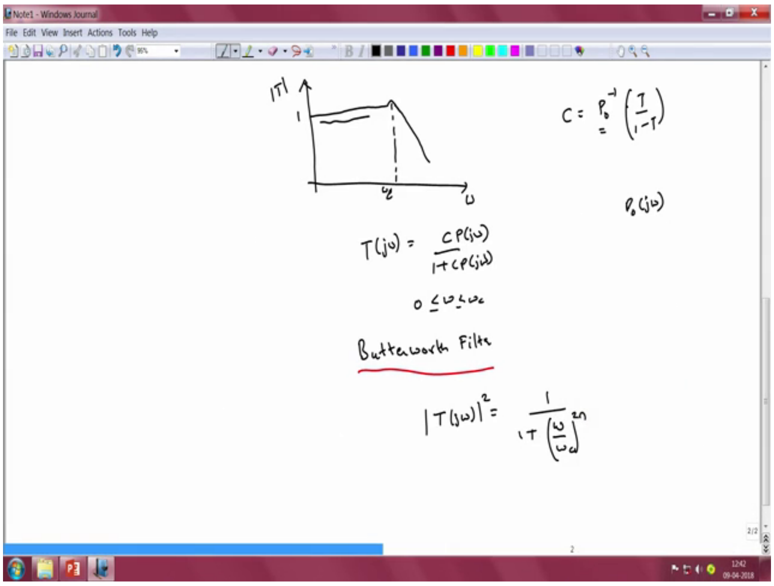

Now, generally as control engineers we want a transmission function to be as close to unity over as wide a frequency range as possible. So, if I were to plot the magnitude of T as function of frequency omega, we want the transmission function to be as close to 1 and as flat as possible or as wide a frequency range is possible. That will ensure that within the bandwidth of this frequency response, we would have x of T to be equal to r of T or we would be achieving the ideal of control to the extent that we possibly can.

Now, of course, we cannot get magnitude of T to be equal to 1 for all frequencies, there will have to be a corner frequency omega C at which we should allow for this transmission function to roll off why is that so? That is so, because if you remember our controller transfer function C was given by C is equal to P naught inverse times T by 1 minus T. Now how do we obtain P naught? P naught is the plant transfer function and that has to be obtained from experimental results. And then those experimental experimentally obtain plant transfer function has to be inverted.

Now, when we experimentally obtain the plant transfer function, you can imagine that when the frequency of excitation to the plant becomes larger and larger, then the response of the plant to excitation of those frequencies correspondingly becomes very small. Therefore, at very large frequencies our plants response will be buried in measurement noise. Therefore, the uncertainty associated with our plant model will be larger at higher frequencies. There is only a certain frequency up to which we know the plant model very well, beyond which we are not very sure about the exact structure of the plant. Therefore, we will not be able to precisely invert the plant dynamics for frequencies beyond that frequency at which our measurement noise starts to dominate over the response of our plant.

Hence, there has to be a frequency omega C, beyond which we will not be able to properly invert the plant. And therefore, we will not be able to exactly get a transmission function that we want.

There is one other reason why we need to limit the transmission functions magnitude to unity up to only a certain frequency omega C that is, because as the frequency increases the plants gain generally tends to drop. So, P naught of j omega is generally a decreasing function of omega beyond the bandwidth of the plant what this implies is that if P naught of j omega is a very small number then P naught inverse will be a very large number. So, therefore, the output of our controller will be a very large magnitude for frequencies that are well beyond the corner frequencies of the plant. And what that in turn implies. Therefore, is that the output of the controller is going to be very large and that is going to leak that is likely going to saturate our electronics.

Hence, for all these reasons we cannot choose arbitrarily large bandwidths for the overall transmission transmission function. As a rule of thumb the upper limit could be approximately one order of magnitude beyond the bandwidth of the plant itself. So, that is the best possible bandwidth for the overall transmission function T. That one can accomplish probably we are able to characterize the model of the plant very well up to one decade beyond the corner frequency the bandwidth of the plant.

So, ideally we want our transmission function to be 1, but we now realize that it cannot be one practically for all frequencies. So, we have chosen to expect our response to be close to one up to some frequency omega C.

Now, the question is what should the specific functional form for T of j omega the overall transmission function? Which is going to be equal to C times P by 1 plus C times P of j omega be between the frequencies 0 and omega C what should T of j omega look like?

So, since we want T of j omega to be as close to one as possible, a good choice for T would be a maximally flat filter or in other words a Butterworth filter. If we choose the transmission function T to be a Butterworth filter, then we would have a maximally flat magnitude response within the pass band of this filter and that will allow our filter to have a gain that is as close to one as is possible within its pass band. If you look at the transfer function of a Butterworth filter you will notice that, the magnitude of T of j omega the square if T of j omega were to have a Butterworth filters characteristics would be given by magnitude of T the square is equal to 1 by 1 plus omega by omega C to the power 2 n where n is the order of that filter.

So, by choosing an appropriately high order filter, you can get very flat pass band transmission function and a fairly steep roll off as per the frequency omega C. This brings us to the end of one degree of freedom control design.

Thank you.