Control System Design
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Lecture - 24
Control of Systems with Some Known Parameters (Part1/2)

Hello in the previous clips we have taken a look at Control System Design using root locus and before that using bode plots and when we get a design, we did not make any particular assumption about the specific nature of the disturbance that was affecting our system or the specific kind of plant that we are dealing with.

So, therefore, we had to rely upon generic controllers; such as the ones are now available of the shelf, namely the proportional time proportional integral or PID or lead lag or one of these controllers, or you might we have to synthesize controllers of a structure different from all of these which satisfy our particular requirements. Since we did not make any assumption about our knowledge of the plant or its environment, there was only so much that we could accomplish as far as control engineering was concerned

So, the ideal of getting x of p to b equal to r of t of course, had to be compromised. It had to be compromised a little bit, but nevertheless compromised, on account of the fact that we did not have any precise information about the plant or its environment

So, one can therefore, expect, but if one knew something about the plant or one knew something about the reference that we are interested to talk or disturbance that we are interested to reject, we should be able to do a little bit better than what we could do with the generic controllers that we have seen so far

So, this lecture, this clip therefore, would focus on discussion of controllers, for systems where something more is known about, either the plant or its environment or the reference that we use to track and the hope is that is extra knowledge will allow us to do better as control engineers and perhaps even accomplish the ideal of perfectly rejecting a disturbance or perfectly tracking a certain reference.

So, what we will see in this clip, is that if we happen to know the precise frequency at which the disturbance might be affecting us, or the precise frequency of reference that we are expected to track or precise set of frequencies of references that we are expected to track, then we can exploit this information do much better then what a generic controllers that we discussed in the previous clips allow us to do. To illustrate the capability of this, let us first revisit the problem that we have taken up when we are doing control design using bode plots

So, in that problem we were interested to reject disturbances and track a certain reference..

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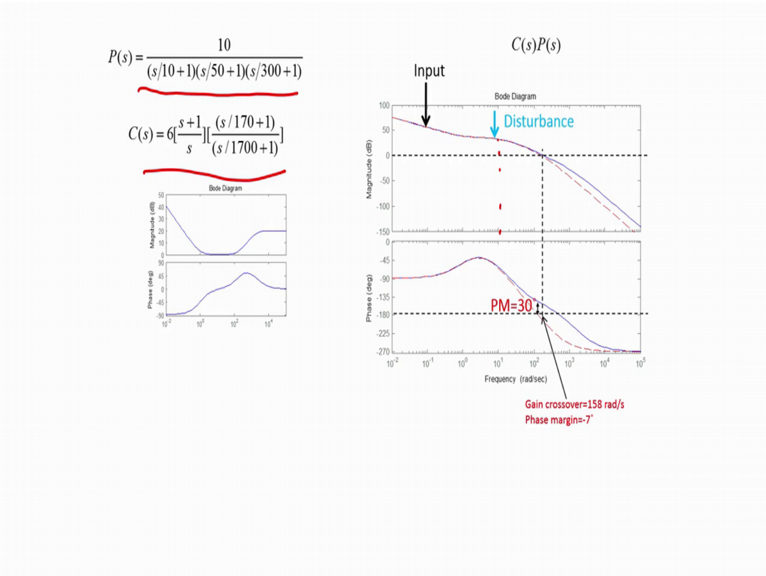

Specifically to refresh your memory we had a plant which was given by P of s is equal to 10 by s by 10 plus 1 times s by 50 plus 1 times s by 300 plus 1, so this was the plant that we had considered. And we were interested to reject a disturbance or frequency 0.2 radian per second, and track a reference of frequency 0.1 gradient per second. Subsequently, we also expressed our desire to design a controller which rejected disturbance at around 10 radiance per second, so at this particular frequency. And we wanted it also to be rejected by 99 percent. So, we wanted only 10 percent of a disturbance to affect our output

Now, let us say we were not interested even in that 1 percent of the disturbance affecting our output; we wanted perfect rejection of this disturbance, is that possible? Well a general if you are told that is disturbance has a certain frequency spectrum, it is not all the energy is not concentrated at a single frequency, but rather it is distributed over a certain set of frequencies and this distribution is not known ahead of time, then there is not much more that you can do then what was done by the PID controller that we design. So, this PID controller allowed us to achieve a sufficiently high loop gain in the frequency range of 10 radian per second and that allowed us to reject a disturbance by the required amount of about 99 percent

But suppose you are told, but this disturbance is actually a sinusoidal disturbance. So, there is only one frequency in this disturbance, and that is 10 radian per second. Of course, this still does not mean it you know everything about a disturbance. For instance we do not know its amplitude, we do not know its phase, but we do happen to know that its frequencies exactly 10 radians per second. The question is can we employ this extra piece of information to do much better then what this particular, the PID controller did. So, even that 1 percent of leakage of disturbance, it is happening into the output can that be plugged, can that be avoided by employing this extra knowledge; that is disturbance e at a single frequency namely 10 radian per second and we want to reject this perfectly.

Now, it is not unreasonable to expect this kind of a scenario to arise in practice, because generally if you think of. For example, the problem of vibration isolation, where you have a sensitive set up, that you want to isolate from the rest of the environment, then generally the vibrations that this set up would feel, would be due to some moto that is running nearby or some footsteps of people and so on and so forth. Footsteps of course, do not have a specific frequency spectrum, so we cannot employ this technique, but if this disturbance, if the set up is being shaken by a motor, then the speed of that motor is generally fixed and therefore, we know precisely the frequency content of a disturbance that is affecting our system, it will be a periodic signal of period equal to the rotation speed of that motor or its higher harmonics

So, therefore, there are occasions in practice where we can be, you might be fortunate enough to find that disturbance has a particular fixed time period or even better, it might be a sinusoidal disturbance of a particular fixed frequency and that is the sper specific problem that we are considering here. So, suppose we know that this disturbance is at 10 radian per second, can we do something to reject it perfectly.

Now, to understand if we can reject this perfectly or not, let us first take a step back and see why this particular controller is not able to reject it perfectly.

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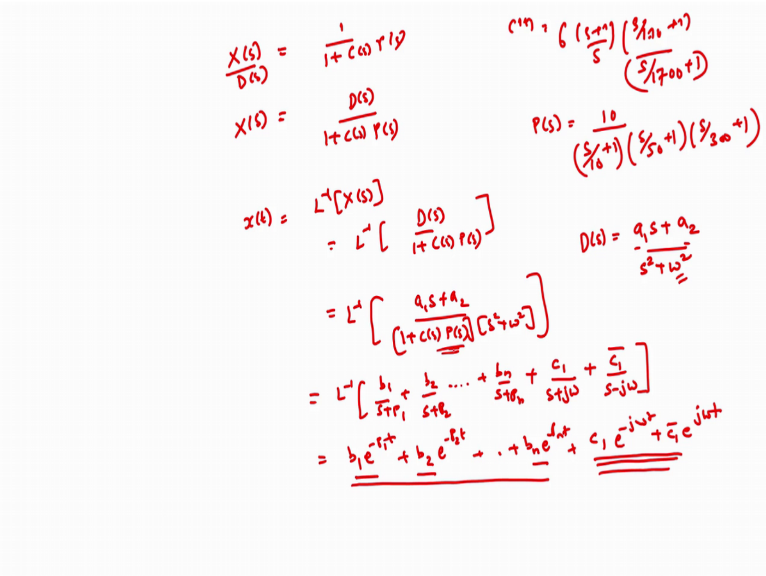

To do that what I would do is first write down the transfer function that relates the output X of s to the disturbance D of s and that as you know is given by 1 by 1 plus C of s times P of s. Therefore, our X of s would be equal to D of s divided by 1 plus C of s P of s. Now our controller transfer function is already been written out in the previous slide, it is given by C of s is equal to 6 times s plus 1 by s times s by 170 plus 1 times s by 1700 plus 1, P of s is the plant, which is given by 10 by s by 10 plus 1 times s by 50 plus 1 times s by 300 plus 1.

Now, X of p; of course, will be the Laplace inverse of X of s, it is going to be the Laplace inverse of this term here on the right, namely D of s by 1 plus C of s times P of s.

Now, if we know that our disturbance D of s is a sinusoidal signal of frequency 10 radian per second or some particular frequency omega, then we know that the Laplace transform of this disturbance would be of the form a 1 s plus a 2 divided by s square plus omega square; that omega is that particular frequency of oscillation of this disturbance. In our particular case it happens to be 10 radian per second

One is constants a 1 and a 2 are unknown, because we do not know the magnitude and the phase of the disturbance, we just happen to its frequency omega namely 10 radian per second. But we can exploit that and write x of t as Laplace inverse of a 1 s plus a 2 divided by 1 plus C of s times P of s times s square plus omega square. So, I have just substituted D of s with the term a 1 s plus a 2 divided by s square plus omega square to get this term.

Now, to obtain the Laplace inverse, we can use the method of partial fractions which we talked about in a few clips back. So, essentially I can write this as Laplace inverse of some constant b 1 by s plus P 1 plus b 2 by s plus P 2 and so on and so forth plus b n by s plus P n, where P 1 2 P n are the n poles of the closed loop system or in other words the zeroes of the term 1 plus C of s times P of s. So, zeroes of 1 plus c times P are given by P 1 to P n. So, I can use partial fractions to write out the entire term within the bracket of this Laplace inverse as this plus a constant, let us we call it c 1 divided by s plus j omega, because the term s square plus omega square can be factorized as s plus j omega times plus minus j omega.

So, when I write it out as a partial fraction, it would be of the form c 1 plus s plus j omega, c 1 by s plus j omega plus c 1 bar by s minus j omega. So, this is going to be the general, partial fractions expansion of the term within the bracket.

Now, if we take the Laplace inverse the first term you would get b 1 e power minus t 1 t and the second term would be b 2 e power minus P to t and so on and so forth up to b n e power minus P nt and we would have c 1 e power minus j omega t plus c 1 bar e power j omega t, so this is going to be the expansion.

To remind you the reason we are undertaking this analysis was to determine why we are having non zero error, why we are unable to get the disturbance to be rejected perfectly by the controller that we have designed early, will be the PID controller. And if we look at the time domain response we see that all these terms b 1 e power minus P 1 t etcetera etcetera are all decaying functions of style why is that so? That is because as control engineers we have designed our closed loop system to be stable, which means by definition that the zeros of 1 plus C of s times P of s will always be by design on the left half of the complex plane.

So, the all have decaying exponential terms therefore, what this indicates is that if I wait wrong enough then all the terms that are related to response due to the plants poles will all decay down to 0. So, if I wait long enough in the limit at time t tends to infinity, every single term b 1 e power minus P 1 t etcetera etcetera all the way up to b n e power minus

P and t will all die down and go to 0, what will remain and persist for all time; however, is this term here, the term that is because of the sinusoidal portion.

The question now then, is can we choose a controller structure that we will make this term go away.

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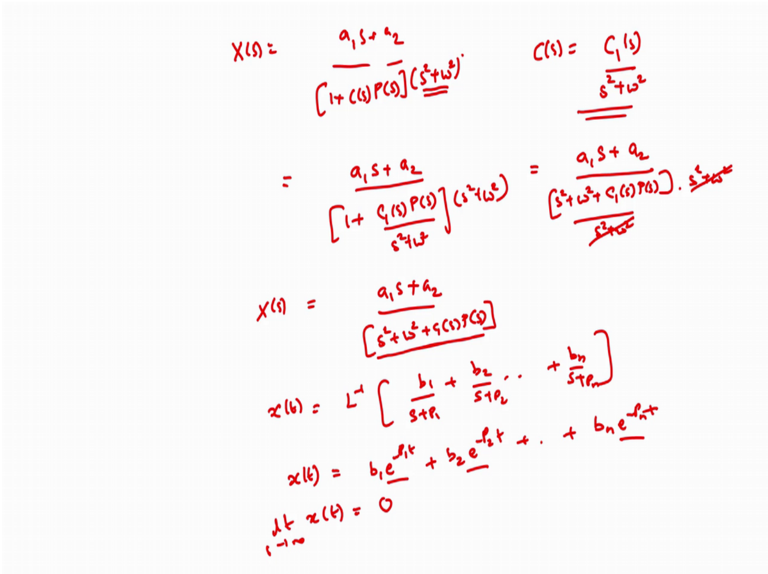

Now to answer that question let us come back to the expression for X of s. The expression for X of s is that X of s is equal to a 1 s plus a 2 divided by 1 plus C of s times P of s times s square plus omega square. Now we discover in the previous slide that the real culprit, the one that is responsible for having that 1 percent leakage of the disturbance at the output was this term s square plus omega square. The response due to the other poles die down, because our close loop system as we design to be stable. Therefore, if at all we can find a way to make this term disappear in the transfer function that relates the input to the disturbance then they are home free.

There is a possibility for us to get the disturbance to be rejected perfectly. So, how do we do that? Let us for instance pick the controller structure C of s to be of the form C 1 of s divided by s square plus omega square plus C 1 of s is a controller that we still need to design, but it would be designed in such a manner that our close loop system would be stable. So, in other words what we have made sure with this choice is that we have ensure that we have that term s square plus omega square in the denominator polynomial of our controller transfer function C of s.

Now, suppose we choose a controller of this form then X of s can be written as X of s is equal to a 1 s plus a 2 divided by 1 plus C 1 of s times P of s divided by s square plus omega square times s square plus omega square, and this upon simplification would become a 1 s plus a 2 divided by s square plus omega square plus C 1 of s times P of s divided by s square plus omega square times s square plus omega square. Now what we note here is that we can cancel out the 2 s square plus omega square terms in this expression and therefore, get X of s to be equal to a 1 s plus a 2 divided by s square plus omega square plus C 1 of s times t of s.

Now, what have we achieved in the process? We have managed to get rid of the term a square plus omega square and the denominator polynomial of X of s. Now what we have as a denominator polynomial instead is this term s square plus omega square plus C 1 of s times P of s, and all we need to do is to design C 1 of s in such a manner that the zeroes of this denominator transfer function s square plus omega square plus C 1 times P are all on the left half of the complex plane. Now if we do that then what can be done is write X of t as Laplace inverse of b 1 by s plus P 1 as b 2 by s plus P 2 and so on and so, forth plus b n by s plus t n.

There will be. Now we will not have the two terms related to s plus j omega and s minus j omega, because that has been removed from the denominator polynomial by our specific choice of our controller structure

Now, when we take the Laplace inverse of this you would get this to be equal to b 1 e power minus P 1 t plus b 2 e power minus P 2 t and so on and so forth plus b n e power minus P n t. Now in the limit that t tends to infinity you would have X of p to be equal to 0, and that is because all of this are decaying functions of Pi. So, is it made long enough all of them will died out and our ultimate x of p will be equal to 0. So, what this indicates is that although we cannot perfectly reject this disturbance from the time t equal to 0. So, instantaneous rejection of disturbance is not possible, if we wait long enough then we will be able to eventually completely get rid of this disturbance. When I say long enough I mean that we have to wait for a time scale that is determined by the closed loop bandwidth of our control system.

So, if we wait for that length of time then we would be able to perfectly reject this disturbance, which is the most attractive feature of this particular strategy. And the central feature of the strategy was to choose a form for the controller as given here C of s is equal to C 1 of s divided by s square plus omega square in other words the included the denominator polynomial of the disturbance as part of the denominator polynomial of the controller.

Now, let us try to implement this technique for the particular control system that we were talking about. So, in our control system I have already shown you the structure of the plant and the controller. So, our plant has this particular transfer function and our controller has that particular transfer function. Suppose we want to implement this technique for this controller.

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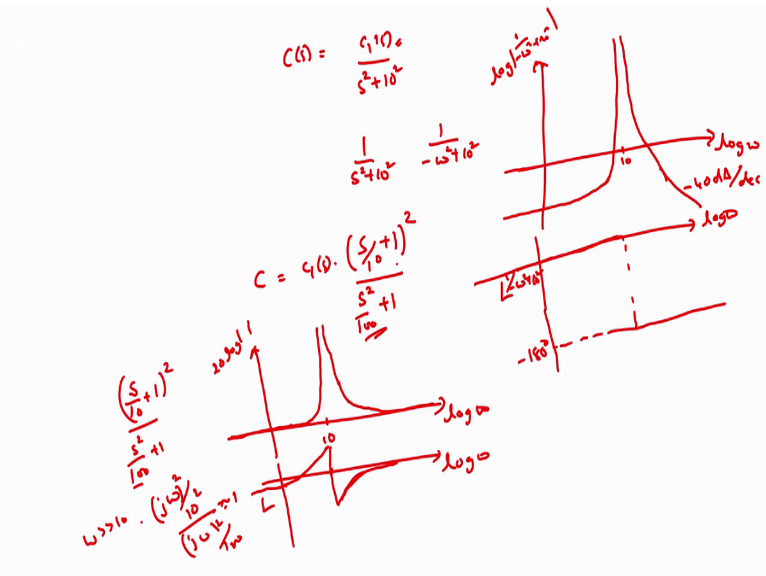

So, what I use to do, is to have a controller C of s to be of the form C 1 of s divided by s square plus 10 square, why did we choose 10, that is because that is the frequency of the disturbance that we want to reject in this particular case

So, can I uncritically choose this as our controller structure, the C 1 is the controller we have already designed. So, we have already designed a PID controller to take care of our disturbance rejection at 0.2 radiance per second reference tracking at 0.1 radian per second and all of that.

Now, can I simply cascade that controller with the term 1 by s square plus 10 square and hope that our close loop system would be able to reject perfectly this disturbance at 10 radian per second. Actually if you think for little, but you will discover that you will run into problems if you directly implement it in this manner why is that so such because of the kind of effect that this term one by s square plus 10 square will have on the (Refer Time:20:01) face characteristics of the overall bode plot

So, let me briefly plot the bode plot of the term 1 by s square plus 10 square. So, if we have s square plus 1 by s square plus 10 square as our transfer function to plot, the bode plot I have to substitute s is equal to j omega, in which case I will get this to be 1 by minus omega square plus 10 square. If I were to draw the bode plot of this transfer function, so our x axis will be log of omega, the y axis will be log of magnitude of 1 by minus omega square plus 10 square, and the phase the second plot would be the phase of 1 by minus omega square plus 10 square and log of omega. Then I will have that for the low frequencies the omega is much less than 10 gradient per second, I would have the gain of this controller to be just 1 by 10 square within 1 by 100. So, which is minus for P d B.

So, its going to be come back here, and as I approach the frequency omega equal to 10 radian per second my magnitude plot will start to increase. And at exactly omega equal to 10 radian per second the gain of this transfer function 1 by minus omega square plus 10 square will be infinity. So, this will blow up at 10 radian per second. Then again it will drop for frequencies beyond that and for very large frequencies omega, it will go down the magnitude will go down as one over omega square.

So, it will go down at the rate of minus 40 decibels per decade. How about the phase characteristics? You see from by instruction that then omega is less than ten radian per second, we would have this transfer function 1 by 10 square minus omega square to always be a positive number positive real number, which means that its phase will be zero degrees

And the omega is greater than 10 radian per second, we would have its phase to be a negative real number, it means that the phase will be 180 degrees minus 180 degrees. Therefore, the phase response will start at 0 degrees and stay at 0 degrees up to 10 radian per second and at 10 radian per second it will abruptly jump from 0 radiance to minus 180 degrees or equivalently minus pi radiance.

So this is the phase characteristics examining the magnitude and a phase characteristics then immediately reveal to you the problem with this controller. Since the phase is changing by 180 degrees at 10 radian per second, if our controller C 1 of s has even a small phase lag then it would cost the overall phase to change from that particular value of phase lag to minus 180 degrees plus that particular value of phase lag, which means that you will have the phase cross over necessarily happening at 10 radian per second, because of addition of its term, and the gain of the controller at 10 radian per second would be multiplied by infinity.

Because a gain of this term 1 by 10 square minus omega square at 10 radian per second is infinity, which means that at the phase cross over frequency which will now be 10 radian per second you will have a very large open loop gain, which means that your close loop system is going to be unstable

So, how do we avoid this problem? We avoid this problem by choosing a controller C of the kind, C 1 of s which is the PID controller we have already designed times s by 10 plus 1, the whole square divided by s square by 100 plus 1. So, I have written 1 by s square plus 10 square in this particular manner and there is no loss of generality in writing it in this particular manner. What is the advantage of choosing this particular structure? We discover that in the numerator we now have the term s by 10 plus 1 and this term will add a phase lead of plus 45 degrees when omega is equal to 10 radian per second and since we have two such terms in the numerator in or in other words we have s by 10 plus 1 the whole square, to get a plus 90 degrees phase lead that is contributed by these two terms

So, therefore, if I want to plot the bode plot of just this part alone, namely s by 10 plus 1 the square divided by s square by 100 plus 1 I would discover that the magnitude plot log frequency the magnitude plot has very low frequencies omega will have a gain of 0 d B, because in the numerator as well as in the denominator I can ignore the terms related to omega and I would have 0 d B and as I approach omega equal to 10 radian per second my gain will increase and it will be infinity as before, and it will come back for frequencies beyond 10 radian per second.

But it will not decade at minus 40 d B per decade as it happened in the previous case, because when for omega much greater than 10 radian per second, we would have this term to be approximately equal to omega square j omega the whole square by 10 square divided by j omega the whole square divided by 100, this once again going to be equal to 1

Which means that these controllers magnitude characteristics will come back to zero degree beyond 10 radian per second. As far as the phase is concerned, the initial phase will be close to 0 degrees as before, and there will be an increase in phase contributed by the term s by 10 plus 1 as we approach the frequency 10 radian per second. So, there will be increase in phase, and an abrupt change in phase of minus 180 degrees introduced by the term s square by 100 plus 1 at the frequency 10 radian per second, where is that observe abrupt change in phase and then subsequently there will be an increase in phase. So, this term s by 10 plus 1 will eventually approach plus 180 degrees phasely so, that will cancel out the minus 180 degrees phase lag contributed by the term s square by 100 plus 1

So, eventually the phase will asymptotically approach zero degrees once again. So, this is the angle of this particular transfer function as function of frequency. So, what we see is that this term here s by 10 plus 1 the whole square divided by s square plus 1 s square by hundred plus 1, modifies the controllers bode plot only in the neighborhood of 10 radian per second. It increases the gain. In fact, at 10 radian per second it makes it infinity and it modifies the phase also in the vicinity of that frequency alone and leaves the rest of the frequency range untouch, and that is precisely what we designed, because we have already designed our PID controller C 1 of s to give us the required stability and performance specifications in the other frequency ranges.

So, with this modification to our controller structure let us see how we are able to fair.

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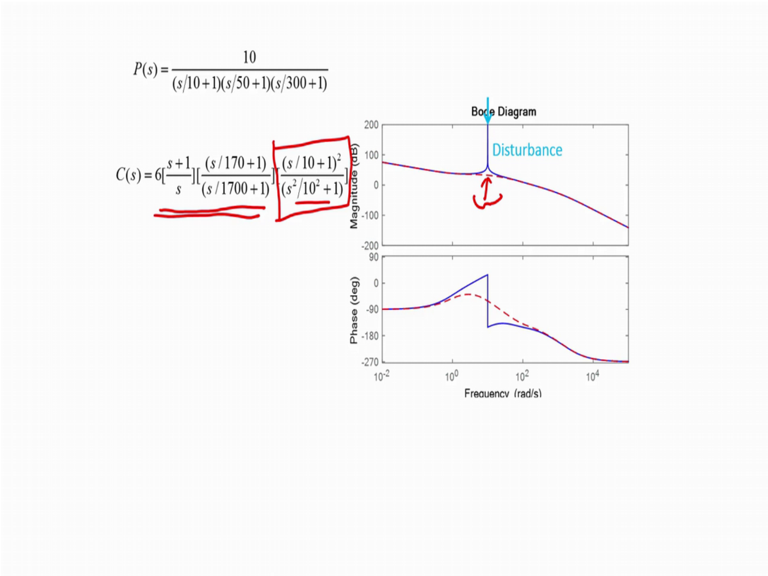

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First I have plotted here the bode plot of this overall system. So, what you see in the red dash line here, is the bode plot of our open loop system, namely the plant times the PID controller or in other words the controller that has only these three terms here and that is given by the red line and this was something that we design a few clips back, when we were discussing design using bode plots. Now what we have done is, we have cascaded the term s by 10 plus 1 the whole square divided by s square by 100 plus 1 to this controller, and as we discussed that modifies the bode plot of the overall open loop system only in the vicinity of 10 radian per second.

And in particular at 10 radian per second, the gain has now become infinite infinitely large. The phase response also is lines that we discussed in the previous slide. So, the phase starts to increase in relation to the original phase characteristics, because of the term s by 10 plus 1 under a sudden change in phase by minus 180 degrees introduced by the term s square by 100 plus 1 and then the phase will continue to increase and asymptotically approach the original phase response of the overall open loop system.

So, this is how it looks and from this we can understand why we can expect perfect rejection of disturbance, because of these controller nuts the cost at 10 radian per second and having a loop gain of infinite d B. An infinite loop gain is the key for us to get perfect control performance, either perfect tracking of references or perfect rejection of the start answers.

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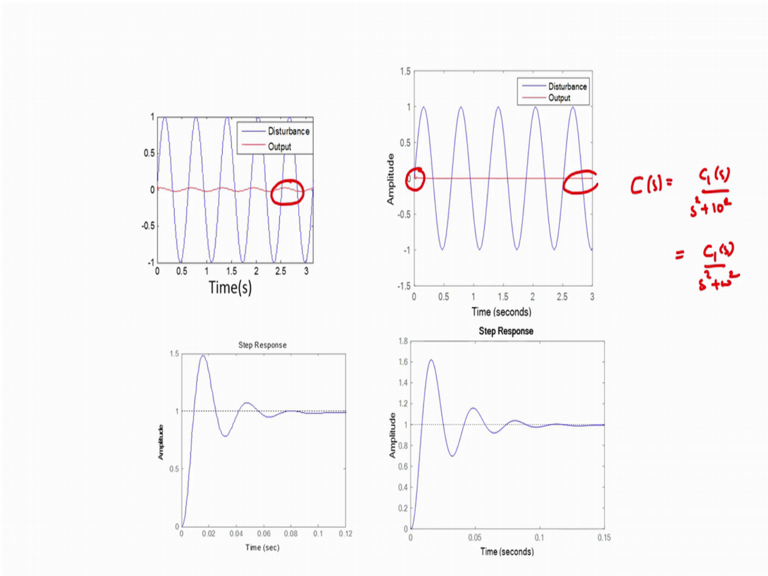

In this case I have carried out a simulation where we have compared the performance of the close loop system in the absence of this controller. So, this is the original PID controller that we had designed a few clips back, and we discussed that you are able to bring down suppress a disturbance by 99 percent, exact 98 percent, exactly as we had desired it to be although it was not completely suppressed.

However now with the new controller where we are, we have included the new term 1 by s square plus omega square in the denominator of the controller and we have also modified the numerator of the controller, so that this new term does not affect the frequency characteristics at frequencies other than 10 radian per second. If I were to plot the close loop response of the system to same disturbance at 10 radian per second what you see is here. So, we see that there is an initial transience, a small transience, but after the transience is passed the system does not at all respond to this disturbance

So, in other words we have succeeded in perfectly rejecting disturbance at 10 radian per second by using this controller. If you look at the close loop transient responses of the system from a first case we had a certain transient response this was something we discussed a few lectures back, I am just reproducing it here for the sake of continuity and with the present controller, you get a similar transient response with a slightly higher overshoot, because a phase margin has dropped a little bit as a result of this controller, but we can always add a leap compensator to improve the phase margin as we have discussed a few clips back and therefore, the slight in increase overshoot is not too much of a problem for us

So, at the heart of what we accomplished, now was our choice of this control structure we chose our controller to be of the form C of s is equal to C 1 of s divided by s square plus 10 square or more generally, if this a disturbance at a particular frequency omega then we choose the controller to be of the form C 1 of s divided by s square plus omega square. So, with this choice you are able to reject this disturbance perfectly and this very neat elegant trick has a name, its called internal model principle.

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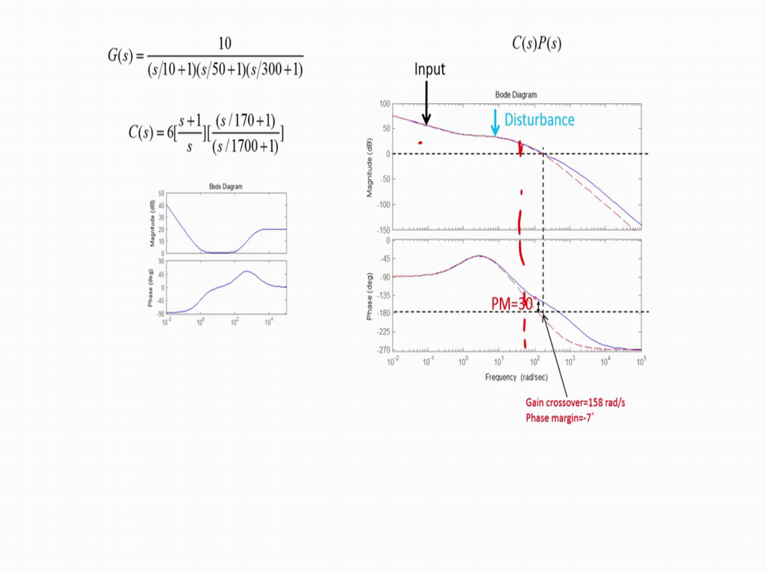

So, the principle states that A signal r of t or a disturbance d of t can be rejected perfectly in steady state, if the denominator polynomial of R of s which is a Laplace transform of R of p or D of s is included as part of the denominator polynomial of the controller C of s. So, if we include the denominator polynomial of r of t of d of t as part of the denominator polynomial of our controller C, then we can in steady state reject the disturbance perfectly, but track that reference perfectly.

So, this is called the internal model principle and this is widely used in industry in scenarios, where you either know the disturbance very well, if its a period disturbance, so that we know that its frequency spectrum is characterized by discrete energy content in discrete set of frequencies, then we can employ internal model principle to perfectly reject a disturbance in steady state.

Similarly, reference starting of sinusoidal inputs or other periodic inputs can also be accomplished with zero steady state error by using internal model principle. Now suppose we wanted to apply internal model principle, so another case namely the case of reference tracking, and suppose we had a reference at around 50 radiance per second that we wanted to track. So, 50 radiance per second would be somewhere here, we see that the loop gain is not high enough in this frequency range, because this is the mid frequency range of our system and therefore, without this internal model controller we will not be able to track this reference perfectly.

So, what I have done therefore is to incorporate this internal model term in as part of the controller.

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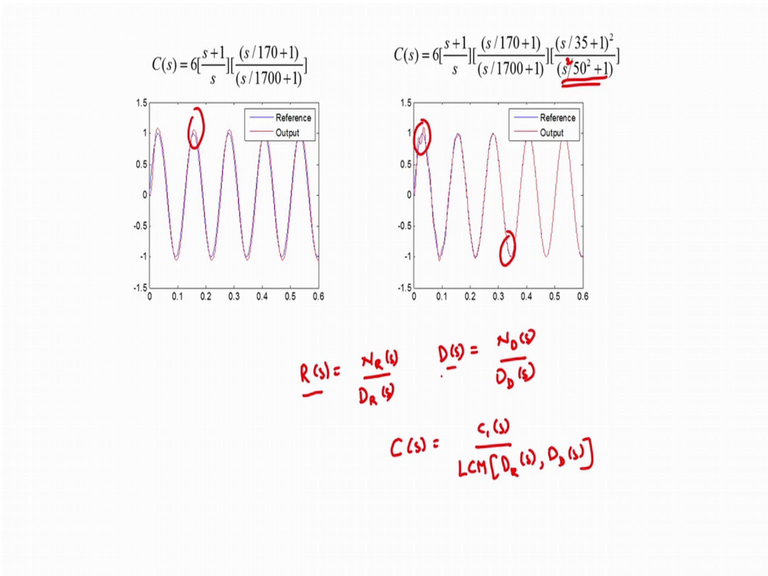

So, this is the tracking performance of the controller without the internal model term, so I do not have a term of the kind 1 by s square by 50 square plus 1 as part the denominator polynomial of my C of s, and without it you see that there is a noticeable error between the reference that you have provided at this frequency, namely 50 radian per second and the response at a frequency

However suppose we were to modify the controller to include this term. So, now, we have the term s square by 50 square plus 1 has part of the denominator polynomial and in this case, I have not chosen to add the 0 exactly 50 radian per second, but I have chosen to be added a slightly lesser frequency, around 35 radians per second with the hope of getting a slightly extra phase lead at 50 radian per second for the overall open loop system. So, with this controller we see that there is an initial transient. So, initially of course, the two are not matching perfectly or another words the reference is not tracked perfectly

But after the transients has dried down, you see that the reference and the output sit almost exactly one on top of another, it indicates that this controller is able to perfectly track this reference to zero steady state error. Now suppose we have let us say a reference R of s which can be written as a numerator polynomial N R of s divided by denominator polynomial D R of s, and suppose we have a disturbance D of s which can be written as the ratio of a numerator polynomial N D of s and a denominator polynomial D D of s. If you want to track this reference perfectly and reject that disturbance perfectly at the same time, then what we need to do is choose a controller C of the form C of s is equal to C 1 of s divided by the Least Common Multiple, another words L C M of D R of s and D D of s.

So, with this choice of a controller structure, the C 1 is designed to ensure that our close loop system is stable and it meets other performance specifications at other frequencies for disturbance rejection and so on. We can make sure that this reference is specific reference R of s and this particular specific disturbance P of s can be tracked perfectly and rejected perfectly respectively by using the internal model principle.