Control System Design

Prof. G. R. Jayanth
Department of Instrumentation and Applied Physics
Indian Institute of Science, Bangalore

Lecture – 23

Control system design using root-locus

Hello in the previous clip, we refresh our memory about root locus and how one would be able to draw the root locus if you are given the open loop transfer function of a system. In this clip we shall take a look at how we can employ the tool of root locus to perform control system design. In order to do this, let us do it through a numerical example.

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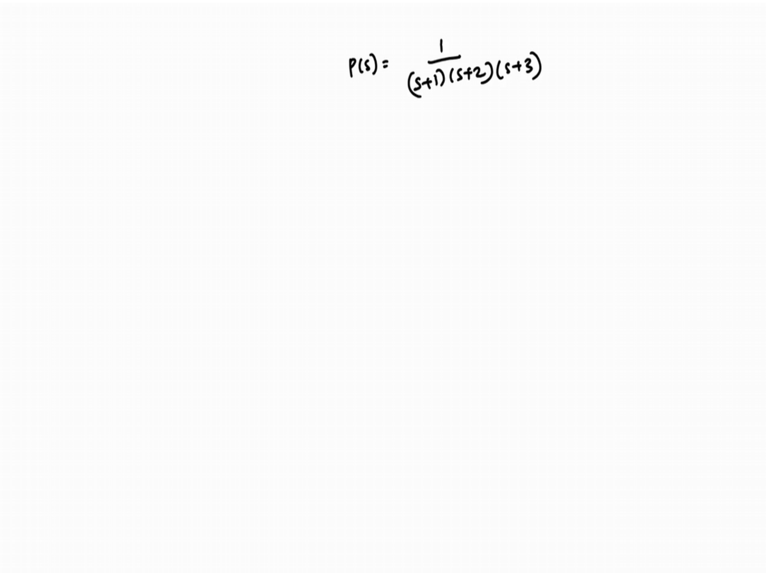

So, what we shall do is consider a plant P of s to be equal to by s plus times s plus 2 times s plus. So, this is a third order plant the root locus of this plant has been sketched by using MATLAB software and the root locus is shown in this slide here

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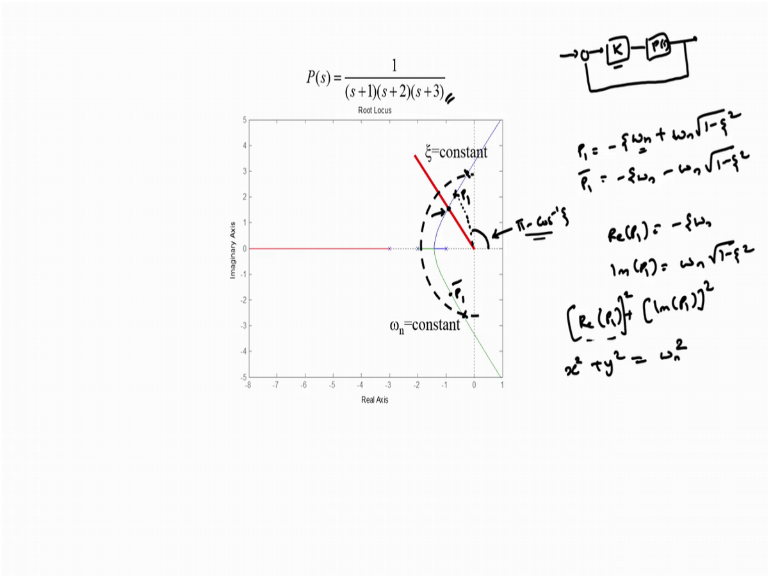

One of the most obvious applications of root locus is to be able to choose a proportional controller's gain such that a pair of poles of the closed loop system would be located where we wanted to be located. So, for instance, the root locus of the plant here tells us all the possible locations of the close loop poles that has this plant P cascaded with a simple proportional controller as part of the open loop system. So, in other words, I am drawing here again K that is cascaded with P of s where P of s is given by this transfer function here and the feedback loop and a feedback loop that is wrapped around this combination essentially will have this root locus.

Now, suppose we want the dominant poles of this closed loop system to be located at some particular position. We know that it is going to be located on or another of these branches of the root locus. Suppose we want the closed loop pole to be located at one particular point on the root locus, then we can use the root locus to determine the gain K that places the closed loop pole other particular location for instance. If let us say we want a pair of dominant closed loop poles; let me call them P1 and P1 bar to have a form P1 is equal to minus zeta omega n plus omega n square root of minus zeta square and P1 bar to be equal to minus zeta omega n minus omega n square root of minus zeta square.

If this is the form that I want my closed loop poles to have, then what we can do is employ root locus to find out the gain K that gives us; for example, a specified zeta. So, if you notice this equation here, the term zeta is related to the angle that this pole makes with respect to the real axis. In fact, this angle that I have shown here will be given by pi minus cos inverse of zeta. So, if I want my closed loop dominant closed loop poles to have a certain specified value of zeta which in effect means that our closed loop system has a certain amount of damping, then what we can do is first we draw a straight line of slope dictated by the particular zeta that is given to us.

So, zeta equal to constant is a straight line that passes through the origin and is inclined with respect to the real axis by this angle pi minus cos inverse zeta. So, the most simple application of root locus is to first draw the straight line and find where the straight line intersects the root locus and that turns out to be this particular point here. Now we determine the gain of the controller K that will allow the closed loop pole to be located at this particular point and that is the gain that our controller needs to have for our closed loop system to have a pair of dominant poles with that particular specified value of zeta. So, this is one very simple application of root locus a very similar application of root locus would be if you specify omega n.

So, you want your dominant closed loop poles to be of this form P equal to minus zeta omega n plus minus omega n times square root of minus zeta square and you wanted to have a certain frequency omega n. Now if omega n is specified to you or in other words omega n is a constant, then what you would notice is that omega n equal to a constant represents a circle. So, the pole P 1 and P1 bar would lie on a circle of radius given by omega n. That is because if I take the real part of P1 it is minus zeta omega n the imaginary part of P1 is omega n square root of minus zeta square the real part of P1 plus or the whole square plus the imaginary part of P1 the whole square which is the square of the magnitude of this complex number is essentially just going be equal to omega n square. Therefore, if I specify omega n, it will move along the curve given by real part of P.

The square plus imaginary part of P the square is equal to omega n square real part of P essentially represents the x co-ordinate. So, this going to be x square plus y square is equal to omega n square and this equation essentially describes a circle of radius equal to omega n. So, the second straight forward application of root locus is to find out that gain K of the proportional controller that allows for the dominant poles of our system to have a specified frequency omega n. To do that all we need to do is draw the circle of radius omega n and find out where it intersects the root locus of our plant. So, it so, happens that in this case is interesting them at these 2 locations.

Now, at these 2 points in the complex plane we determine the gain K and once we have determine the gain K, then we know that our proportional controller needs to have the particular gain for our dominant poles P 1 and P 1 bar to have that particular omega n. So, designing a closed loop system using the proportional controller such that the closed loop system has a certain specified damping or a specified omega n for its dominant poles is straight forward application of the notion of root locus. However, its scope is for bigger than what we have just discussed.

Now, in this case you notice that we specified either omega n or zeta for our closed loop system, the problem is if you want to specify both omega and zeta in other words both omega and zeta are decided ahead of time, then you run into problems. So, we shall look at that in the next example.

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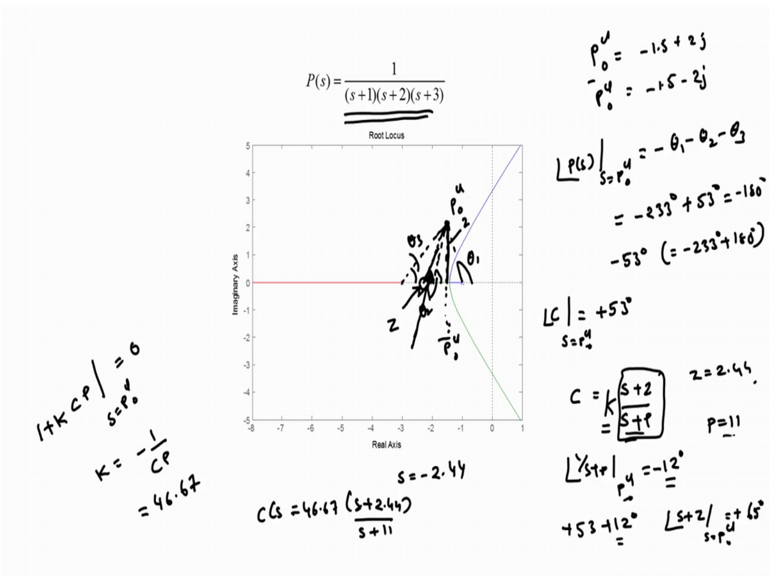

So, we come back to this plant p of s is equal to by s plus times s plus 2 times s plus 3. And in step insists that its closed loop poles be located at P cl naught equal to minus 1.5 plus 2 j and P cl naught bar equal to minus 1.5 minus 2 j. So, you want to design a closed loop system for this particular plant that as its dominant poles minus 1.5 plus minus 2 j.

Now, in this graph in this root locus if I want to locate minus 1.5 plus minus 2 j, I would notice that along the x axis I need to have minus 1.5 which is somewhere here and minus 1.5 plus 2 j would be this point here that is P cl naught and minus 1.5 minus 2 j would be a point here this is P cl naught complex conjugate. Now we see that in this particular case, it is impossible for a proportional controller to get us to have this particular closed loop dominant dynamics for our system.

Simply because by just using a proportional controller, our closed loop poles can only lie on the root locus of our plant. So, if I want to change the gain k the closed loop poles can vary along this particular curves; however, for this case the location where we want our closed loop pole to lie the dominant closed loop pole to lie is outside of the root locus. So, it is not possible to use a simple proportional controller as we did in the previous examples in order for us to realize this particular pair of dominant poles for our closed loop system. So, what should we do? The first thing that we need to do is to get the root locus of the overall open loop system namely, the controller times the plant to pass through the point at which the closed loop pole has to lie.

So, that is the first thing the root locus has to pass through P cl naught and P cl naught bar of course, it if it passes through P P cl naught naught it will automatically pass through P cl not bar because the root locus is symmetric about the horizontal axis. So, that is the first thing that we need to do. So, we need to choose such a controller c of s that it allows the root locus to pass through the point P cl naught. Having chosen that controller, we have to then decide on the gain of the controller such that the closed loop pole is actually located exactly at P cl naught.

So, the first step of choosing such a controller that the root locus passes through P cl naught P cl naught will not automatically guarantees that the closed loop pole will be exactly at P cl naught. We have to now choose the gain of the controller appropriately to make sure that the closed loop dominant pole will be located at P cl naught P cl naught and P cl naught bar. Of course, there will be one more pole on the other branch of the root locus which we are not we are not going to discuss because its go to be to the left of these 2 poles and is therefore, not going to be a dominant pole of our closed loop system.

So, we have to therefore, first get the root locus to pass through the point P cl naught. What does that mean? In practice what; that means, is that the angle subtended by all the open loop poles and 0 of our system at the point P cl naught P cl naught should be equal to minus pi or its cyclic equivalents. So, at the moment we have a plant with 3 poles namely pole at minus 2 and minus 3. Let us compute the angle subtended by all these 3 poles put together at the point P cl naught. The angle subtended by the point minus at P cl naught P cl naught is given by this particular angle. I shall call it theta.

The angle subtended by the pole minus 2 at P cl naught is given by this particular angle I shall call it theta 2. The angle subtended by the third pole of the plant at minus 3 at the point P cl naught P cl naught is given by this angle and I shall call that theta 3. So, the net angle of P of s at the point s is equal to P cl naught will be equal to minus of theta minus theta 2 minus theta 3. The reason we have this negative signs in front of these angles is because they represent the angle subtended by the poles of our system and poles are terms of appear in the denominator of this transfer function and therefore, the angles of these complex numbers s plus this complex number s plus and the angle of by s plus is minus of theta the angle of min[us]- by s plus 2 is minus of theta 2 and the angle by s plus 3 is minus of theta 3. Hence we have this negative signs here.

Now if we compute these angles from by using trigonometry, we know P cl naught P cl naught it is equal to minus 1.5 plus 2 j and we know the locations of these poles. So, it is quite straight forward for us to compute these angles. If you compute them, we get the sum of these angles to be equal to minus 233 degrees. Now for the point P cl naught to be a point on the root locus the angle has to actually be minus 180 degrees or its cyclic equivalents. Therefore, what you see is that there is a deficit of minus 53 degrees which is essentially equal to minus 233 degrees plus 180 that needs to be compensated.

So, I need to add plus 53 degrees to this phase, in order for the net phase of our open loop system at this point P cl naught to be equal to minus 180 degrees. So, if I want to add plus 53 degrees here somehow, then I would have minus 233 plus 53 to be equal to minus 180 degrees and I would be able to get the root locus to pass through the point P cl naught. Now we need to choose a controller that provides to us a phase lead of plus 53 degrees at the point P cl naught. So, in other words the angle of c whichever controller that is it is that we choose at the point s is equal to P cl naught P cl naught should be equal to plus 53 degrees.

Now, the question is what controller do we pick. Here again we have a infinite number of controller structures that we can choose all of which can potentially give us 53 degrees face at the point P cl naught by choosing different combinations of poles and 0s and appropriately relocating the poles and 0s of the closed loop of the controller.

We can get the net phase contributed by this combination of poles and 0s to be equal to 53 degrees and there are infinitely many controller structures that will give us that same angle.

So, how do we pick a suitable controller from among these infinitely many controllers? Once again as we did in the case of bode plots. We use simplicity as our guide for picking a suitable controller structure we notice that we have to get positive angle of 53 degrees to be subtended at the point P cl naught. So, the simplest controller structure that allows us to do that is a simple 0. So, if I choose the controller C to be of a form S plus z. Then let us say I locate the point z in the complex plane. Let us say this is that point z ok. Then I choose this point z in such a manner that the angle subtended by this point z at the point P cl naught which is given by this particular angle here which I am shading here for the sake of clarity that angle should be equal to plus 53 degrees.

Now, I know what P cl naught is. So, P cl naught is minus.5 plus minus 2 j. So, the height its height from the real axis is essentially 2 units. So, I can use once again trigonometry to determine and I know that this angle the shaded angle has to be plus 53 degrees. So, I can determine therefore, the location of z from trigonometry and I get that location to essentially be equal to z 3 units. So, z should be placed at s is equal to minus 3 this is all well and good, but there is simple problem with this control structure namely that it is not a causal controller. So, we have a transfer function here which has no denominator polynomial at all and just a numerator polynomial.

So, it cannot be physically realized if we recollect we need to have physical controllers or physical plants to be strictly proper transfer functions or at best proper transfer functions or in other words the degree of the numerator polynomial should be either less than or equal to the degree of the denominator polynomial it cannot be greater than the degree of the denominator polynomial. Therefore, for the sake of causality what we have to do is you to also choose to have a pole for the plant. So, our controller structure inter in structure would looks something like this we are not done with the design yet.

So, this is not our final controller, but if we choose a controller of this structure, then if we place a 0 z at s is equal to minus 3 then we would get plus 53 degrees at the point P cl naught from that 0 and that will cause the net angle subtended by the controller times the plant at that particular point to be equal to minus 80. However, because you have introduced this pole the pole will also contribute to a small phase negative phase. Now if we choose to place the pole at a fairly distant location. So, in this particular case I have chosen to place the pole at p equal to1.

So, or in other words s is equal to minus 1 that is the place where I chosen to place the pole. Now if we place the pole at that location we can. So, it that point would be somewhere outside of this graph. Then we can compute the angle subtended by that pole at the location P cl naught. So, we can compute angle of by s plus p at the point P cl naught. And if we do that we find that that angle is equal to2 degrees with a minus sign because this is a pole. So, essentially therefore, we have an extra negative angle that has been added on to the already existing negative angle of minus 230 degrees by this extra controller pole that we had to incorporate. And this controller pole had to be incorporated in the interest of causality of the controllers transfer function.

Now, our 0s position has to be fine tuned a little bit because it has to now also compensate for this extra negative angle that has been supplied by this by the controller pole. So, now, instead of having to compensate for minus 53 degrees difference between the angle at the point P cl naught and angle necessary for P cl naught to be a point on the root locus namely minus 80 degrees has to have a controller whose 0 subtends an angle of not plus 53 degrees, but instead plus 53 plus2 degrees. So, this plus2 degrees is to compensate for the extra negative phase that has been contributed by the term s plus p.

On other words the 0 has to subtend an angle of plus 65 degrees at the point P cl naught at s angle of s plus z at the point s is equal to P cl naught has to be equal to plus 65 degrees.

Now if once again employs trigonometry to compute the location z at which the angle subtended by that 0 at the point P cl naught. In other words the angle of the shaded angle in this figure here which I am once again indicating by means of this arrow here for angle to be equal to plus 65 degrees can compute that the 0 has to be placed at s is equal to minus 2.44.

Or in other words the term z in this particular controller would be equal to 2.44. The new value of z has to be equal to 2.44. Now when we have the 0 at s is equal to minus 2.44 and a pole at s is equal to minus1. Notice that the pole location was rather arbitrary. We chose it to be at minus 1, we could have chosen it to be at other locations also in practice.

We choose it on the basis of available bandwidth for implementation of our overall control system. So, in this case we are only discussing the theory of control designs.

So, we shall not look at the practical aspects. So, here we have chosen to place it arbitrarily this controller structure allows the root locus to pass through the point P cl naught and P cl naught bar. So, now, there is hope for us to place our closed loop pole at P cl naught and P cl naught bar by choosing an appropriate gain k for our controller C of s.

Now how do we determine the gain K what we need to do is to apply the condition for root locus? So, if a point P cl naught is a point on the root locus then plus k times C times P where c is the controller transfer function p is the plant transfer function evaluated at the point s is equal to P cl naught should be equal to 0.

This by definition is valid for all points on the root locus and therefore, is also going to be valid for point P cl naught because it is now a point on the root locus. So, to determine the controller gain K therefore, all we need to do is to compute K as K being equal to minus by C times P where C is given by this particular transfer function s plus z by s plus p where z is 2.44 and P is 1. And the plant transfer function of course, already been specified to us and I am underlining it there for you. So, if we use this equation and compute the gain k we find that we get it to be equal to 46.67.

Therefore the controller that allows our dominant closed loop poles to lie at minus.5 plus minus 2 j is given by C of s is equal to 46.67 times S plus 2.44 divided by S plus1 and this completes our design.

Note that our closed loop poles will not be located only at these 2 locations you have another branch of the root locus also on this side and therefore, you will have a closed loop pole also lying on that branch, but since that pole is much farther away from the origin compared to P cl naught and P cl naught bar it will not dominate a dynamics of the overall closed loop system. And therefore, we do not need to worry about its exact position.

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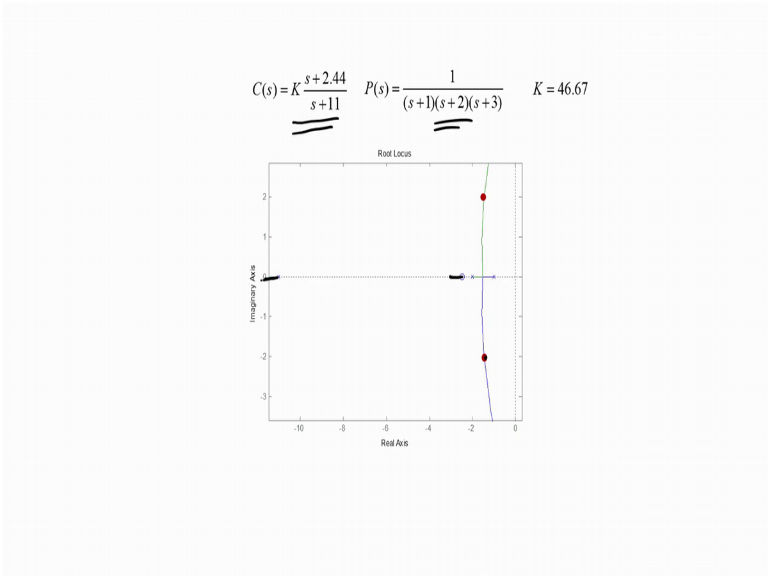

So, what I have done here is to show you the root locus of the system that employs this particular controller, the that we designed in the previous slide along with this plant. And what you see therefore, is that we have the root locus passing through the 2 points minus 1.5 plus minus 2 j and for this particular gain is going to be the closed loop pole actually going to be located exactly at minus 1.5 plus minus 2 j. So, we would have more pole on the on this branch of the root locus between this pole and that 0. And another pole on the other branch of the root locus that is somewhere here.

So, since these poles are not as close to the origin as the 2 poles minus 1.5 plus minus 2 j they will not dominate the response of the closed loop system.

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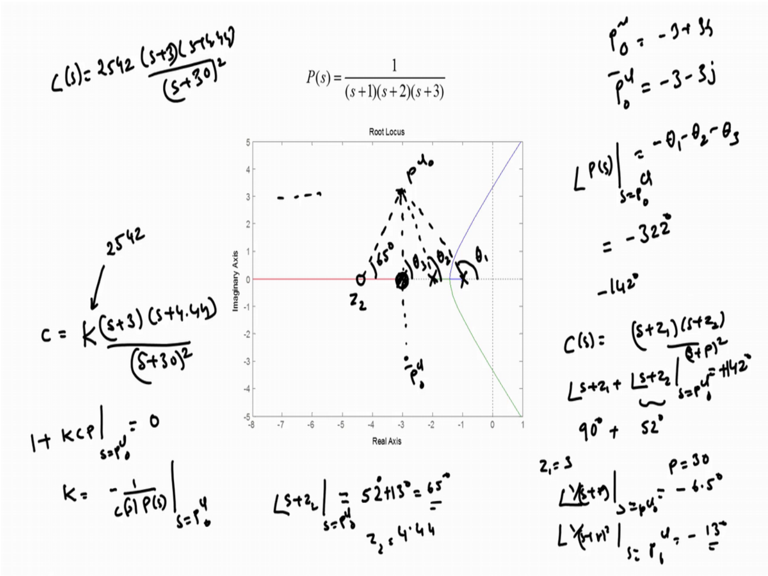

Now, how it seen how we can employ root locus in order to get a certain pair of points in the complex plane to be the dominant pole locations. Even when those points do not on their own lie on the root locus of the plant, we shall now take another example to further explore this particular aspect of root locus based design. So, we shall come back to this same plant P of s is equal to by s plus times s plus 2 times s plus 3.

And this time earlier in the previous example we wanted the closed loop poles to be at minus 1.5 plus minus 2 j. Here we should expect the closed loop pole P cl naught to be at minus 3 plus 3 j and its complex conjugate P cl naught bar to be minus 3 minus 3 j. So, in this complex plane if I want to represent minus 3 plus 3 j it would be somewhere here. So, this would be P cl naught and this would be its complex conjugate would be P cl naught bar.

So, we want the closed loop pole to be located at P cl naught the dominant closed loop poles to be located at P cl naught and P cl naught bar. How do we design a controller that gets this to happen.

If I look at this problem it does not look substantially different from the problem that we considered in the previous example. Once again this is a case where the points P cl naught and P cl naught bar are not lying on the root locus of the plant. So, what we have to do is to adopt essentially a 2 step based approach. In the first step we choose such a controller that it gets the root locus of the open loop system namely the controller times the plant to pass through the point P cl naught and the second step is to choose the controller gain k such that the closed loop pole actually lies at the point P cl naught and P cl naught bar.

So, these are the 2 steps that we have to adopt and that is what we have to do even here, but there is a small clash in this particular problem and hence I am separately discussing it. So, if you compute once again the angle subtended by the open loop poles the 3 open loop poles of the plant namely at minus 3 and minus 2 and minus 1 at the point P cl naught. So, once again we can do that from trigonometry. So, this is the angle theta 1 that is subtended by the complex number s plus at the point P cl naught and this is theta 2 and this is theta 3. We would have the angle of P of s at the point s is equal to P cl naught to be equal to minus of theta minus of theta 2 minus of theta 3.

And if one uses trigonometry in compute these angles then we find that the net angle is going to be equal to minus 322 degrees. Now for this point P cl naught to be a point on the root locus of the open loop system, we need the angle of course, to be equal to minus 180 degrees. So, that is a difference a very large difference of minus 142 degrees between what the net angle at the point P cl naught has to be and what it actually is. Now if we want to choose to go with a single 0 for our controller in order to supply this positive angle of 142 degrees, then that 0 would probably have to be located on the right half of the complex plane.

So, our 0 might have to be located here somewhere on the right half of the complex plane for the angle subtended by that 0 at the point P cl naught to be equal to plus 142 degrees and a problem with locating a 0 on the right half of the complex plane is that it becomes a non-minimum phase 0 and it is something that we wish to avoid for reasons that we will get to in other lectures in later lectures. So, for the moment let us assume that it is not wise and desirable to have a 0 on the right half of the complex plane. So, let us (Refer Time: 29:508) choose to place a single 0 there. So, we have to find other ways by which we can compensate for this difference of 142 degrees between what we desire as a net angle and what we have.

One way to do it is to go with 2 0s. So, if you have a controller C of s to be equal to s plus z 1 times s plus z 2 as the starting initial structure for the controller of course, this is not a causal controller, but for the moment let us assume that we use the 2 0s to get the angle of 142 degrees at the point P cl naught then we have to pick z 1 and z 2 such that the angle of s plus z 1 plus the angle of s plus z 2 at the point s is equal to P cl naught is equal to plus 142 degrees. Now we have freedom in locating z 1 and z 2 wherever we want. If you pick z1 that we can the, we need to choose z 2 such that the angles subtended by the two together add up to 142 degrees.

So, what we can do is we can choose to place z1 exactly at s is equal to minus 3. So, at this location here I am shading it there. So, in other words this controller 0 z 1 essentially cancels the plant pole at s is equal to minus 3. Now if you notice this 0 subtends an angle of plus theta 3 at the point P cl naught and a simple observation will reveal that theta 3 is equal to 90 degrees and therefore, the angle s plus z 1 at the point P cl naught will be equal to 90 degrees if I choose z 1 to be equal to 3. So, what that means, is that my angle s plus z 2 has to be such that the sum should add up to 142 degrees and that will reveal that my angle s plus z 2 has to be equal to 52 degrees.

So, now that I know what angle z 2 has to subtend at the point P cl naught. I can use trigonometry to locate z 2 at some suitable location to the left of the point z 1 such that it subtends a angle of so, I can choose to locate it here and use trigonometry to determine its exact position. So, that this angle is going to be equal to plus 52 degrees. However, exactly as with the previous controller we have a non-causal implementation with just the 2 0s being ones that are supplying the phase necessary to make the point P cl naught a point on the root locus. Therefore, we have to also choose to have 2 poles for this controller. So, that it becomes a causal controller whose denominator polynomial is at least of the same degree as the numerator polynomial.

In this case again we have freedom in placing these poles wherever we want. And I shall choose to place them at the same position. So, I shall choose the denominator to be of the S plus P the whole square. And I shall choose also to locate the point p at 30 on other words the pole of the controller is at s is equal to minus 30.

Now once again this point s is equal to minus 30 where I have chosen to locate my controller pole is rather arbitrary in practice this location has to be decided on the basis of the available bandwidth for the controller, but in general we chose it to be sufficiently far away in the hope that that far away pole will not contribute to a very large negative angle at the point P cl naught.

So, the farther away it is the smaller is angular contribution of the point P cl naught. And in this case now assumed that s is equal to minus 30 is adequately for a way for our particular purpose. So, if you want to compute the phase contributed by a single pole in other words if I want to contribute calculate by S plus p at the point P cl naught this is equal to P cl naught. I would get that to be equal to minus 6.5 degrees. So, the 2 poles put together they are. So, contribute to minus 6.5 angle of by S plus p the whole square therefore, at the point s is equal to P cl naught will be equal to minus 13 degrees.

So, what you know how to do is to fine tune the positions of z 1 and z 2. So, that they now compensate for this extra phase deficit let us come about because of the 2 poles that we have added to the controller in the interest of causality. So, earlier the 2 were subtending angles 90 degrees and 52 degrees respectively at the point P cl naught. Now they have to subtend 90 plus 52 plus 13 degrees. Now we can fine tune both the positions or of only of the positions of the 0. In this case I shall choose not to move the 0 z1 I shall choose to continue to locate it at z 1 is equal to 3. I shall choose the location of z 2 in such a manner that the net phase of z 2 is no going to be not 52 degrees.

But instead 52 plus 13 degrees so, the angle of s plus z 2 at the point s is equal to P cl naught has been chosen to be equal to 52 plus 13 degrees which is going to be equal to 65 degrees. If I choose this to be the angle of s plus z 2 then angle of s plus z 2 plus s plus z 1 is going to be equal to142 plus3 degrees which is 155 degrees. And with that I will be able to get the net angle at the point P cl naught to be equal to minus180 degrees. So, from trigonometry I can find out that locations z2 at which this angle is going to be equal to 65 degrees. And I find that to be equal to 4.44. So, z 2 has to be at equal to 4.44 of the 0 has to be placed at s is equal to minus 4.44 for the angle subtended by the 0 at the point P cl naught to be equal to 65 degrees. So, our tentative controller therefore, is going to be of the form s plus 3 times s plus 4.44 divided by s plus 30 the whole square.

Now, this controller when cascaded with this plant will cause the root locus to pass through the point P cl naught and P cl naught bar and, but this controller as it exist will not guarantee that the closed loop pole will lie exactly at P cl naught and P cl naught bar. It may be somewhere else on the root locus. What has to be done in order to get it to be exactly at P cl naught and P cl naught bar is to choose an appropriate gain K for the controller in order to get the closed loop poles to be located at these 2 particular locations. To find the value of the gain k we employ the standard equation for root locus namely plus k times CP at the point s is equal to P cl naught to be equal to 0 or in other words the gain K necessary would be equal to minus by C of s times p of s at the point s is equal to P cl naught.

So, if you use this equation to compute the gain K. I would the I would get the gain to be equal to 2542 therefore, our controller C of s is going to be given by c equal to 254 2 times s plus 3 times s plus 4.44 divided by s plus 30 the whole square. Now this controller not only ensures that the root locus passes through P cl naught, but actually make sure that the closed loop poles would be located pair of closed loop poles would be located at P cl naught and P cl naught bar. There will be other closed loop poles on other branches of the root locus.

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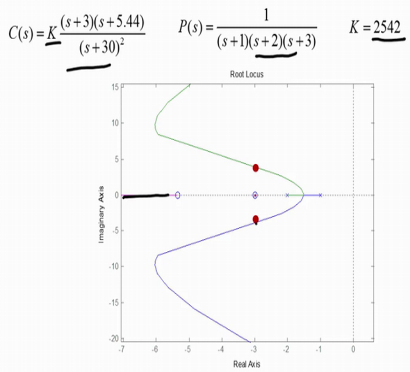

So, in the next slide I have plotted the root locus for the overall system namely this particular controller C of s and this particular plant P of s with the gain 2542 as the controller gain. And you notice, but the closed loop poles would be located exactly where you want them to be situated. The other closed loop pole would be located on this particular branch of the root locus. And since it is significantly to the left of these two poles that will that pole will not be a dominant pole of the closed loop system and hence we will not significantly affect the dynamics of the closed loop system.

Thus for you have seen how root locus can be used as a tool to design for stability of the closed loop system or other words to get the dominant poles of the closed loop system to be exactly where we want them to be. If you move the dominant pole significantly to the left of the imaginary axis where improving the stability of the closed loop system or if it is slightly to the right, then or if it is closer to the imaginary axis essentially resulting in a closed loop system is slightly reduce damping.

But we cannot focus only on the stability considerations as control engineers you also have to look at the performance considerations. How do we engineer control systems to have a desired amount of rejection of disturbances might be input disturbance output disturbance or how can we get it to track references with the desired level of an accuracy. That is not very easy to do in root locus because unlike in bode plots we cannot specify in a very transparent manner the performance specifications and the necessary gain that a controller needs to have in order for it to meet those particular performance requirements.

Nevertheless for simple cases it is possible to use root locus to also satisfy performance requirements. For instance if you want the same plant P of s to have 0 steady state error to constant inputs or in other words dc inputs. Then you note that in order for any system control system to have 0 steady state error to dc references or steady references then we need to have an integrator as part of our open loop control structure that is because an integrator results in infinite loop gain at omega equal to 0 which essentially corresponds to study inputs. And thereby enables achieving 0 steady state error to tracking to dc references.

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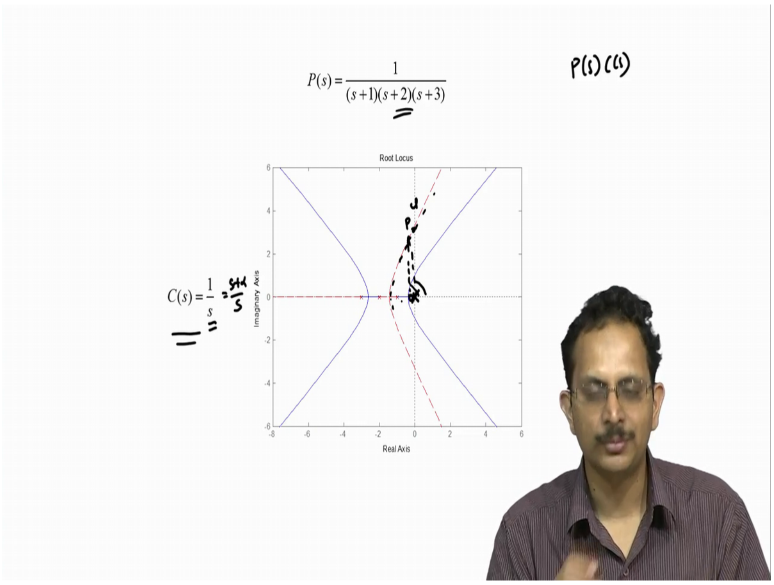

However if we were to introduce a simple integrator as I have done this particular example what you notice is that the root locus will change dramatically. In fact, this is one of the drawbacks of root locus as a tool. In the case of bode plots when we had the bode plot of a plant and a controller. In the bode, plot of the overall open loop system simply the summation of the bode plots of the plant in the controller.

So, if I had the bode plot of the controller and I modify the, bode plot of the controller a little bit I can easily anticipate how the overall bode plot would change. In the case of root locus; however, such a super position or summation property is not valid.

So, the bode plot of a new controller times an existing open loop system can be extremely different from the bode plot of the original open loop system itself and the precisely what has been shown in this graph here. So, the red dotted curve here shows the root locus for the plant P of s. And the blue curves here to the show the root locus for the open loop system P of s times C of s where C of s is given by an integral controller. So, what you see is that the blue curve is substantially different than when compared to the red dash curve.

And therefore, if we had taken the trouble to design the dominant dynamics of our closed loop system or in other words we have already placed the dominant closed loop poles at P cl naught and P cl naught bar inclusion of this integration integrator term will completely spoil the design. Because root locus will now no longer passed through that original point P cl naught and P cl naught bar and therefore, we will not able to realize those particular dominant dynamics.

So, how do we address this issue? In our time to ensure good performance in terms of tracking dc references well we have now spoiled our dominant dynamics design.

In order to address this issue what we can do is instead of adding a single pole at the origin. As what we would do if we have to if we use a simple integrator as the controller. Let us say we add a pole 0 pair. So, you addition to this pole at the origin let us say I also add a 0 very close to the origin. So, when I say very close to the origin I mean that I am adding the 0 at a distance that is much smaller than the distances of the other plants poles and 0s from the origin.

Now what happens is that because this pole and 0 pair are very close to one another or in other words I am having a control of the form C of s is equal to s plus alpha divided by s that is the kind of controller structure that I have picked where alpha is very close to 0. Now because alpha is very close to 0 if I want to pick a point on the root locus of the original plant or in other words or the root locus of the open loop system before introduction of this integrator term. What we notice is that for a point on that root locus the angle subtended by the 0 and angle subtended by the integrator or very close to one another and because a very close to one another and angle subtended by the 0 is going to be the negative of the angle subtended by the pole namely the integrator that algebraic sum of these two angles will essentially go to 0 or may be very close to 0.

What does essentially means therefore, is that if we choose a pole 0 pair that is very close to the origin then that pole 0 pair will not affect the overall phase of points on already existing root locus. So, if already we are getting the next phase of our open loop system at some point P cl naught to be equal to minus 180 degrees the phase because of these two this combination of pole zole will not be very different from minus 180 degree it maybe a little bit larger or little bit smaller depending on the exact positions of these two terms.

And what that in turn would mean is that the root locus of the new system that incorporates the 0 and this pole will be very nearly the same as the root locus of the original system itself. There will be a small deviation depending on how far away we have place the 0 from the integrator, but it will not be substantial enough for us to be concerned that our dominant dynamics design would have been spoilt.

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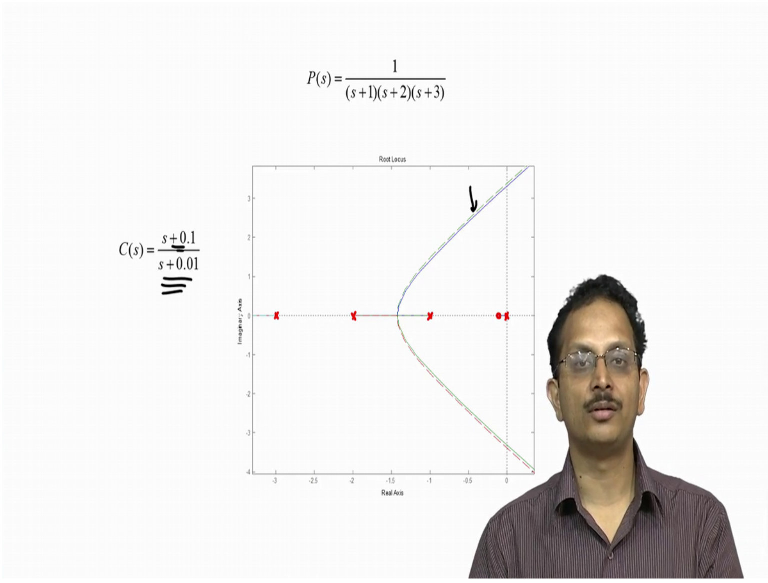

And that is precisely what has been done in this example that have considered here we have chosen to place a pole at s is equal to 0.01 and this was intended to improve the performance in the low frequency equation.

And in order to prevent the pole are at s is equal to minus 0.01 from affecting the overall root locus if we have chosen to also add a 0 closed by at s is equal to 0.1. So, the two together will result in amplification in gain of by a factor of 10 for frequencies below omega equal to 0.01 radian per second.

As per as the root locus is concerned what I have ported here is the root locus of the original plant in the dashed curve and the root locus of the plant times this controller in the blue curve and what you see is that these two curves are very close to one another which indicates that this particular controller allows us to improve no frequency performance by adding a gain of 10 in the low frequency region.

But without changing the without significantly changing the dominant dynamics that we might have already designed for this particular system and if one notices this controller structure is essentially that of a black compensator. So, having talked about how one would go about doing design using root locus a couple of comparisons between root locus and bode plots are in order. So, the first is the utility of one with respect to the other. As we discussed in the previous clip the main purpose of control engineers is to contend with uncertainties either in the plant or an it is environment.

Or in other words we have to ensure robustness of the system to variation in plant parameters and prevent disturbances occurring either at the input or at the output from affecting the overall response of the system. With this as the primary objective of control engineering, we notice that our performance specifications are more important we have to satisfy our performance specification. We have to reject disturbances by the specified amount due to track references with the desired degree of accuracy that is really what me get paid for, but in our effort to do that we might end up destabilizing our closed loop system.

And hence the importance of naique stability theory and phase margin and gain margin and so, one in performing control design. Therefore, performance specifications are more important meeting performance specifications are more important than designing a closed loop system with the desired dominant poles or the desired dominant dynamics. As we saw in this particular clip representing the constraints on the overall open loop system that would arise due to the performance specifications in terms of disturbance rejection or reference tracking is difficult in case of root locus.

Whereas, one can readily represent in a bode plot the necessary loop gain or the necessary controller gain for us to be able to track references or reject disturbances in certain frequency ranges.

And hence root locus is not as power full a tool as bode plots when we are performing control design, but still it is a very useful complementary tool to analyze the overall response of control systems once you are done with the design of a controller. So, although one might not always want to use root locus as the tool of choice for performing control design hovering to the difficulties associated with satisfying performance specifications even though stability specifications are very easy to satisfy because you know exactly where the closed loop poles are going to be located. The root locus still performs a very valuable service by allowing us to check where the closed loop poles where all the closed loop poles of our system are.

This brings us to a problem that we encountered when we were doing bode plot based design.

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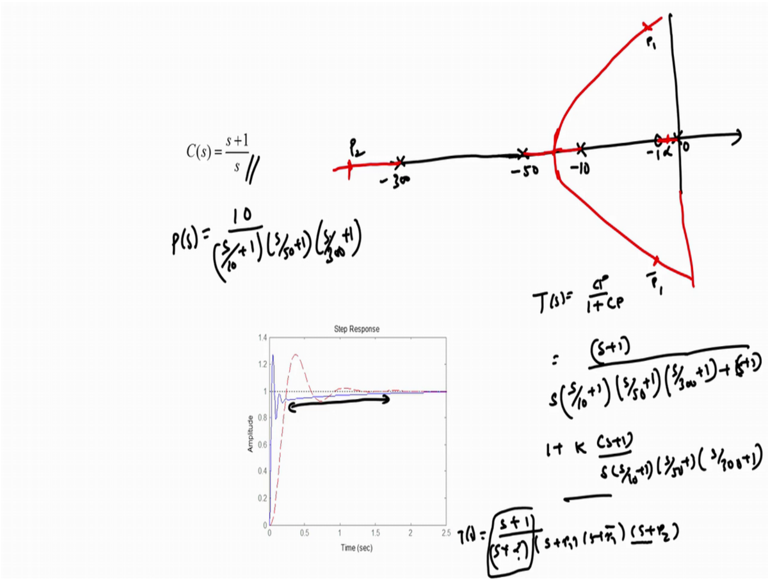

So, to refresh your memory we had design a proportional integral controller C of s is equal to s plus 1 by s for the plant P of s which was nominally given by P of s was equal to 0 by s by 0 plus times s by 50 plus times s by 300 plus. So, this was the plant that we had considered in the clip where we had performed control design using bode plots and the Pi controller was chosen in order for us to improve the mid frequency performance.

When we plotted a step response of the closed loop system, we noticed that there was a long inexplicable tail in a response of the closed loop system that could not be explained by using bode plots. There was nothing in the body plot of the open loop system that could hint at the possibility of such a tail existing in the response of the closed loop system. However, since root locus allows us to locate all the closed loop poles in principle we should be able to answer this question by using root locus.

How do we explain the existence of this pole? So, if this is the controller of that we are using and this is the plant transfer function in the overall transmission function whose step response we are plotting here is given by T of s is equal to C times p divided by 1plus c times p.

And if one going to simplify this one would get this to be equal to s plus divided by s by 10 plus times s by 50 plus times s by 500 plus. So, these are all the plant poles times s which is the controller pole plus s plus. So, this is the denominator polynomial of our system. Now just inspection of this will not reveal to us how we are getting this long type to understand this let us plot the root locus for the transfer function plus k times s plus divided by s times s by 10 plus times s by 50 plus times s by 300 plus.

Anyway it is a 0 of this particular transfer function that are going to be the poles of our closed loop system. And that is what is going to dictate the step response of our closed loop system. So, if one want to plot the root locus for this particular denominator transfer function. Then one would notice that we have a pole at the origin at s is equal to 0 and then, we have poles at s is equal to minus0 and then minus 50 and a very far away pole at s is equal to minus 300 minus 10 minus 50 and minus 300. And then we have a zero also at s is equal to minus.

Now, as far as the root locus is concerned you would have a branch of the root locus. Starting from this pole the integrator and ending in the 0 for this is branch. Then we would have one more branch starting at minus 10 a second branch starting at minus 50 these two branches will meet at a certain point and break away into the complex plane. And then you would have a third branch starting at minus 300 and then I am going towards negative infinity. And the branches that broke away will eventually asymptotically approach straight lines that are inclined at plus minus 60 degrees to the real axis.

So, this is how the root locus of the overall open loop system could appear. So, we would have one closed loop pole on this branch you would have one closed loop pole on that branch you would have one closed loop pole on that branch you would have one closed loop pole on this branch on the branch near the origin. And it is that closed loop pole which is of interest to us because if I would represent T of s in terms of these closed loop poles I would have T of s to be equal to s plus divided by s plus alpha where alpha is a point here in between the points minus 1 and 0 times. If I call this point p1 and call that point p1 bar and if I want to call this point p2 then we would have S plus p1 times s plus p1 bar times S plus p2.

This would be the structure of our closed loop transfer function. So, let us focus on this term s plus by s plus by s plus alpha. We note that this term alpha is a very slow pole because it is situated much closer to the origin compared to the other poles p 1 p 1 bar p 2 or all on branches of a root locus at a quit far away from the origin compared to the point alpha. And it is this slope pole that is responsible for this long tail in the response of the closed loop system, but we do see that this long tail does not end of dominating the step response of the closed loop system how does one explain that? One can usually understand that by noting that you have this term S plus 1 also in the numerator. And when the gain is high the point alpha will be close to the point minus 1 because the root locus always ands in a 0 therefore, when gain is high when the open loop gain is high the point alpha will be close to the point.

So, the numerator s plus will essentially cancel this pole and therefore, will prevent it from dominating the response of the overall system. In this particular case however, the gain of our controller C of s simply equal to 1. Which means that for this particular case our alpha was not really ask close to the point s is equal to minus 1 as you wanted it to be. And as a consequence of that the effect of this slope pole was not adequately suppressed by the 0 S plus and the fact that it was not suppressed was what contributed to the existence of this long tail in the closed loop response of our system. So, what you see in this example is a situation that one could not readily explain by using bode plot for analysis. Whereas, one can easily explain with great clarity one when one employees root locus as a tool for analysis.

Thank you.