Control System Design

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Lecture – 22
Introduction to root-locus

Hello, in this clip we shall took take a look at a new tool for Control Design namely the Root Locus. I expect that the people who are viewing these lectures would have already had one course on feedback control. And therefore would have already been exposed to the notion of root locus and how one might draw root locus of a given system. Therefore, what we would do in this clip is to primarily refresh our memory about all the important aspects related to drawing root locus, so that we can use them for design purposes later on.

So, the motivation for drawing the root locus is essentially it stems essentially from some of the limitations with Bode plot based design as versatile and as intuitive as Bode plot based control design is. There are a couple of limitations; one of the most important limitations is the fact that the Bode plot does not reveal the locations of all the closed loop poles of our system. It thus however reveal the approximate locations of the dominant poles, because the gain crossover frequency and the phase margin together can give us information about the approximate location of the closed loop dominant poles. But as far as all the poles are concerned, it is not available from the Bode plots.

And this brought us some trouble as we saw in the previous clip, because there were certain aspects of the closed loop response and we could not explain by simply looking at the Bode plot of the open loop system. Hence, the hope is that the new tool namely that of root locus would reveal to us the reasons why the closed loop response might behave in a certain manner, if we choose a certain structure for the open loop controller and the plant. So, let us get started with defining root locus, what is root locus? Essentially, root locus is the set of points the set of all possible locations in the complex plane, where the closed loop poles can lie, when the gain parameter of the open loop system changes.

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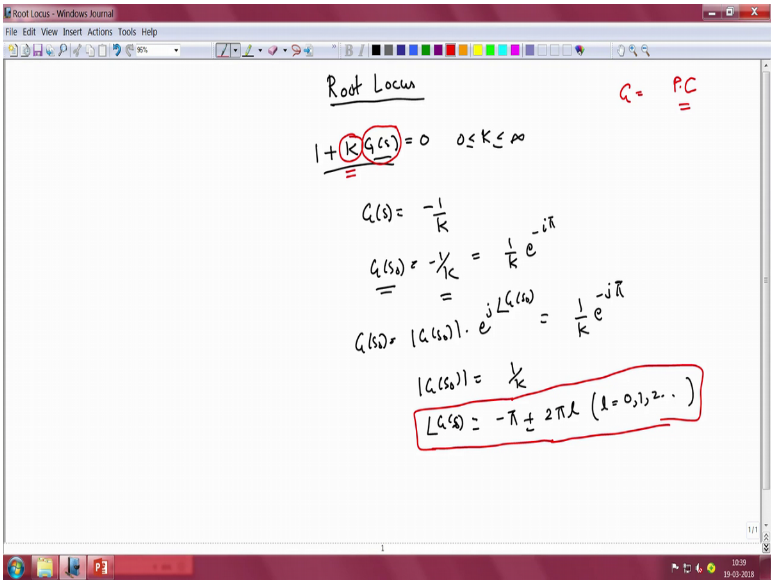

In other words, if I have a denominator polynomial for the closed loop system to be of the form 1 plus K times G of s equal to 0, then for each particular value of the gain K, we would have a certain set of points as the roots this a polynomial or the 0s of this polynomial would be at certain specific locations depending on the specific value of the gain K that we pick. Now, suppose we were to let the gain K to vary between 0 and infinity, then the question we ask ourselves is where all would the closed loop poles of this system of our system or equivalently the 0s of this particular transfer function 1 plus K times G of s equal to 0, where all would they lie.

So, the root locus is the set of all points in the complex plane where the 0s of this particular transfer function 1 plus K times G of s lie as the parameter K is varied from 0 to infinity. It is worth noting that in this definition we assume that none of the parameters of the transfer function G of s which is like the open loop transfer function of our system are dependent on the gain K. So, it is assumed that the gain K only multiplies G of s and does not in any way and is not contained in any of the parameters of G of s.

Now, stated as such it is not very useful, because we cannot employ this definition to draw the root locus. A slightly more useful definition of the root locus would arise, if we rearrange this particular equation. For instance, I can write G of s to be equal to minus 1 by K, for those points which satisfy this particular equation. This implies that if s naught is a point that is a closed loop pole of our system or equivalently a 0 of the denominator polynomial denominator transfer function namely 1 plus K time G of s equal to 0, then we would have G of s naught to be equal to minus 1 by K. And on the left hand side of course, I have a complex number so, I have a complex number also on the right hand side.

But, if I have to represent this in the form of magnitude and phase, I would get this to be equal to 1 by K e to the power minus i pi, because 1 by K e to the power minus i pi is essentially equal to minus 1 by K. Now, I can write the left hand side of this equation also as G of s naught equal to magnitude of G of s naught times e to the power j times the angle of G of s naught and that is going to be equal to 1 by K e to the power j times minus j times pi. So, what this means is by comparing the magnitudes and the phases on the left hand side and the right hand side, we would get the magnitude of G of s naught to be equal to 1 by K and the phase of G of s naught to be equal to minus pi radians.

For those points s naught that are the 0s of our denominator transfer function. Of course, minus pi is not the only possible solution. All cyclically equivalent angles in other words minus pi plus minus 360 degrees or 720 degrees and so on and so forth, in other words minus pi times plus minus 2 pi l, where l is any integer, one can assume values of 0, 1, 2 and so on are all the possible locations where our closed loop poles can lie. Now, so therefore we have a more useful definition of the root locus, namely it is a set of points in the complex plane, where the angle of the open loop transfer function, in other words G of s naught is equal to minus pi or it is cyclic equivalents minus pi plus minus 2 pi l.

This is really useful in helping us to determine the locations where the closed loop poles can lie or essentially the 0s of the denominator transfer function can lie. Now, the way we have to talking about root locus it appears as though, it is useful only in the context of us having a proportional controller of gain K cascaded with our open loop system.

So, may be this the transfer function G can represent the plant transfer function times some controller. And cascaded with this controller, let us say we have this gain K. Then it appears as though this technique of plotting the root locus allows us to study, where the closed loop poles of our system would wander about in the complex plane, when this proportional gain K is varied given this particular plant and controller transfer functions.

Although it appears that it is therefore rather restrictive in it is use to being able to study the effect of proportional controllers on the location of the closed loop poles it is a little bit more general than that. It can also be used to study the variation the effect of variation of other parameters of our open loop system on the location of the closed loop poles, we should illustrate that by means of a numerical example.

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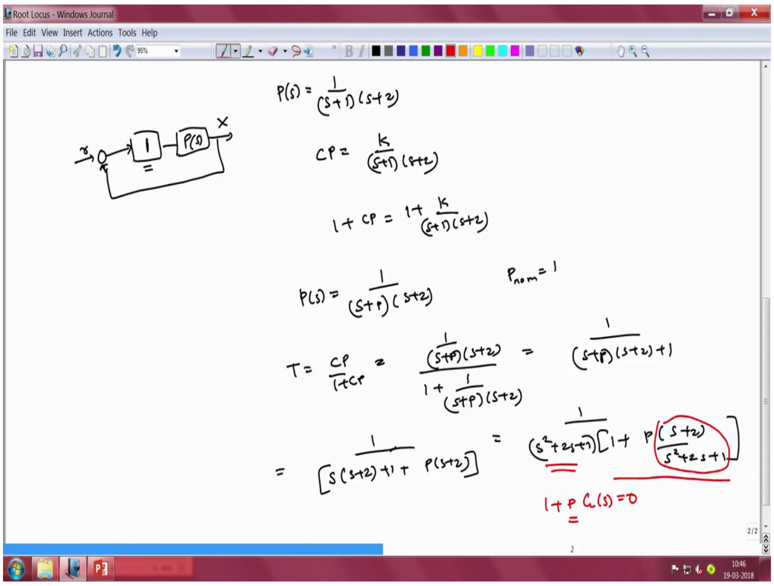

So, let us assume that we have a plant P of s equal to 1 by s plus 1 times s plus 2. And let us assume that we actually have a proportional controller cascaded with our plant, in other words our closed loop system would have a controller gain k cascaded with P of s and the output of P of s is compared with the reference and the errors is regulated by means of this proportional controller. So, if this were the case, then our open loop transfer function which is going to be c times p would be equal to k by s plus 1 times s plus 2. And the denominator transfer function for this closed loop system would be 1 plus c p which would be equal to 1 plus k by s plus 1 times s plus 2.

So, in this case, we see that root locus does exactly what we interpreted it to do in the previous slide, namely that it allows us to study, how the closed loop poles of this system change when a proportional controllers gain k over to vary between the limits 0 and infinity. But, let us assume that in our particular problem we do not have any variation of this gain k or other words, let us say it is always equal to 1, but we had actually have a plant whose structure is uncertain. In other words, let us say P of s were to be of the form 1 by s plus p small p times s plus 2, where p is the open loop pole nominally let us assume that the open loop pole is at 1, P nominal is equal to 1 in which case we would have the plant transfer function to be 1 by s plus 1 times s plus 2.

However, let us say that this pole the open loop pole might a position might change from one experiment to the other so, might change slowly or it might be uncertain. Now, one might be interested to know how the closed loop poles of our system vary on the basis of the position of the open loop poles.

So, it can be shown that we can rearrange this problem and be able to use root locus to attack this problem as well. In this particular case the overall transmission function relating the output X of my system to the reference r which is given by T is equal to c p by 1 plus c p that is going to be given by 1 by s plus p times s plus 2 divided by 1 plus 1 by s plus p times s plus 2 and that is equal to 1 by s plus p times s plus 2 plus 1.

Now, I can write this as T equal to 1 by s times s plus 2 plus 1 plus p times s plus 2. And this in turn can be written as 1 by s square plus 2 s plus 1 times 1 plus p times s plus 2 divided by s square plus 2 s plus 1. Now, if we focus on the term within the bracket 1 plus p times s plus 2 by s square plus 2 s plus 1. We notice that we have this parameter p outside multiplying a particular transfer function none of whose parameters are dependent on p.

The other transfer function that is multiplying this term has it is roots already known to us. But, in this case suppose p were to be uncertain, we would have the term within the bracket to be of the kind 1 plus p times G of s equal to 0, where G of s is this particular transfer function. And once again we can think of this p as some kind of a gain. And study therefore how the closed loop poles of the system would change when the parameter p varies. So, in this case, kindly note that the parameter p is not really some controller gain that is varying and therefore we are looking at the variation of the closed loop pole in response to this controller gain.

In this case the variable p is actually the pole of the plant, but because we can write the closed loop transfer function in the form in the standard form necessary for us to plot the root locus, namely 1 plus some constant times some transfer function is equal to 0. We are now able to look at how changing this constant p allowing it to assume different values will change the closed loop poles of our system.

So, therefore the lesson here is that the tool of root locus is not restricted only to looking at the variation of the closed loop poles in response to some controller gain. It can also be used to look at the other parameters of the open loop system such as the plant pole or the plant gain would have on the closed loop poles of the system for performing control design.

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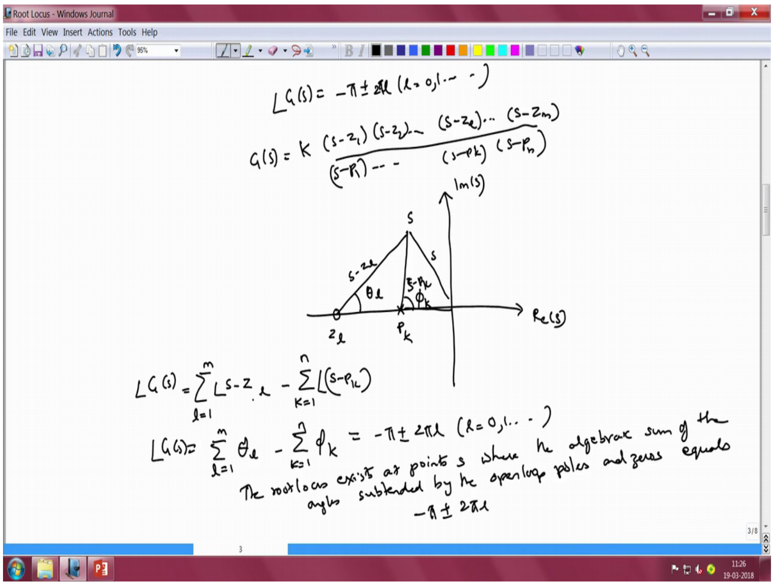

Before we look at the different rules for drawing the root locus, it is worthwhile to look at the geometric interpretation of the equation that we wrote in the previous slide namely that the angle of G of s is equal to minus pi plus minus 2 pi l, where l goes from 1 to 0, 1 and so on up to infinity. So, typically as we have seen in our previous clips our transfer function G of s will be of the form some constant times s minus z 1 times s minus z 2 and so and so forth. A general term would be of the from s minus z l and let us say we have m 0s, then we would have s minus z m as the last term divided by s minus p 1 and so on and so forth up to s minus p n assuming that we have n poles.

Now, let us look at a general kth pole s minus p k in this expansion and locate it in the complex plane. So, this is the real part of s, this is the imaginary part of s. And let us say the point p k is located here and the point z l is located there. Now, let us consider a general point s in the complex plane, we note that the complex number s minus p k essentially given by this particular complex number, because this is s and that is minus p k therefore, this guy will be s minus p k. Likewise, this complex number will be s minus z l so, this complex number, therefore represents s minus p k while that complex number represents s minus z l.

Now, we know that if our G of s is written in this particular form the angle of G of s will essentially be equal to the angle of s minus z l summed over different values of l going from 1 to m minus the angles of s minus p k, where the index k can assume value from 1 to n. Now, what is the angle s minus p k geometrically; geometrically the angle s minus p k is essentially the angle that this particular complex number s minus p k makes with the real axis I shall call that phi k. Likewise, the angle s minus z l is the angle that the complex number s minus z l makes with the real axis I shall call that theta l.

Therefore, the angle of G of s is equal to sigma theta l, where l goes from 1 to m minus sigma phi k where k goes from 1 to n. So, we note that if our transfer function G of s is represented in this particular form we have this very nice geometric interpretation for the points on the root locus. Because, there essentially the set of points in the complex plane, where the algebraic sum of the angles subtended by the open loop poles and 0s at that point add up to minus pi or it is cyclic equivalents.

So, when sigma theta l minus sigma phi k is equal to minus pi plus minus 2 pi l, where l goes from 1 2 and so on and so forth. Then all those points are points on the root locus or in other words, the root locus exist at points s where the algebraic sum of the angles subtended by the open loop poles and 0s equals minus pi plus minus 2 pi l minus pi or it is cyclic equivalents. We will notice later on when we are doing design that is geometric interpretation is very useful in performing control design.

And making sure that our closed loop system has provided we can cast the problem in the standard form necessary for plotting the root locus namely, 1 plus some constant times some transfer function is equal to 0. Now, let us briefly refresh our memory on how one might draw the root locus for a given open loop transfer function. You might remember that we adapt a set of rules to draw the root locus for the given transfer function. So, let us lay down all these rules the important rules one by one, because you would be subsequently using one or more of these rules for plotting our root locus.

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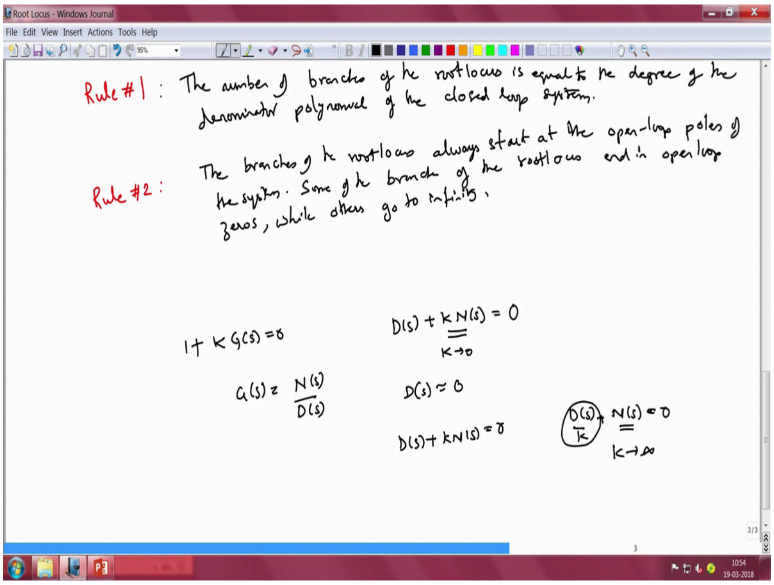

So, the first rule; rule number 1 is that you have as many branches to a root locus as the degree of the denominator polynomial of your closed loop system. That is because if you have a denominator polynomial for the closed loop system to have a degree n, then you have n 0s for this polynomial or in other words you will have n closed loop poles. And each of this closed loop poles in general will change, when the gain open loop gain k were to change. Therefore, as the gain k is varied, each of these closed loop poles will trace their own particular trajectory in the complex plane.

And therefore, we would have n branches to the root locus of this system. So, the first rule is that the number of branches of the root locus, which essentially refers to the trajectories of each of these n closed loop poles of our system. The number of branches of the root locus is equal to the degree of the denominator polynomial of the closed loop system so, this is the 1st rule.

The 2nd rule has to do with where these branches start and where they end so, let us come to rule number 2. So, in order to look at where the root locus starts, in other words where the closed loop poles would be located when the gain k is close to 0 and where it ends or in other words where the closed loop poles would be located, when the gain K tends to infinity. We need to first look at the denominator transfer function 1 plus K times G of s equal to 0. If we were to represent G of s the transfer function G of s as the ratio of a numerator polynomial N of s and a denominator polynomial D of s, then we would have the 0s of 1 plus K times G of s to essentially be the 0s of the polynomial D of s plus k times n of s equal to 0.

Now, in the limit that k tends to 0, we would notice that this term k times N of s will get weighted down enormously to the point that in the vicinity of k equal to 0. Our closed loop denominator polynomial will essentially be D of s is equal to 0. And D of s equal to 0 has n solutions which are the poles of the open loop system itself. Therefore, the first point to note is that the root locus all the N branches of the root locus start at the location of the open loop poles of our system.

Similarly, what happens when k tends to infinity? So, if we return to this equation D of s plus k times N of s equal to 0 and rearrange it somewhat to look like this namely D of s by k plus N of s equal to 0, in other words we have divided this equation by k and obtained D of s by k plus N of s equal to 0. We note that in the limit k turns to infinity. The term D of s by k gets weighted down dramatically.

And therefore the sum of the branches of the root locus essentially converts to the 0s of N of s, in other words some of the branches of the root locus end up in the open loop 0s of the system. It is worth noting that the degree of N of s is in general not equal to the degree of D of s. And indeed for all physical systems N of s is strictly lesser in degree compared to that of D of s. And therefore, not all of the branches of the root locus end up in the open loop 0s some of the branches actually turn to infinity.

The way they tend to infinity is something that we would look at in so one of the later rules. Therefore, in rule 2 we confine ourselves to those branches that stink inside the open loop 0s. And we can then state that the branches of the root locus always start at the open loop poles of the system which are essentially the 0s of D of s. However, not all of these branches end up in the open loop 0s, some of them end up in open loop 0s and a rest go to infinity. Some of the branches of the root locus end in open loop 0s while others go to infinity, in what manner, they go to infinity, at what directions along what curves is something that we will look at shortly so, this is the 2nd rule.

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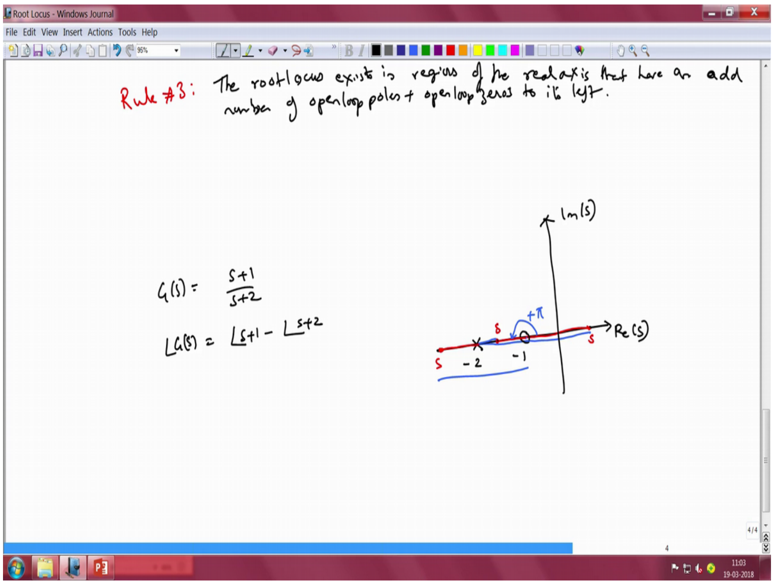

The 3third rule is intended to help us determine the location of the root locus on the real axis of the complex plane. So, rule number 3 has to do with where the root locus would be situated on the real axis of the complex plane. Now, let us try to determine where the closed loop poles could lie on the real axis by means of a numerical example. So, let us take the transfer function G of s to be equal to s plus 1 s plus 2. So, if we were to represent the location of the open loop pole and 0 in the complex plane so this is the real part of s and this is the imaginary part of s, then we would have the 0 to be located at s is equal to minus 1 and the pole to be located at s is equal to minus 2.

In future, we shall adopt the notation that has been shown here. The open loop 0 would be represented by a circle and an open loop pole would be represented by a cross mark. Now in order to answer this question of where on the real axis would the root locus lie. Let us, say we pick a point s on the real axis. Then we note that s plus 1 essentially represents this particular complex number. Likewise, we note that s plus 2 represents that other complex number.

Therefore, the angle of G of s is going to be equal to the angle of s plus 1 minus the angle of s plus 2. You get a minus sign a term s plus 2 is in the denominator. And if you look at the angles that each of these complex numbers make with the real axis, you find that the complex number s plus 1 and s plus 2 both make angle 0 degrees with respect to the real axis. Therefore, the difference between these 0 angles is also going to be 0 degrees. Therefore, all points s that are to the right of this 0, and this pole cannot be points on the root locus.

In contrast let us say we pick another point s that is somewhere midway between these two 0 and the pole pair. So, if this is the point s, then we note that the complex number s plus 1 represents this particular complex number. And a complex number s plus 2 represents that particular complex number. Now, the angle that the complex number s plus 1 makes with respect to the real axis is going to be equal to plus pi radians. Likewise, the angle that the other complex number s plus 2 makes with respect to the real axis is going to be equal to once again 0 radians.

Therefore, when I take the difference between the angle of s plus 1 and plus 2 for points that are between this open loop pole and 0, we notice that it is going to be equal to pi. And this indicates that all points that are between this open loop pole and 0 are possibly are points on the root locus. Now, likewise if we consider a third point that is to the left of both the open loop 0 as well as the open loop pole, we see that the complex number s plus 2 refers to this particular complex number.

The complex number s plus 1 refers to that particular complex number. And they both subtend an angle of 180 degrees with respect to the real axis. And therefore the difference between the angles subtended by each of them will be equal to 0. Therefore, all the points s that are to the left of the open loop pole and 0 also cannot be points on the root locus.

Now, we can take other examples and clearly see that there would a trend as far as where the closed loop poles where the root locus can exist on the real axis and that is given by rule number 3. The root locus exist in regions of the real axis that have an odd number of poles open loop poles plus open loop 0s to it is left. So, you can verify that in this case for complex numbers here, you have 2 namely 1 pole and 10 to the left of this complex number and therefore these points cannot be points on the root locus.

Likewise, for points here in between the open loop pole and 0 we have just 1 pole to its left. And therefore, we have an odd number of poles plus 0s to it is left. And therefore all such points in between this open loop pole and 0 can be points on the root locus. And likewise for points s to the left of both these pole as well as the 0. We see that we have no poles and 0s to the left of s and since we have 0 poles and 0s to the left of s these points s also cannot be points on the root locus. This essentially this conclusion is essentially derived by looking at the phase that is added by the open loop poles and 0s on the real axis at any point s on the real axis.

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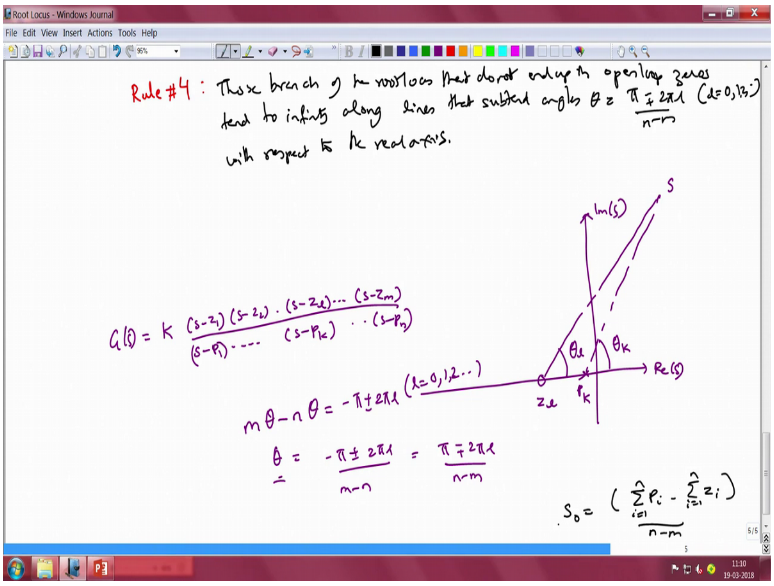

The 4th rule addresses itself to the location of the root locus for points that are very far away from the origin. So, as s tends to infinity where will the how will the root locus look? So, to address this question, let us once again draw the complex plane so, this is real part of s and this is imaginary part of s. And let us assume that our open loop transfer function G of s would be of the form some constant times s minus z 1 times s minus z 2 and so on and so forth. Some general term would be of the kind s minus z l. And the last term would be s minus z m so, there are m 0s and likewise there are n poles. So, s minus p 1 is the first pole and so on and so forth s minus p k is the general kth pole and s minus p n is the last pole.

So, let us assume that this is the structure of our open loop system so, we can locate all the poles and 0s in the vicinity of the origin. So, our let us for instance for the present locate just one of the representative poles p k and one of the representative 0s z l. Now, let us consider a point s which is very far away from the origin. Now, if this point s is sufficiently far away, you would note that angle that the complex number s plus p k makes with respect to the real axis is going to be equal to theta k. And the angle that s plus z l makes with respect to the real axis is going to be equal to theta l are these two angles are going to be very nearly equal to one another, when this point s is extremely far away.

Therefore, if I have m 0s then for a point s that is sufficiently far away from the origin, then I would have the angle contribution from all the 0s of the open loop system to be equal to m times theta. Likewise, the n terms s plus p 1 all the way up to s plus p n will result in a net angle contribution of minus n times theta. And for this point s to be a point on the root locus we should have this to be equal to minus pi plus minus 2 pi l, where l is any integer 0 1 2 and so on.

So, this therefore, tells us where the root locus can lie in the complex plane for points s that are very far away from the origin. We would have from this equation that theta would be equal to minus pi plus minus 2 pi l divided by m minus n or equivalently it is going to be equal to pi minus or plus 2 pi l divided by n minus m. So, what it indicates therefore is that very far away from the origin the root locus will look like straight lines. And these straight lines are tilted with respect to the real axis of the complex plane at these particular values of theta.

So, if I want to substitute different values of l here, I would get different values of theta. And these are the inclinations of these straight lines along with the closed loop poles would move for branches of the root locus that are very far away from the origin. Now, therefore, if I am going to state rule number 4, those branches of the root locus that do not end up in 0s in open loop 0s tend to infinity along lines that subtend angles theta equal to pi minus or plus 2 pi l divided by n minus m where l can be assumed l can assume value 0, 1, 2, 3 etcetera with respect to the real axis.

So, the root locus of course asymptotically approaches these straight lines. And when one extends these asymptotes, so the straight line asymptotes when one extends is asymptotes, then these asymptotes can be shown to meet at a particular point near the origin and that point can be shown to be s naught equal to sigma of the open loop poles p i where i goes from 1 to n minus sigma of the open loop 0s z i where I goes from 1 to n divided by n minus m. So, this is a location where all these straight line asymptotes to which the root locus converges when s tends to infinity will all meet in their extended backwards towards the origin. The last rule has to do with the break away or the break in points of the root locus.

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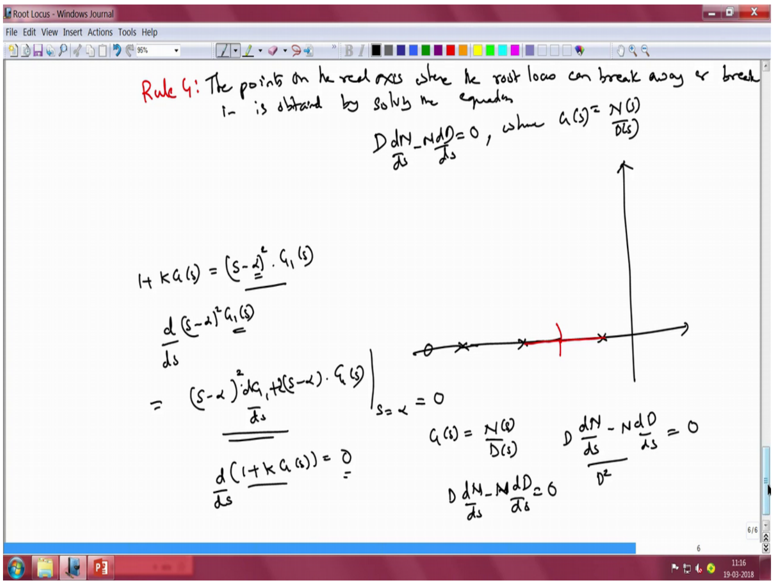

So, rule 4 so break away and break in points are the locations where the root loci break away into the complex plane or break in back from the complex plane on to the real axis. So, in order to obtain the locations where break away or break in happens, we note that at the points where break away or break in happens we would have repeated roots for the closed loop system that is because, if for instance we were to again consider the complex plane.

So, suppose we have a set of open loop poles and 0s along the real axis. Let us, say we have one branch of the root locus starting from one of the poles open loop poles another branch starting from another other open loop poles. The point where they meet, and they break away is a point where you would have two closed loop poles of identical position. So, in other words you would have repeated roots for the closed loop system. Since, at the break away or break in points you would have repeated roots for the closed loop system, we can therefore represent 1 plus k times G of s which is the closed loop transfer function denominator transfer function to be of the form s minus alpha the whole square times G 1 of s where s minus alpha where alpha represents the particular repeated root.

Now, if you have the structure of the denominator transfer function to be of the form s minus alpha the whole square times G 1 of s, you can clearly you can easily show that if we take the derivative of the right hand side, then we would have d by d s of s minus alpha the whole square times G 1 of s to be equal to s minus alpha the whole square times d G 1 by d s plus s minus alpha 2 times s minus alpha times G 1 of s.

Therefore, regardless of what is other transfer function G1 of s is we note that this right hand side evaluated at s equal to alpha will always go to 0 simply because, we have repeated roots at this particular point alpha. Therefore, to determine where the break away and the break in points happen we essentially have to solve the algebraic equation which is given by d by d s of 1 plus k times G of s equal to 0. Because, at the location of break away or break in the right hand side will be equal to 0 and the left hand side essentially is equal to 1 plus k times G of s.

So, if we were to represent the open loop transfer function G of s is the ratio of a numerator polynomial and a denominator polynomial, we would get that d by d s of 1 plus k time G of s is equal to 0. Essentially implies that d N by d s times D minus d D by d s times N divided by D square is equal to 0 or equivalently we would have D times d N by d s minus N times d D by d s equal to 0. So, this essentially represents an algebraic equation and when we determine the 0s of the algebraic equation, we would we will be able to determine the possible locations in the complex plane where break away or break in of the root locus can happen.

Therefore, the points on real axis real axis where the root locus can break away or break in is obtained by solving the equation D times d N by d s minus N times d D by d s is equal to 0, where G of s which is the open loop transfer function is equal to N of s by D of s. Now, this concludes the important rules that we would need in order to draw the root locus for the systems that we would be considering in this lecture. We shall now take a look at some of the typical root loci.

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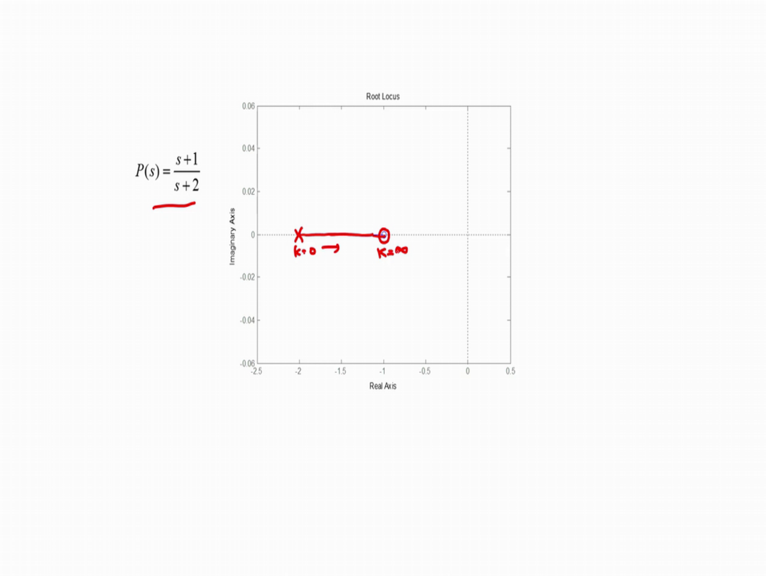

So, the first example that I have considered is P of s is equal to s plus 1 by s plus 2. In this case, we note that we have a 0 at s is equal to minus 1 and a pole at s is equal to minus 2. And we can easily verify that the angle criterion is not met on either the right side of both the pole and 0 as well as on the left side of both the pole and 0. It is only in between these two that the angle criterion is met so; the root locus for this case is a straight line that connects the pole to the 0. So, k tends to 0 k is equal to 0 at the location of the open loop pole. And k increases as one moves towards the open loop 0 and at the location, where the open loop 0 is located k will be equal to infinity.

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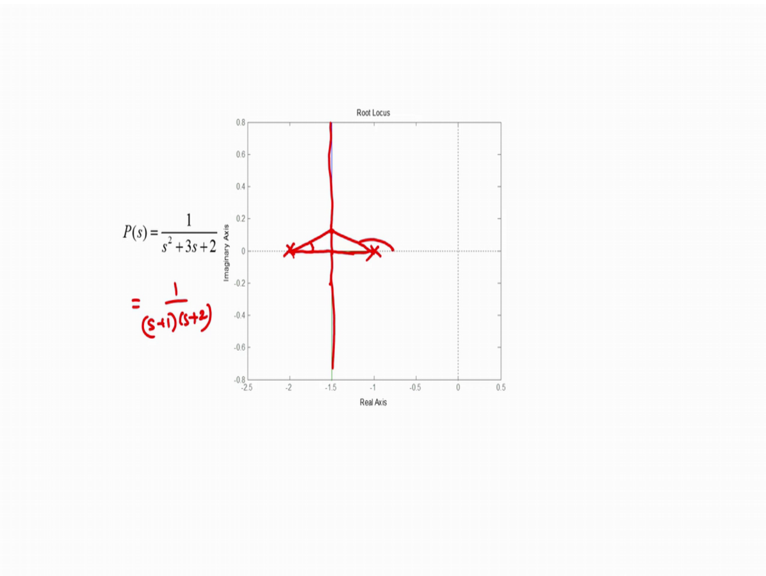

In the 2nd example, we have considered a second order system P of s is equal to 1 by s square plus 3 s plus 2. And one can easily see that this can be written as 1 by s plus 1 times s plus 2. Therefore, the open loop system has 2 poles one at minus 2 and the other at minus 2. So, we would have root loci starting from each of these open loop poles, root loci will meet at a certain point and then break away into the complex plane.

Now, if we were to consider any point in the general complex plane, we note that if this point lies on the perpendicular bisector of the line that connects the 2 poles, then the angle subtended by this the first pole namely at s is equal to minus 2 will be given by this particular angle. And the angle subtended by the other pole will be given by that particular angle and by symmetry this angle plus that angle will be equal to 180 degrees.

In other words, therefore in the complex plane in the general complex plane the root locus essentially will be the perpendicular bisector of the line segment that connects the 2 poles. Therefore, the root locus looks has shown here. It is worth noting in this case that for a second order system, you can never have instability, because the root locus at best parallel to the imaginary axis and it never crosses the imaginary axis.

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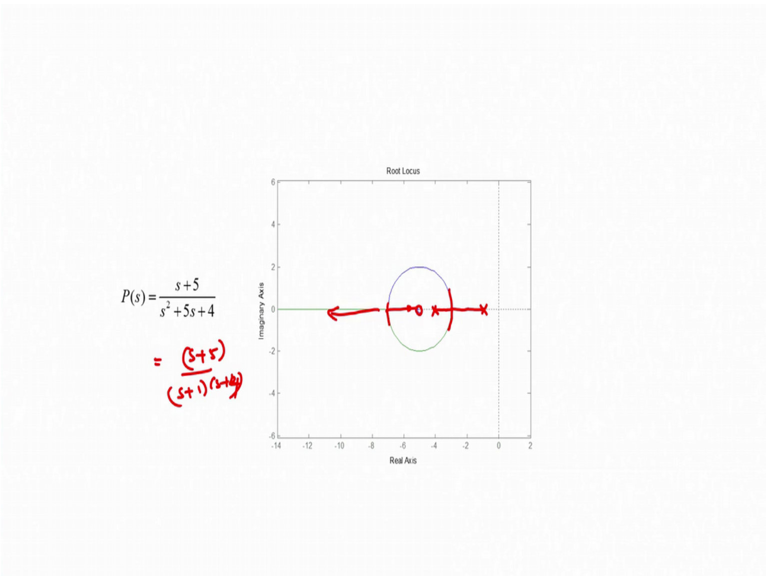

In the 3rd example, we have considered a second order system with one - zero and 2 poles. So, we can write P of s as s plus 5 divided by s plus 1 times s plus 4. So, you have 2 poles s plus 4 so, you have 2 poles one at minus 1, the other at minus 4, and a 0 at s is equal to minus 5. We note that the root locus starts from the open loop poles. They meet at a particular point the break away into the complex plane and move along this particular circular trajectory, one of the they both break back into the complex plane, one of the trajectories then sinks into the open loop 0, and the other one goes off along a negative real axis towards minus infinity.

Now, these are just a small set of examples that we have considered. Just to refresh our memory about drawing the root locus for typical plants and controllers what we might come across while doing our control design. In the next clip, we shall look at the application of root locus.

Thank you.