Control System Design
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Lecture – 21
Bode plot-based control design (Part 2/2)

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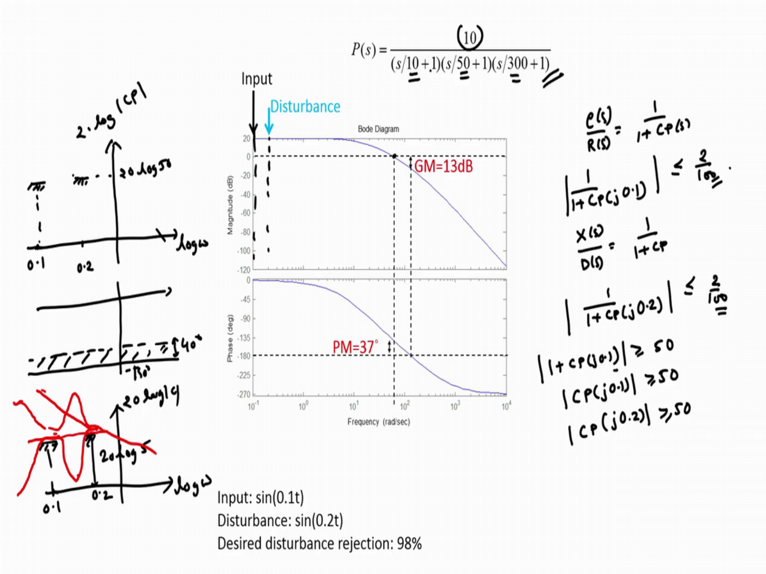

Before we answer this question, let us first ask ourselves how many controllers are there that allow us to get a gain of 5 at 0.1 radian per second, and gain of 5 also at 0.2 radian per second. Well for one thing, I can have a controller with has a constant magnitude characteristic, and that will give me a gain of 5 at both these frequencies. I can have a controller with a decreasing magnitude characteristic such that at the frequency 0.2 radian per second, the magnitude is 5; and at lesser frequencies the magnitude is even greater. And therefore, our performance specification at 0.1 radian per second is met even better than what was specified to us. So, this is one possible trend.

I can have a variety of trends, I can even have a trend, where it was higher there, it comes down, it goes up again, and then it comes down back and so on and so forth. It can even be lower than 5 at other frequencies just come up to 5 at this frequency, and then come down below 5 at intermediate frequencies, come back up to 5 at 0.2 radian per second and then do something else after that.

So, as you can see, there are infinitely many you know controller structures that are possible, because their performance has been specified at only two frequencies. So, as long as all these controllers have gain of at least 5 at 0.1 and 0.2 radian per second, they are allow to have any gain they might wish to have at other frequency. And there is no constraint on what gain they can have at these other frequencies. Therefore, in principle, we have infinitely many controllers as candidates that satisfy these two performance requirements. And that puts us in a spot of bother, because we need to now pick a suitable controller from these infinitely many controllers.

How do we choose a suitable one among the ones that satisfy these performance requirements? To choose a suitable one, we use a principle that is routinely and repeatedly used in all of engineering, namely we choose one, which allows very simple implementation or its mathematical structure is very simple. So, simplicity will become the guide for us to pick a suitable controller among the infinitely many that satisfy the two performance requirements one at 0.1 and the other at 0.2 radian per second.

Now, if we focus on the specific frequencies at which were interested performance namely 0.1 and 0.2, we see that these frequencies are much smaller than the corner frequencies of the plant, which are at 10 radian per second, 50 radian per second and 300 radian per second. Even the gain crossover frequency is around 50 or 60 radians per second, and that two is much greater than the two frequencies at which we are interested in performance.

So, what is essentially tells us that the two frequencies 0.1 and 0.2 fall within the low frequency range of our open loop system. And what we are expecting therefore is good performance in the low frequency range, low frequency regime. And we know from our previous discussions that there is one of the self controllers, which readily gives us good performance in the low frequency regime, and that is the integral controller.

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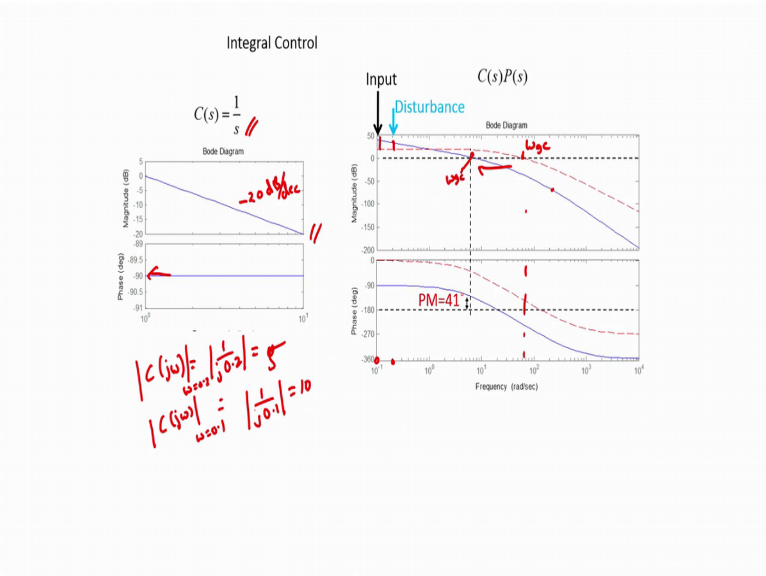

So, one possible candidate for the controller structure would be the integral controller C of s equal to 1 by s. We note that at 0.2 radian per second C of j omega will be equal to the magnitude of C of j omega will be equal to magnitude of 1 by j times 0.2 and that is going to be equal to 5. Therefore, we are getting the requisite amount of gain simply by using an integrator as our controller at 0.2 radian per second.

At 0.1 radian per second C of j omega at omega equal to 0.1, here it is omega was equal to 0.2, but omega equal to 0.1 is going to be equal to magnitude of 1 by j times 0.1 that is going to be equal to 10. So, the gain at 0.1 radian per second is even better than what we were looking out for. What is means is that while we wanted 98 percent accuracy in tracking a signal at 0.1 radian per second. We can actually do better than that by using this integral controller as the one to satisfy the performance requirements.

The Bode plot of the integral controller is shown here, its magnitude characteristics has a typical minus 20 dB per decade role off. And a phase characteristic is a constant at minus 90 degrees. If we were to cascade the controller with the plant, what we see is that the Bode plot of the open loop system looks as shown by the blue curve here.

We see that the gain in the frequency range of 10 power minus 1, and two times 10 power minus 1 or 0.1 and 0.2 radiance per second has increased somewhat compared to that of the original plant. So, the original plants gain characteristics have been indicated by the red dotted dashed curve here. And blue curve is higher than the red dashed curve at these two frequencies. And it is higher by the desired extent for us to be able to rejected disturbance at 0.2, and track the reference at 0.1 by the desired levels of accuracy.

Now, because we have chosen to go with an integral controller, we see that our gain crossover frequency omega g c, which was earlier somewhere near 50 radians per second has now become much smaller, it has moved to the left. The new gain crossover frequency is given by omega g c prime, which is actually less than 10 radian per second. And what this means is that our closed loop transient response is going to be significantly slowed down as a consequence of the limited bandwidth of our closed loop system.

However, as far as this particular problem is concerned, we have no problems with having a small closed loop bandwidth, because we were asked only to track that particular reference namely at 0.1 radian per second, and reject that particular disturbance at 0.2 radian per second, and this controller does a good job of satisfying both those specifications.

As for as the phase margin requirement, which is which is essential for stability is concerned, we see that we do not have to do any special work in order to achieve the desired phase margin that is because, when we cascade the controller with the plant the phase response drops by minus 90 degrees, because we are adding minus 90 degrees to the phase response of the plant. And the new phase response has results in a phase margin of about 41 degrees at the gain new gain crossover frequency. And this is greater than the phase margin that we had set for ourselves namely 40 degrees.

And therefore, our performance specifications have been met, and our stability specifications have also been met without having to do anything special, and this therefore completes the design. Now, that we have completed the design using Bode plots, and come up with a controller structure namely C of s is equal to 1 over s. It is only prudent for us to check whether the closed loop system satisfies the specifications correctly, this is because as we discussed there are approximations that one makes in going from time domain, to closed loop frequency domain, and from closed frequency domain, to open loop frequency domain. So, in order to be sure that the actual time domain specifications are being met by the controller that we have design, it is prudent to look at that time domain specifications.

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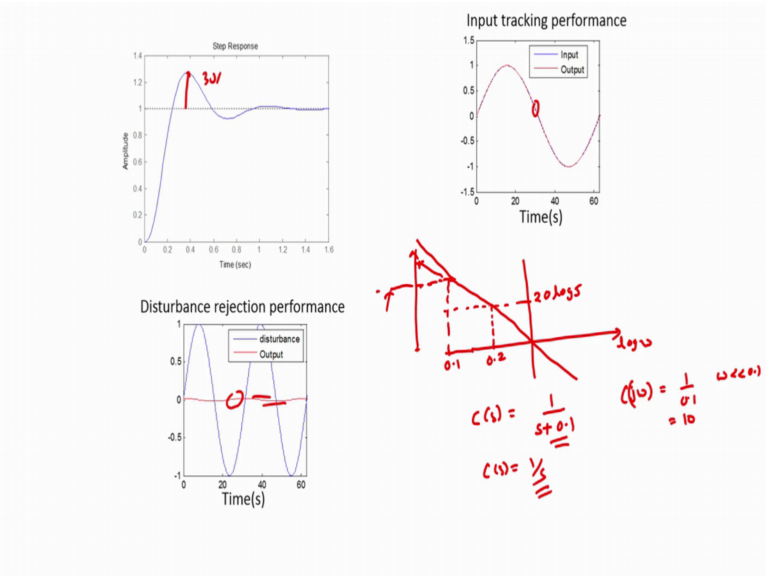

First, I have plotted in this slide, the response of the closed loop system to step inputs. And we see that the overshoot here is exactly what one would expect. If you have a phase margin of about 40 degrees, and overshoot here is about 30 percent perhaps a little bit less than 30 percent. And this is typically, what one would see if the phase margin was 40 degrees or equivalently the closed loop damping was 0.4, but what we are really paid for is not to look at the response of the system to step inputs, what we are paid for is to see to it that the closed loop system rejects disturbances at 0.2, and tracks a reference at 0.1 with adequate accuracy.

As far as reference tracking is concerned, I have plotted in this graph on the right the input in blue and the output in red. And you see that the two are sitting very nearly on top of one another. This indicates that the error between the two is extremely small. If one were to zoom in, and look at the maximum error between these two time domain waveforms, one would discover that this error is within 2 percent. Actually it is better than 2 percent, because the controllers gain at 0.1 radian per second was more than what was required in order for us to satisfy the specifications.

The second specification was disturbance rejection. So, here I have provided an output disturbance at 0.2 radian per second of amplitude 1 and phase equal to 0, just for the sake of simulation. And the red curve here plots the response of the closed loop system to this disturbance. And what one sees is that there is a huge attenuation of the disturbance at the output as a consequence of employing feedback control.

Once again if one were to zoom in, and find the peak to peak value of the response in on account of this disturbance, one would find that this response has been suppressed by a factor of 50 exactly as what was desire or in other words there 98 percent attenuation in the of the disturbance of the output disturbance signal at the output of the plant.

So, in principal therefore we have completed our design, but I wanted to make a couple of points in connection with the controller structure that we have come up with. We have come up with an integrator, so the Bode plot of the integrator as we saw in the previous slide had this characteristic, where the magnitude of the integrator increases with decreased frequency. And at 0.2 radian per second, the gain was 5; and at 0.1 radian per second, the gain was 10.

Now, we entered up meeting the requirements at both these frequencies, but is it really necessary for us to continue this integrator characteristics at frequency is below 0.1 radian per second as well. A movement start would reveal that it is actually not necessary, because there are no specifications on performance at any frequency other than 0.1 and 0.2. So, we are needlessly ensuring that our controller gain is high at frequency is below 0.1 radian per second also. Why is this a problem, this is a problem, because if you have an integrator, you would have to deal with the issues of integrator wind up.

So, one could therefore alternately choose not to continue to have this increasing characteristic with reduced frequency for frequency is below 0.1 radian per second. Indeed even at 0.1 radian per second, it is the gain of the controller is already much more than what is desired, because the gain that was desired was five units or in the logarithmic scale 20 log 5 units. The x axis is log of omega.

And at 0.1 radian per second, it is already the gain is already double of this particular value. So, what we can do in order to avoid the problems of integrator wind up is to choose to add a pole at 0.1 radian per second. Instead of employing an integrator one can employ a first order system, whose pole is at 0.1 radian per second. And the Bode plot of such a system would look as shown by this dotted curve here.

In other words the gain of this controller would level of at 0.1 radian per second, because at would be the corner frequency of our controller. In other words, we could choose to have a controller C of s equal to 1 by s plus 0.1. Now, if you want to go with this controller instead of the controller C of s equal to 1 by s, then for frequencies omega less than 0.1 radian per second, the gain of the controller C of j omega would be equal to 1 by 0.1 for omega much less than 0.1 radian per second, and that is equal to 10.

Now, at the corner frequency of 0.1 radian per second, this would have a this will start to reduce. But, it will at 0.2 radian per second, you can this compute and find that the gain is pretty close to 5 units. So, our controller 1 by s plus 0.1 behaves like an integral controller in the frequency range of 0.1 and 0.2 radian per second. And therefore, satisfies the performance requirements that are already being satisfied by the integrator that we designed just a few minutes. But, but for frequencies below 0.1 radian per second, the gain is not allowed to monotonically increased, but rather we are leveling it off by choosing this as a structure 1 by s plus 0.1 as a structure for the controller instead of 1 by s. So, we have chosen to level it off in the with the intention of avoiding problems associated with integrator wind up.

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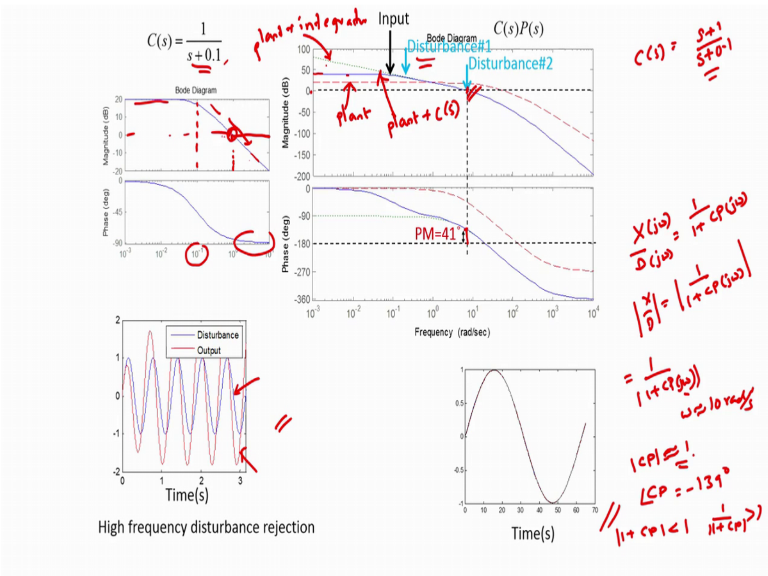

So, if one were to look at the Bode plot of this transfer function, as we discussed its corner frequency will be 0.1 radian per second, it is the corner frequency. And its gain at frequency is below its corner frequency is 20 dB, which corresponds to a linear gain of a factor of 10. And beyond 0.1 radian per second, it drops down just as an integrators characteristic drops down. The phase response starts at a value close to zero unlike in case of the integrator, we have phase a constant equal to minus 90 degrees at all frequencies. And it monotonically reduces and approaches minus 90 degrees at frequencies much beyond it is corner frequency.

So, the further away we go from the corner frequency of 0.1 radian per second, we find this controller to resemble an integrator more and more. So, if we look at the overall Bode plot of the open loop system namely that of the plant and the controller, what I have plotted with the red dashed here is the Bode plot of the plant alone plant. The blue curve is the Bode plot of the new controller times the plant. The black dashed curve here is the plant plus integrator. So, this is plant plus integrator. And the blue curve here is the plant plus the new controller C of s, where C of s is given by this particular transfer function.

So, what you see by comparing the Bode plots of the plant plus integrator and the plant plus C of s is that for frequencies beyond 0.1 radian per second. These two Bode plots overlap as one would expect, because the magnitude characteristics would resemble that of an integrator beyond the corner frequency of this controller namely 0.1 radian per second, but at frequencies less than 0.1 radian per second.

The gain characteristic of the overall system levels off at a participating values, so it is constant. Unlike in case of the plant plus integrator, where the integrator results in the magnitude characteristic continuing to increase as the frequency is reduced. And therefore, with this controller 1 by s plus 0.1 we have manage to avoid the problems of integrator wind up while simultaneously being able to meet our performance specifications at both 0.1 and 0.2 radian per second as well as having the same phase margin of 41 degrees that our original controller had.

Now, I want to make two points in connection with this new controller structure C of s is equal to 1 by s plus 0.1. The first point I want to make is that there is no unique solution to the problem of design of a control system, we saw that C of s is equal to 1 over s was a reasonably good design. And we were motivated to choose that as our controller structure, because of the mathematical simplicity of our controller structure. It was a simple integrator, and that was why we choose to go with that structure. But, then we had this after thought that it an integrator though simple in it is mathematical appearance still has this practical problem of integrator wind up.

And to avoid integrator wind up, we decided not to have the integrated at frequencies below those atom of interest was and namely 0.1 radian per second. So, we replaced the integrator with C of s is equal 1 by s plus 0.1. And both these controllers 1 by s and 1 by s plus 0.1 give us very similar performances as far as disturbance rejection and tracking of references are concerned at the two frequencies of interest. And give as very similar stability specifications as well. But, they happen to be different in structure. And one has happens to have lesser practical problems compared to the other one. So, the first point is I wanted to make therefore is a non- uniqueness of a solution to the control design problem.

The second point, I want to make is the fact that this controller structure C of s is equal to 1 by s plus 0.1 is not any does not match any of the off the shelf controller structures that we introduced in the previous clips. So, we talked about for instance the proportional controller, the integral controller the PI, the PID, the lead compensator, the lag compensator none of them actually have this particular structure 1 by s plus 0.1.

And what this indicates to us is that there is nothing really sacrosanct about any of those standard controller structures, it is greater there it is greater, we understand what they do, but if our particular control problem, demands a controller structure that is different from them, then so be it. One does not need to stick mandatorily to one of those of the shelf readily available control structures for us to realize our controller.

Just by looking at the Bode plot of the controller and the kind of problems and the problems specifications that have been given to us, we might be able to synthesize controllers that have a structured different from the commonly used, commonly available controller structures, and there is nothing wrong with it. As long as these controller structures meet our particular requirements in terms of robust tracking, and disturbance rejection we are good. And indeed if one looks at robust tracking at 0.1 radian per second, one notices that the controller that we have designed does exactly as well as an integral controller as far as tracking is concerned. It is equally well as far as disturbance rejection is concerned as well.

However, let us say in addition to this disturbance at 0.1 radian per at 0.2 radian per second, which for which we have designed a controller already. There was an additional disturbance close to 10 radian per second. So, in other words, there is an additional disturbance at this particular frequency. And we want reject this disturbance as well by a factor of 50 or in other words by 98 percent. Is it possible for the same controller that we have already designed namely 1 by s plus 0.1 to reject this disturbance by 98 percent Well if one were to look at the gain characteristics of the open loop system, it is not very promising, because the gain of the open loop system close to the frequency, where we are interested to reject the disturbance is closed to 0 dB or in other words the gain is very low.

And we know that the mantra feedback control is to achieve high gains, and thereby ensure the disturbances are rejected, and references are tracked. So, we think, therefore that this disturbance may not be rejected quite well. And when we actually do the simulation, to see the extent by which we are variable to reject this disturbance at 10 radian per second by using this particular controller, we are in for a rude shock. What I have plotted in this graph is output disturbance in blue color, and the response of the closed loop system to the output disturbance, and that is in red color.

Now, what we see from the response is that the output has increased the effect of disturbance. So, the response is therefore larger than the disturbance itself. And this is a huge shock, because we were out to suppress this disturbance by a factor of 50 or suppress it by 98 percent instead of doing that we have actually ended up amplifying this disturbance, amplifying the effect of this disturbance.

Now, why did that happen? The transfer function that relates the output disturbance to the output transfer namely X by D is given by 1 by 1 plus C times P. Therefore, the magnitude of X by D would be given by the magnitude of 1 by 1 plus C times P of j omega, and that is going to be equal to the magnitude of 1 by 1 plus C P of j omega.

Now, what we notice is that in the vicinity of 10 radian per second. In other words, omega is close to 10 radian per second, our magnitude of C P is close to 1 is approximately equal to 1. So, you we can see that at around 10 radian per second the magnitude is 0 dB, and 0 dB corresponds to a gain of 1, and therefore a magnitude of C P is close to 1, while the angle of C P is close to minus 140 degrees, in fact it is minus 139 degrees.

Therefore, if one were to compute 1 plus C P, one would notice that the magnitude of 1 plus C P would actually be less than 1. When the magnitude of C P is equal to 1, and the angle of C P is close to minus 180 degrees, in this case it is around minus 139 degrees. And what this means is that we would have 1 by magnitude of 1 plus C P to be greater than 1 or in other words the output by the disturbance at this particular frequency would have a gain greater than 1. And that is what explains the amplification that one sees of the disturbance at the output of the closed loop system.

So, our first goal as control engineers is to make sure that this disturbances at least not amplified. If even if you are not able to suppress it by the desired extent certainly, we should not allow it to get amplified. So, how does one accomplish this? In order to accomplish this one has to come back to this particular controller structure. So, this structure 1 by s plus 0.1 has an integrating characteristic at frequencies beyond 0.1 radian per second as evident here. The slope is minus 20 dB per decade and a phases close to minus 90 degrees.

And as we discussed in our previous clip, an integrator tends to limit the bandwidth and it tends or in other words it reduces the gain crossover frequency of the open loop system. And it is because our gain crossover frequency has reduced, but we are now having a gain crossover close to 10 radian per second, whereas earlier it was closed to 50 radian per second. And therefore, the gain at this gain crossover frequency has become small, and that was what has resulted in amplification of the disturbance.

Therefore, if one were to curve this integrating characteristic, at frequencies beyond the frequency at which the gain of the integrator falls below 0 dB, in this case we see that it happens at 1 radian per second. So, at 1 radian per second the magnitude of C of s is close to 0 dB. And beyond 1 radian per second, the magnitude of C of s comes to negative decibel values on other words the gain becomes less than 0 dB. And it is this attenuation of gain that has resulted in reduction of the gain crossover frequency. And has resulted in a poor gain at the frequency that we are interested in namely at 10 radian per second.

So, to avoid this problem of amplification of disturbance, what one could do is to somehow curve this attenuating characteristic of an integrator beyond 1 radian per second. And one can do that by introducing a 0 at 1 radian per second or in other word choosing a controller C of s, which has s plus 1 in the numerator, and the denominator would be exactly as it was before namely s plus 0.1. Now, this controller has the exact same Bode plot as that of the original controllers C of s is equal to s plus 0.1. So, frequencies up to 1 radian per second. So, in other word it will be flat with a gain of 20 dB for frequencies less than 0.1 radian per second. And between 0.1 and 1 radian per second, it will reduce at minus 20 dB per decade exactly as before.

However, at 1 radian per second since, we have introduced this 0; the magnitude characteristic will level off. It will not continue to decay, as it happened in case of the original controller. And it will level of at 0 dB or in other words, the gain the linear been will be 1. And what does means is that the overall loop gain namely the gain of the plant and the controller beyond 1 radian per second would simply be equal to the gain of the plant itself, because the controller gain is simply going to be close to 1.

And therefore, the Bode plot of the open loop system beyond 1 radian per second, we will simply resemble the body plot of the plant itself. Although below 1 radian per second, the Bode plot will be significantly modified on account of the increased gain that results per using this controller. Therefore, if one were to plot the Bode plot of the controller C of s is equal to s plus 1 by s plus 0.1, then one would notice, but the Bode plot would resemble that of s plus 1 by s plus 0.1 of frequency is below 0.1 radian per second or in other words it will look similar to this. And for frequencies above 0.1 radian per second, it will be flat.

As far as the phase characteristics are concerned, it will resemble that of the phase characteristics of the original controller 1 by s plus 0.1 for frequencies below 1 radian per second. And instead of the phase going to minus 90 degrees as it happened in the previous case, the phase will now gradually go back to 0 degrees. Now, one could either choose to use this as a controller and if one notices here, we have the location of 0 at 1, and location of the pole at 0.1. And therefore, this is this structure resembles a structure of a lag compensator.

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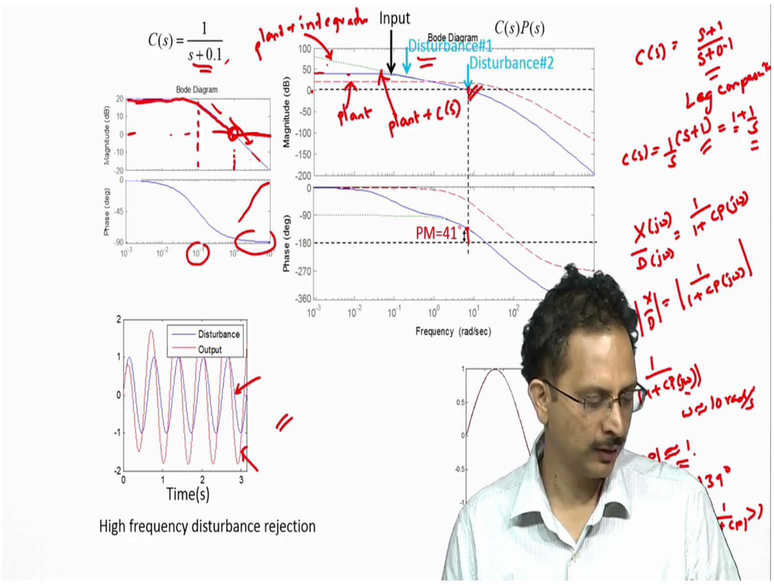

So, indeed this is a lag compensator. One could alternately go with a PI controller as well. So, instead of having 1 by s plus 0.1 as the controller that we would modify, we could choose to have C of s equal to 1 by s as the controller that we would modified. On other words we would include a 0 introduce a 0 at 1 radian per second, so that the integrator here does not attenuate the gain of an open loop system for frequencies beyond 1 radian per second. So, this structure is that of a PI controller, because it can be written as 1 plus 1 by s, a proportional term plus an integral term.

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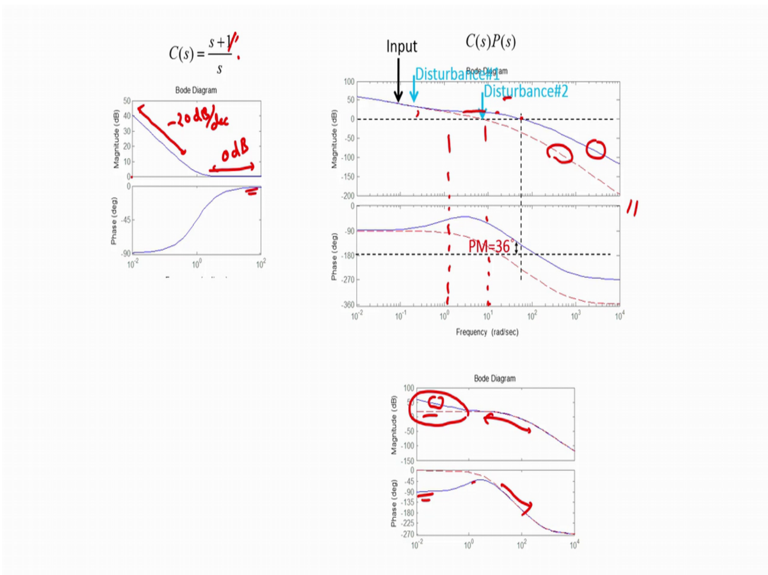

If one were to plot the Bode plot of this controller C of s is equal to s plus 1 by s, then it looks as shown on the graph on the left. So, it has the integrating characteristic for frequencies less than 1 radian per second and it roles of at minus 20 decibels per decade. And at frequencies for frequencies beyond 1 radian per second, it flatten soft and the gain is going to be equal to 0 dB or in another words it does nothing to the magnitude characteristics of the plant. So, this will going to be 0 dB.

As far as the phase is concerned, we see that the phase at frequencies well below the corner frequency of the zero namely 1 radian per second is close to minus 90 degrees, which is what one would expect, because we have an integrator there. And then the term s plus 1 introduces a phase lead of plus 90 degrees as omega is monotonically increased from 0 to infinity. And therefore, these phases lead of plus 90 degrees cancels the phase lag of minus 90 degrees that is added by the integrator. And at high frequencies, therefore the net phase of this controller would be close to zero.

So, if we were plot the Bode plot of the overall open loop system namely that of the plant and the new controller the PI controller, it has been done on this graph on the right. What I have shown in this red dashed curve is the Bode plot of just an integrator cascaded with the plant. What is shown by the blue curve here is the Bode plot of the new controller namely the PI controller cascaded with that of the plant. What we say is that at low frequencies these two Bode plots coincide, because at low frequencies this term s plus 1 does not modify the gain in any particular manner.

But, for frequencies beyond 10 power 0 or equivalently 1 radian per second, we see that the integrated continues to attenuate the gain of the open loop system. Whereas, the PI controller flattens of the gain, and for frequencies beyond 1 radian per second, the gain of open loop system will simply be the gain of the plant. And we see that the gain plant is reasonably high it is 10. And therefore, at 10 in radian per second, which is the frequency at which we are interested in performance, we would get 10 open loop magnitude of magnitude that is supplied by the plant itself, and that is going to be close to 10.

So, since the open loop gain is now greater than now significantly greater than 0 dB, we will be able to attend our disturbance, though we may not be able to do it by the extent that we desired to do so. As far as the stability specification is concerned at the new gain crossover frequency, which is close to 50 radians per second. It is close to 50 radians per second, because the Bode plot of the overall system looks similar to that of the plant for frequencies beyond the corner frequency of this controller beyond 1 radian per second.

So, the gain crossover frequency will be very similar to the gain crossover frequency of the plant itself, which was which we saw was close to 50 radians per second. And at that frequency the phase will be also similar to the phase of the plant itself, because the phase added by the controller will be close to 0. Therefore, the phase margin we discover is about 36 degrees.

We are set for ourselves a phase margin of 40 degrees. And with this two controller, we have got a phase margin of 36 degrees. It is a little bit lesser than what we had set for ourselves, but the small difference is not very significant. So, we shall not introduced any additional complexity to this controller structure to correct for this small difference in phase margin, we shall stop the design at this point.

Before, we look at the time domain disturbance rejection performance, I have also plotted here. The Bode plot of the original plant and the Bode plot of the new controller so, this is the Bode plot of the new controller, and this red dashed curve is the bode flat the original plant. I plotted this explicitly to show that this two controller modifies the magnitude characteristic of the plant only at frequencies below 1 radian per second. It is only in this frequency range that the overall Bode plot differs from the plant. At frequencies beyond 1 radian per second, these two are very nearly coincident in this frequency range the two are very nearly coincident or in other words the controller has a gain close to 1 and a phase close to 0.

And that is also revealed by the phase characteristic at very low frequencies this behaves like an integrator, because the term 1 over s is going to dominate over the response of s plus 1. And a frequency increases; the overall phase of the controller goes to 0. So, the net phase of the open loop system, which is the sum of the phases of the plant, and the controller tends to the phase of the plant itself. So, for frequencies somewhat to the right of 1 radian per second, the phase characteristics of the plant, and a phase characteristics of the controller plus the plant are very nearly identical.

So, having established that this controller helps to restore the gain of the plant in the mid frequency region or in the frequency region around 10 radian per second, where we begin to see the plants dynamics (Refer Time: 36:22). We shall now see how your time domain response of our closed loop system would be to this particular disturbance, the disturbance at 10 radian per second.

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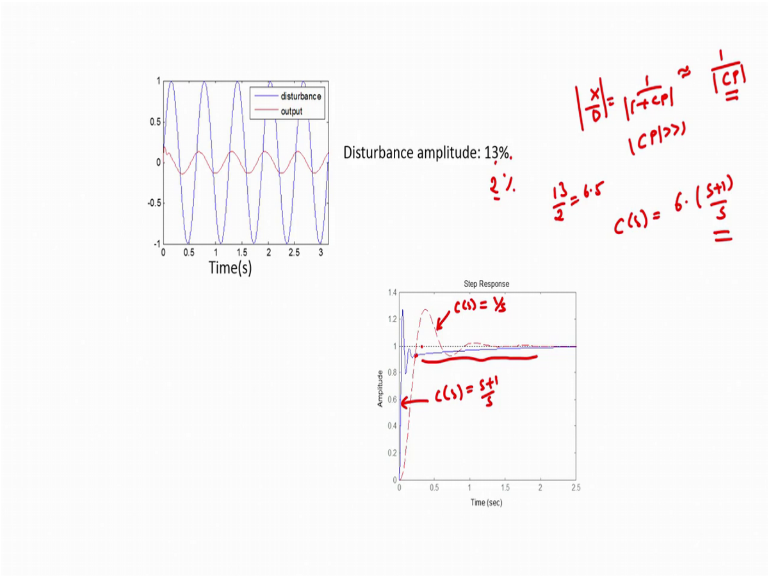

If one were to look at that, one would see that the disturbance now has been attenuated, which is great news, because in the previous because with the previous controller we actually had an amplification of disturbance, which is the exact opposite of what we as control engineers wish to do. And that is not a problem in this case. So, the disturbance has been attenuated. And the disturbance amplitude is now merely 13 percent of the amplitude of the output disturbance, which in the simulation has been assume to be 1 unit.

The step response of our closed loop system is shown here. And what I have done here is to superpose the step response of the closed loop system, when we had C of s as a simple integrator with the step response that we have with the PI controller. So, for in this case C of s is equal to s plus 1 by s. Now, we see that our settling time has improved significantly, and that is because our gain crossover frequency now has increase substantially in comparison with the gain crossover frequency, you have pick would accomplish for the overall system, when we had an integrator as our controller.

Our overall system had a lesser gain crossover frequency with an integrator, and therefore lesser bandwidth, and therefore associated with it was higher settling time in response to step inputs. So, the settling time was reduced dramatically, but what we also see is this long slope tail. So, our closed loop response very quickly comes to a value close to its steady state value, but it takes quite a bit of time. Before, this small remaining error between the steady state value, and the value at this time gets reduced, and comes close to 0. Now, example why we have this long tail is somewhat of a mystery, because it is not evident to us from the Bode plot that we drew for the open loop system. We shall return to this mystery towards the end of this discussion.

So, if we look at the disturbance rejection performance, we see that it has reduced disturbance, but not by the amount that we desired. What we desired was 98 percent rejection in disturbance or the disturbance had to be merely 2 percent of the actual value. Now if one were to look at the transfer function that relates the output to the output disturbance, it would be X by D equal to 1 by 1 plus C P. Therefore, the magnitude of the ratio would be equal to 1 by the magnitude of 1 plus C P.

Now, in the limit that the magnitude of C P is much greater than 1, the magnitude of 1 by 1 plus C P would simply be approximately equal to the magnitude of 1 by C P. Therefore, if we want to improve the disturbance rejection performance at 10 radian per second from 13 attenuation to 10 percent attenuation, we have to increase the gain of the open loop system namely C times P by a factor of 13 by 2, which is 6.5. So, if we were to increase the gain of C P by a factor of 6.5, then we would be able to suppress the disturbance not to 13 percent of its actual value, but actually it would to just 2 percent of its value.

Now, for the purpose of calculations I have assumed that it is adequate for us to increase the gain by a factor of 6. So, if we were want to choose a controller C of s to be equal to 6 times the controller that we had earlier namely s plus 1 by s. Then the gain of this new controller will be 6 times larger, at every frequency compared to the gain of the original controller namely s plus 1 by s. And that ensures that at 10 radian per second, we will be able to attenuate a disturbance by a factor of 6 or in other words by 13 percent divided by 6, which will be a little bit more than 2 percent. But, since it is just a little bit more, we shall assume that it is adequate for our particular application.

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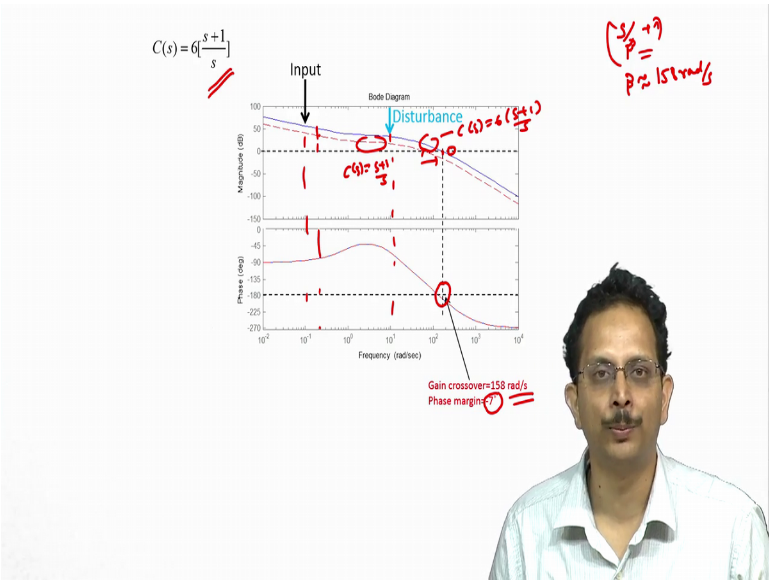

Now, suppose we were to adopt this as our control structure, and were to plot the Bode plot of the overall system namely C of s times P of s. The Bode plot of the overall system looks as shown in this graph. What I have plotted with the red dashed curve is the Bode plot overall system namely C times P when we had C of s to be equal to s plus 1 by s or in other word we just a PI controller. But, we notice that the PI controller cannot reject the disturbance by the specified amount, so we increased its gain by a factor of 6. And the blue curve is what you would get with C of s equal to 6 times s plus 1 by s.

Now, this controller enables us to achieve all our performance specifications near 10 radian per second, we would have a gain of about of over 50. And therefore, we can reject the disturbance in this frequency range by the desired amount at 0.1 radian per second and at 0.2 radian per second, the gain is much higher than what we want it to be. In fact, at 0.2 radian per second earlier the gain was exactly 5 with the controller s plus 1 by s, now the gain is now 5 times 6 or 30.

So, we will be able to suppress the disturbance at 0.2 radian per second much better than what was specified for us. And our input tracking performance will also be significantly better as a result of this controller. So, all our performance specifications have been met. However, there is one small, but actually a fatal problem with this control structure.

So, if one were to look at the gain crossover frequency, when one adopts this new controller, we see that the gain crossover frequency has shifted to the right. And the gain crossover frequency is around 158 radian per second. Now, what is gain crossover frequency? If you look at the phase margin of our closed loop system, we see that the phase margin is negative. There is no meaning to saying that the phase margin is negative. Accept to point out that our closed loop system would be unstable, if our overall phase lag is greater than minus 180 degrees at the gain crossover frequency.

So, therefore if one more to adopt this controller, we will be meeting our performance requirements, but we have an unstable closed loop system on our hands. So, on the whole, we will not be able to implement a stable closed loop system. And this is a problem that requires immediate attention. Now, as we discussed the key problem that one has to address here is the problem of this poor phase lag near the gain crossover frequency. If you can address not this one problem alone, then we are done, because our performance specifications have already been met. And it is in this context that we introduced the zero in the previous discussions. So, if you going to add a 0 here, slightly to the right of this gain crossover frequency of 158 radian per second.

Then or another word when I say add a 0, I mean multiply the existing transfer function C of s by a term of the kind s by beta plus 1, where beta is of the order of 158 radians per second. Then in the vicinity of the gain crossover frequency this term does not significantly modify the magnitude characteristics, because if beta is slightly higher than 158 radian per second. The amplification that occur because of this term at frequencies below beta will be rather small. So, the amplification will be so the gain will be of the of this term will be close to one.

However, the phase contributed by this term s by beta plus 1 would be plus 45 degrees. So, this phase response of this term essentially pulls up the overall phase response of the open loop system without significantly modifying the magnitude characteristics below the gain crossover frequency.

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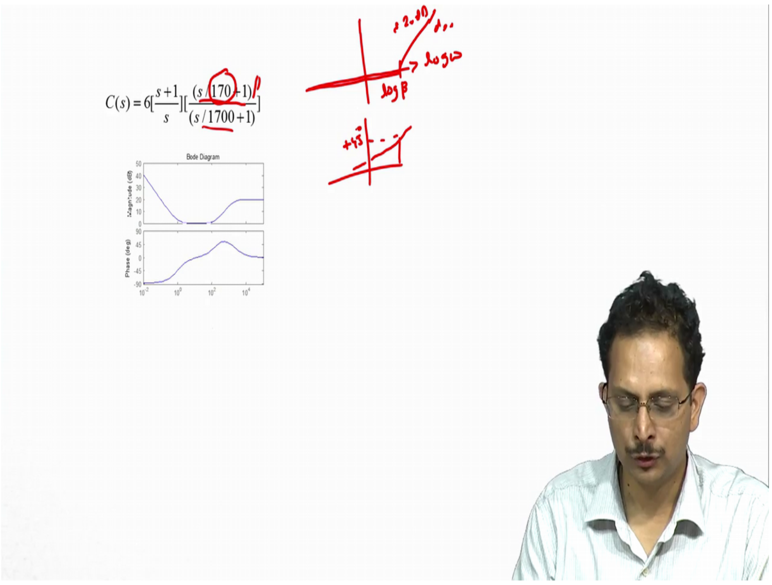

So, what I have done, therefore is I have chosen to add the 0 not at 158, but slightly to the right of it namely at 170 radians per second. But, since we would need a causal implementation of this controller, I have chosen to add a pole very far away about 10 times further away. This number 1700 is arbitrary I could have chosen to add it even further away or a little bit closer. It is there only to ensure that the numerator polynomial and a denominator polynomial or either of similar degrees or the denominator polynomial is slightly of higher degree. In this case, I have chosen it to be of identical degrees both of them are second degree polynomials.

The term that we are interested in though is this term s by 170 plus 1. And this term and supplying plus 40 degrees phase close to 45 degrees phase around the gain crossover frequency namely 148 158 radians per second. And since it does not affect the magnitude characteristics below 170 radian per second that is because the Bode plot of this term would look something like this at the frequency omega equal to beta. So, this is log omega let us say.

So, what omega equal to beta, the frequency log omega will be equal to log beta. The asymptotic Bode plot would have a gain of close to 0 dB, and it is only beyond beta will there be a plus 20 dB per decade rise. So, it does not affect the magnetic characteristics, and therefore also does not affect the gain crossover frequencies significantly.

The phase characteristics however, we will supply result in plus 45 degrees phase at beta. So, since beta is very close to omega g c, we would have close to plus 45 degrees phase lead brought about by this controller. So, when the cascade the, this term s plus 170 s by 170 plus 1 by s by 1700 plus 1 with the existing controller namely 6 times s plus 1 by s.

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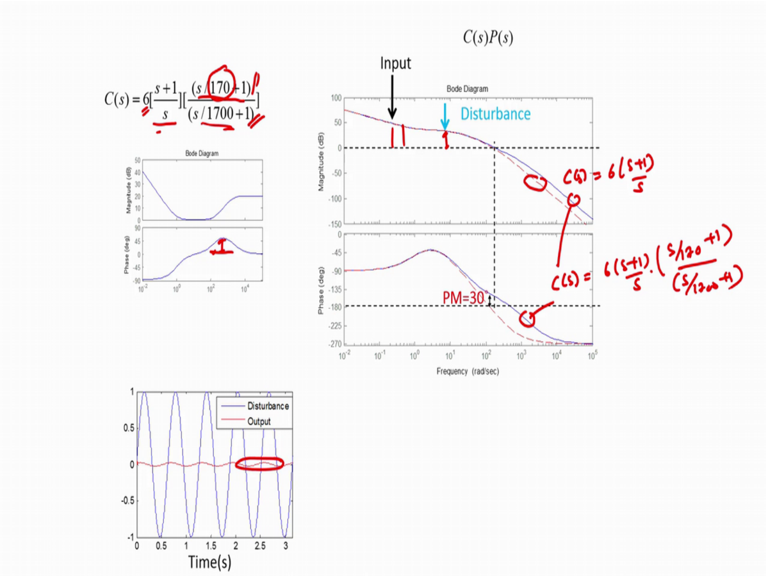

The overall Bode plot of our open loop system would look as shown here. Here the red dashed curve represents the Bode plot for C of s equal to 6 times s plus 1 n by s. This satisfies all our performance requirements of input and disturbance input tracking and disturbance rejection, but it has the problem of a negative phase margin or an unstable un stable closed loop system.

So, by cascading it with this term, we note that this term is essentially a lead compensator, because the location of the 0 is less than the location of the pole. It adds up it results in adding a certain positive phase in the vicinity of 170 radians per second. Therefore, the overall phase response of this open loop system has improved significantly in the neighborhood of this frequency 170 radian per second.

So, the blue phase response here. And the blue magnitude response together or for the controller C of s is equal to 6 times s plus 1 by s times, this lead compensator namely s by 170 plus 1 divided by s by 1700 plus 1. Now, what we see here is that we have managed to get the phase margin to now be 30 degrees. Of course, this is still a little bit smaller, then what we wished it to be namely 40 degrees. We shall see a little while later how this problem can be fixed, but what is worth celebrating at this point is at our closed loop system is now stable, because the phase margin is positive.

And the magnitude characteristics are such that our disturbance and input tracking specifications either met or often exceeded. In case of the disturbance at 10 radian per second, it is exactly met. In case of the disturbance at 0.2 radian per second, and the input at 0.1 radian per second the loop gains are much higher, then what one would wish for them to be. And therefore, the performance is better than what we expected it to be.

So, if one were to plot the response of this closed loop system to an output disturbance of unit amplitude and frequency 10 radian per second, we notice that the response has actually ended up attenuating the disturbance to close to 2 percent of its initial amplitude. Having if you were, if we had chosen slightly `higher gain instead of this being 6 is if you are closer it to be 6.5, then we would have exactly two person attenuation, but that design modification is a trivial one.

So, in this clip, I shall not discuss that modification. This concludes the design of controllers in response to certain specifications for tracking as well as disturbance rejection. These examples for taken in order to highlight the different steps involved in the design by means of a numerical examples.

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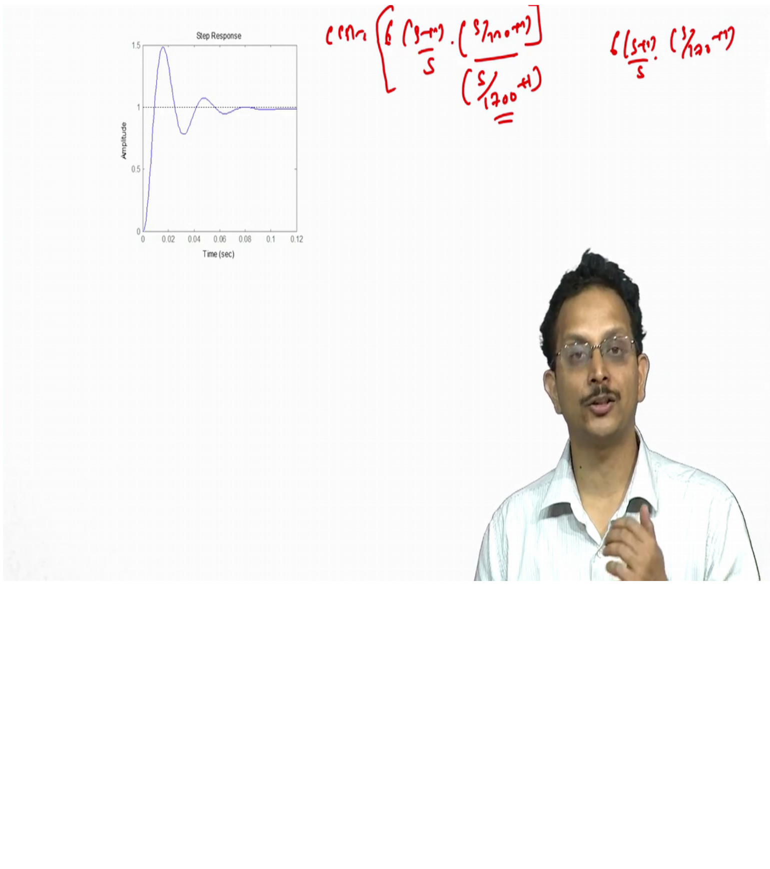

So, if one were to look at the step response of the present system, which has the new controller namely C of s is equal to 6 times s plus 1 by s times s by seventy 170 plus 1 n divided by s by 1700 plus 1, the step response would be something like this.

Now, we note that this is essentially an PID controller, because the terms that out of interest to us. As far as control performance is concerned are these terms 6 times s plus 1 by s times a s by 170 plus 1. This term has been added only for causality set, and it does not affect either the magnitude characteristic or the phase characteristic in the frequency ranges it is of interest to us, namely in the frequency range up to the gain crossover frequency of the open loop system. And if we note this term 6 times s plus 1 divided by s times s by 170 plus 1, and expand it, we can write it as a proportional term plus a derivative term plus an integral term, on other words this would be a PID controller.

We note that the gain crossover frequency of this PID controller is close to 160 radians per second. And whereas, the gain crossover frequency of the PI controller was much lesser, it was close to 50 radians per second. And the gain crossover frequency of the high controller even lesser, it was close to 10 radians per second. Therefore, we expect that the transient response of the PID controller would be much faster than that of the PI controller, and transient response of the PI controller will correspondingly be much faster than that of the integral controller, and that is revealed by the step response of the three controllers.

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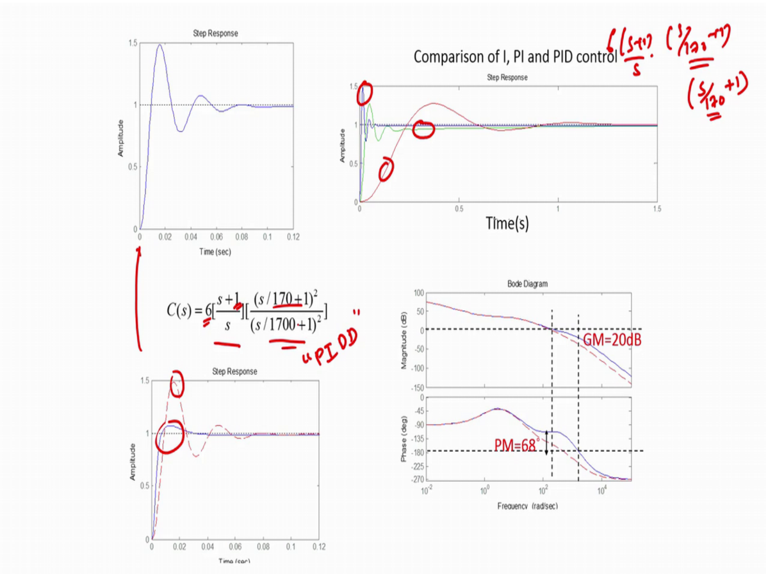

So, this is the step response of the closed loop system can be employed an integrator. This is the step response of the closed loop system, when we employed a PI controller. And the fastest one here is the step response of the close loop system, when we employed a PID controller. On account of increased gain crossover frequency and the increased corresponding closed loop bandwidth as on progresses from integral control to PID control.

Now, in the previous slide, we discuss that our phase was slightly less than what we desired it to be. And this can be addressed by adding another 0. For example, instead of adding a just a single 0 near the gain crossover frequency, which gave us a phase lead of plus 45 degrees near the gain crossover frequency. If we add another 0, on other words we were to multiply this term with another term of the kind just by 170 plus 1. We would note that these two together will give us even higher phase lead. So, each term gives us plus 45 degrees phase lead. So, the two put together give us plus 90 degrees phase lead.

And if one were to do that you would get much better phase margin, then what was accomplished with just a single term. In this case, the phase margin is close to 68 degrees. We know that the gain crossover frequency here still not change much, because neither of these terms affect the magnitude characteristic at frequencies below their corner frequency. And since the gain crossover frequency is 158 radian per second and is much is less than these two numbers, these two terms do not affect the gain crossover frequency much, but the phase margin has been input improved dramatically as a result of using two of these terms.

And the consequence of this improved phase margin is the fact that the transient response now for a controller of the kind 6 C of s is equal to 6 times s plus 1 by s times s by 170 plus 1 the whole square divided by s by 1700 plus 1 the whole square. The transient response is much better than that of the original PID controller, whose transient response we looked at here.

Now, I want to make a comment connection with this particular controller structure. If we look at this controller, once again you recognize that this structure is not really recognizable as any of the standard of the shelf readily, widely employed controller structures, we introduced in the previous clip. So, it is neither a PI controller nor a PID controller nor an integrator nor a proportional controller. So, this once again reinforces the point that I made earlier in this clip, but there is nothing really sacrosanct about the off the shelf controller structures that we talked about. If there if the work for you that is well and good, we can easily adopt them, but we can also choose controller structures that are different from the structures at we looked at in the previous clip.

If anything this controller might resemble a PIDD controller, because you have two differential terms, you have 1 0 that has been added, and a second 0 that has been added take together result in improved phase margin. Also what I want to point out is that despite the factor this is not a standard structure, the reason for choosing this structure was very obvious to us as control designers.

We know, why each of these terms were chosen the way that they were. We know, why we had to have a gain of 6. We know, why we had to have a 0 at s plus at s is equal to minus 1. We know, why we introduce a 0 at 170 radian per second, we also know we introduce second 0 at the same frequency 170 radian per second. So, what does reveals therefore is that if one were to look at the specifications, and look at the resulting performance in terms of tracking disturbance rejection, and instability.

One can organically develop controller structures that satisfy these requirements without taking records to simply trying to borrow an already existing structure such as a PID or a PI or a or a lag compensator. And try to work with it. And try to get it to fit our particular requirements. This organic growth of the controller structure is very useful, because this structure is intended to satisfy the particular requirements that we have.

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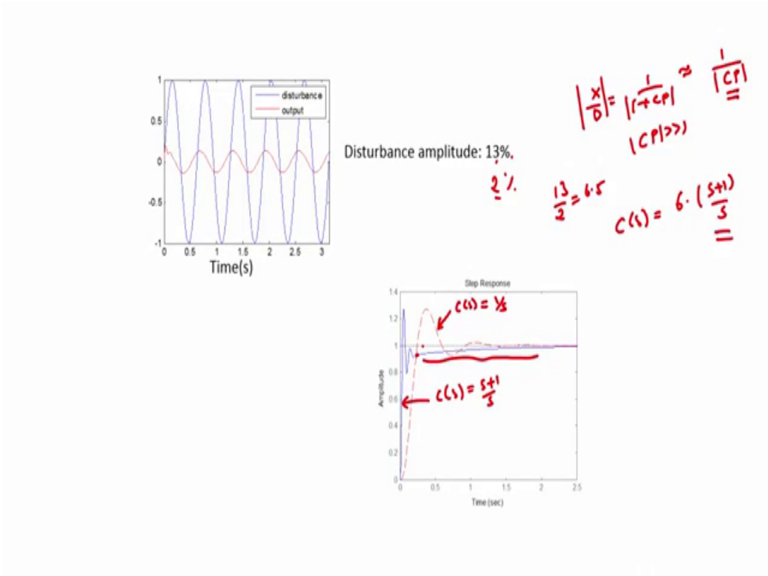

So, before I conclude this clip, I want to revisit the problem that we associated with the transient response of the PI controller. I have now come back to the slide, where we looked at the transient response of the PI controller. And we noted this fairly long tail in the response. So, although the gain crossover frequency was better than that of the integrator and therefore, our settling time was actually better. We had this long undesirable tail in the response of our closed loop system to step inputs. Now, exactly why we have this tail is not evident by looking at the Bode plot of the open loop system with the PI controller, because nothing in the Bode plot that will give us a hint us to why this tail has to exist. And this is one of the problems of Bode plot based design.

As transparent and advantages as it is from the point of view of engineering for performance that is because we are able to see the magnitudes at different frequencies clearly, and see whether our open loop transfer functions have the desired magnitudes in order to meet the performance specifications, it still has one important problem. The problem is that the Bode plots do not tell us, where all the closed loop poles are. This long tail clearly in the closed loop response of our system has to do with the presence of a slow pole in our closed loop transfer function.

But, why do we have this slope pole, where did it come from, is it, and how do we address the problems associated with it are not revealed by the Bode plot. The Bode plot at best can give us an approximate idea us together dominant poles are located, but not all the poles of our closed loop system. And it is this lack of information about the location of the other poles that is preventing us from coming up with a good explanation for the presence of this long tail.

So, what we shall do in the next clip is to take a look at another design tool which allows us to look at where all the closed loop poles are located, and this design tool is the root locus.

Thank you.