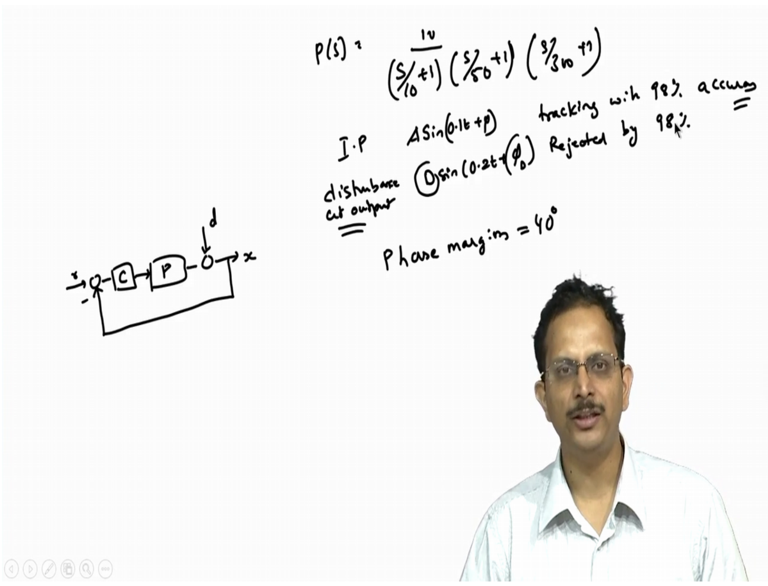
Control System Design
Prof. G.R. Jayanth
Department of Instrumentation and Applied Physics
Indian Institute of Science, Bangalore

Lecture - 20
Bode plot-based control design (Part 1/2)

In the previous clip, we took a look at some of the common of the shelf controllers that are available. I called them off the shelf because they are very frequently used by the controls community and by the industry in order to feedback control various plants. What we shall do now is take a numerical example and go through the steps of design and look at all the considerations that lead us to develop our particular controllers.

So, I shall write out the problem statement first and then, decide on how to go about doing control.

(Refer Slide Time: 00:57)



So, let us say we have a plant which is given by P of S equal to 10 by S by 10 plus 1 times S by 50 plus 1 times S by 300 plus 1. Now, we want this plant to track an input at 0.1 radian per second. So, input signal is a signal at 0.1 radian per second. So, it will be of the form sin 0.1 t plus phi with some amplitude. This signal has to be tracked with 98 percent accuracy, tracking with 98 percent accuracy.

The plant also is assumed to experience a disturbance at the output. So, disturbance at output at 0.2 radians per second. So, the disturbance therefore would be of the kind d sin 0.2 t plus phi d, where phi d represents some phase associated with the disturbance signal and we want this to be rejected by 98 percent. So, we have this plant that is required to track a reference namely a sin 0.1 t plus phi a signal at 0.1 radian per second with 98 percent accuracy, and it should at the same time also reject an output disturbance at 0.2 radian per second again by 98 percent.

Now, this is often how a problem is posed to a control engineer. Nothing is given about the kind of controller that one needs to employ, the kind of tool that one needs to employ, to do control design the kind of stability specifications that are required to be set. In other words, there are lot of things that are not stated in this problem which become the responsibility of the engineer, the design engineer to state and subsequently complete the design indeed. If one were to look at this problem, it is not even told to you that you are supposed to employ feedback control or in other words, supposed to use the sensor to measure the output of your plant and then, compare it with the reference and then, manipulate this error by means of some function in order to get the reference to be tracked.

So, the first thing that we need to do is see whether feedback control is necessary at all for this case or not. What we do notice when we look at the plant structure is that we have not been specified any uncertainty in the plants dynamics. So, to some extent therefore feedback control is not necessary, but there is a disturbance that affects our plant and this disturbance has an amplitude and a phase both of which are not known. Therefore, on account of the presence of this disturbance, we cannot employ open loop based solution to this control problem and one has to therefore employ feedback control.

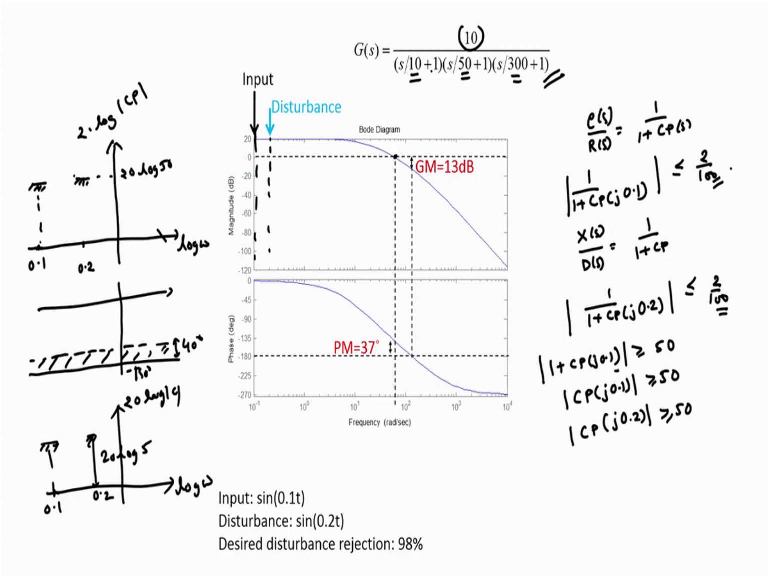
In other words, if we have this plant P, we know that this plant P has an output disturbance d that is afflicting its output. So, overall output is x 1 has to invest in a sensor to measure x and then, compare it with the reference r and any error has to be manipulated by means of some controller c and then, feedback to the plant. So, feedback control is necessary in this particular case because we have an uncertain signal that is affecting our plant namely this disturbance at 0.2 radians per second. Now, that we have discovered that feedback control is essential, we can now chose to employ bode plots in order to execute our design, but feedback control innovatively comes with the problem of stability considerations.

Once again you note in this problem that the people who pose this control problem to the control engineer are not as worried about stability. Stability is an issue that we as control engineers have to automatically address what we are paid to do, however to accomplish performance specifications namely to track references or reject disturbances or to achieve a certain amount of robustness to variation in plant parameters and so on and so forth.

So, stability considerations are something that we have to set for ourselves before proceeding with the design. It will not be specified to us by the engineers who require signals to be tracked or disturbances to be rejected. Therefore, let us go ahead and set for ourselves a certain stability margin. In this particular case, we shall choose to design a control system whose phase margin is about 40 degrees. So, this is a specification that was not given to us, but as control engineers we know that it is important for us to achieve a certain extent of stability for our close loop system and therefore, we have to set for ourselves a certain phase margin that allows us to achieve a certain degree of stability for our closed loop system.

So, we have chosen to have a phase margin for our open loop bode plot to be 40 degrees. So, whenever we do design where we are trying to satisfy these two performance requirements, we pay attention to the phase lag at the gain crossover frequency and see if the difference between minus 180 degrees and this phase lag is less than or greater than 40 degrees. If it is less than 40 degrees, we need to modify the controller structure to make sure that it exceeds or is at least equal to the specified and set phase margin.

(Refer Slide Time: 07:39)



So, with these initial statements of the problem, let us now look at the bode plot of the plant. This slide here shows the bode plot of the plant which has been plotted using Matlab. So, the plant transfer function is given here and the bode plot shows that the gain crossover frequency which is given by this particular frequency is around 50 radians per second and the phase margin is about 37 degrees. So, already our phase margin is close to what we desire it to be namely 40 degrees.

This plant experiences an input at 0.1 radian per second. So, at this particular frequency and an output disturbance at 0.2 radian per second, which is at this particular frequency we are expected to reject the input. So, we are expected to reject disturbance by 98 percent and tract the input with 98 percent accuracy. So, the transfer function that relates the error in tracking to the reference is given by E of S divided by R of S to be equal to 1 by 1 plus C times P of S. Therefore, what we want in order to track our reference with the desired degree of accuracy is that at omega equal to 0.1 radian per second, the magnitude of this transfer function should be less than or equal to 2 percent because we want to track this reference with 98 percent accuracy.

In other words, magnitude of 1 by 1 plus CP of j omega in this case omega is 0.1. So, j 0.1 the magnitude of this should be less than or equal to 2 by 100 which is 2 percent. Likewise we want an output disturbance that affects our plant to also be rejected by 98 percent, which means that the transfer function that relates the output X of S to the disturbance D of S which is given by 1 by 1 plus CP should also assume a value of less than 2 percent at the frequency at which the disturbance affects our system.

So, in other words the magnitude of 1 by 1 plus CP at 0.2 radian per second. So, j 0.2 should be less than or equal to 2 by 100. So, this allows for us to reject the output disturbance by 98 percent at 0.2 radian per second, at this condition allows the reference to be tracked with 98 percent accuracy at 0.1 radian per second. If we take the first inequality, we can rewrite it as 1 plus CP at j 0.1. The magnitude of this should be greater than or equal to 100 by 2 which is 50 and what is been approximately is that is ensure that the magnitude of CP at 0.1 radian per second is greater than or equal to 50, then automatically 1 plus magnitude of CP will also be greater than or equal to 50 and that allows us to meet our performance requirement at 0.1 radian per second.

Similarly, in order for us to meet our performance requirement at 0.2 radian per second, we expect the magnitude of the loop gain namely the controller times, the plant transfer function at 0.2 radian per second. So, magnitude of this should be greater than or equal to 50 for us to meet our performance requirement at 0.2 radian per second. So, if one were to draw a bode plot for the open loop system, so log of omega and this is 20 log magnitude of CP, then at 0.2 radian per second, our gain should be at least 20 log 50. So, our gain should be greater than 20 log 50.

Likewise at 0.1 radian per second, our gain should be again greater than 20 log of 50. As far as the phase characteristics are concerned, we are specified for ourselves a certain phase margin of 40 degrees. So, if this is minus pi or minus 180 degrees, then I shall draw a band here of width 40 degrees. So, whenever in our attempt to design the controller, whenever the open loop transfer function crosses 0 dB, the phase lag associated with that should be not less than minus 140 degrees because if it is less than minus 140 degrees, it will enter within this forbidden band and we will end up with the phase margin that is less than what is required.

So, the specifications in terms of time domain tracking have been converted to corresponding specifications for the open loop system. We now have to come up with a controller that allows for the open loop system to meet the specifications and then, we would be done with the design. So, let us see how we are going to do that next. If we pay attention to these frequencies, we note that both 0.1 and 0.2 radians per second are much less than the corner frequencies of the plant itself. The corner frequencies are at 10 radian per second, 50 radian per second and 300 radian per second and the gain of the plant itself in the frequency range of 0.1 and 0.2 is obtained by setting S is equal to j omega.

Now, the gain of the plant itself in this, in the frequency range of 0.1 and 0.2 radian per second can be easily obtained by ignoring the terms S by 10 plus 1, S by 10, S by 50 and S by 300 because these terms will be very small in comparison with 1 when omega is 0.1 or 0.2 radian per second and we would get therefore the low frequency gain of the plant to simply be equal to 10. So, as things are the open loop system without the controller has a gain of 20 log 10 dB which is less than what is required namely 20 log 50 dB. In other words, whatever controller we choose to come up with should increase the gain of the open loop system by a factor of 5 both at 0.2 as well as at 0.1 radian per second in order for us to be able to meet the performance specifications.

So, if one were to draw the required gain for the controller C and this is log omega, this is 20 log magnitude of C 20 log magnitude of C, we note that the controller should have a gain of at least 20 log 5 at 0.2 radian per second and likewise a gain of 20 log 5 also at 0.1 radian per second. If our controller has this gain and our plant already has a gain of 10, the two together will have a gain of at least 50 at these two frequencies and therefore, we will be able to meet our performance requirements. Now, the question is what controller to go with in order to obtain these performance requirements.