Control System Design
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Lecture - 19
General Controllers (Part 3/3)

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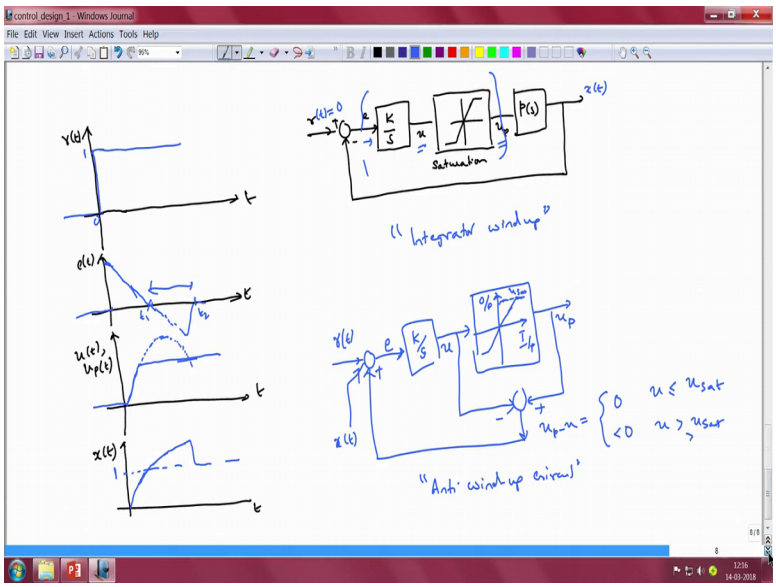

Despite the versatility of all these shelf-controllers that we talked about, namely the proportional controller, integral controller, the proportional integral controller, the PID controller, there is a peculiar problem that is associated with the use of integrators. In any of these, any of the 3 control techniques that we talked about by your the integral controller or PI or PID control architectures and thisunique issue arises on account of the presence of a saturation block in any control system. So, the output of any controller is limited to a certain saturation voltage and therefore, one has to inevitably deal with the fact that we have, we would have finite voltage outputs from all of these controllers and that is captured by a saturation block here.

So, the saturation block has an input output relationship that is linear or that is a straight line with the input being equal to the output in this linear region, but then beyond a certain magnitude of the input, we will response of the output of the block will not change. It will stay put at a particular value. So, we assume a symmetric saturation block to be cascaded with our controller.

In this case, I have considered purely an integral controller although the issue that would be discussing associated with the integrator is also present in other control architecture that include the integrator as part of their structure. So, in order to understand special issue associated with integral controllers in the presence of saturation, let us take an example where initially our control system is tracking a reference let us say a study reference or even a reference r of t being equal to close to 0.

We have already read steady state, therefore our initial r of t. Let us say was close to 0 or some steady value t naught b exactly 0 and corresponding to this since we have a integrator, we get 0 steady state error to constant references. Therefore, our error signal will also be 0. Now, let us assume that at time t equal to 0, there is sudden change in reference. So, the reference suddenly changes from 0 to 1 and this represents let us say a fairly large change in reference. So, once again I want to make this point that initially r of t need not be 0, it can be some constant reference and there is a sudden change in the set point that happens at time t equal to 0.

In an example, I have assumed that this change in magnitude is 1 unit. So, because there is a sudden change in reference, our error e of t which is equal to r of t minus x of t will also suddenly increase. It will instantaneously go from 0 to 1 at time t equal to 0. Now, in response to this fairly large change in error which is the input to the integrator block, we see that the output of integrator block also changes rather quickly. So, that results in a corresponding increased input to the plant and the plant also correspondingly responds to this increased input.

So, what I have plotted at the bottom is the input to the plant and the input to the plant is initially let us assume it was equal to 0, then it starts to rise. So, because you have this non-zero error, that error gets integrated and that results in a certain output. Now, we have labeled the input to the saturation block as u and the output of the saturation block as u p because u p is 1. That is eventually going to be fact of the plant. Now, as long as u p is less in magnitude than the saturation value, then our u and u p signals are both going to be identical.

It is only when u becomes equal to saturation voltage will there be no further change in voltage and u p will stay put, but a fixed value namely the saturation voltage of that control electronics. So, initially let us assume we are not in that saturation regime, but in

the linear regime of the saturation block and those results in an increase in the output of the saturation block and this increased output correspondingly change the output of the plant itself. The plant output also starts to increase.

Now, because the plant output has increased, we see that the error between the reference and the output will now start to drop. So, error will now start to drop, but the error is still non-zero. So, the plant output will continue to increase and there will come a point at which the controller output u becomes equal to the value of the saturation voltage in which case our signal u p will not change any further. It will flatten out and the signal u, however, which is the output of integrator, will continue to increase. Now, our plant is going to see only u p. So, it is going to see a constant reference as the input to the plant and our plant will therefore continue to respond to this constant reference and will continue to increase in its magnitude.

Now, as it continues to increase and reaches some steady state value in response to this constant input to the plant, the error will also start to drop further and there will come a time at which the error goes to 0. Now, at the instant the error goes to 0, the output u which is the integral of the error will flatten out. It will stop integrating the error because an error has gone to 0. Now, in principle we expect the control system to stop its control action at this instant of time, I shall call it t 1 because it is at this instant of time that error has actually come to 0 and therefore, we do not need not any longer change the output of the plant. However, the tragedy is that at the time t 1, the input to the plant is a constant and does not change in response to the output because the output of the saturation block stays put at u p. The error will not stop at e equal to 0 at the time t 1 because the input to the plant still stays put at its saturation value which is at constant and that causes the plant's output to continue to increase in response to this steady input and that causes the error to continue to drop and when the error continues to drop, the integral of error will also start to reduce in magnitude.

So, u signal u will start to reduce in magnitude, but still the output of the saturation block is going to stay put at a constant value because the value of u is still greater than the saturation value. This will continue until a time when the signal u becomes less in magnitude compared to saturation and at this instant there within the linear regime of the saturation block and therefore, our feedback control kicks in. So, once our feedback control kicks in, we would have the error come to 0.

According to the time constant dictated by the closed loop band width of our control system and our x of t come back to the value that it was supposed to be at namely 1 and therefore, makes the error equal to 0. So, we note that it is at a later time t 2 that the error comes to 0 and stays put at 0 and there is an extra time between t 1 and t 2 that is taken by the control system in order to bring the error to 0 and this phenomenon where this extra time is taken by the control system in order to regulate the output at a particular set point especially in response to fast changing set point, it is called integrator wind up.

It occurs for 2 reasons. Firstly, that we have a saturation block and the output of the saturation block cannot exceed a certain fixed value and despite the fact that the integrator continuous to integrate even though the output of the saturation block does not change, once the input to the saturation block exceeds the saturation value. So, how do we avoid this problem of integrator windup? There is a control block diagram called anti-windup circuit that allows us to avoid integrator windup altogether. So, suppose we focus on just this part alone, the part that is responsible for integrator windup and therefore, is responsible for extra time that is taken by the control system to bring the error to 0.

So, I shall draw the saturation block here. This is the input to the block, it is the output of the block and the input to the saturation block is the control referred to the output is u p and we have the integrator here. So, in order to make sure that this windup does not happen, we need to make sure that the moment there is a difference between u and u p. Remember that if we are in the linear region of our saturation block u and u p will be identical to one another. It is only when the output of the saturation block is at its saturation value, there is going to be a difference between u and u p.

Therefore, what one can do is to look at the difference between u and u p and whenever this difference is non-zero, we take an action in order to prevent you from increasing any further. In order to do that, therefore we first take a look at the difference between u and u p. So, if we look at the difference namely u p minus u which is given by the signal here, this is u p minus u. We note that u p minus u is equal to 0. When u is less than or equal to the saturation value of this block which I shall call as u sat, however when you exceed u sat this signal u p minus u becomes less than 0. So, it becomes negative when u becomes greater than u sat to avoid the integrator from continuing to integrate.

Once the output of the integrator has exceeded the saturation value of the saturation block, what we do is we take the signal and feed it back to the integrator in this particular manner. Now, the moment u p minus u becomes negative, we would have a negative input to the integrator block in addition to whatever input our integrated block already gets in the form of the difference between power of t and x of t. So, in addition to this difference, we add this extra difference namely u p minus u and this being negative will minimize the error and will prevent wind up from happening.

So, this is called as an anti-windup circuit. So, one can therefore continue to use integrator and reap the benefits of it, high gain at low frequencies and at the same time avoid these issues associated with integrator, wind up by incorporating an anti-windup circuit as part of their control implementation, however one might also wonder if there are ways where by this wind up can be avoided all together. Therefore, one can avoid the use of anti-windup circuits altogether to see if this is possible.

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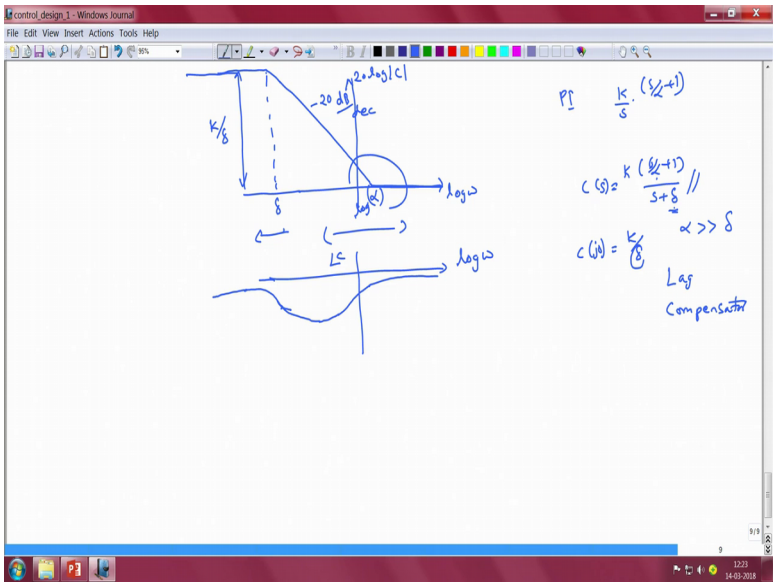

Let us return to the bode plot of let us say PI controller, a PI controller has a bode plot that looks something like this. So, the PI controller has a transfer function of the kind K by S times S by alpha plus 1. So, at frequency alpha, so, x axis is log omega. So, frequency will be log alpha and y axis is 20 log magnitude of C. We note that at low frequencies the characteristic is that of an integrator. So, the slope will be minus 20 dB

per decade and at the corner frequency omega equal to alpha, there is a change in slope and beyond omega equal to alpha our PI controller has a constant gain.

In this particular example that have considered it has 0 gain, now as the frequency reduces we note that the gain of the integrator increases and indeed at omega equal to 0, the gain of the integrator is infinite, but it has one really need infinite gain. Is it really necessary to ensure that tracking errors to constant references or slow varying references is extremely close to 0? As engineers it may not be necessary for us to track references and reject disturbances DC disturbances or DC references with exactly P error.

It may still be acceptable for us to reject disturbances or track references with very small error although the error might not be 0. So, on account of the fact that as engineers you do not really expect exactly zero error in tracking references or rejecting disturbances. We can settle for a certain finite, but small error. We can exploit this realization by leveling of the magnitude characteristic beyond below a certain frequency delta. Now, if you want to write down the transfer function that allows for this kind of a flattening in the magnitude characteristic to happen, it would look something like this. We would have K times S by alpha plus 1 as before it is responsible for this trend for the magnitude characteristic in this particular frequency range, but for frequencies below delta our gain should not increase and in other words, we do not want the integrator action to happen and that can be achieved by choosing a term of the kind S plus delta in the denominator.

So, we note that we will choose the denominator to be S plus delta. Instead of S, we would have a magnitude characteristic like this and gain at low frequencies will not tend to infinity, but will stay constant given by K by delta because if 1 volt to set S tending to 0 in this transfer function C of S, we would have C of j 0 to be equal to K by delta.

By choosing the delta, the term delta to be a sufficiently small, we can choose this gain in low frequencies to be high enough for our particular requirement. What this controller allows us to do is to achieve adequately high gain at low frequencies though not infinite gain in the vicinity of omega equal to 0. This is not a problem because as engineers, we are generally happy with adequately high gain as far as tracking differences or rejecting disturbances is concerned. So, if you look at this controller, this controller has a 0 as well as a pole, the location of the 0 is greater than the location, the location of the 0 alpha is greater than the location of the pole and such a controller is called as a lack compensator.

To understand why it is called a lack compensator, one has to look at its phase characteristics. So, if one volt to draw the phase of C as function of frequency, we note that at very low frequencies, we have the phase being dominated by the term S plus delta and it is going to be close to 0 and as we approach the frequency omega equal to delta, you start to see a phase lag and the phase lag will continue and as we start to approach 0, alpha as omega tends to alpha the term S by alpha plus 1 will start to provide positive phase or phase lead.

Therefore, the phase response will start to improve and at very high frequencies, the phase is going to go back to 0. So, therefore between the frequencies delta and alpha, this controller contributes to a net negative phase and hence, this is called as a lag compensator, but the terminology of lag compensation is rather unfortunate because it does not highlight the real purpose of this controller. The real purpose of a lag compensator is to improve the magnitude characteristics in the low frequency range. It is not to in anyway modify the phase characteristics of the open loop system.

So, the name appears to indicate that it has to do, it does something to the phase characteristics of the plant because it is adding a certain phase lag in a certain frequency range, but its true utility is to add gain in the low frequency range. In other words, it is intended to act as a good replacement for PI controller and the reason we want to find a good replacement for PI controller is because of the issues associated with the integrating integral control action namely that of integrator wind up just as a lag compensator contributes to a phase lag in a certain frequency range.

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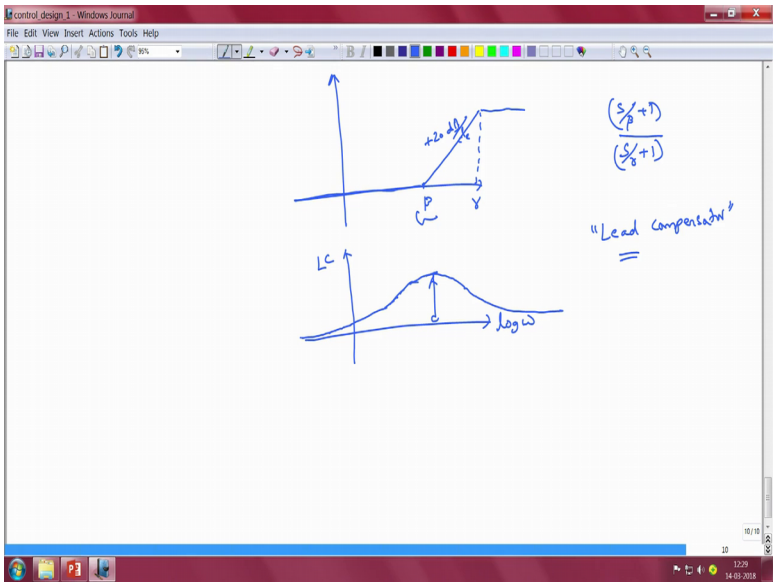

We can also have a controller structure that contributes to a phase lead. So, if we choose to add 0 of a control at some frequency beta, then the magnitude characteristics of the term S by beta plus 1, for instance would look something like this. Its magnitude would be 0 dB for frequencies up to beta and would increase at plus 20 dB per decade for frequencies beyond omega equal to beta now, however the problem with this term is that it is a non-causal transfer function. Its numerator polynomial has a degree 1 and the denominator polynomial has a degree 0 and therefore, it cannot be physically realized to address this issue. We add a pole at a sufficiently large frequency gamma, so that the magnitude characteristics taper of and our transfer function would look like S by beta plus 1 by S by gamma plus 1 if 1 were to look at the phase characteristics of such a transfer function.

So, if I were to plot the angle of C verses log omega, we see that at very low frequencies the phase contributed by both these terms going to be 0, but as one approaches the frequency omega, the phase response will start to increase and it will continue to increase beyond omega and as you approach gamma, the term 1 by S by gamma plus 1 will start to add start to provide phase lag and therefore, the phase, the net phase of this transfer function will start to reduce and at very high frequencies, you have the phase going back to 0. Therefore, there is certain frequency range between beta and gamma in which this controller adds phase and hence, this controller is called as a lead compensator. The purpose of a lead compensator is accurately captured by its name. The purpose of lead

compensator is to add phase. It is not intended to modify the magnitude characteristics at frequencies below beta. Generally, the lead compensator is employed in order to provide a phase lead near the gain crossover frequency.

So, typically the value of beta that we choose would be close to the gain crossover frequency of the plant. Therefore, in this case unlike in the case of the lag compensator, the name is appropriate because the function of this controller is to provide phase lead in the vicinity of the gain crossover frequency or in other words, the high frequency regime of the open loop system and thereby prevent instability that might occur on account of increased gain in the mid-frequency or in the low frequency regions.

So, with these two controllers we conclude our overview of the standard controller that are available for dealing with control problems. We have reviewed proportional controllers, integral controllers, PI proportional integral controllers, and PID controllers. We concluded that PID controllers are very versatile and therefore, very popular in industry because the enable addressing performance requirements in the low frequency, in the mid-frequency regions and stability requirements in the high frequency regions. Now, we subsequently look, took a look at the lag compensator. This controller was introduced because PI controller had an integrator in it and integrators come with the problem of integrator windup which we discussed a couple of slides back and to avoid the problem of integrator windup, we can either have an anti-windup circuit or modify the controller structure all together, so that its output does not go to infinity as omega tends to 0 and that structure was that of the lag compensator.

We also understood the fact that the name lag compensator is rather unfortunate because it refers to its phase characteristics, by the true purpose of a lag compensator is to improve the magnitude characteristics of the open loop system in particular to improve the low frequency gain of the system. In contrast, the last controller that we introduced was the lead compensator whose primary purpose was to supply phase lead and phase lead needs to be supplied primarily, primarily near the gain crossover frequency because it is in the vicinity of this frequency, but our stability becomes a concern. So, of the phase margin is dangerously small or if the phase lag of the plant has come below minus 180 degrees, then one can employ a lead compensator to improve the phase response and rescue the closed loop system from instability.

In the next clip, we should take a numerical example and look at how one can progress through the design considerations and the design steps.

Thank you.